Coimisiún na Scrúduithe Stáit
State Examinations Commission

Scéim Mharcála	*Scrúduithe Ardteistiméireachta, 2005*
Matamaitic	*Gnáthleibhéal*

Marking Scheme	*Leaving Certificate Examination, 2005*
Mathematics	*Ordinary Level*

Contents

MARKING SCHEME

LEAVING CERTIFICATE EXAMINATION 2005

MATHEMATICS – ORDINARY LEVEL – PAPER 1

GENERAL GUIDELINES FOR EXAMINERS – PAPER 1

1. Penalties of three types are applied to candidates' work as follows:
   - Blunders - mathematical errors/omissions (-3)
   - Slips - numerical errors (-1)
   - Misreadings (provided task is not oversimplified) (-1).

   Frequently occurring errors to which these penalties must be applied are listed in the scheme. They are labelled: B1, B2, B3,…, S1, S2,…, M1, M2,…etc. These lists are not exhaustive.

2. When awarding attempt marks, e.g. Att(3), note that
   - any *correct, relevant* step in a part of a question merits at least the attempt mark for that part
   - if deductions result in a mark which is lower than the attempt mark, then the attempt mark must be awarded
   - a mark between zero and the attempt mark is never awarded.

3. Worthless work is awarded zero marks. Some examples of such work are listed in the scheme and they are labelled as W1, W2,…etc.

4. The phrase "hit or miss" means that partial marks are not awarded – the candidate receives all of the relevant marks or none.

5. The phrase "and stops" means that no more work is shown by the candidate.

6. Special notes relating to the marking of a particular part of a question are indicated by an asterisk. These notes immediately follow the box containing the relevant solution.

7. The sample solutions for each question are not intended to be exhaustive lists – there may be other correct solutions. Any examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his/her advising examiner.

8. Unless otherwise indicated in the scheme, accept the best of two or more attempts – even when attempts have been cancelled.

9. The *same* error in the *same* section of a question is penalised *once* only.

10. Particular cases, verifications and answers derived from diagrams (unless requested) qualify for attempt marks at most.

11. A serious blunder, omission or misreading results in the attempt mark at most.

12. Do not penalise the use of a comma for a decimal point, e.g. €5.50 may be written as €5,50.

QUESTION 1

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 marks**	**Att 6**
Part (c)	**20 marks**	**Att 7**

Part (a)	**10 marks**	**Att 3**

Express 35 cm as a fraction of 1 m.   Give your answer in its simplest form.

(a)	**10 marks**	**Att 3**

1 m = 100 cm …3m

$\frac{35}{100}$ …7m

$= \frac{7}{20}$ ..10m

* Correct answer without work: full marks.
* For 35/1000 and stops (with or without work): award 4m. Also 35/1 merits 4m (B2+B4).
* For 7/100 without work: award 4m. (B2 + B4).
* Treat .35 and 35% as 35/100. Treat other decimal answers in the same way, e.g. 100/35 = 2.85 … 4 marks.

Blunders (-3)
B1 An incorrect numerator, e.g. 350/100 and continues, or 65/100 and continues.
B2 An incorrect denominator, e.g. 35/1000 and continues, or 7/100 without work.
Note: 65/1000 and continues is B1+B2 (= 4m); if stops, it's B1+B2+B4 => Att 3.
B3 100/35 and continues. [If 100/35 and stops, award 4m].
B4 Incorrect or no simplification (e.g. 35/100 = 7/100), or simplification is not possible.

Slips (-1)
S1 A fraction simplified but not to its simplest form, e.g. 350/100 = 35/10 is B1 + S1.

Attempts (3 marks)
A1 Mentions 100 and stops, or 1000 mm and stops.
A2 1/35 and stops.

Worthless (0)
W1 65 and stops, or 135 and stops.

Part (b) **20 (10, 10) marks** **Att (3, 3)**

(i) The approximation 50×80 was used for the calculation 51×79.
Find the percentage error, correct to one decimal place.

(ii) Express the ratio $\frac{1}{2} : \frac{1}{3} : \frac{1}{4}$ as a ratio of natural numbers.

Divide 325 in the ratio $\frac{1}{2} : \frac{1}{3} : \frac{1}{4}$.

(i) **10 marks** **Att 3**

50 x 80 = 4000 51 x 79 = 4029 } …3m Difference = 29 …4m $\frac{29}{4029} \times 100$ …7m = 0.71 or 0.719 …9m = 0.7 ..10m	50 x 80 = 4000 51 x 79 = 4029 } …3m $\frac{4000}{4029} \times 100$ ...7m = 99.28 or 99.3 ..still 7m 100 - 99.28 = 0.72 …9m = 0.7 ..10m	4029 = 100% <u>or</u> 40.29 = 1% } … 3m 1 = 1 / 40.29 or 0.02482 29 = …4m 29 / 40.29 <u>or</u> 29(0.0248)…7m = 0.719.. …9m = 0.7 ..10m

* Answers without work: 0.7 merits 10m; 0.71 merits 9m; 29 merits 4m; and 4000 and/or 4029 merits 3m. Other incorrect answers without work and stops: 0m

Blunders (-3)
B1 Incorrect ratio, e.g. (29 ÷ 4000)x100 = 0.725 = 0.7 in I, or (4029 ÷ 4000)x100 = 100.725 leading on to 0.7 in II.
B2 Not multiplying by 100 (in methods I and II), e.g. 0.007 with work..

Slips (-1)
S1 Incorrect or no rounding.

Attempts (3 marks)
A1 % error = (error / true value) x 100.
A2 Line 1 and/or line 2 of method I or II.

(b)(ii) **10 marks** **Att 3**

$\frac{6}{12}:\frac{4}{12}:\frac{3}{12} = 6:4:3$	$\frac{6}{12}:\frac{4}{12}:\frac{3}{12} = 6:4:3$	$\frac{1}{2}+\frac{1}{3}+\frac{1}{4}=\frac{13}{12}$	…3m
and 6 + 4 + 3 = 13	and 6 + 4 + 3 = 13	$325 \div \frac{13}{12} = 300$	…4m
$\frac{325}{13} = 25$	(6/13)(325) = 150	$\frac{1}{2}(300) : \frac{1}{3}(300) : \frac{1}{4}(300)$	…7m
6(25) : 4(25) : 3(25) 150 : 100 : 75	(4/13)(325) = 100 <u>and</u> (3/13)(325) = 75	150 : 100 : 75	..10m

* Correct answer without work: full marks. Incorrect answer without work: no marks.
Note: ½ (325), 1/3 (325), ¼(325) = 162.5, 108, 81 => B4 .. 7 marks; but, 162.5, 108, 81 without work is an incorrect answer without work and merits 0 marks.
* Order of ratios unimportant, apart from B1 case below. Needn't present answer as ratios.
* May multiply by 24 (or any multiple of 12) to get 12 : 8 : 6.
* 0.5: 0.33:0.25 is not necessarily B3. See Note after B3.

Blunders (-3)

B1 Multiplies by incorrect ratio involving denominators 2, 3 and 4, e.g. multiplies by 9.
B2 Inverts the ratios, i.e. 2 : 3 : 4. Note: May continue to get 72 : 108 : 144, merits 7m.
B3 0.5 : 0.33 : 0.25. Note: 0.5(325) = 150.46 merits 7m, and may continue to get 10m.
B4 Doesn't sum ratios, e.g. ½(325) **:** (325) **:** ¼ (325), and continues, e.g. (6/12)(325) : (4/12)(325) : (3/12)(325). Note: Must have two numbers at least or a ratio (but not necessarily in correct order). Can have 6 + 4 + 3 = 13 and still incur B4.
B5 325 multiplied by (13/12) or 13 or 12, and continues along correct lines afterwards.

Attempts (3 marks)

A1 Mentions 12 or multiples of 12 (the CD).
A2 6: 4:3 and stops, or ½ + + ¼ = 13/12 and stops, or 0.5 + 0.33 + 0.25 = 1.08 and stops.
A3 (325 ÷ ½) **:** (325 ÷ ⅓) **:** (325) ÷ ¼, and continues.
A4 ½(325) = 162.5 or 163 and stops, or (325) = 108 and stops, or ¼ (325) = 81 and stops.
A5 2 + 3 + 4 = 9 and stops; or (1/9)(325) = 36 and stops. See B2.

Worthless (0)

W1 2 : 3 : 4 and stops
W2 ½ + + ¼ = 3/9 and stops.

Part (c) **20 (5, 10, 5) marks** **Att (2, 3, 2)**

At the start of the year 2000 the population of a particular town was *P*.
During the year 2000, the population of the town increased by 10%.

(i) Express, in terms of *P*, the population of the town at the end of the year 2000.

(ii) During the year 2001 the population of the town increased by 4%.
During the year 2002 the population increased by 2%.
Find the total percentage increase in the population of the town over the three years.

(iii) The actual increase in the population was 8344. Find the value of *P*.

(i) **5 marks** **Att 2**

$p \times \frac{110}{100}$ **or** $\frac{110p}{100}$ **or** 1.1p	$p + \frac{p}{10}$ **or** $\frac{11p}{10}$	p + 10% of p **or** 110% of p	} ..5m	10% of p **or** 0.1p …2m => answer = 1.1p …5m

* Correct answer without work : full marks. Incorrect answer without work: no marks.

Blunders (-3)

B1 p x (100/110).
B2 Gets p - p/10.
B3 0.1p or 10% of p and stops.

Attempts (2 marks)

A1 p + 10%, or an incorrect answer containing p.
A2 10% = 10/ 100 or 1/10, and stops.
A3 110% or 10% of 2000 (or any number) correctly calculated.
A4 Compound interest or simple interest formula stated correctly, and stops.

(ii) **10 marks** **Att 3**

$1.1P \text{ x } \dfrac{104}{100}$	$1.1P$ x 1.04	1.1 P x 1.04	1.1 x 1.04	…3m
$1.1P \text{ x } \dfrac{104}{100} \text{ x } \dfrac{102}{100}$	1.1 P x 1.04 x 1.02	1.144 P x 1.02	1.144 x 1.02	…4m
$= 1.16688P$	=1.16688 P	1.16688 P	1.16688	…7m
$\Rightarrow \%\ inc. = 16.688$	=> % inc. = 16.688	=> % inc. = 16.688	% inc. = 16.688	..10m

* Correct answer without work: full marks (10m). Incorrect answer without work: no marks.

* May go on with 2000 (or any number of their own choice, getting 10%, 4% and 2% of their correct amounts) , e.g. 2200 → 2288 (..3m) → 2333.76 (..7m) => inc = 16.688% (10m). Mark according the steps in methods above (with p = 2000) for 3m, 4m, 7m and 10m.

Blunders (-3)

B1 Decimal error. Each time.

B2 Leaves answer at 1.16688P, or candidate's equivalent.

B3 Percentage error, e.g. 4% = ¼ . Each time.

B4 Find 4% (and 2%) but doesn't add on the extra percentage, or subtracts it. Each time.

B5 Incorrect percentage rate used in the interest formula, each time. e.g. $A = 1.1(1.04)^2 = 1.18976$ => 18.976% is B5. And, $A = 1.1(1.1)^2 = 1.331$ => 33.1% is B5 + B5.

B6 Incorrect principal used in the interest formula, each time. *Incorrect* means *not ans (c)(i)*, e.g. 2001(1.04) + 2002(1.02) is B6 + B6 at this stage, while $2001(1.04)^2$ is B6 + B5 + B5.

Attempts (3 marks)

A1 Tries to use answer in (c)(i), e.g. multiplies it by ?/100 other than 104/100.

A2 Mentions 104/100 or 1.04, or gets 1% or 2% or 4% of P or of candidate's value.

A3 16% and stops, or 10% + 4% + 2% = 16% and stops. Oversimplified (rather than B4 twice).

A4 Compound interest or simple interest formula stated correctly, and stops.

(iii) **5 marks** **Att 2**

16.688% = 8344 $1\% = \dfrac{8344}{16.688} = 500$	16.688% = 8344 $100\% = \dfrac{8344 \text{ x } 100}{16.688}$	8344 ÷ 0.16688	…2m
100% = 50 000	$= 50\,000$	= 50 000	…5m

* Allow candidate to use answer from (ii), e.g. if 16% then 16% = 8344 => 52150 merits 5m.

* Correct answer (using candidate's previous data) without work: 5 marks.
Incorrect answer without work: no marks.

* 1.16688P = 8344 => P = 7150.6 merits 5 marks.

Blunders (-3)

B1 .16688 % = 8344, and continues.

B2 Gets a relevant percentage of 8344, correctly; e.g. 16% or percentage from (ii).

B3 Decimal error in calculations.

Slips (-1)

S1 16.688% rounded to 17%, which gives an answer of 49 082.

Attempts (2 marks)

A1 Mentions 1.16688P or similar.

A2 16.688% (or candidate's figure) = 8344 and stops.

QUESTION 2

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 marks**	**Att 7**
Part (c)	**20 marks**	**Att 7**

Part (a) **10 marks** **Att 3**

Find the value of $x^2 - 5xy$ when $x = 3$ and $y = -2$.

(a) **10 marks** **Att 3**

$3^2 - 5(3)(-2)$ …3m	x(x - 5y) or 3[3 - 5(-2)] …3m
= 9 + 30 …7m	= 3[3 + 10] …7m
= 39 ..10m	= 39 ..10m

* Correct answer without work: full marks (10). Incorrect answer without work: no marks.

Blunders (-3)

B1 Substitution error, once for x and once for y, but if x and y swapped then once overall.
B2 Squaring error, i.e. $(3)^2 \neq 9$.
B3 Sign error, e.g. when multiplying -5(3)(-2).
B4 Algebraic error, e.g. adds or subtracts instead of multiplies, e.g. 3 – 5(3)(–2) = 3 – 15 –2

Attempts (3 marks)

A1 Some correct substitution, and stops.
A2 Some correct relevant multiplication.

Worthless (0)

W1 Solve an equation, unless marks obtained for substitution.
W2 $x^2 = 5xy$ and stops.

Part (b)	**20 marks**	**Att 7**

Solve for x and y

$$x+3=2y$$
$$xy-7y+8=0.$$

(b)	**20 marks**	**Att 7**

I	II	III / IV
I: $x=2y-3$ ...7m $=>(2y-3)y-7y+8=0$ ...8m $\Rightarrow 2y^2-10y+8=0$ ..11m or $y^2-5y+4=0$ $\Rightarrow (y-1)(y-4)=0$ ..14m $=> y=1$ or $y=4$ ..17m $\Rightarrow x=-1$ or $x=5$ ..20m	**II**: $\frac{x(x+3)}{2}-7\frac{(x+3)}{2}+8=0$ ..7m $x(x+3)-7(x+3)+16=0$ ...8m $x^2+3x-7x-21+16=0$ ...11m **or** $x^2-4x-5=0$ $(x+1)(x-5)=0$ ..14m $x=-1, x=5$ ..17m $y=1, y=4$ ..20m	**III**: $xy-2y^2=-3y$ …7m $xy-7y=-8$ …7m sub. $-2y^2+7y=-3y+8$ …8m or $2y^2-10y+8=0$ ..11m and continuing as in I. **IV**: $7x-14y=-21$ …7m $-2xy+14y=16$ …7m add: $7x-2xy=-5$ …8m .: $7(2y-3)-2(2y-3)y=-5$ …8m => $-4y^2+20y-16=0$ ..11m and continuing as in I.

* Sets of coordinates found by trial and error, or without work:
Two correct sets: if verified in both equations, 20 marks; if not verified in both, Att 7.
One correct set: if verified in both equations, Att 7; if not verified in both, no marks.
Both sets incorrect: no marks, whether tried to verify of not.

* No additional marks from the point where the equation becomes linear. But see A3 below.

Blunders (-3)

B1 Error multiplying out brackets, each of brackets.
B2 Sign error, each time, e.g. when totting terms, e.g. -3y - 7y = 10y.
B3 Quadratic formula error (in formula, substitution or simplification). Each time to a maximum of 2 blunders (equivalent to two steps).
B4 Incorrect factors. Apply once.
B5 Incorrect root(s) from candidate's factor(s). Apply once.
B6 One value for x when two available, or one value for y when two available.
B7 Fails to find values of second variable. (B6 and B7 could both apply).
B8 Finds x but substitutes back into y (or vice versa)
B9 Transposition error, e.g. signs. Each time.

Attempts (7 marks)

A1 x = 2y - 3 and stops, or y = (x + 3)/2 and stops.
A2 Correct quadratic formula and stops.
A3 An effort to find the *second* variable, having found the first variable with work of no value (other than inventing the value). i.e. letting x = 0 and y = 0, etc., won't lead to the att.mark.

Worthless (0)

W1 Incorrect values without work.
W2 Invented values substituted, and continues, e.g. x + 3 = 2y => x = 0, y = 1.5 or some such.
However x = 2y – 3 or y = 1/2(x + 3), before inventing value(s) merits Att 7 marks.

Part (c)	20 (10, 5, 5) marks	Att (3, 2, 2)

(i) Write $\sqrt{x} + \dfrac{1}{\sqrt{x}}$ as a single fraction.

(ii) Hence, or otherwise, simplify $\left(\dfrac{2\sqrt{x}}{1 + x}\right)\left(\sqrt{x} + \dfrac{1}{\sqrt{x}}\right)$.

(iii) Solve for x

$$\left(\frac{2\sqrt{x}}{1 + x}\right)\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right) = x - 3.$$

(i)	10 marks	Att 3

$L.C.D. = \sqrt{x}$ ...3m

$= \dfrac{\sqrt{x}.\sqrt{x} + 1}{\sqrt{x}}$ ...7m

$= \dfrac{x + 1}{\sqrt{x}}$ ..10m

Blunders (-3)

B1 Incorrect L.C.D.

B2 Incorrect use of correct L.C.D.

B3 $\sqrt{x} . \sqrt{x} \neq x$, or other index error, e.g. $\sqrt{x}.\sqrt{x} = 2\sqrt{x}$.

B4 $\sqrt{x} + \dfrac{1}{\sqrt{x}} = 0 \Rightarrow x + 1 = 0$, or $x + 1$, or $x = -1$. Compare A1 re $x + 1$.

Attempts (3 marks)

A1 $\dfrac{\sqrt{x}+1}{\sqrt{x}}$ or $\dfrac{\sqrt{x}+1}{x}$ or $\dfrac{\sqrt{x}+x}{x}$ or $\dfrac{x+1}{x}$ or $x + 1$ or $\dfrac{2\sqrt{x}}{\sqrt{x}}$, without supporting work.

A2 $\sqrt{x} = x^{1/2}$ and stops, or $\dfrac{1}{\sqrt{x}} = x^{-1/2}$ and stops.

A3 $\sqrt{x}.\sqrt{x} = x$ anywhere in (i), or any correct multiplication involving x in (i).

A4 Invented value for x and finished correctly.

Worthless (0)

W1 Cancels the $\sqrt{x}$ terms in the question to get 1 + 1 = 2, or cancels them to get 1.

(ii)	**5 marks**	**Att 2**	
$\left(\frac{2\sqrt{x}}{1+x}\right).\left(\frac{x+1}{\sqrt{x}}\right)$	$\frac{2\sqrt{x}}{1+x}.\frac{\sqrt{x}}{} + \frac{2\sqrt{x}}{1+x}.\frac{1}{\sqrt{x}}$ $= \frac{2x}{1+x} + \frac{2}{1+x}$ ...still 2m	$\frac{2\sqrt{x}}{1+x}.\frac{\sqrt{x}}{} + \frac{2\sqrt{x}}{1+x}.\frac{1}{\sqrt{x}} =$	...2m
$= \frac{2\sqrt{x}}{\sqrt{x}}$ *or* $\frac{2(x+1)}{1+x}$ *or* 2	$= \frac{2x+2}{1+x}$ *or* 2	$\frac{2x\sqrt{x}+2\sqrt{x}}{(1+x)\sqrt{x}}$ *or* $\frac{2\sqrt{x}(1+x)}{(1+x)\sqrt{x}}$ *or* 2	...5m

* Accept candidate's answer from (c)(i) for $\sqrt{x}+\frac{1}{\sqrt{x}}$. See A1.

Blunders (-3)
B1 Cross-multiplies instead of cancels.
B2 After cancellation, adds instead of multiplies, e.g. $\frac{2+1}{1+1}=\frac{3}{2}$.

Attempts (2 marks)
A1 Substitutes answer for $\sqrt{x}+\frac{1}{\sqrt{x}}$ from (c)(i) and stops.
A2 Using answer from (c)(i), candidate replaces $\sqrt{x}+\frac{1}{\sqrt{x}}$ with -1 or 1. [Relates to B4 in (i)].
A3 Using answer from (c)(i), candidate replaces every x in both terms of (ii) with -1 or 1.
[A3 relates to B4 in (i)].
A4 An effort to multiply out or to square.
A5 $\sqrt{x}=x^{1/2}$ and stops, or $\frac{1}{\sqrt{x}}=\mathrm{x}^{-1/2}$ and stops.
A6 An invented value for x substituted into the left or both brackets (of question) and finishes correctly -- but A1 might apply.

(iii)	**5 marks**	**Att 2**
2 = x - 3	… 2m	
x = 5	… 5m	

* Accept candidate's answer from (c)(ii) if used in (c)(iii).

Blunders (-3)
B1 Transposition error, e.g. sign error.
B2 Finds value of - x and stops.

Attempts (2m)
A1 Substitutes answer from (c)(ii) and stops.
A2 An effort to multiply out or to square, e.g. squaring the x- 3.
A3 $\sqrt{x}=x^{1/2}$ and stops, or $\frac{1}{\sqrt{x}}=\mathrm{x}^{-1/2}$ and stops.
A4 Any correct operation.

QUESTION 3

Part (a)	**10 marks**	**–**
Part (b)	**20 marks**	**–**
Part (c)	**20 marks**	**–**

Note: The marking of Question 3 is not based on slips, blunders and attempts. In the case of each part, descriptions or typical examples of work meriting particular numbers of marks are given. The mark awarded must be one of the marks indicated. For example, in part (a), descriptions are given for work meriting 0, 4, 7 or 10 marks. It is therefore not permissible to award 1,2,3,5,6,8 or 9 marks for this part.

Part (a) **10 marks** **–**

Given that $ax + b = c$, express x in terms of a, b and c, where $a \neq 0$.

(a) **10 marks** **–**

transpose $ax = c - b$ *transpose* $x = \dfrac{c-b}{a}$	$x + \dfrac{b}{a} = \dfrac{c}{a}$ $x = \dfrac{c}{a} - \dfrac{b}{a}$	$ax + b - c = 0$ $x + \dfrac{b}{a} - \dfrac{c}{a} = 0$ $x = -\dfrac{b}{a} + \dfrac{c}{a}$

* Do not penalise a misreading that does not change the nature of the question, e.g. $ax + 6 = c$, or $ax + c = b$.

10 marks Fully correct answer (i.e. a and b transposed correctly).

7 marks: a or b transposed correctly, e.g. $x = \dfrac{c+b}{a}$, **or** $ax = c - b$, **or** $x = c - b - a$.

4 marks: Correct but unhelpful transposition, e.g. $ax + b - c = 0$ and stops;
or values assigned to a, b and c, and continues with at least one correct transposition.

0 marks: No correct transposition, e.g. $x = b + c - a$ and stops.

Part (b) **20 marks (10, 5, 5)** **–**

(i) Find A, the solution set of $3x - 2 \leq 4$, $x \in Z$.

(ii) Find B, the solution set of $\dfrac{1 - 3x}{2} < 5$, $x \in Z$.

(iii) List the elements of $A \cap B$.

(i) **10 marks** **–**

Transpose: $3x \leq 6$ Transpose: $x \leq 2$ OR List/graph A: $\{\ldots -2,-1,0,1,2\}$	Transpose : $x - \dfrac{2}{3} \leq \dfrac{4}{3}$ Transpose : $x \leq \dfrac{6}{3}$ *or* 2 OR List/graph A: $\{\ldots -2,-1,0,1,2\}$

* Allow $<$ for $\leq$ in (i) but penalise in (iii).

10 marks: Fully correct answer ($x \leq 2$, or set listed)
7 marks: Any correct transposition, e.g. line 1 of either method;
or one correct value found by solving $3x - 2 = 4$ correctly.

4 marks: Tests one or more values;
or solves the equation $3x - 2 = 4$ with error(s);

or any three values correct without work, e.g. {0, 1, 2};

or some work of merit.

0 marks: No work of merit.

(ii) **5 marks** **–**

$1 - 3x < 10$ $-3x < 9$ $x > -3$ <u>OR</u> listed/graphed: {-2,-1,0,1,2,3…}	$1 - 3x < 10$ $-9 < 3x$ $-3 < x$ <u>OR</u> listed/graphed: {-2,-1,0,1,2,3…}	$x > -3$ without work <u>OR</u> listed/graphed without work: {-2,-1,0,1,2,3…}

* Allow $\leq$ for < in (ii) but penalise in (iii)

5 marks: Fully correct answer ($x > -3$ or set listed).

2 marks: Any correct relevant step, e.g. a correct transposition,
or tests at least one value
or_ solves the equation $\frac{1-3x}{2} = 5$.
or any three values correct without work, e.g. {0, 1, 2};
or some work of merit.

0 marks: No correct relevant step.

(iii) **5 marks** **–**

A = {…-2,-1,0,1,2} and B = {-2,-1,0,1,2,3…} => $A \cap B = \{-2,-1,0,1,2\}$.

5 marks Fully correct list of numbers for $A \cap B$ from candidate's A and B,
i.e. from inequalities such as $x \leq 2$ and $x > -3$, not from listed sets in (i) and (ii).

4 marks: Incorrectly includes and/or excludes the endpoints of the set $A \cap B$ in an otherwise correct solution. [See Note 1 in (i) and (ii)].

2 marks: Any list containing ONE correct element of the candidate's $A \cap B$;
or some effort to combine sets A and B;
or correct intersection of the candidate's sets indicated on a number line (showing integers or real numbers); (Question asked for a *list.*)
or draws set A or B or both on a number line.

0 marks: No work of merit.

Part (c)	**20 marks**	–

Let $f(x) = 2x^3 - 3x^2 - 11x + 6$.
(i) Verify that $f(3) = 0$.
(ii) Solve the equation: $2x^3 - 3x^2 - 11x + 6 = 0$.

(c)(i)	**10 marks**	–

$f(x) = 2(3)^3 - 3(3)^2 - 11(3) + 6$ $= 2(27) - 3(9) - 11(3) + 6$ $= 54 - 27 - 33 + 6.$	May sccessfully divide x – 3 into the cubic, using algebra or synthetic division, and then conclude that x –3 a factor => x = 3 is a root or f(3) = 0.

10 marks: Fully correct answer, (i.e. f(3) evaluated fully and correctly).

7 marks: Fully evaluates f(3) but with one error. Note: treat $3^3= 9$ and $3^2=6$ as one error;
or correctly verifies that x – 3 is a factor of the cubic,
but has no conclusion re f(3) = 0 or x = 3 being a root.

4 marks: Correctly substitutes x = 3, or $2(3)^3 - 3(3)^2 - 11(3) + 6 = 0$ and stops;
or f(–3) or f(any other number) evaluated correctly.

0 marks: No work of merit.

(ii)	**10 marks**	–

$$\begin{array}{r} 2x^2 + 3x - 2 \\ x-3\,\overline{)\,2x^3 - 3x^2 - 11x + 6} \\ \underline{2x^3 - 6x^2} \\ 3x^2 - 11x \\ \underline{3x^2 - 9x} \\ -2x + 6 \\ \underline{-2x + 6} \end{array}$$ $2x^2 + 3x - 2 = (2x-1)(x+2)$ $x = 3$ and $x = 1/2$, x = –2.	$$\begin{array}{cc|ccc} 3 & 2 & -3 & -11 & 6 \\ & & 6 & 9 & -6 \\ \hline & & 3 & -2 & 0 \end{array}$$ $2x^2 + 3x - 2 = (2x-1)(x+2)$ $x = 3$ and $x = 1/2$, x = –2.	x - 3 is a factor => $(x - 3)(2x^2 + ax - 2)$ = $2x^3 - 3x^2 - 11 + 6$.: $2x^3 - 6x^2 + ax^2 - 3ax - 2x + 6$ $= 2x^3 - 3x^2 \quad -11x \quad + 6$ - 6 + a = -3 => a = 3 or - 3a - 2 = -11 => a = 3 .: $(x - 3)(2x^2 + 3x - 2) = 0$ = (x - 3)(2x -1)(x + 2) = 0 => x = 3, ½ , -2.

* If (c)(i) has been done, then x = 3 need <u>not</u> be mentioned in (c)(ii)

* If (c) (i) not done, but (ii) fully correct award 10 + (10 or 7) depending on whether the conclusion re x -3 a factor => f(3) = 0 or x = 3being a root is stated or not.
If (c)(i) not done, and (c)(ii) is attempted but x-3 is unsuccessfully divided into the cubic, then mark from (ii) only, with 0m for (i).

10 marks: Fully correct solution (i.e. find the remaining two roots, -2 and ½).

7 marks: Forms the quadratic, by division or method III, with one or more errors, then solves own quadratic correctly;
or forms correct quadratic, by division or method III, and finishes with the correct method but with error(s);
or finds roots- 2 <u>or</u> ½ by trial and error (substitution);

4 marks: Says x - 3 is a factor or shows intent to divide cubic by x - 3, i.e. $x - 3\,\overline{)\text{cubic}}$;
or an unsuccessful effort to find roots by trial and error;
or incorrectly formed quadratic with an effort to solve it by quad formula or factors;
or quadratic formula correct, and stops;
or incorrect quad formula (at most one error) and some substitution from a quadratic.

0 marks No new work of merit, e.g. e.g. states f(3) = 0 again, or states x = 3 is a root.

QUESTION 4

Part (a)	**15 marks**	**Att 6**
Part (b)	**20 marks**	**Att 6**
Part (c)	**15 marks**	**Att 5**

Part (a) **15 (5; 5, 5) marks** **Att (2; 2, 2)**

Let $u = 4 - 2i$, where $i^2 = -1$.
Plot
(i) u
(ii) $u - 4$
on an Argand diagram.

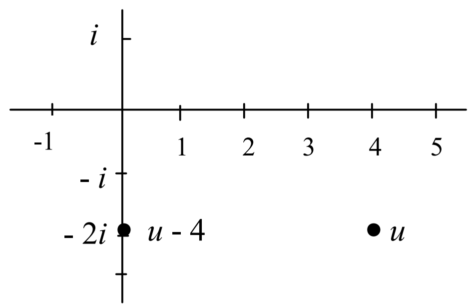
(a) **15 (5; 5, 5) marks** **Att (2; 2, 2)**

(i) $u = 4 - 2i$ plotted … 5m

(ii)
$u - 4 = 4 - 2i - 4$ … 2m
$= -2i$ … 5m

---

and $-2i$ plotted … 5m



* In marking terms, there are three sections in (a), each worth 5 marks, att 2m
* If the axes are reversed they must be identified, or B1 applies.
* Unlabelled axes: assume horizontal axis is real, e.g. (2, - 4) plotted on unlabelled axes: B1.
* Attempt marks *for drawing axes* can not be awarded twice, e.g.. if marks in (i) are not for drawing axes (e.g. 5m, or B1=> 2m), then candidate can earn *att 2m* for the axes in (ii).
* If $u - 4 = -2i$ is not found and/or $-2i$ is not clearly identified on imaginary axis as the answer to (ii), allow the attempt marks for axes, if not already awarded in (i).
* One unnamed point plotted: assume it is u.
* If two unnamed dots in correct positions on graph, apply benefit of doubt: award 5+10m.
* If $u - 4$ is not calculated, award 5m + 5m for correct plotting of $u - 4$, and 0m + 5m for incorrect plotting of $u - 4$. Or, without work, u plotted at $4 + 2i$ and $u - 4$ plotted at $2i$: treat as M(-1) => 4 + 5 + 5 = 14 marks.

Blunders (-3)
B1 Incorrect plotting of u, e.g. at $2 - 4i$.
B2 Incorrect calculation of u - 4, e.g. $2i$.
B3 Incorrect plotting of u - 4.
B4 Incorrect or no scales on axes (i.e. tick marks if not using graph paper). If scales *incorrect* apply B4 once, but if *no scales* apply B4 each time.

Misreadings (-1)
M1 u and $u - 4$ correctly plotted but labels swapped. Apply misreading once. (4+10m).

Attempts (2 marks)
A1 A correct set of scaled axes (ticks sufficient). Apply once.
A2 $u - 4$ incorrectly calculated (i.e. att 2m), but if result plotted correctly: att 2 + 5m in (ii).

Part (b)	**20 (10,10) marks**	**Att (3, 3)**

Let $w = 1 + 3i$.

(i) Express $\dfrac{2}{w}$ in the form $x + yi$, where $x, y \in \mathbf{R}$.

(ii) Investigate whether $|iw + w| = |iw| + |w|$.

(i)	**10 marks**	**Att 3**

I: $\left(\dfrac{2}{1+3i}\right)\left(\dfrac{1-3i}{1-3i}\right)$	…3m	II: $\left(\dfrac{2}{w}\right)\left(\dfrac{\overline{w}}{\overline{w}}\right)$	…3m
$2 - 6i$ <u>or</u> $1 - 9i^2$	…4m	$2\overline{w} = 2-6i$ *or* $w\overline{w} = 1-9i^2$	…4m
$2-6i$ *and* $1-9i^2$; **or** $\dfrac{2-6i}{1-9i^2}$	…7m	$2-6i$ *and* $1-9i^2$; **or** $\dfrac{2-6i}{1-9i^2}$	…7m
$= \dfrac{2-6i}{10}$ *or* $\dfrac{2}{10} - \dfrac{6i}{10}$	..10m	$= \dfrac{2-6i}{10}$ *or* $\dfrac{1}{5} - \dfrac{3i}{5}$	..10m
III: $\dfrac{2}{1+3i} = a+bi$ (3m), cross mult, compare coeffs and solves for *a, b*: mark on slip/blunder.			

Blunders (–3)
B1 Incorrect conjugate.
B2 $i^2 \neq -1$. Apply once in (b)(i).
B3 Each omitted or incorrect term when multiplying out. max of 2 (1 on num., 1 on denom.).
B4 Real and imaginary parts mixed up, e.g. when adding.
B5 Inverts in the last step, e.g. $10/(2 - 6i)$.
B6 Denominator not real after multiplication, or forgets to multiply denom. by conjugate.
B7 Multiplies out numerators and denominators and stops (i.e. 7m if correctly done).

Misreading(-1)
M1 $w = 1 - 3i$.

Slips (–1)

S1 Numerical slip when adding real to real, or imaginary to imaginary.

Attempts (3 marks)
A1 Substitutes for *w* and stops.
A2 Correct conjugate and stops.
A3 Any correct and relevant multiplication.

(ii) **10 marks** **Att (3)**

$iw = i(1+3i) = 1i + 3i^2 = i - 3$ or $-3 + i$

$iw + w = -3 + i + 1 + 3i = -2 + 4i$

$|iw + w| = |-2+4i| = \sqrt{(-2)^2 + 4^2} = \sqrt{4+16} = \sqrt{20}$

$|w| = \sqrt{1^2 + 3^2} = \sqrt{1+9} = \sqrt{10}$

$|iw| = \sqrt{(-3)^2 + 1^2} = \sqrt{9+1} = \sqrt{10}$

$\therefore \sqrt{20} \neq \sqrt{10} + \sqrt{10}.$

* Any modulus found correctly, 3m. Two correct, 6m. Three correct, 9m. And 1 mark for the correct conclusion, using candidate's data.
* If no effort made to find a modulus, maximum possible marks are att 3 for substitution or multiplication, i.e. a mod formula must be stated or used to earn more than 3 marks.
* Accept correct Coordinate Geometry distance method; apply the usual slips and blunders.
* B1, B2, B3, B4 and B6: each applies <u>once</u> <u>overall</u> in (b)(ii) <u>if consistent.</u>

Blunders (–3)

B1 Incorrect modulus formula (with at most one error), i.e. $\sqrt{a^2 - b^2}$ evaluated correctly, or $a^2 + b^2$ evaluated correctly. These only. See notes above.

B2 Incorrect substitution into correct formula, e.g. $\sqrt{-2^2 + 4i^2}$.

B3 Square root error, e.g. $\sqrt{4+16} = 2+4$.

B4 Adds real to imaginary parts.

B5 $|-3 + i + 1 + 3i| = \sqrt{(-3)^2 + 1^2 + 1^2 + 3^2}$

B6 Sign error involving i^2.

Slips (–1)

S1 Numerical slip, e.g. in concluding $\sqrt{20} = \sqrt{10} + \sqrt{10}$, or candidate's equivalent.

S2 No final conclusion.

Attempts (3 marks)

A1 Substitutes correctly for w (or iw, or $w + iw$), and stops. (Needn't be in mod form).

A2 $\sqrt{a^2 + b^2}$ and stops, or Coordinate Geometry distance formula correct and stops.

A3 $\sqrt{a^2 - b^2}$ *with* some correct substitution, or $a^2 + b^2$ *with* some correct substitution or distance formula with *one* error and *with* some correct substitution, and stops.

A4 $|z|^2 = z.\bar{z}$ (or equivalent).

Worthless (0)

W1 $\sqrt{a^2 - b^2}$ *without* substitution, or $a^2 + b^2$ *without* substitution.

W2 Other incorrect formula with/without substitution.

Part (c) **15 (5, 10) marks** **Att (2, 3)**

Let $z = 1 - 2i$.

(i) Write down $\bar{z}$, the complex conjugate of z.

(ii) Find the real numbers k and t such that

$$kz + t\bar{z} = 2z^2.$$

(c)(i) **5 marks** **Att 2**

$\bar{z} = 1 + 2i$

Blunders (-3)

B1 $\bar{z}$ = -1 - 2i or $\bar{z}$ = -1 + 2i.

B2 $\bar{z}$ = 2 + i, or 2 - i, or -2 + i, or -2 - i.

B3 z = 1 - 2i = - i and *continues*, e.g. $\bar{z}$ = i merits attempt 2 marks.

Misreadings (-1)

M1 z = 1 + 2i, or 1 + 3i. Apply once overall in (c).

Attempts (2 marks)

A1 1 + 2i appears in the candidate's answer, e.g. $\dfrac{1-2i}{1+2i}$ *or* $\dfrac{1+2i}{1-2i}$.

Worthless (0)

W1 All other incorrect $\bar{z}$ apart from B1 and B2 above, e.g. $\bar{z}$ = 1 - 2i (i.e. no change).

(c)(ii) **10 marks** **Att 3**

$k(1-2i) + t(1+2i) = 2(1-2i)^2$ ...3m

$\Rightarrow k - 2ki + t + 2ti = 2 - 8i \overbrace{-8 \text{ or } +8i^2}$ ...4m

=> k + t = - 6 *or* k + t = - 6
-2ki + 2ti = - 8i -2k + 2t = - 8 ...7m (for one or both equations)

.: 2k + 2t = -12
4t = - 20 => t = - 5
=> k - 5 = - 6 => k = - 1 ..10m

* Accept candidate's $\bar{z}$ from (i) unless it oversimplifies the question, e.g. if candidate uses z = - i and $\bar{z}$ = i, then this question is oversimplified: award a max. of att 3 for (ii).

* Correct k and t found without work: if both verified, 10m; but if both not verified, 0m.

Blunders (-3)

B1 $i^2 \neq -1$.

B2 Sign errors, each time.

B3 Error multiplying out brackets, once if consistent.

B4 Error squaring brackets, once only — but may also incur B1.

B5 Real and imaginary parts confused, e.g. real ≠ real, imaginary ≠ imaginary.

B6 Error solving simultaeous equations, e.g. finding one value only.

Attempts (3 marks)

A1 Substitutes for z and/or $\bar{z}$, and stops.

QUESTION 5

Part (a)	**10 marks**	**Att 4**
Part (b)	**20 marks**	**Att 8**
Part (c)	**20 marks**	**Att 7**

<u>*The following apply to all parts of Question 5*</u>

* Correct answers without work: full marks. Incorrect answers without work: no marks.
* No penalty for incorrect notation, e.g. Sn written instead of Tn.
* No retrospective marking. (i.e. work done for later parts doesn't yield marks for earlier parts.)
* Allow candidate to use answers found in previous part(s) within (a), (b) and (c).
* Decimals may be used instead of fractions. Reasonable accuracy required.

B Incorrect relevant formula (at most one error) with some correct substitution: B(-3).
A Correct relevant formula and stops: award the attempt marks.
A Correct but *irrelevant* formula merits the attempt mark if it has some correct substitution, e.g. correct GP formula in an AP question merits att marks IF some substitution is correct.
W Correct but *irrelevant* formula and *stops*: no marks, e.g. GP formula in AP question, stops.
W Incorrect and irrelevant formula, and stops: no marks.

Part (a)	**10 (5, 5)marks**	**Att (2, 2)**

The first term of an arithmetic sequence is 9 and the second term is 13.
(i) Find the common difference.
(ii) Find the third term.

(i)	**5 marks**	**Att 2**

$$d = T_2 - T_1 \text{ or } 13 - 9 \quad \ldots 2m$$
$$= 4 \quad \ldots 5m$$

Blunders (-3)
B1 Adds instead of subtracts, e.g. $T_2 + T_1$ or $13 + 9 = 22$.
B2 $T_1 - T_2 = 9 - 13 = -4$, but treat $9 - 13 = 4$ as an error rectified and do not penalise.

Attempts (2 marks)
A1 $d = T_2 - T_1$ and stops.
A2 9 - 13 and stops, or 13 - 9 and stops.
A3 $T_1 = 9$, or $T_2 = 13$, or $a = 9$, or $a + d = 13$, or $T_3 = a + 2d$, or $T_n = a + (n - 1)d$ and stops. (i.e some correct relevant statement, and stops).

Worthless (0)
W1 Multiplies or divides 9 and 13.
W2 Writes $T_1 - T_2$ and stops.

(ii)	**5 marks**	**Att 2**

$T_3 = T_2 + d$ *or* $13 + 4$ … 2m	$T_3 = a + (n-1)d$ *or* $a + 2d$ *or* $9 + (3-1)(4)$ *or* $T_3 = 9 + 2(4)$ … 2m
$T_3 = 17$ … 5m	$T_3 = 17$ …5m

Blunders (-3)
B1 Incorrect d, e.g. d correct in (i) but a different d used in (ii).
B2 Subtracts instead of adds, i.e. 13 - 4 instead of 13 + 4..
B3 $T_1 + d = 9 + 4 = 13$.
B4 Incorrect substitution (for a or d or n) into T_n or S_n of AP formula.
B5 Swaps a and d, one blunder, e.g. $a + (n - 1)d = 4 + (3 - 1)9 = 4 + 2(9) = 22$.

Attempts (2 marks)

A1 Correct a or d values stated or substituted into an AP or GP formula.

Worthless (0)

W1 13 x 4 = 52, or 13 / 4 = 3.25, with no specific mention of a and d values.

Part (b) **20 (5, 5, 5, 5) marks** **Att (2, 2, 2, 2)**

> The sum of the first n terms of an arithmetic series is given by
> $S_n = n^2 + n$.
> **(i)** Find a, the first term.
> **(ii)** Find S_2, the sum of the first two terms.
> **(iii)** Find d, the common difference.
> **(iv)** Write down the first five terms of the series.

(i) **5 marks** **Att 2**

I: a or $S_1 = 1^2 + 1$ …2m $= 1 + 1$ …4m $= 2$ …5m	II: Substitute into $T_n = S_n - S_{n-1}$ …2m to find $T_n = 2n$ …4m $\Rightarrow T_1 = 2(1) = 2$ …5m
III: Substitute into $T_{n+1} = S_{n+1} - S_n$ …2m to find $T_{n+1} = 2n^2 + 2$ …4m $n = 0 \Rightarrow T_1 = 2(0)^2 + 2 = 2$ …5m	

Blunders (-3)

B1 Squaring error, e.g. $1^2 = 2$, or $(n+1)^2 \neq n^2 + 2n + 1$, or doesn't square (stops at $1^2 + 1$).

B2 Incorrect relevant formula, e.g. $T_n = S_n + S_{n-1}$ or $T_n = S_{n-1} - S_n$, *with some correct subs.*

B3 Substitution error, e.g. incorrect value of n.

Attempts (2 marks)

A1 n = 1 and stops, or S_1 and stops.

A2 Incorrect n substituted and continues.

A3 $T_2 = S_2 - S_1$ and stops, or $S_2 - T_2 = S_1$ and stops.

Worthless (0)

W1 Incorrect formula and stops.

(ii) **5 marks** **Att 2**

$S_2 = 2^2 + 2$ …2m $= 4 + 2$ … 4m $= 6$ …5m	From (i) method II above , $T_n = 2n = 4$ … 2m $\Rightarrow S_2 = T_1 + T_2 = 2 + 4$ … 4m $= 6$ …5m

* If method II is used, accept candidate's answer from (i).

* Refer to notes at start of Q5 when necessary.

Blunders (-3)

B1 Squaring error, or doesn't square (stops at $2^2 + 2$).

B2 Incorrect relevant formula (with at most one error) with *some correct substitution.*

B3 Incorrect substitution, e.g. $Sn = n^2 + n \Rightarrow S_2 = 2^2 + 1$, but if $Sn = n^2 + 1 \Rightarrow S_2 = 2^2 + 1$ then it's a *misreading* (-1m).

Note: $Sn = n^2 + 1$ generates the Sn sequence 2,5,8,11,14, and Tn sequence 2,3,4,5,6.

Attempts (2 marks)

A1 $n = 2$ stated and stops.

A2 $T_3 = S_3 - S_2$ and stops, or $S_3 - T_3 = S_2$ and stops.

Worthless (0)

W1 Incorrect formula and stops.

(iii) **5 marks** **Att 2**

I: $T_2 = S_2 - S_1$ or $6 - 2 = 4$ …2m $d = T_2 - T_1$ or $4 - 2 = 2$ …5m	II: $T_2 = S_2 - S_1$ or $6 - 2 = 4$ …2m $a + 1d = 4 \Rightarrow 2 + d = 4 \Rightarrow d = 2$ …5m
III: Candidate may use $T_n = 2n$, from (i), and $d = T_{n+1} - T_n$ or $d = T_n - T_{n-1}$ to find $d = 2$ …5m	

* If method III used, mark on slip and blunder. Refer to notes at start of Q5 when necessary.

Blunders (-3)

B1 $T_2 = 4$ and stops. Or, takes d to be $S_2 - S_1$, or 6 - 2, or 4, or the candidate's equivalents.

B2 $T_2 = S_2 + S_1$ and continues, e.g. $T_2 = 6 + 2 = 8$.

B3 $S_2 = (2^2 + 2) + (1^2 + 1) = (4 + 2) + (1 + 1) = 8$.

Attempts (2 marks)

A1 Incorrect relevant formula *used*, $T_n = S_n + S_{n-1}$, $T_n = S_{n-1} - S_n$, with some correct substittion.

A2 $S_2 - S_1$ and stops, or $T_n - T_{n-1}$ and stops

A3 Mentions 2n, but not 2n +1 (i.e. differentiation)

A4 Some correct substitution, and stops.

(iv) **5 marks** **Att 2**

I: $a = 2$, $d = 2$ $\Rightarrow$ 2, 4, 6, 8, 10	II: $Tn = a + (n-1)d$, with $a = 2$, $d = 2$, $n = 1, 2, 3, 4, 5$ $T_1 = 2 + (0).2 = 2$ $T_2 = 2 + (1.)2 = 4$ $T_3 = 2 + (2).2 = 6$ $T_4 = 2 + (3).2 = 8$ $T_5 = 2 + (4).2 = 10$

* Correct five terms without work: 5 marks. All terms incorrect without work: 0m. See A1.

* See Note 4 of notes at start of Q5.

Blunders (-3)

B1 Subtracts instead of adds, e.g. 2, 0, -2, -4, -6. (one blunder)

B2 Incorrect or missing term without work, each time. T_1 may be implied e.g. $T_2 = 2 + 2$.

Misreadings (-1)

M1 Misreads as different five terms, e.g. T_2 to T_6, or T_5 to T_9, etc.

Attempts (2 marks)

A1 Any one term correct, using candidate's data, or without work.

A2 $a = 2$, $T_1 = 2$, or $d = 2$ and stops

A3 Correct Tn of AP and stops, or Sn of AP with some correct substitution.

Worthless (0)

W1 Sn formula of AP (even if correct) unless some substitution redeems it.

Part (c) **20 (10, 5, 5) marks** **Att (3, 2, 2)**

In a geometric sequence of positive terms, the third term is $\frac{1}{4}$ and the fifth term is $\frac{1}{16}$.

(i) Find r, the common ratio.
(ii) Find a, the first term.
(iii) How many terms of the sequence are greater than 0.01?

(i) **10 marks** **Att 3**

$T_3 = ar^2 = \frac{1}{4}$	$T_5 = ar^4 = \frac{1}{16}$	$a, \ ar, \ ar^2$	¼, T_4, 1/16	…3m
$T_5 = ar^4 = \frac{1}{16}$	$T_3 = ar^2 = \frac{1}{4}$	¼ , ¼ r, ¼ r^2	By inspection, T_4 =	…4m
$\frac{ar^2}{ar^4} = \frac{1/4}{1/16} = \frac{16}{4}$	$\frac{ar^4}{ar^2} = \frac{1/16}{1/4} = \frac{4}{16}$	¼ r^2 = 1/16 => $r^2 = \frac{1/16}{1/4}$	$r = \frac{\;}{1/4}$	…7m
$\Rightarrow \frac{1}{r^2} = 4$	$\Rightarrow r^2 = \frac{1}{4}$	=> r^2 = ¼		
$\Rightarrow r = \frac{1}{2}$	$\Rightarrow r = \frac{1}{2}$	=> r = ½	=> r = ½	..10m

* Correct answer, without work: full marks. Incorrect answer without work: no marks

* If (c) is unlabelled, then for 1, ½, ¼, , 1/16 without work, award 10m for (i), 5m for (ii), 0m for (iii); but, if ¼, , 1/16 without work, award 4m for (i), 0m for (ii) and 0m for (iii).

Blunders (-3)

B1 Incorrect substitution into correct Tn of GP formula.
B2 Does not divide equations, or error in division.
B3 Square root error.
B4 Gives r = -1/2 only.
B5 Indices error.
B6 1/16 ÷ ¼ = ¼ or ¼ ÷ 1/16 = 4, and stops. (Last step missing, 7m). [continued…]

Misreading (-1)

M1 Gives $r = \pm$ ½.

Attempts (3 marks)

A1 Correct Sn of GP formula with some correct substitution
A2 Incorrect Tn of GP formula with correct substitution, e.g. $Tn = ar^n - 1$ substituted.
A3 ¼ . ¼ = 1/16 => r = ¼ [This is not B6].

(ii) **5 marks** **Att 2**

$ar^2 = a(\frac{1}{2})^2 = \frac{1}{4}a = \frac{1}{4}$ $a = 1$	$ar^4 = a(\frac{1}{2})^4 = 1/16\, a = 1/16$ $a = 1$	$T_2 = T_3 \div r = \frac{1}{4} \div \frac{1}{2} = \frac{1}{2}$ $T_1 = T_2 \div r = \frac{1}{2} \div \frac{1}{2} = 1$	…2m …5m

Blunders (-3)
B1 Incorrect substitution into correct relevant formula, e.g. substituting value of *r* into *a*.
B2 Division error
B3 Indices error.

Attempts (2 marks)
A1 Correct Sn of GP formula with some correct substitution.
A2 Some correct substitution
A3 $1/16 \div \frac{1}{4}$ or $1/16 \div \frac{1}{2}$, and stops.
A4 $1/16 \div 1/16$ or $\frac{1}{4} \div \frac{1}{4}$, and stops.

(iii) **5 marks** **Att 2**

$ar^{n-1} > 0.01 = \dfrac{1}{100} \Rightarrow \dfrac{1}{2^{n-1}} > \dfrac{1}{100}$ $\Rightarrow 2^{n-1} < 100 \Rightarrow n = 7$	$1, \dfrac{1}{2}, \dfrac{1}{4}, \dfrac{1}{8}, \dfrac{1}{16}, \dfrac{1}{32}, \dfrac{1}{64}, \dfrac{1}{128}$ or decimal equivalents = 7 terms

* Mark the candidate's sequence on slip and blunder.

Blunders (-3)
B1 Decimal error, e.g. $1 \div .01 = 10$.
B2 Indices error, e.g. $2^{n-1} = 2^n - 1$
B3 Transposition error.
B4 Sign error.
B5 Inversion error, e.g. in the inequality sign.

Slips (-1)
S1 In method II, miscounts terms, e.g. n = 6 or 8.

Attempts (2 marks)
A1 Sn of GP formula used.
A2 Some correct substitution of *a* and/or *r*.

QUESTION 6

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 marks**	**Att 7**
Part (c)	**20 marks**	**Att 7**

Part (a) **10 marks** **Att 3**

Let $g(x) = \dfrac{x+5}{2}$, $x \in \mathbf{R}$.
Find $g(0) + g(2)$.

(a) **10 marks** **Att 3**

g(0) = (0 + 5) / 2 …3m = 2.5 or 5/2 …4m g(2) = (2 + 5) / 2 …7m = 3.5 or 7/2 …9m = 6 ..10m	$g(0)+g(2)=\dfrac{0+5}{2}+\dfrac{2+5}{2}$ ...7m = 2.5 + 3.5 ...9m = 6 ..10m

* Correct answer without work: full marks. Incorrect answer without work: no marks.
* If g(2) is done first, then g(2) = (2 + 5)/2 gets the 3m, and '=3.5' gets the 4m, and so on.

Blunders (-3)
B1 Cancellation error, for example $\dfrac{2+5}{2}=1+5=6$.
B2 Decimal error, each time.

Misreadings (-1)
M1 Consistently takes $\dfrac{x+5}{2}$ as $x+\dfrac{5}{2}$.

Attempts (3 marks)
A1 Any correct substitution and stops.
A2 (x + 5)/2 = 0 or (x + 5)/2 = 2 solved, or an effort (with work) to solve the equation.

Part (b) **20 marks** **Att 7**

Differentiate $3x - x^2$ with respect to x from first principles.

(b) **20 marks** **Att 7**

$f(x+h) = 3(x+h) - (x+h)^2$	7m	$y + \Delta y = 3(x + \Delta x) - (x + \Delta x)^2$
$f(x+h) = 3x + 3h - (x^2 + 2hx + h^2)$	8m	$y + \Delta y = 3x + 3\Delta x - \{x^2 + 2x\Delta x + (\Delta x)^2\}$
$f(x+h) = 3x + 3h - x^2 - 2hx - h^2$	11m	$y + \Delta y = 3x + 3\Delta x - x^2 - 2x\Delta x - (\Delta x)^2$
$f(x+h)-f(x) = 3x + 3h - x^2 - 2hx - h^2 - 3x + x^2$		$\underline{y = 3x - x^2}$
$f(x+h)-f(x) = 3h - 2hx - h^2$	14m	$\Delta y = 3\Delta x - 2x\Delta x - (\Delta x)^2$
$\dfrac{f(x+h)-f(x)}{h} = 3 - 2x - h$	17m	$\dfrac{\Delta y}{\Delta x} = 3 - 2x - \Delta x$
$\lim\limits_{h \to 0} \dfrac{f(x+h)-f(x)}{h} = 3 - 2x$	20m	$\lim\limits_{\Delta x \to 0} \dfrac{\Delta y}{\Delta x} = 3 - 2x$

* Overlook $\Delta x = 0$ or $h = 0$ in limit; and use of dy/dx instead of lim $\Delta y/\Delta x$.
* May begin with *[f(x + h) - f(x)] /h* and work out RHS down to 2nd last line, and take limit.
* If first mention of LHS is in last line, then B5 and B6 apply; i.e. 14m for RHS correct.
* Perfectly correct RHS but <u>no</u> LHS => B5 + B6 + B7 apply.
* After substit. and further work, B(-3) for each major step omitted; see steps in central col.
* f(x – h) used: no penalty.
* Sign error in early part of solution (e.g. when removing brackets) but switches back at/near end to 3 - 2x in final line: treat as one blunder.

Blunders (-3)

B1 Error multiplying out $(x + \Delta x)^2$ or $(x + h)^2$. Apply B1 once, but B3 may also apply.
B2 Error multiplying out , e.g. $3(x + \Delta x) = 3x + \Delta x$, or omits the multiplication by 3.
B3 Sign/bracket error in line 3. Apply once.
B4 $(\Delta x)^2 = \Delta^2.x^2$ (if it affects the solution); or $2x\Delta x = 2\Delta x^2$, <u>but</u> allow Δx^2 for $(\Delta x)^2$.
B5 Omits $y + \Delta y$, or Δy, or f(x + h), or f(x + h) - f(x) on LHS. Apply once.
B6 Omits $\Delta y/\Delta x$ or { f(x + h) - f(x)}/h on L.H.S.
B7 Omits limiting idea (word "lim" unnec.) or has other than $\Delta x \to 0$ or $h \to 0$ on L.H.S., i.e. should have "lim", or "$\Delta x \to 0$" (allow "$\Delta x = 0$" instead), or "dy/dx".
B8 Evaluates limit where Δx or h will not divide, e.g. no Δx on R.H.S. at that stage.
B9 Limit error, e.g. $\Delta y/\Delta x = 3 - 2x - \Delta x$ but lim $\Delta y/\Delta x \neq 3 - 2x$.
B10 Differentiates from first principles $3x + x^2$.
B11 $3x - x^2 = - 2x^2$ and differentiates this from first principles, but if $3x - x^2 = 2x^2$ apply two blunders (as risk of B2 and B3 is avoided).

Slips (-1)

S1 Correct term such as $2x\Delta x$ subsequently "becomes" $2\Delta x$. (Misreads own work.)

Misreadings (-1)

M1 A misreading which retains the $- x^2$, e.g. $5x - x^2$. (See B10). If $3 - x^2$ then B2 would apply.

Attempts (7 marks)

A1 $y + \Delta y$ or f(x + h) on LHS; or $x+\Delta x$ or x + h or x – h substituted somewhere on RHS.
A2 Linear function differentiated from first principles: even if correct, award Att 7. e.g. 3 - 2x.

Worthless (0)

W1 Answer 3 - 2x without work (i.e. not from first principles).

Part (c)	**20 (5, 10, 5) marks**	**Att (2, 3, 2)**

Let $f(x) = x^2 + px + 10, \quad x \in \mathbf{R}, \quad$ where $p \in \mathbf{Z}$.
(i) Find $f'(x)$, the derivative of $f(x)$.
(ii) The minimum value of $f(x)$ is at $x = 3$. Find the value of p.
(iii) Find the equation of the tangent to $f(x)$ at the point (0, 10).

(c)(i)	**5 marks**	**Att 2**

$$f'(x) = 2x + p$$

Blunders (-3)

B1 Differentiation error, once per term. Three terms to check.

Attempts (2 marks)

A1 Mentions dy/dx, or tries to differentiate.
A2 Effort at first principles, e.g. $y + \Delta y$, $x + \Delta x$, or x + h mentioned.

(ii)	**10 marks**	**Att 3**
$f'(x) = 0$ or $2x + p = 0$ At $x = 3$, $2(3) + p$ $2(3) + p = 0$ $\Rightarrow p = -6$	$2x + p = 0$ … $x = \frac{-p}{2}$ $\frac{-p}{2} = 3$ $\Rightarrow p = -6$	…3m …4m ..7m . .10m

* No marks if derivative is not used or mentioned, e.g. solves $f(3) = 0$ instead of $f'(3) = 0$.

Blunders (-3)
B1 Sign error.
B2 Transposing error.

Attempts (3 marks)
A1 Mentions $f'(x)$, or dy/dx, or $2x + p$, or candidate's derivative in (i), and stops.

(iii)	**5 marks**	**Att 2**
Slope at $x = 0$ is $f'(0) = 0 - 6 = -6$. $y - 10 = -6(x - 0)$ <u>*or*</u> $6x + y - 10 = 0$	<u>y = mx + c</u> m = $f'(0)$ = 2(0) + p = p .: <u>m = - 6</u> => y = - 6 x + c …. and (0,10) $\in$ T => 10 = - 6(0) + c => <u>c = 10</u> <u>*or*</u> y = - 6x + 10	

* Mark on slip and blunder, using candidate's value of p.

Blunders (-3)
B1 Sign error.
B2 Transposing error.
B3 Slope error, e.g. $m = f(0)$, or $f'(x) = 0 \Rightarrow 2x - 6 = 0 \Rightarrow m = 3$, or $y - 10 = m(x - 0)$ and stops.
B4 Incorrect formula for equation of line, with correct substitution of (0, 10) or the value of m.
B5 Substitution error, e.g. (10, 0) for (0, 10). Apply once.

Attempts (2 marks)
A1 Slope = dy/dx or $f'(x)$, or $m = f'(x)$, or $m = f'(0)$, stops.
A2 Equation of line formula correct, either version, and stops.
A3 $f(x) = x^2 - 6x + 10$ or candidate's equivalent, and stops.

Worthless (0)
W1 $m = \frac{y_2 - y_1}{x_2 - x_1}$.

QUESTION 7

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 marks**	**Att 6**
Part (c)	**20 marks**	**Att 7**

Part (a) **10 marks** **Att 3**

Differentiate $9+3x-5x^2$ with respect to x.

(a) **10 marks** **Att 3**

3 - 10x

* Correct answer without work or notation: full marks, 10m.
* If done from first principles, ignore errors in procedure – just mark the answer.
* Only one term correctly differentiated, award 4 marks.

Blunders (-3)
B1 Differentiation error. Once per term. (Three terms to check. See note 3 above).

Attempts (3 marks)
A1 Unsuccessful effort at first principles, e.g. $y + \Delta y$ on L.H.S., or x replaced by $x + \Delta x$ on R.H.S., 'limit' mentioned, $\Delta x \rightarrow 0$, $f(x+h)$, etc.
A2 Writes down the notation 'dy/dx' or '$f'(x)$' and stops.

Worthless (0)
W1 No term differentiated correctly, but check attempts first.

Part (b) **20 (10,10) marks** **Att (3, 3)**

(i)	Differentiate $(3x^2-2)(x^2+4)$ with respect to x.
(ii)	Given that $y = \dfrac{x^2}{x-1}$, find $\dfrac{dy}{dx}$ when $x = 3$.

(i) **10 marks** **Att 3**

I:	II:
$(3x^2-2)(2x) \quad + \quad (x^2+4)(6x)$ ..10m	$y = 3x^4 - 2x^2 + 12x^2 - 8$ …3m $dy/dx = 12x^3 - 4x + 24x$ ..10m
	III: $y = 3x^4 + 10x^2 - 8$ …3m $dy/dx = 12x^3 + 20x$ ..10m

* In method I, no penalty for omission of brackets as long as multiplication is implied.
* If u/v used, apply B2 twice (central sign, division by v^2). There may be other errors.
* dy/dx = (6x)(2x) merits 3 marks, i.e. 10 – B1 – B1 – B2.

Blunders (–3)

B1 Differentiation error. Once per term. Two terms to check in I, three/four terms in II / III.
B2 Error in u.v formula, e.g. central sign.
B3 Does u.(du/dx) + v.(dv/dx), i.e. dy/dx = $(3x^2 – 2)(6x) + (x^2 + 4)(2x)$. Apply once.
B4 In II and III, each omitted or incorrect term in the expansion (line 1) to a max of 2 blunders.

Attempts (3 marks)

A1 Any correct derivative, e.g. an implied "0".
A2 $u = 3x^2 – 2$ or $v = x^2 + 4$, or vice versa, and stops.
A3 One or more terms multiplied correctly in method II or III.

Worthless (0)

W1 u.v or u/v rule written down (from Tables) and stops.

(ii) **10 marks** **Att 3**

	$y = x^2.(x - 1)^{-1}$ …3m
$\frac{dy}{dx} = \frac{(x-1)(2x) - x^2.1}{(x-1)^2}$ ...7m	dy/dx $= x^2\{-(x-1)^{-2}.1\} + (x - 1)^{-1}.2x$ …7m
or $\frac{(3-1).2.3 - 3^2.1}{(3-1)^2}$ ...still 7m	or $3^2\{-(3-1)^{-2}.1\} + (3 - 1)^{-1}.2.3$ ..still 7m
$= \frac{2.6 \;-\; 9}{2^2} = \frac{3}{4}$ ..10m	$= 9.\{- ¼\} + ½ . 6 = \frac{3}{4}$ ..10m

* No penalty for omission of brackets if multiplication implied. (Decide by later work).
* If *u.v* used (even if u/v identified initially) apply B2 + B3, and others if necessary.
* No marks for writing down *u/v* or *u.v* formula from Tables, and stopping.
* Errors simplifying *dy/dx* before evaluation at x = 3: penalise from final 3m.
* $y = x^2.(x – 1)^1$: apply B5 + B3.

Blunders (-3)

B1 Differentiation error. Once per term. (Two terms to check in both methods).
B2 Central sign incorrect in formula.
B3 No ÷ by v^2 in method I.
B4 Vice versa substitution for *du/dx* and *dv/dx* in *u/v* formula. Apply once.
B5 $y \neq x^2.(x – 1)^{-1}$ in method II. Also apply B3 if power of (x – 1) is 1.
B6 Bracket error multiplying out numerator. Apply once if necessary. See Note 1.
B7 Mathematical error in last step, e.g. $(3 - 1) - 2.3 - 3^2$ in numerator. See S1.

Slips (-1)

S1 Numerical error in last step (evaluation).

Attempts (3 marks)

A1 $u = x^2$ and $v = x – 1$ and stops.
A2 *du/dx* or *dv/dx* correct , and stops; *e.g.* *dy/dx* = 2x /1 or $x^2/1$ or 2x or x^2 or 2/1.

Worthless (0)

W1 Substitutes into original (y) function -- no differentiation.

Part (c)	**20 (10, 5, 5) marks**	**Att (3, 2, 2)**

A car begins to slow down at p in order to stop at a red traffic light at q.

The distance of the car from p, after t seconds, is given by

$$s = 12t - \frac{3}{2}t^2$$

where s is in metres.

(i) Find the speed of the car as it passes p.
(ii) Find the time taken to stop.
(iii) The car stops exactly at q. Find the distance from p to q.

(i)	**10 marks**	**Att 3**

ds/dt or *dy/dx* …3m

$= 12 - 3t$, or $12 - 2\left(\frac{3}{2}\right)t$, or $12 - \left(\frac{3}{2}\right)2t$ …7m

$= 12 - 3(0)$ …9m

$= 12$ ..10m

* Correct answer without work: 10 marks. (i.e. we assume differentiation was done).
* Marks are non-transferable between parts of (c), i.e. no retrospective marking allowed.
* No differentiation or reference to differentiation: no marks..
* In (c)(i), $ds/dt = 12 - 3t = 0 \Rightarrow t = 4$: award 7 marks for (i). Do not assume it is (c)(ii).
* No penalty for incorrect notation.
* If the parts of (c) are unlabelled, and the context doesn't identify which part is which, assume the questions were answered in sequence from (c)(i) to (c)(iii).
* Applied Maths approach: $s = ut + ½ at^2 \Rightarrow ut = 12t \Rightarrow u = 12$. Mark on slip and blunder.

Blunders (-3)
B1 Differentiation error, once per term. (Two terms to check).
B2 Incorrect or no value for t substituted into speed (*ds/dt*) equation.

Slips (–1)
S1 Numerical slip, e.g. 3(0) = 3.

Attempts (3 marks)
A1 *ds/dt* or *dy/dx* or $f'(x)$ mentioned.
A2 Speed = $d^2s/dt^2 = -3$ without work. (Candidate may rectify error in this part).

Worthless (0)
W1 t = 0 substituted into the s equation. (See note 3 above).
W2 Incorrect answer without work.
W3 States speed = d^2s/dt^2 and stops.
W4 Effort to use Speed = Distance ÷ Time.

(c)(ii) **5 marks** **Att 2**

$\dfrac{ds}{dt} = 0$ or $12 - 3t = 0$ ...2m $\therefore\ t = 4$ ...5m

* Correct answer without work: 5 marks. Incorrect answer without work: no marks.
* If derivative not used, found or mentioned: no marks. But see A2.
* Accept candidate's *ds/dt* from (c)(i) provided it was a derivative.
* No retrospective award of marks from (c)(ii) to (c)(i).
* "Applied Maths" technique: v = u + at => 0 = 12 - 3t (...2m) => t = 4 (…5m).

Blunders (-3)
B1 Solves *ds/dt* = *n* where $n \neq 0$.
B2 t = 0 substituted into *ds/dt* = 12 - 3t (getting 12 seconds).
B3 Transposition error.

Attempts (2 marks)
A1 Mentions ds/dt in this part, or ds/dt found again.
A2 *ds/dt* = 0 and stops, or candidate's *ds/dt* from (c)(i) = 0 and stops, or speed = 0 and stops.
A3 Using 12 - 3t or candidate's derivative, an effort to tabulate, graph or trial and error.

Worthless (0)
W1 Solves s = 0, i.e. $12t - (3/2)t^2 = 0$, or substitutes t = 0 into s.

(iii) **5 marks** **Att 2**

$t = 4 \Rightarrow s = 12(4) - \frac{3}{2}(4)^2$ ...2m $= 48 - 24$ ...4m $= 24$ ...5m

* Accept candidate's value of *t* from (c)(ii).
* If distance formula is not substituted in this part, award no marks unless A1 applies.
* "Applied Maths": $s = ut + ½ at^2 \Rightarrow s = 12(4) + ½ (-3)(4)^2$ [..2m] => s = 48 -24 = 24 [..5m].
or: $v^2 = u^2 + 2as \Rightarrow s = (v^2 - u^2) /2a$ [..2m] =>$s = (0 - 12^2) / (2(-3))$ =- 144 / (-6) = 24 [..5m]

Blunders (-3)
B1 Incorrect *t* substituted into distance formula, i.e. $t \neq$ ans (c)(ii) substituted.
B2 Mathematical errors, e.g. $\frac{3}{2}(4)^2 = \frac{(12)^2}{2}$

Slips (-1)
S1 Numerical slips, e.g. 48 - 24 = 16.
Attempts (2 marks)
A1 $12t - \frac{3}{2}t^2$ = *some number*, i.e. attempt marks for using the distance formula.
Worthless (0)
W1 Derivative used.

QUESTION 8

Part (i)	**15 marks**	**Att 5**
Part (ii)	**10 marks**	**Att 3**
Part (iii)	**10 marks**	**Att 3**
Part (iv)	**5 marks**	**Att 2**
Part (v)	**10 marks**	**Att 3**

Part (i) **15 marks** **Att 5**

Let $f(x) = \frac{1}{x-1}$, $x \in \mathbf{R}$, $x \neq 1$.

(i) Find $f(-3)$, $f(-1.5)$, $f(0.5)$, $f(1.5)$, $f(5)$.

(i) **15 marks** **Att 5**

$f(-3) = \frac{1}{-3-1}$ $= -0.25$ or $\frac{-1}{4}$	$f(-1.5) = \frac{1}{-1.5-1}$ $= -0.4$ or $\frac{-2}{5}$	$f(0.5) = \frac{1}{0.5-1}$ $= -2$	$f(1.5) = \frac{1}{1.5-1}$ $= 2$	$f(5) = \frac{1}{5-1}$ $= 0.25$ or $\frac{1}{4}$	**2m** H/M + **1m** H/M

* Each unsimplified fraction merits 2 marks if correct and 0 marks if incorrect. Hit/Miss. Each such fraction (*correct or not*) merits 1m if it is then simplified correctly. Hit/Miss. Incorrect value without work: 0 + 0m (out of 2m + 1m), e.g. *f(-3)* = ½ merits 0 marks.

* Partial evaluation doesn't earn final 1 mark, e.g. $f(-1.5) = \frac{-1}{2.5}$ and stops, merits 2m + 0m.

* Correct value without work: full marks (3m) each time, e.g. *f(-3)* = -1/4 merits 3m.

* Blunders do not apply in (i).

* If a candidate earns 1,2,3 or 4 marks, replace with Att 5.

* Values of f(x) = x - 1 calculated (all/some correct):misreading which oversimplifies, att 5m.

Misreading (-1)

M1 *Consistently* treats the function as $f(x) = \frac{1}{x} - 1$, even if $f(x) = \frac{1}{x-1}$ written.

The relevant values for $\frac{1}{x} - 1$ are -1.3, -1.6, 1, -0.3 and -0.8, respectively.

Or, $\frac{1}{x+a}$ or $\frac{1}{x-a}$, where a ≠ 0, used consistently.

Part (ii) **10 marks** **Att 3**

(ii)	Draw the graph of the function f from $x = -3$ to $x = 5$.

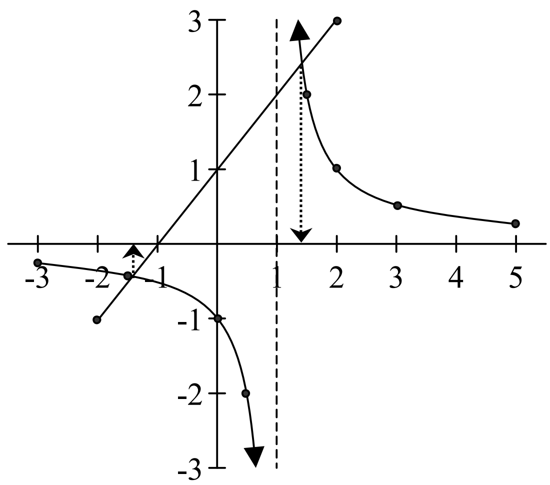
(ii) **10 marks** **Att 3**

I: Using a Table from (i):

x	-3	-1.5	0.5	1.5	5
y	-0.25	-0.4	-2	2	0.25

or: **Using a new table:**

x	y
-3	-0.25
-2	-0.33
-1	-0.5
0	-1
1	no
2	1
3	0.5
4	0.33
5	0.25



II: Sketching in relation to asymptotes (Table not used):

<u>**Award 3m**</u> <u>(Hit/Miss)</u> for correct asymptote (drawn/implied, not intersected by curve).
<u>**Award 4m**</u> <u>(Hit/Miss)</u> for one side of the curve (approx. shape). } First check if the 'better' side merits 4m or 0m.
<u>**Award 3m**</u> <u>(Hit/Miss)</u> for the other side of the curve (approx. shape).

* Graph may be drawn from a table or in relation to the asymptotes x = 1 and x axis.
* If new table is drawn up, mark accordingly: <u>do not refer back</u> to the earlier values in (i). However, candidates may import values from (i) to the table or direct to the graph.
* Be lenient in regard to approximate scaling of axes. (Ignore incorrect scale if possible).
* Asymptote x = 1 need not be drawn: an implied vertical asymptote (or visible gap) is fine.
* If left and right branches of the curve are joined, apply B1. e.g. (0.5,–2) joined to (1.5,2). If curve cuts horizontal asymptote *elsewhere* (in the domain $-3 \le x \le 5$), apply B1 again only if method II is being used.
* Graph of x - 1: oversimplified, att 3m at most. (See A1, A2).
* Straight lines instead of curves; apply S(-1) once, unless B2 supersedes it.

Table Method:

Blunders (-3)

B1 Asymptote error. See notes 4 and 5 above.
B2 Very poor scaling, e.g. -0.25, -0.4, -2 equally scaled on y axis (str. line graphs).
B3 f(x) = (1/x) - 1 calculated and graphed in (ii). Other errors may also occur.
B4 Points plotted but not joined. (Max available 7m).

Slips (-1)

S1 Check the points on the left and on the right of the vertical asymptote (x = 1) and penalise <u>each</u> incorrect point, subject to a max penalty of –3m on each side.
In this case, an incorrect point means one which is wrongly calculated and/or plotted.
Allow candidate to leave out a few points if shape of graph is not affected.
S2 -0.25, -0.4, -2 incorrectly scaled if causing a blip in the graph. Apply once. (See B2)

Attempts (3 marks) (<u>for either method</u>):

A1 One point correctly calculated and/or plotted, e.g. point correct in Table in (ii) but no graph.
A2 Draws axes and stops
A3 x = 1 an asymptote and stops, or states that x axis is an asymptote and stops.

Part (iii) **10 marks** **Att 3**

(iii) On the same diagram, draw the graph of the function $$g(x) = x+1$$ in the domain $-2 \le x \le 2$, $x \in \mathbf{R}$.

(iii) **10 marks** **Att 3**

I

x	-2	-1	0	1	2
g(x)	-1	0	1	2	3

II: Endpoints calculated, plotted and joined.

See linear graph in (ii).

III: x = 0 => y = 1 …(0, 1)
y = 0 => x = -1 … (-1, 0) …plotted, joined and extended from x = 2 to x = -2.

* No penalty if g(x) is drawn on a separate graph.

Blunders (-3)

B1 Points plotted but not joined.

Slips (-1)

S1 Each point calculated incorrectly, to a max penalty -2m.
S2 Each point plotted incorrectly, to a max penalty -2m.
S3 Each endpoint (i.e. points at x = 2 and x = -2) omitted on graph. (See method III).

Attempts (3 marks)

A1 Only one point correctly calculated or correctly plotted, or a table but no graph.
A2 Draws new set of axes for this part, and stops.

Part (iv) **5 marks** **Att 2**

(iv) Use your graphs to estimate the values of x for which $f(x) = g(x)$.

(iv) **5 marks** **Att 2**

$f(x) = g(x)$ at $x = \pm 1.4$ (or x values of candidate's points) with a tolerance of ± 0.3

* One x value correctly named (within tolerance) using candidate's graphs: 2m HIT / MISS
2^{nd} value correctly named (within tolerance) using candidate's graphs: 3m HIT / MISS
* If candidate's graphs have only one intersection point, final 3 marks is lost.

Attempts (2 marks)

A1 Marks point(s) of intersection, with or without lines drawn to the x axis, and stops

Part (v) **10 marks** **Att 3**

(v) Find, using algebra, the values of x for which $f(x) = g(x)$.

(v) **10 marks** **Att 3**

$\dfrac{1}{x-1} = \dfrac{x+1}{1}$	...3m	$\dfrac{1}{x-1} = \dfrac{x+1}{1}$	...3m
$(x-1)(x+1) = 1$	...4m	$(x-1)(x+1) = 1$	...4m
$x^2 - 1 = 1$ or $x^2 = 2$	...7m	$x^2 - 2 = 0$	...7m
$x = \pm\sqrt{2}$ or ± 1.4	..10m	$\left(x - \sqrt{2}\right)\left(x + \sqrt{2}\right) = 0 \Rightarrow x = \pm\sqrt{2}$ or ± 1.4	..10m

* If candidate uses the quadratic formula to solve $x^2 - 2 = 0$, mark on slip and blunder for the final 3 marks.

* No additional marks from the point where the equation becomes linear.

Blunders (–3)

B1 Inversion error.
B2 Transposition error.
B3 Error multiplying out brackets.
B4 Incorrect factors. Apply once
B5 Incorrect roots from factors. Apply once.
B6 Only one root found, e.g. positive root.
B7 Quadratic formula error (in formula, substitution or simplification).

Attempts (3 marks)

A1 *f(x) = g(x)* substituted and stops, as in line 1.
A2 Quadratic formula correct, and stops.