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LEAVING CERTIFICATE EXAMINATION, 1999

MATHEMATICS — HIGHER LEVEL
PAPER 2 (300 marks)

FRIDAY, 11 JUNE — MORNING, 9.30 to 12.00

Attempt **five** questions from Section **A** and **one** question from Section **B.**
Each question carries 50 marks.

Marks may be lost if necessary work is not clearly shown or if you do not indicate where a calculator has been used.

SECTION A

1. *(a)* Find the Cartesian equation of the circle

$$x = 6 + \cos\theta, \quad y = 4 + \sin\theta,$$

where $0 \leq \theta \leq 2\pi$.

(b) The equation of a circle with radius length 7 is

$$x^2 + y^2 - 10kx + 6y + 60 = 0$$

where $k > 0$.

(i) Find the centre of the circle in terms of k.

(ii) Find the value of k.

(iii) The line $3x + 4y + d = 0$ is a tangent to the circle, where $d \in \mathbf{Z}$.
Show that one value for d is 17.
Find the other value for d.

(c) Two circles intersect at the points $a(1, 2)$ and $b\,(7, -6)$. The line joining the centres of the circles is the perpendicular bisector of $[\,ab\,]$.

The distance from the centre of each circle to the midpoint of $[\,ab\,]$ is 10.

Find the midpoint of $[\,ab\,]$ and the radius length of each circle.

Find the equation of each circle.

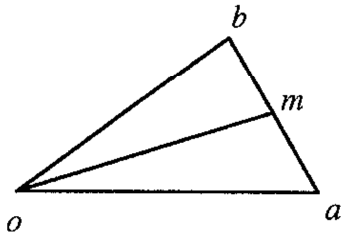
2. *(a)* $\vec{p} = 3\vec{i} + 2\vec{j}$, $\vec{q} = 5\vec{i} - 6\vec{j}$ and $\vec{p} = \vec{rq}$.

Express $\vec{r}$ in terms of $\vec{i}$ and $\vec{j}$.

(b) **(i)** *oab* is a triangle where *o* is the origin.

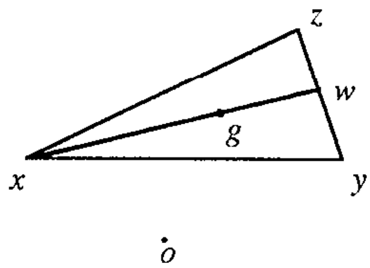
m is the midpoint of [*ab*].

Express $\vec{m}$ in terms of $\vec{a}$ and $\vec{b}$.



(ii) In the triangle *xyz*, *w* is the midpoint of [*yz*], *g* is a point in [*xw*] such that $|xg| = \frac{2}{3}|xw|$ and *o* is the origin.

Express $\vec{g}$ in terms of $\vec{x}$, $\vec{y}$ and $\vec{z}$.



(c) **(i)** Show for all vectors $\vec{r}$ and $\vec{s}$ that $\vec{r}.\vec{s}^{\perp} = -\vec{r}^{\perp}.\vec{s}$.

(ii) *a*, *b* and *c* are distinct points.

If

$$\vec{ad} = t\left(\frac{\vec{ab}^{\perp}}{|\vec{ab}|} - \frac{\vec{ac}^{\perp}}{|\vec{ac}|}\right),$$

where $t \in \mathbf{R}$ and $t \neq 0$, show that

$$|\angle bad| = |\angle cad|.$$

3. *(a)* Show that the line $6x - 8y - 71 = 0$ contains the midpoint of [*ab*] where *a* has coordinates $(8, -6)$ and *b* has coordinates $(5, -2)$.

(b) The line *L* has equation $5x - 3y + 10 = 0$.
The point *k* has coordinates $(6, 2)$.
Show that the perpendicular distance from *k* to *L* is $\sqrt{34}$.

f is the transformation $(x, y) \longrightarrow (x', y')$ where

$$x' = 7x - 2y$$
$$y' = -4x + y.$$

The image of *L* under *f* is the line *f*(*L*). Find the equation of *f*(*L*).

Show that the perpendicular distance from *f*(*k*) to *f*(*L*) is $\frac{\sqrt{34}}{\sqrt{5}}$.

(c) A line containing the point $(-4, -2)$ has slope *m*, where $m \neq 0$.

This line intercepts the *x* axis at $(x_1, 0)$ and the *y* axis at $(0, y_1)$.

Given that $x_1 + y_1 = 3$, find the slopes of the two lines that satisfy this condition.

Find the measure of the acute angle between these two lines and give your answer to the nearest degree.

4. *(a)* Find the value of k for which

$$\sin 75^\circ - \sin 15^\circ = \frac{1}{\sqrt{k}}, \quad k \in \mathbf{N}_0.$$

See Tables page 9.

(b) (i) Express $\sin 5x - \sin x$ as a product of sine and cosine.

(ii) Find all the solutions of the equation

$$\sin 5x - \sin x = 0$$

in the domain $0^\circ \leq x \leq 180^\circ$.

(c) Prove that

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}.$$

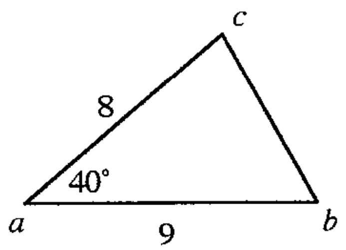
Find, in the form $p \pm \sqrt{q}$, $p \in \mathbf{Z}$, $q \in \mathbf{N}$,

(i) $\tan 75^\circ$

(ii) $\tan 15^\circ$.

5. *(a)* In the triangle abc, $|ab| = 9$, $|ac| = 8$ and $|\angle cab| = 40^\circ$.

Find the area of triangle abc, correct to two places of decimals.

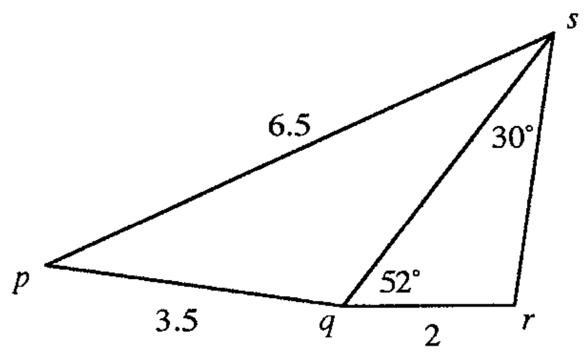


(b) In the triangles pqs and qrs, $|pq| = 3.5$, $|qr| = 2$, $|ps| = 6.5$, $|\angle qsr| = 30^\circ$ and $|\angle sqr| = 52^\circ$.

Calculate

(i) $|qs|$, correct to two places of decimals

(ii) $|\angle pqs|$, correct to the nearest degree.



(c) Express $\sin(135^\circ - A)$ in terms of $\sin A$ and $\cos A$.

Express $\sin(135^\circ - A)\cos(135^\circ + A)$ in the form $k(1 + \sin pA)$, where $k, p \in \mathbf{R}$.

Find the values of A for which

$$\sin(135^\circ - A)\cos(135^\circ + A) = -\frac{3}{4}$$

where $0^\circ \leq A \leq 180^\circ$.

6. *(a)* In how many ways can a group of five people be selected from four women and four men?

In how many of these groups are there exactly three women?

(b) Solve the difference equation

$$u_{n+2} - 2u_{n+1} - 6u_n = 0, \quad \text{where } n \geq 0,$$

given that $u_0 = 0$ and $u_1 = 14$.

(c) In a class of 24 students, there are 14 boys and 10 girls.

In a particular week (Monday to Sunday inclusive), three students celebrate their birthdays. Assume that the birthdays are equally likely to fall on any day of the week and that the birthdays are independent of each other.

What is the probability that these three students

(i) are three boys or three girls

(ii) have birthdays falling on different days of the week or on the same day of the week other than Monday?

7. *(a)* Six discs of equal size are stacked one on top of the other. There are two identical red discs and one each of blue, yellow, green and white.

In how many different ways can the six discs be stacked so that the two red discs are either at the top or at the bottom?

(b) Two balls are at thc same time taken at random from a box containing three black, three red and three yellow balls.

Find the probability that

(i) both balls are yellow

(ii) neither of the two balls is yellow

(iii) at least one of the two balls is yellow.

(c) The numbers a, $3a$, b, $2b$ have mean $2b$ and standard deviation σ.

(i) Express b in terms of a.

(ii) Express σ in terms of a.

(iii) Find the range of values of a for which $\sigma^2 < 18.5$.

SECTION B

Answer ONE question from this section.

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8. *(a)* Use integration by parts to find $\int xe^x dx$.

(b) $f(x) = f(0) + \frac{f'(0)x}{1!} + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} + \ldots$ is the Maclaurin series.

Derive the Maclaurin series for $f(x) = \cos x$ up to and including the term containing x^6.

Hence write down the general term of the Maclaurin series for $f(x) = \cos x$.

Use the ratio test to show that the series converges for all $x \in \mathbb{R}$.

(c) A solid cylinder of radius r and height h has a fixed volume K.

(i) Express h in terms of r, π and K.

(ii) Find the ratio $r : h$ when the total surface area of the cylinder is a minimum.

Give your answer as a ratio of natural numbers.

9. *(a)* An unbiased die is thrown twice. Find the probability of getting a total less than four.

(b) Bag A contains three gold coins and two silver coins. Bag B contains four gold coins and five silver coins.

(i) A coin is picked at random from each bag and then returned to the bag from which it was picked. What is the probability that one of the picked coins is gold and the other silver?

(ii) A coin is picked at random from bag A and placed in bag B.

A coin is then picked at random from bag B and placed in bag A.

These two operations are performed again, in the same order.

What is the probability that bag A now contains five gold coins?

(c) A coin is tossed 500 times resulting in 270 heads.

Test at the 5% level of significance if the coin is biased in favour of heads.

10. *(a)* Let $p*q = \dfrac{p+q}{1+pq}$, $p, q \in \mathbf{R}$, $p,q \geq 0$.

Find the identity element.

(b) The group $G, \circ$ may be represented by the following table

$\circ$	e	a	b	c	d	g
e	e	a	b	c	d	g
a	a	b	e	g	c	d
b	b	e	a	d	g	c
c	c	d	g	e	a	b
d	d	g	c	b	e	a
g	g	c	d	a	b	e

(i) Write down the elements of G of order two.

(ii) Find the set $C(b) = \{x \in G : x \circ b = b \circ x\}$.

(c) (i) Prove that any group of prime order is cyclic.

(ii) Prove that the order of any element of a finite group G divides the order of G.

11. *(a)* Find the equation of the ellipse with centre (0, 0), eccentricity $\dfrac{1}{3}$ and one focus at (3, 0).

(b) f is an affine transformation.

(i) The midpoint of the line segment $[ab]$ is m.

Show that $f(m)$ is the midpoint of $[f(a)f(b)]$.

(ii) f maps the triangle pqr to the triangle $p'q'r'$.

The centroid of triangle pqr is g and the centroid of triangle $p'q'r'$ is h.

Show that $f(g) = h$.

(c) Under the affine transformation f, the circle $C : x^2 + y^2 = 1$ is mapped to the ellipse E.

The circle C is circumscribed by the square $abcd$ where a and c are opposite vertices.

Prove that the ellipse E is circumscribed by the parallelogram $f(a)f(b)f(c)f(d)$.