

## MARKING SCHEME

### LEAVING CERTIFICATE EXAMINATION 2005

### MATHEMATICS – ORDINARY LEVEL – PAPER 2

#### GENERAL GUIDELINES FOR EXAMINERS – PAPER 2

1. Penalties of three types are applied to candidates' work as follows:

- Blunders - mathematical errors/omissions (-3)
- Slips - numerical errors (-1)
- Misreadings (provided task is not oversimplified) (-1).

Frequently occurring errors to which these penalties must be applied are listed in the scheme. They are labelled: B1, B2, B3,..., S1, S2,..., M1, M2,...etc. These lists are not exhaustive.

2. When awarding attempt marks, e.g. Att(3), note that

- any *correct, relevant* step in a part of a question merits at least the attempt mark for that part
- if deductions result in a mark which is lower than the attempt mark, then the attempt mark must be awarded
- a mark between zero and the attempt mark is never awarded.

3. Worthless work is awarded zero marks. Some examples of such work are listed in the scheme and they are labelled as W1, W2,...etc.

4. The phrase “hit or miss” means that partial marks are not awarded – the candidate receives all of the relevant marks or none.

5. The phrase “and stops” means that no more work is shown by the candidate.

6. Special notes relating to the marking of a particular part of a question are indicated by an asterisk. These notes immediately follow the box containing the relevant solution.

7. The sample solutions for each question are not intended to be exhaustive lists – there may be other correct solutions. Any examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his/her advising examiner.

8. Unless otherwise indicated in the scheme, accept the best of two or more attempts – even when attempts have been cancelled.

9. The *same* error in the *same* section of a question is penalised *once* only.

10. Particular cases, verifications and answers derived from diagrams (unless requested) qualify for attempt marks at most.

11. A serious blunder, omission or misreading results in the attempt mark at most.

12. Do not penalise the use of a comma for a decimal point, e.g. €5.50 may be written as €5,50.

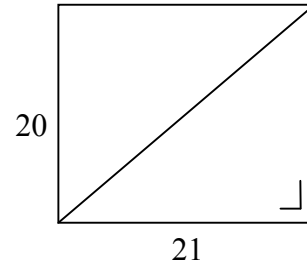
## QUESTION 1

<b>Part (a)</b>	<b>10 marks</b>	<b>Att 4</b>
<b>Part (b)</b>	<b>20 marks</b>	<b>Att 7</b>
<b>Part (c)</b>	<b>20 marks</b>	<b>Att 7</b>

**Part (a)** **10 (5, 5) marks** **Att (2, 2)**

A rectangle has length 21 cm and width 20 cm.

- (i) Find the area of the rectangle.
- (ii) Find the length of the diagonal.



**(a)(i)** **5 marks** **Att 2**

<p><b>I</b></p> <p>Area of rectangle = <math>l \times w</math>          = <math>21 \text{ cm} \times 20 \text{ cm}</math>          = <math>420 \text{ cm}^2</math></p>	<p><b>II</b></p> <p>Area = <math>2 \times \text{area } \Delta</math> <span style="float: right;">2 marks</span>          = <math>2(210) \text{ cm}^2</math> <span style="float: right;">4 marks</span>          = <math>420 \text{ cm}^2</math> <span style="float: right;">5 marks</span></p>
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- \* Accept correct answer without work.
- \* Each step presupposes the previous ones.
- \* Not more than 1 mark may be deducted for errors in calculations.

*Blunders (-3)*

- B1 Incorrect relevant formula, e.g. forgets to double area of triangle or  $21 + 20$ .
- B2 Each arbitrary dimension, subject to attempt mark.

*Slips (-1)*

- S1 Numerical slip.
- S2 Leaves answer as  $21 \times 20$ .

*Misreadings (-1)*

- M1 Misreading of a dimension, e.g. 12 for 21.

*Attempts (2 marks)*

- A1 Some relevant work, e.g.  $20$  or  $21 = \text{base of triangle}$  and stops.
- A2 Correct relevant formula not transcribed from the tables.
- A3 Substitution of one of the given dimensions into a reasonable formula.
- A4 Perimeter, e.g.  $20 + 21 + 20 + 21$ .
- A5  $82$  or  $210$  without work.

*Worthless (0)*

- W1 Incorrect answer without work, except A5.

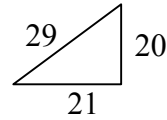
(a)(ii)

5 marks

Att 2

$$d^2 = 21^2 + 20^2 = 400 + 441 = 841 \Rightarrow d = \sqrt{841} \text{ cm or } 29 \text{ cm.}$$

- \* Accept correct answer without work.
- \* Accept Pythagorean triple 20, 21, 29, or diagram.



*Blunders (-3)*

- B1 Blunder in Pythagoras' Theorem.
- B2 Blunder in indices, e.g.  $20^2 = 40$ .
- B3 Incorrect substitution, if not an obvious misreading.

*Slips (-1)*

- S1 Each numerical slip, to a maximum of 3.

*Attempts (2 marks)*

- A1 Statement of, or use of, any relevant theorem.
- A2 Attempt at scaled diagram
- A3 Some relevant step, e.g.  $20^2$ .

*Worthless (0 marks)*

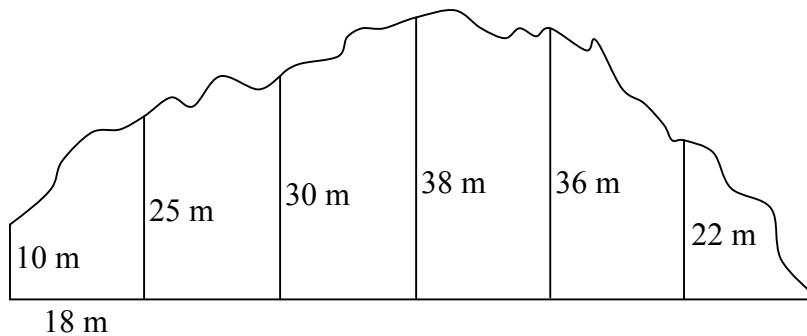
- W1 Area of triangle in this part.

**Part (b)**

**20 (10, 5, 5) marks**

**Att (3, 2, 2)**

The sketch shows a lake bounded on one side by a straight dam.



At equal intervals of 18 m along the dam, perpendicular measurements are made to the opposite bank, as shown on the sketch.

- (i) Use Simpson's Rule to estimate the area of the lake.
- (ii) If the lake contains  $15\,000 \text{ m}^3$  of water, calculate the average depth of water in the lake, correct to the nearest metre.

**(b) (i) Use of formula**

**10 marks**

**Att 3**

**Calculations**

**5 marks**

**Att 2**

<p>I Area = <math>\frac{h}{3} \{F + L + 2(\text{odds}) + 4(\text{evens})\} \dots\dots\dots 3 \text{ m}</math>  <math>= \frac{18}{3} \{10 + 0 + 2(30 + 36) + 4(25 + 38 + 22)\} \dots\dots 10 \text{ m}</math>  <math>= \frac{18}{3} \{10 + 2(66) + 4(85)\}</math>  <math>= 6\{10 + 132 + 340\}</math>  <math>= 6\{482\}</math>  <math>= 2892 \text{ m}^2 \dots\dots\dots 15 \text{ m}</math></p>	<p>II <math>\frac{h}{3} \{F + L + \text{TOFE}\} \dots\dots 3 \text{ m}</math></p> <table style="margin-left: 20px;"> <tr><td>F/L</td><td>O</td><td>E</td></tr> <tr><td>10</td><td>30</td><td>25</td></tr> <tr><td>0</td><td>36</td><td>38</td></tr> <tr><td></td><td></td><td><u>22</u></td></tr> </table> <p><math>\frac{18}{3} \{ \dots \times 2 \dots \times 4 \} \dots\dots 10 \text{ m}</math>  <math>= \frac{18}{3} \{ 10 + 2(66) + 4(85) \}</math>  <math>= 6\{10 + 132 + 340\}</math>  <math>= 6\{482\}</math>  <math>= 2892 \text{ m}^2 \dots\dots 15 \text{ m}</math></p>	F/L	O	E	10	30	25	0	36	38			<u>22</u>
F/L	O	E											
10	30	25											
0	36	38											
		<u>22</u>											

- \* Candidate must not lose more than 3 marks for calculations.
- \* Allow  $\frac{h}{3} = \{F + L + \text{TOFE}\}$  and penalise in calculations if formula not used correctly.
- \* Marks for part (i) may be awarded for work done in part (ii).
- \* Accept correct TOFE or TOFE consistent with candidates F and L.

*Blunders (-3)*

- B1 Incorrect  $\frac{h}{3}$  (once).
- B2 Incorrect F and / or L or extra terms with F and / or L (once).
- B3 Incorrect TOFE (once), if not consistent with candidates F and L.
- B4 E **or** O omitted (once).
- B5 Mathematical blunder, e.g. distribution error (once) or divides instead of multiplies by 6.

*Slips (-1)*

- S1 Each numerical slip to a maximum of 3.

*Attempts (3 marks for substituting into formula, 2 marks for calculations)*

- A1 Some relevant step, e.g. identifies F and / or L or odds or evens and stops. (3 m)
- A2 Statement of Simpson's Rule not transcribed from tables. (3 m)
- A3 E **and** O omitted (candidate may be awarded attempt at most). (Max. 3 m and/or 2 m)
- A4 Completes all rectangles but no calculations. (3 m)
- A5 Completes all rectangles and adds areas. (3 m + 2 m)
- A6 Correct answer without work. (3 m + 2 m)
- A7 Some correct calculation only. (2 m)

*Worthless (0)*

- W1 Incorrect answer without work.
- W2 Formula transcribed from tables and stops.

**(b)(ii)**

**5 marks**

**Att 2**

<p>Volume = area by average depth <math>\Rightarrow 15000 = 2892d \Rightarrow d = \frac{15000}{2892}</math> or 5.18</p> <p style="text-align: center;">2 marks</p>	<p style="text-align: right;"><math>= 5 \text{ m}</math></p> <p style="text-align: center;">4 marks</p>	<p style="text-align: right;"><math>= 5 \text{ m}</math></p> <p style="text-align: center;">5 marks</p>
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- \* Accept an answer consistent with candidate's answer from part (i).
- \* Accept correct or consistent answer without work.
- \* 5.18 or similar consistent with part (i), without work merits 4 marks.

*Blunders (-3)*

B1 Mathematical error in calculation, e.g. multiplies instead of divides.

*Slips (-1)*

S1 Failure to round off, or rounds off incorrectly.

S2 Numerical slip.

*Attempts (2 marks)*

A1 Some relevant step, e.g. 15000 \* Area, where \* is not division, or work with his area.

**Part (c)**

**20 (10, 5, 5) marks**

**Att (3, 2, 2)**

A steel-works buys steel in the form of solid cylindrical rods of radius 10 centimetres and length 30 metres.

The steel rods are melted to produce solid spherical ball-bearings. No steel is wasted in the process.

(i) Find the volume of steel in one cylindrical rod, in terms of  $\pi$ .

(ii) The radius of a ball-bearing is 2 centimetres.  
How many such ball-bearings are made from one steel rod?

(iii) Ball-bearings of a different size are also produced.  
One steel rod makes 225 000 of these new ball-bearings.  
Find the radius of the new ball-bearings.

**(c)(i)**

**10 marks**

**Att 3**

$\text{Volume} = \pi r^2 h = \pi(10)^2(3000) = 300\,000\pi \text{ cm}^3 \quad \text{or} \quad 0.3\pi \text{ m}^3$ <p style="text-align: center;">7 marks                                  10 marks</p>
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\* Candidate may not lose more than 3 marks for calculations.

*Blunders (-3)*

B1 Incorrect relevant cylinder formula, e.g.  $2\pi rh$ ,  $\pi rh$ ,  $\frac{1}{3}\pi r^2 h$ ,  $r^2 h$  and continues.

B2 Incorrect substitution.

B3 Mathematical error.

B4 Misplaced decimal point, e.g. confuses metres and centimetres.

B5 Correctly fills in formula and stops.

B6 Obvious value of  $\pi$  outside tolerance, subject to full marks for volume = 300 000 $\pi$  or  $\pi$  omitted from final answer.

*Slips (-1)*

S1 Each numerical slip to a maximum of 3.

S2 Inserts a value for  $\pi$  where  $3.1 \leq \pi \leq 3.2$ .

*Attempts (3 marks)*

A1 Some relevant step, e.g. diagram with a correct dimension shown.

A2 Correct answer without work.

**(c)(ii)**

**5 marks**

**Att 2**

I: Volume of ball-bearing = $\frac{4}{3}\pi(2)^3 = \frac{32}{3}\pi \text{ cm}^3$	II: $\frac{4}{3}(3.141592654)2^3 = 33.51032164$
Number of ball-bearings = $\frac{300\,000\pi}{\frac{32}{3}\pi} = 28\,125$ .	$\frac{942\,477.7961}{33.51032164} = 28\,125$

\* Allow candidate to use values from part (i).

\* Accept volume of sphere read as  $\frac{4}{8}\pi r^3$ .

*Blunders (-3)*

B1 Any error other than S1.

*Slips (-1)*

S1 Early or incorrect rounding off that affects the answer. (once only).

*Attempts (2 marks)*

A1 Some relevant step.

**(c)(iii)**

**5 marks**

**Att 2**

I: $225\,000\left(\frac{4}{3}\pi r^3 = 300\,000\pi\right) \Rightarrow r^3 = \frac{300\,000 \times \pi \times 3}{225\,000 \times 4 \times \pi} = 1 \Rightarrow r = \sqrt[3]{1}$ or $r = 1 \text{ cm}$ .
or
II: $225\,000\left(\frac{4}{3} \times 3.141\dots \times r^3 = 300\,000 \times 3.141\dots\right) \Rightarrow r^3 = \frac{2\,827\,433.388}{2\,827\,433.388} = 1 \Rightarrow r = \sqrt[3]{1}$ or $r = 1 \text{ cm}$ .

\* Accept candidate's values from previous parts, if used.

*Blunders (-3)*

B1 Any error other than S1.

*Slips (-1)*

S1 Early or incorrect rounding off that affects the answer.

*Attempts (2 marks)*

A1 Some relevant step, e.g. some correct substitution.

A2 Correctly fills in formula and stops.

A3 Correct answer without work.

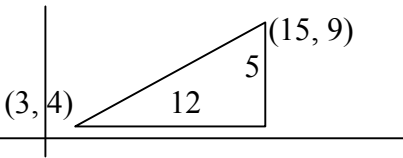
## QUESTION 2

<b>Part (a)</b>	<b>10 marks</b>	<b>Att 3</b>
<b>Part (b)</b>	<b>25 marks</b>	<b>Att 9</b>
<b>Part (c)</b>	<b>15 marks</b>	<b>Att 5</b>

**Part (a)** **10 marks** **Att 3**

Find the distance between the two points (3, 4) and (15, 9).

**(a)** **10 marks** **Att 3**

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(15 - 3)^2 + (9 - 4)^2}$ <p style="text-align: center;">3 marks</p>	$\text{or } \sqrt{12^2 + 5^2} \text{ or } \sqrt{144 + 25} = \sqrt{169} \text{ or } 13$ <p style="text-align: center;">7 marks</p>	<p style="text-align: center;">10 marks</p>
<p>or</p> <div style="display: flex; align-items: center;">  <div style="margin-left: 20px;"> <p>Distance = <math>\sqrt{12^2 + 5^2}</math> or <math>\sqrt{144 + 25}</math> <span style="float: right;">7 marks</span></p> <p style="margin-left: 100px;">= <math>\sqrt{169}</math> or 13 <span style="float: right;">10 marks</span></p> </div> </div>		

- \* Correct substitution into a correct formula and fails to finish merits 7 marks.
- \* Second step presupposes the first step.
- \* If a formula for distance is not written, any sign or substitution error is at least a blunder,
  - e.g. Distance =  $\sqrt{(15 - 3)^2 - (9 - 4)^2} = \sqrt{119}$  ..... one blunder,
  - Distance =  $\sqrt{(15 + 3)^2 + (9 + 4)^2} = \sqrt{493}$  ..... one blunder,
  - Distance =  $\sqrt{(15 - 3)^2 - (9 + 4)^2} = \sqrt{313}$  ..... one blunder,
  - Distance =  $\sqrt{(3 + 4)^2 + (15 + 9)^2} = \sqrt{625}$  ..... two blunders B1 and B2.

### Blunders (-3)

- B1 Incorrectly treats couples as  $(x_1, x_2)$  and  $(y_1, y_2)$  or switches  $x$  and  $y$ . [Penalise once].
- B2 Incorrect relevant formula,
  - e.g.  $\sqrt{(x_2 - x_1)^2 - (y_2 - y_1)^2}$  or  $\sqrt{(x_2 + x_1)^2 + (y_2 + y_1)^2}$  or  $\sqrt{(x_2 + x_1)^2 - (y_2 + y_1)^2}$
- B3 Two or more incorrect substitutions, if formula is written.
- B4 Mathematical error, e.g.  $12^2 = 24$ .
- B5 Incorrect use of  $\sqrt{\quad}$ , e.g. Distance = 169.
- B6 Last step omitted.

### Slips (-1)

- S1 One incorrect sign in  $(x_2 - x_1)$  or  $(y_2 - y_1)$  part of formula. [Note: central sign error is B-3].
- S2 One incorrect substitution, if formula is written.
- S3 Obvious misreading of coordinate.
- S4 Each numerical slip to a maximum of 3.

### Attempts (3 marks)

- A1 Some relevant step, e.g. (3, 4) with  $x_1$  and  $y_1$  identified.
- A2 Correct relevant formula and stops.
- A3 Formula with  $(x_2 - x_1)$  and/or  $(y_2 - y_1)$  and some correct substitution.
- A4 Oversimplifies, e.g.  $\sqrt{(x_2 - x_1) + (y_2 - y_1)}$  with some correct substitution, even if completed.
- A5 Plots (3, 4) and/or (15, 9) reasonably well.

- A6 States Pythagoras' Theorem or  $\sqrt{a^2 + b^2}$  .  
 A7  $\sqrt{169}$  or 13 without work.  
 A8 Uses translation, e.g. (12, 5) and stops.

*Worthless (0 marks)*

W1 Irrelevant formula and stops.

**Part (b)**

**25 (10, 5, 5, 5) marks**

**Att (3, 2, 2, 2)**

$L$  is the line  $3x - 4y + 12 = 0$ .

$L$  intersects the  $x$ -axis at  $a$  and the  $y$ -axis at  $b$ .

- (i) Find the co-ordinates of  $a$  and the co-ordinates of  $b$ .  
 (ii)  $K$  is the line that passes through  $b$  and is perpendicular to  $L$ .  
 Show  $L$  and  $K$  on a co-ordinate diagram.  
 (iii) Find the equation of  $K$ .  
 (iv) The point  $(2t, 3t)$  is on the line  $K$ . Find the value of  $t$ .

**(b)(i)**

**10 marks**

**Att 3**

$$\left. \begin{array}{l} y = 0 \\ 3x + 12 = 0 \\ 3x = -12 \end{array} \right\} \text{ or } \left. \begin{array}{l} x = 0 \\ -4y + 12 = 0 \\ -4y = -12 \end{array} \right\} \quad 3 \text{ marks}$$

$$x = -40 \quad \text{or} \quad y = 3 \quad 7 \text{ marks}$$

$$x = -4 \quad \text{and} \quad y = 3 \quad 10 \text{ marks}$$

- \* If  $b$  is misread for  $a$  and vice versa, penalise in part (ii).
- \* Accept correct points without work.
- \* If part (i) is not shown and points  $a$  and  $b$  are correctly indicated in part (ii), award 10 marks for part (i).

*Blunders (-3)*

- B1 Fails to find second intercept.  
 B2 Mathematical error, e.g.  $4y = 12 \Rightarrow y = 8$ .  
 B3 Transposing error, e.g.  $3x + 12 = 0 \Rightarrow 3x = 12$ .

*Slips (-1)*

S1 Each numerical slip to a maximum of 3.

*Attempts (3 marks)*

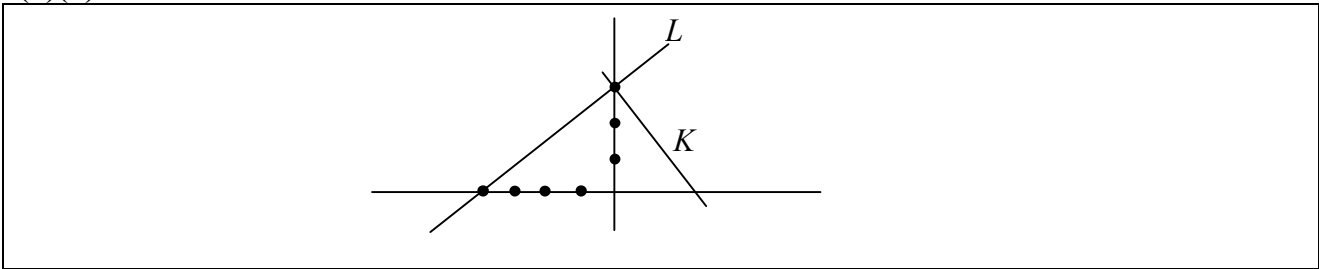
- A1 Some relevant step, e.g.  $x = 0$  and stops.  
 A2 Finds another point on  $L$ , e.g. (20, 18).



(b)(ii)

5 marks

Att 2



- \* Accept candidate's points from (i).
- \* Accept a vertical  $x$  axis and a horizontal  $y$  axis.
- \* Intervals should be indicated or implied.
- \* Allow a tolerance of  $\pm 10^\circ$  in the angle between  $L$  and  $K$ .

*Blunders (-3)*

- B1 Plots  $a$  or  $b$  as  $(-4, 3)$ .
- B2 Fails to plot line  $K$ .
- B3 Measure of angle between  $L$  and  $K$  outside tolerance.
- B4  $K$  intersects  $L$  at an arbitrary point.

*Misreadings (-1)*

- M1 Plots  $a(0, 3)$  and  $b(-4, 0)$ .
- M2  $L \cap K = \{a\}$  instead of  $\{b\}$ .

*Attempts (2 marks)*

- A1 Plots  $a$  and/or  $b$  and stops.
- A2 Sketches an irrelevant line  $L$  and line  $K$  perpendicular to it.
- A3 Draws axes and stops.
- A4 Plots some other correct point on  $L$  and stops.

(b)(iii)

5 marks

Att 2

<p>I</p> <p>Slope of <math>L = \frac{3-0}{0+4} = \frac{3}{4}</math></p> <p>Slope of <math>K = -\frac{4}{3}</math></p> <p><math>K : y - 3 = -\frac{4}{3}(x - 0)</math></p> <p>or <math>3y - 9 = -4x</math></p> <p>or <math>4x + 3y - 9 = 0</math></p>	<p>II</p> <p>Slope of <math>L = \frac{3}{4}</math></p> <p><math>K: 4x + 3y = c</math> or <math>4(0) + 3(3) = c</math></p> <p><math>c = 9</math> or <math>4x + 3y = 9</math></p>	<p>III</p> <p>Slope of <math>L = \frac{3}{4}</math></p> <p>Slope of <math>K = -\frac{4}{3}</math></p> <p>or <math>y = -\frac{4}{3}x + c</math></p> <p><math>3 = -\frac{4}{3}(0) + c</math> or</p> <p><math>c = 3</math> or <math>y = -\frac{4}{3}x + 3</math>.</p>
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- \* Step 2 presupposes step 1 in each method.
- \* Errors in simplifying the equation of  $K$  to be penalised in part (iv) if used.
- \* Incorrect slope of  $L$  warrants a penalty of -3 at most. Allow candidate to use it correctly.
- \* Accept correct answer without work.

*Blunders (-3)*

- B1 Incorrectly switches  $x$  and  $y$ , e.g.  $y - 0 = -\frac{4}{3}(x - 3)$ .
- B2 Incorrect relevant formula, e.g.  $y + y_1 = m(x + x_1)$ . [Both signs incorrect]
- B3 Blunder in slope of  $L$ , e.g.  $-\frac{3}{4}$  or  $\frac{4}{3}$ .
- B4  $m_1 \times m_2 \neq -1$  or blunder in slope of  $K$ .
- B5 Mathematical error, e.g. transposing error in method III.
- B6 Two or more incorrect substitutions or signs in formula.
- B7 Uses an arbitrary point.

*Slips (-1)*

- S1 One incorrect sign in  $(x_2 - x_1)$  or  $(y_2 - y_1)$  part of formula.
- S2 One incorrect sign in substitution, if formula is written.
- S3 Each numerical slip to a maximum of 3.

*Attempts (2 marks)*

- A1 Some correct step.
- A2 Correct formula and stops, e.g.  $m_1 \times m_2 = -1$  or  $-\frac{a}{b}$ .
- A3 Formula with  $(x_2 - x_1)$  and/or  $(y_2 - y_1)$  and some correct substitution.

*Worthless (0 marks)*

- W1 Tests  $(0, 3)$  in line  $L$ .

**(b)(iv)**

**5 marks**

**Att 2**

I $K : 4x + 3y - 9 = 0$ $(2t, 3t) \in K \Rightarrow 4(2t) + 3(3t) - 9 = 0$ $\Rightarrow 17t = 9 \Rightarrow t = \frac{9}{17}$	II Slope = $-\frac{4}{3}$ Equation $y - 3t = -\frac{4}{3}(x - 2t)$ $\Rightarrow 4x + 3y - 17t = 0$ But $K$ is $4x + 3y - 9 = 0 \Rightarrow t = \frac{9}{17}$
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- \* Accept  $t = \frac{9}{17}$  correctly verified.
- \* Incorrect answer without work merits zero marks.

*Blunders (-3)*

- B1 Mixes up  $x$  and  $y$  entries, e.g.  $4(3t) + 3(2t) - 9 = 0$ .
- B2 Error in more than one sign when substituting.
- B3 Incorrect relevant formula, e.g.  $x - x_1 = m(y - y_1)$ .
- B4 Incorrect slope without work and continues with it.
- B5 Mathematical error, e.g.  $8t + 9t = 17t^2$ .
- B6 Transposing error.

*Slips (-1)*

- S1 Error in one sign when substituting into  $(x_2 - x_1)$  or  $(y_2 - y_1)$  or similar.
- S2 Each numerical slip to a maximum of 3.

*Attempts (2 marks)*

- A1 Some relevant work, e.g. effort at substituting and stops or writes  $x = 2t$  and/or  $y = 3t$ .
- A2 Substitutes one value and stops.
- A3 Verifies or tries to verify a wrong value for  $t$ .
- A4 Correct relevant formula and stops, e.g. equation of line or slope formula.
- A5 Correct answer without work.
- A6 Formula with  $(x_2 - x_1)$  or  $(y_2 - y_1)$  or similar and some correct substitution.

**Part (c)****15 (10, 5) marks****Att (3, 2)**

The lines  $2x - y + 3 = 0$  and  $4x - y + k = 0$  intersect at a point.

- (i) Find, in terms of  $k$ , the co-ordinates of the point of intersection of the lines.  
 (ii) For what value of  $k$  is the point of intersection on the  $y$ -axis?

**(c)(i)****10 marks****Att 3**

<p>I</p> $2x - y + 3 = 0$ $4x - y + k = 0$ $-2x + 3 - k = 0 \quad \Rightarrow x = \frac{1}{2}(3 - k)$ $2x - y + 3 = 0 \Rightarrow 2\left(\frac{1}{2}(3 - k)\right) - y + 3 = 0$ $\Rightarrow 3 - k + 3 = y \Rightarrow y = 6 - k.$ <p>Point of intersection <math>\left(\frac{1}{2}(3 - k), 6 - k\right)</math></p>	<p>II</p> $2x - y + 3 = 0 \quad y = 4x + k$ $2x - (4x + k) + 3 = 0$ $2x - 4x - k + 3 = 0$ $-2x - k + 3 = 0$ $\Rightarrow x = \frac{1}{2}(3 - k)$ $\Rightarrow y = 6 - k$ <p>Point of intersection <math>\left(\frac{1}{2}(3 - k), 6 - k\right)</math></p>
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\* Award 7 marks for one coordinate found correctly.

*Blunders (-3)*

- B1 Finds only one coordinate.  
 B2 Transposing error, e.g.  $-2x - k + 3 = 0 \Rightarrow -2x = 3 + k$ .  
 B3 Failure to deal with both sides of the equation in the same way.  
 B4 Mathematical error, e.g.  $-(4x + k) = -4x + k$ .

*Slips (-1)*

- S1 Each numerical slip to maximum of 3.

*Attempts (3 marks)*

- A1 Some relevant step, e.g. some transposition towards solving the equations.  
 A2 Verifies an irrelevant point in  $2x - y + 3 = 0$  or calculates a point on the line  $2x - y + 3 = 0$ .  
 A3 Correct answer without work.

*Worthless (0 marks)*

- W1 Incorrect answer without work.

**(c)(ii)****5 marks****Att 2**

$$p \text{ on the } y\text{-axis i.e. } \frac{1}{2}(3 - k) = 0 \Rightarrow 3 - k = 0 \Rightarrow k = 3$$

\* Accept correct answer without work shown or accept  $k = 3$  substituted to give  $x = 0$ .

*Blunders (-3)*

- B1 Uses  $y = 0$ .  
 B2 Mathematical error, e.g.  $\frac{1}{2}(3 - k) = 0 \Rightarrow 6 - 2k = 0$ .  
 B3 Transposing error, e.g.  $3 - k = 0 \Rightarrow k = -3$ .

*Slips (-1)*

- S1  $\frac{1}{2}(3 - k) = 0 \Rightarrow 3 - k = 2(0) = 2$ .

*Attempts (2 marks)*

- A1 Writes  $x = 0$  and/or  $y = 0$  and stops.  
 A2 Any relevant step.

### QUESTION 3

<b>Part (a)</b>	<b>10 marks</b>	<b>Att 4</b>
<b>Part (b)</b>	<b>20 marks</b>	<b>Att 6</b>
<b>Part (c)</b>	<b>20 marks</b>	<b>Att 7</b>

**Part (a)** **10 (5, 5) marks** **Att (2, 2)**

The circle  $C$  has equation  $x^2 + y^2 = 49$ .

- (i) Write down the centre and the radius of  $C$ .  
(ii) Verify that the point  $(5, -5)$  lies outside the circle  $C$ .

**(a)(i)** **5 marks** **Att 2**

centre  $(0, 0)$ , radius  $\sqrt{49}$  or  $7$

\* Accept  $r = 7$  or  $r^2 = 7^2$  or  $r = \sqrt{49}$  without work.

Centre and radius written correctly, award 5 marks.  
Centre or radius written correctly, award 4 marks.  
Centre and radius drawn correctly (not written), award 4 marks.  
Otherwise award attempt mark 2 for relevant work.

*Attempts (2 marks)*

- A1 Some relevant step e.g. mentions origin or draws graph with centre at  $(0,0)$  or mentions  $(h, k)$ .  
A2 Correct relevant formula and stops, e.g.  $x^2 + y^2 = r^2$  or  $(x - h)^2 + (y - k)^2 = r^2$ .  
A3 Names a point that is on the circle, e.g.  $(7, 0)$  and stops.  
A4 Writes down the distance formula  $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ .  
A5 Any mention of  $x_1 = 0$  or  $y_1 = 0$ .  
A6  $r = 49$  with or without work.

**(a) (ii)** **5 marks** **Att 2**

I:  $(5)^2 + (-5)^2 = 25 + 25 = 50 > 49 \Rightarrow$  outside circle

II:  $\sqrt{(5-0)^2 + (-5-0)^2} = \sqrt{50} > 7 \Rightarrow$  outside

- \* Accept "distance from  $(5, -5)$  to  $(0, 0)$  is  $\sqrt{50}$  which is greater than the radius".  
\* Accept candidate's centre and radius from (i).

$50$  or  $\sqrt{50}$  and conclusion, award 5 marks.  
 $50$  or  $\sqrt{50}$  without conclusion, award 4 marks.  
Otherwise award attempt mark for relevant work.

*Attempts (2 marks)*

- A1 Some relevant step, e.g. draws a circle and plots the point  $(5, -5)$ .  
A2 Correct relevant formula  $x^2 + y^2 = r^2$  and stops.  
A3 Any formula with  $(x_2 - x_1)$  or  $(y_2 - y_1)$  and some correct substitution.  
A4 Identifies  $x_1 = 5$  and/or  $y_1 = -5$ .  
A5 States or refers to Pythagoras' Theorem or writes  $5^2$  or similar.

The line  $y = 10 - 2x$  intersects the circle  $x^2 + y^2 = 40$  at the points  $a$  and  $b$ .

- (i) Find the co-ordinates of  $a$  and the co-ordinates of  $b$ .  
 (ii) Show the line, the circle and the points of intersection on a co-ordinate diagram.

(b)(i)

10 marks

Att 3

$$y = 10 - 2x$$

$$x^2 + y^2 = 40$$

$$\Rightarrow x^2 + (10 - 2x)^2 = 40$$

$$\Rightarrow x^2 + 100 - 40x + 4x^2 - 40 = 0$$

$$\Rightarrow 5x^2 - 40x + 60 = 0$$

$$\Rightarrow x^2 - 8x + 12 = 0$$

$$\Rightarrow (x - 2)(x - 6) = 0$$

$$\Rightarrow x = 2, \quad x = 6$$

$$x = 2 \Rightarrow y = 10 - 2x = 10 - 4 = 6 \quad \text{giving} \quad (2, 6)$$

$$x = 6 \Rightarrow y = 10 - 2x = 10 - 12 = -2 \quad \text{giving} \quad (6, -2)$$

$$\text{or } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{8 \pm \sqrt{64 - 48}}{2}$$

\* Accept two correct points verified correctly in both line and circle.

\* Case I:  $y = 10 - 2x \Rightarrow 2x + y = 10 \Rightarrow 4x^2 + y^2 = 100$   
 $\frac{x^2 + y^2 = 40}{3x^2 = 60}$

merits A1 for  $2x + y = 10$ , even if candidate continues.

*Blunders (- 3)*

B1 Transposing error.

B2 Blunder in expanding  $(10 - 2x)^2$ .

B3 Blunder in quadratic formula or application, e.g.  $\frac{-b + \sqrt{b^2 - 4ac}}{2a}$

B4 Error in factorising.

B5 Having found, say,  $x$  coordinates, substitutes into circle instead of line equation to find  $y$  coordinates and fails to find the correct points.

B6 Mathematical error.

B7 One correct point verified correctly into both line and circle, without solving simultaneous equations.

*Slips (- 1)*

S1 Each numerical slip to a maximum of 3.

*Attempts (3 marks)*

A1 Some relevant step, e.g. a relevant formula.

A2 Correct substitution of any value of  $x$  or  $y$  into one of the given equations.

A3 Accurate graphical solution. [Graph may merit more marks in part (ii).]

A4  $(2, 6)$  and/or  $(6, -2)$  without work shown.

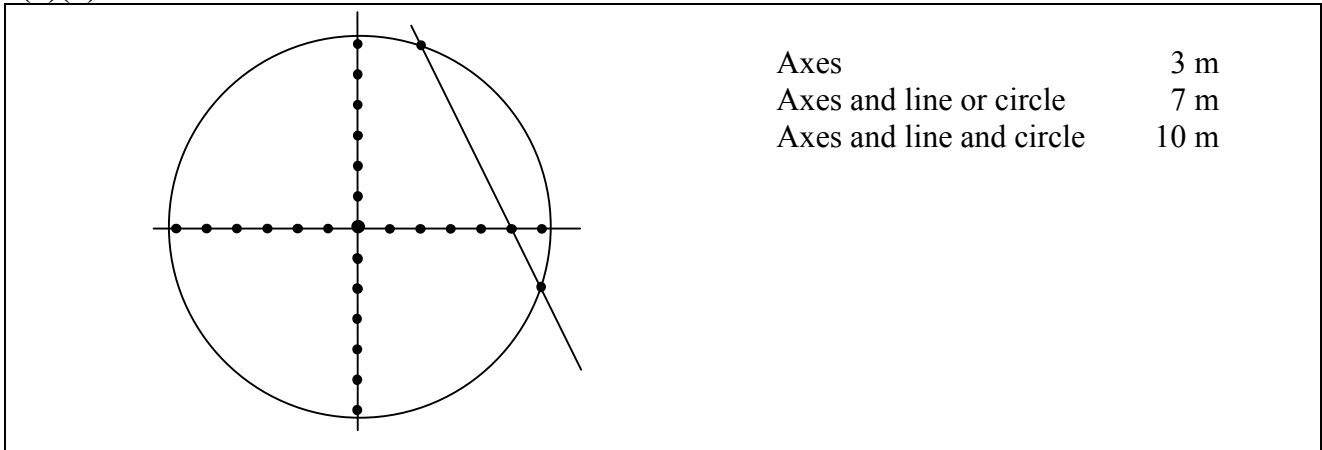
A5  $(0, 0)$ , and/or  $\sqrt{40}$  or  $r^2 = 40$  written.

A6 Work that leads to a linear equation merits attempt 3 at most.

**(b)(ii)**

**10 marks**

**Att 3**



Axes	3 m
Axes and line or circle	7 m
Axes and line and circle	10 m

- \* Accept candidate's points from (i).
- \* Accept horizontal  $y$  axis and vertical  $x$  axis.
- \* Scale must be indicated or implied for full marks.
- \* Accept reasonable freehand drawing of circle.

*Blunders (-3)*

- B1 Confuses  $x$  and  $y$  coordinates, e.g.  $(6, -2)$  plotted as  $(-2, 6)$ , if not consistent with 2<sup>nd</sup> \*.
- B2 Scale unreasonably inconsistent.
- B3 Inconsistent centre and/or radius of circle. [Radius to the eye.]
- B4 Line inconsistent with  $y = 10 - 2x$  or points from part (i).

*Attempts (3 marks)*

- A1 Some relevant step, e.g. effort at finding a point on the line or circle.
- A2 Axes drawn and stops.
- A3  $(0, 0)$  or  $\sqrt{40}$  for this part.

**Part (c)**

**20 (5, 5, 10) marks**

**Att (2, 2, 3)**

The circle  $K$  has equation  $(x + 4)^2 + (y - 3)^2 = 36$ .

- (i) Write down the co-ordinates of the centre of  $K$ .
- (ii) The point  $(2, 3)$  is one end-point of a diameter of  $K$ . Find the co-ordinates of the other end-point.
- (iii) The point  $(-4, y)$  is on the circle  $K$ . Find the two values of  $y$ .

**(c)(i)**

**5 marks**

**Att 2**

I Centre $(-4, 3)$ or $h = -4, k = 3$	II $x^2 + y^2 + 8x - 6y - 11 = 0$ Centre $(-4, 3)$	III May plot and name $(-4, 3)$ and use it as the centre of a circle.
---------------------------------------------	----------------------------------------------------------	--------------------------------------------------------------------------

- \* Accept correct answer without work shown.

Both coordinates correct, award 5 marks.

One sign incorrect, award 4 marks.

*Slips (-1)*

S1 Clearly plots  $(-4, 3)$  but does not write the coordinates, e.g. in method III.

*Attempts (2 marks)*

A1 Answer given is  $(4, -3)$  or  $(3, -4)$ .

A2 Correct relevant formula, e.g.  $(x-h)^2 + (y-k)^2 = r^2$  and stops.

A3 Some relevant work.

**(c)(ii)**

**5 marks**

**Att 2**

I  $(2, 3) \rightarrow (-4, 3) \rightarrow (-10, 3)$	II Uses midpoint $\frac{2+x}{2} = -4, \quad \frac{3+y}{2} = 3$ $x = -10 \quad y = 3$
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\* Accept correct answer without work shown.

\* Accept candidate's answer consistent with part (i) for this part.

*Blunders (-3)*

B1 Uses wrong translation or error in applying translation, if not an obvious slip.

B2 Incorrect relevant formula and continues.

B3 Two or more incorrect substitutions, if formula is written.

B4 Incorrectly treats couples as  $(x_1, x_2)$  and  $(y_1, y_2)$ .

B5 Transposing error.

B6 Mathematical blunder, e.g. in dealing with fractions.

*Slips (-1)*

S1 Each numerical slip to a maximum of 3.

S2 One incorrect sign in formula or substitution, if formula is written.

S3 One incorrect substitution, if formula is written.

S4 Required point clearly indicated but coordinates not written.

*Attempts (2 marks)*

A1 Some relevant step, e.g.  $(2, 3)$  with  $x_1$  and  $y_1$  identified.

A2 Plots point  $(2, 3)$  and/or  $(-4, 3)$  for this part.

A3 Correct relevant formula and stops.

(c)(iii)

10 marks

Att 3

I	II	III
$K: (x+4)^2 + (y-3)^2 = 36$	$(x+4)^2 + (y-3)^2 = 36.$	Centre $(-4, 3)$ , point $(-4, y)$
$(-4, y)$ is on $K$ .	$x^2 + y^2 + 8x - 6y - 11 = 0$	Radius = 6
$(-4+4)^2 + (y-3)^2 = 36$	$16 + y^2 - 32 - 6y - 11 = 0$	$\sqrt{(-4+4)^2 + (y-3)^2} = 6$
$\Rightarrow (y-3)^2 = 36$	$y^2 - 6y - 27 = 0$	$(y-3)^2 = 6^2$
$\Rightarrow y-3 = \pm 6$	$(y-9)(y+3) = 0$	$y-3 = \pm 6$
$\Rightarrow y = 6+3 = 9,$	or $y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	$y = 6+3 = 9$
or $y = -6+3 = -3.$	$y = 9$ or $y = -3$	$y = -6+3 = -3$

*Blunders (-3)*

- B1 Blunder in expanding  $(x+4)^2$  or  $(y-3)^2$ .
- B2 Transposing error.
- B3 Mathematical error, e.g.  $(-4)^2 = -8$ .
- B4 Incorrect relevant formula, e.g.  $\sqrt{(x_2 - x_1)^2 - (y_2 - y_1)^2}$  or  $\sqrt{(x_2 + x_1)^2 + (y_2 + y_1)^2}$  or  $\sqrt{(x_2 + x_1)^2 - (y_2 + y_1)^2}$ .
- B5 Two or more signs incorrect in substitution, if formula is written.
- B6 Error in factorising or in application of quadratic formula.
- B7 Finds only one value of  $y$ .
- B8 Confuses  $x$  and  $y$  coordinates, e.g. treats point as  $(y, -4)$ .

*Slips (-1)*

- S1 Each numerical slip to a maximum of 3.

*Attempts (3 marks)*

- A1 Some relevant step, e.g.  $x = -4$  and/or  $y = y$  or  $(x+4)^2$  expanded partly correctly.
- A2 States centre  $(-4, 3)$  and/or radius = 6 for this part.
- A3 Any formula with  $(x_2 - x_1)$  or  $(y_2 - y_1)$  and some correct substitution.
- A4 One or both correct values of  $y$  without work or verification shown.

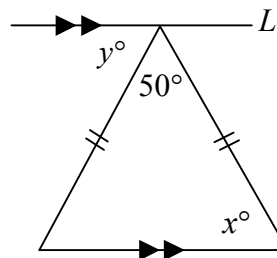


## QUESTION 4

<b>Part (a)</b>	<b>10 marks</b>	<b>Att 4</b>
<b>Part (b)</b>	<b>20 marks</b>	<b>Att 7</b>
<b>Part (c)</b>	<b>20 marks</b>	<b>Att 8</b>

**Part (a)** **10 (5, 5) marks** **Att (2, 2)**

In the diagram, the line  $L$  is parallel to the base of the isosceles triangle.



- (i) Find  $x$ .  
(ii) Find  $y$ .

**(a)(i)** **5 marks** **Att 2**

$$x + x + 50 = 180$$

$$\Rightarrow 2x = 130 \Rightarrow x = 65$$

**(a)(ii)** **5 marks** **Att 2**

$$y = x \text{ or } 65$$

- \* Accept correct answer without work shown.
- \* Allow candidate's value for  $x$  in finding  $y$ .

### *Blunders (-3)*

- B1 Sum of three angles  $\neq 180^\circ$ .
- B2  $x = 50$  or measure of unmarked angle in triangle = 50. [Part (i)]
- B3 Transposing error, e.g.  $2x = 130 \Rightarrow x = 128$ .

### *Slips (-1)*

- S1 Each numerical slip to a maximum of 3.

### *Attempts (2 marks each part)*

- A1 Some relevant step or statement, e.g. unmarked angle in triangle =  $x$  and stops.
- A2  $x + y + 50 = 180$  or similar and stops.
- A3  $y = x$  without finding values and stops.

### *Worthless (0 marks)*

- W1 Incorrect answer, other than those allowed above, without work.

**Part (b)**

**20 marks**

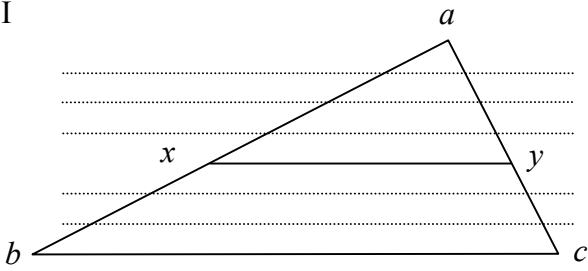
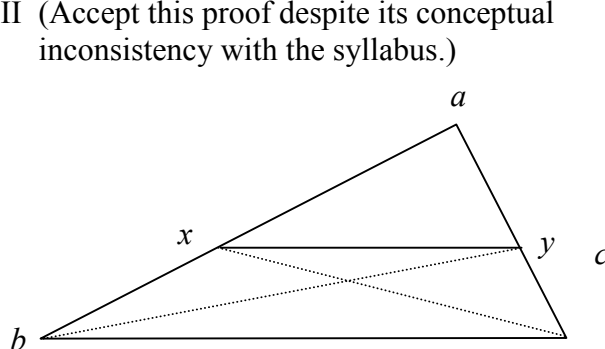
**Att 7**

Prove that a line which is parallel to one side-line of a triangle, and cuts a second side, will cut the third side in the same proportion as the second.

**Part b**

**20 marks**

**Att 7**

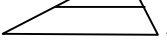
<p>I</p>  <p>Construction:          Let <math> ax : xb  = m:n</math>. Divide <math>[ax]</math> into <math>m</math> and <math>[xb]</math> into <math>n</math> equal parts.          Through each point of division draw a line parallel to <math>bc</math>. <span style="float: right;">7m</span></p> <p>Proof:          The parallel lines make intercepts of equal length along <math>[ac]</math> <span style="float: right;">10m</span>  <math>[ay]</math> is divided into <math>m</math> of these parts. <span style="float: right;">13m</span>  <math>[yc]</math> is divided into <math>n</math> of these parts. <span style="float: right;">16m</span>  <math> ay  :  yc  = m : n</math> <span style="float: right;">19m</span>  <math> ax  :  xb  =  ay  :  yc </math>. <span style="float: right;">20m</span></p>	<p>II (Accept this proof despite its conceptual inconsistency with the syllabus.)</p>  <p>Construction: Join <math>x</math> to <math>c</math> and <math>y</math> to <math>b</math>. <span style="float: right;">7m</span></p> <p>Proof:  <math>\frac{\Delta axy}{\Delta xby} = \frac{\frac{1}{2} ax h_1}{\frac{1}{2} xb h_1} = \frac{ ax }{ xb }</math> <span style="float: right;">10m</span></p> $\frac{\Delta axy}{\Delta xcy} = \frac{\frac{1}{2} ay h_2}{\frac{1}{2} yc h_2} = \frac{ ay }{ yc }$ <span style="float: right;">13m</span> <p>Area <math>\Delta xby = \text{Area} \Delta xcy</math> <span style="float: right;">16m</span></p> $\Rightarrow \frac{ ax }{ xb } = \frac{ ay }{ yc }$ <span style="float: right;">20m</span>
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- \* Proof without a diagram merits att 7, if proof can be reconciled with a diagram.
- \* Other proofs to be marked using similar steps as outlined above.
- \* If a candidate's work is not worthless, he/she must be awarded **at least** att 7.

*Blunders (-3)*

- B1 Each step omitted or incorrect.
- B2 Steps written in an illogical order. [Penalise once only.]  
 [Note: Some of the steps above may be interchanged.]

*Attempts (7 marks)*

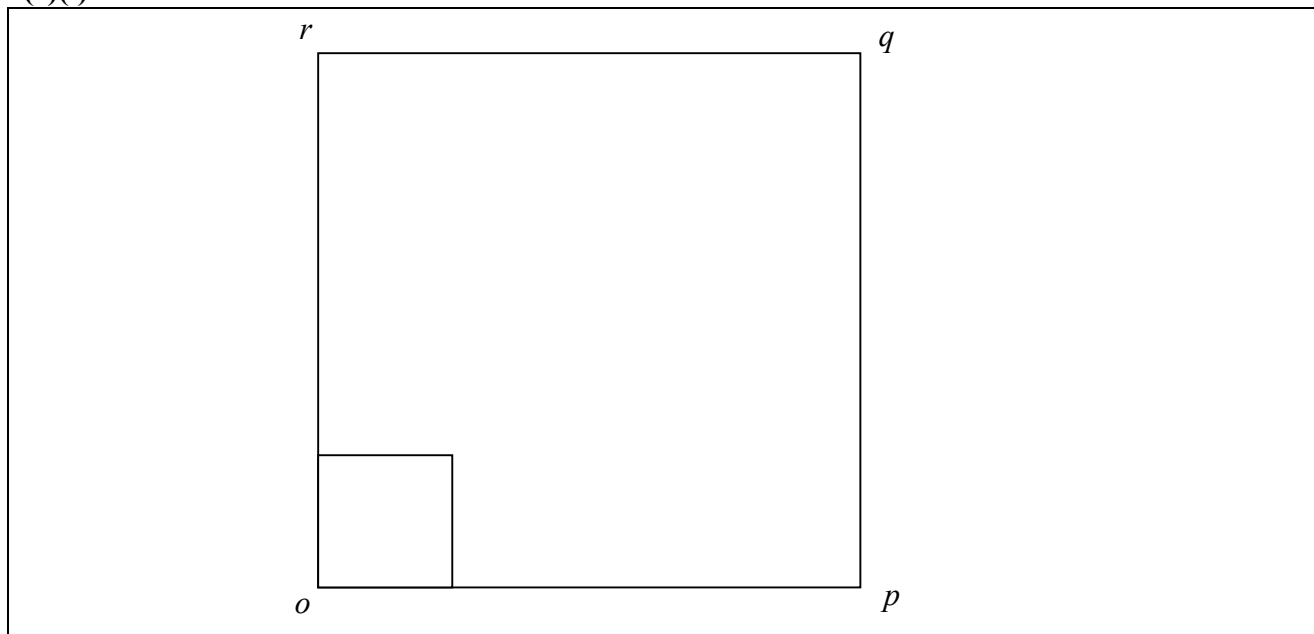
- A1 Any relevant step stated or indicated, e.g. .
- A2 States or illustrates a special case, e.g. gives values to the line segments.
- A3 A relevant theorem stated or proved, e.g. theorem dealing with equal intercepts.
- A4 No construction merits, at most, attempt marks.

*Worthless (0 marks)*

- W1 Any irrelevant theorem, subject to the attempt mark.
- W2 Triangle only.

**Part (c)****20 (5, 5, 5, 5) marks****Att (2, 2, 2, 2)**

- (i) Draw a square  $opqr$  with sides 8 cm.  
(ii) Draw the image of this square under the enlargement with centre  $o$  and scale factor 0.25.  
(iii) Calculate the area of this image square.  
(iv) Under another enlargement the area of the image of the square  $opqr$  is  $100 \text{ cm}^2$ . What is the scale factor of this enlargement

**(c)(i)****5 marks****Att 2**

\* Accept a quadrilateral with sides within  $\pm 1 \text{ cm}$  and angles within  $\pm 5^\circ$ .

*Slips (-1)*

- S1 Sides not straight (no straight edge) but measurements within tolerance.  
S2 Square but with measure of sides outside tolerance.  
S3 Does not label all vertices or labels vertices incorrectly e.g. order not cyclic..

*Attempts (2 marks)*

- A1 Some relevant step, e.g. draws a line segment, within tolerance, even if freehand.  
A2 Indicates some knowledge of a square.

**(c)(ii)****5 marks****Att 2**

See solution box for (c)(i).

- \* Apply the scheme of (c)(i) where appropriate.
- \* Do not demand labels on the vertices of the image.
- \* Allow the candidate to enlarge the figure drawn for (c)(i) by a scale factor.
- \* Allow candidate to work on the diagram of (c)(i), i.e. to do (c)(i) and (c)(ii) on the same diagram.

Image is right size, right place, award 5 marks.

Image is right size, wrong place, award 2 marks.

Image is wrong size, right place, award 2 marks.

Otherwise, award 0 marks.

*Attempts (2 marks)*

- A1 Some relevant step, e.g. joins  $q$  to  $o$ .

**(c)(iii)**

**5 marks**

**Att 2**

I Area = $ op' ^2 = 2^2 = 4$	II Area of original square = $8 \times 8 = 64$ Area of image = $(0.25)^2 \times 64 = 4$
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- \* Accept area of candidate's image in (c)(ii).
- \* Accept a correct answer without work shown.

*Blunders (-3)*

- B1 Blunder in area formula for image, e.g.  $0.25 \times 8 = 2$  and stops.  
B2 Leaves answer as 64 or finds an inconsistent fraction of 64, e.g.  $64 \times 0.25$  or  $64 \div 0.25$ .  
B3 Mathematical blunder, e.g.  $(0.25)^2 = 0.5$ .

*Slips (-1)*

- S1 Each numerical slip to a maximum of 3.

*Attempts (2 marks)*

- A1 Correct relevant formula, not transcribed from the tables.  
A2 64 with or without work.  
A3 Some relevant step.

**(c)(iv)**

**5 marks**

**Att 2**

I Area $ op' ^2 = 100 \Rightarrow  op'  = 10$ Scale factor $\frac{10}{8}$ or 1.25	II Area of original square = $8 \times 8 = 64$ $(\text{Scale factor})^2 = \frac{\text{Image}}{\text{Original}} = \frac{100}{64} \Rightarrow \text{Scale factor} = \frac{10}{8}$
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- \* Accept correct answer without work.
- \* Accept answer consistent with earlier parts.
- \* Accept  $-10/8$ .

*Blunders (-3)*

- B1 Mathematical blunder.  
B2 Misplaced decimal point.  
B3  $64 * k^2 = 100$ , where \* is not "multiply" and continues.  
B4  $k = 100 \div 64$  or 1.5625.

*Attempts (2 marks)*

- A1 Some relevant step, e.g. side of image = 10.

## QUESTION 5

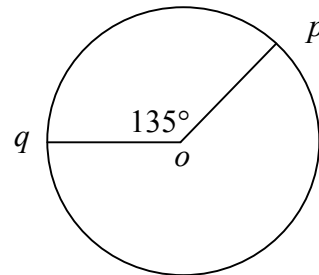
<b>Part (a)</b>	<b>10 marks</b>	-
<b>Part (b)</b>	<b>20 marks</b>	-
<b>Part (c)</b>	<b>20 marks</b>	-

*Note:* The marking of Question 5 is not based on slips, blunders and attempts. In the case of each part, descriptions or typical examples of work meriting particular numbers of marks are given. The mark awarded must be one of the marks indicated. For example, in part (a), descriptions are given for work meriting 0, 3, 7 or 10 marks. It is therefore not permissible to award, for example 6, or 8 marks for this part.

**Part (a)** **10 marks**      -

A circle has centre  $o$  and radius 14 cm.

$p$  and  $q$  are two points on the circle and  $|\angle qop| = 135^\circ$ .



Find the length of the shorter arc  $pq$ .

Take  $\pi = \frac{22}{7}$ .

**(a)** **10 marks**      -

<p>I</p> $\begin{aligned} \text{Arc length} &= \frac{135}{360} \times (2\pi r) \\ &= \frac{135}{360} \left( 2 \times \frac{22}{7} \times 14 \right) \\ &= 33 \text{ cm.} \end{aligned}$	<p>II</p> $135^\circ = \frac{135\pi}{180} \text{ rads or Arc} = 14\theta$ $\begin{aligned} \text{Arc} &= \frac{14 \times 135 \times 22}{180 \times 7} \\ &= 33 \text{ cm.} \end{aligned}$	<p>3 m</p> <p>7 m</p> <p>10 m</p>
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- \* A serious error is one that fundamentally alters the nature or complexity of a task and normally results in 3 marks at most.
- \* Incorrectly using radians or gradients or vice versa should be penalised only once.

10 marks: Correct answer with work shown.  
Ignore the obvious slip of the pen e.g.  $\frac{135}{360} \times 2\pi r = \frac{135}{160} \times 2 \times \frac{22}{7} \times 14 = 74.25$ .

7 marks: Has a correct formula with fully correct substitutions and stops or has one error in the formula or substitution and continues to the end without further errors of significance, e.g.  $14 \times \frac{135}{180} = 10.5$  [i.e. mishandles one of the digits in the fraction] or find  $135(2\pi r)$  or finds the large arc or work such as  $\frac{135}{360} \times \pi r^2 = \frac{135}{360} \times \frac{22}{7} \times 14 \times 14 = 231$  or answer in terms of  $\pi$ .

3 marks: Has correct answer without work, or mishandles two of the digits in the second line of solution or has a relevant step, e.g.  $\frac{135}{360}$  or  $360^\circ$  or substitutes for  $r$  in  $2\pi r$  or writes  $14 \times 135$  or work such as  $\pi r^2$  substituted.

0 marks: Nothing relevant written.

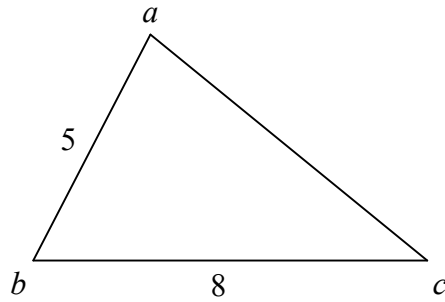
**Part (b)****20 (10, 10) marks**

-

In the triangle  $abc$ ,  $|ab| = 5$  cm and  $|bc| = 8$  cm.  
The area of the triangle is  $16.58$  cm<sup>2</sup>.

(i) Find  $|\angle abc|$ , correct to the nearest degree.

(ii) Find  $|ac|$ , correct to the nearest centimetre.

**(b)(i)****10 marks**

$$\frac{1}{2} ab \sin C = 16.58$$

$$0.5 \times 5 \times 8 \times \sin \angle abc = 16.58$$

3 marks

$$\Rightarrow \sin \angle abc = \frac{16.58}{0.5 \times 5 \times 8} \quad \text{or} \quad \frac{16.58 \times 2}{5 \times 8} \quad \text{or} \quad 0.829$$

7 marks

$$\Rightarrow |\angle abc| = 55.99^\circ$$

$$= 56^\circ.$$

9 marks

10 marks

10 marks: Correct answer with work shown.

9 marks: Round-off omitted or incorrect.

7 marks:  $\sin \angle abc$  isolated correctly, e.g.  $\frac{16.58}{\frac{1}{2} \times 5 \times 8}$  but fails to finish or

one error in isolating  $\sin \angle abc$  e.g.  $\frac{16.58}{2 \times 5 \times 8} = 0.20725 \Rightarrow |\angle abc| = 11.9$  or  $12^\circ$  and finishes without further errors of significance.

3 marks: A correct area formula not transcribed from the tables, e.g.  $\frac{1}{2}$  base  $\times$  height or an incorrect relevant formula with some correct substitutions, e.g.,  $\frac{5}{\sin C} = \frac{8}{\sin A}$  or correct answer without work, or

a relevant step, e.g.  $\frac{1}{2} \times 5$  or  $\frac{16.58}{2.5}$  from which candidate cannot finish.

0 marks: Nothing relevant written.

e.g. treats triangle  $abc$  as right angled,  $\cos A = \frac{5}{8}$  or  $5^2 + 8^2 = h^2$ .

**(b)(ii)****10 marks**

-

$$|ac|^2 = |ab|^2 + |bc|^2 - 2|ab||bc|\cos 56^\circ$$

$$|ac|^2 = 5^2 + 8^2 - 2(5)(8)\cos 56^\circ \quad \text{or} \quad |ac|^2 = 5^2 + 8^2 - 2(5)(8)(0.5592)$$

7 marks

$$\Rightarrow |ac|^2 = 25 + 64 - 44.736 = 44.264$$

$$\Rightarrow |ac| = 6.65$$

$$= 7 \text{ cm.}$$

10 marks: Correct or consistent answer with work shown.

9 marks: Round-off omitted or incorrect.

7 marks: A correct formula with fully correct substitutions but fails to finish or a blunder in signs in the formula or application and finishes correctly without further errors of significance or substitutes incorrectly one term into a correct formula and finishes accurately without further errors of significance

3 marks: Correct answer without work, or an incorrect relevant formula with substitution e.g. attempt at sine rule with work or squares omitted and cannot finish work such as  $\cos 56 = 5/8$  or A relevant step, e.g.  $5^2$ .

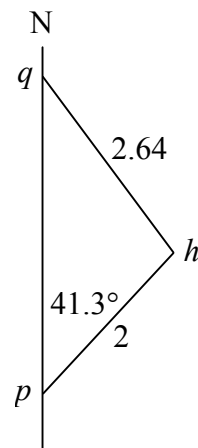
0 marks: Nothing relevant written e.g.  $\tan 56$ .

**Part (c)**

**20 (10, 10) marks**

-

A lighthouse,  $h$ , is observed from a ship sailing a straight course due North.  
The distance from  $p$  to  $h$  is 2 km and the bearing of the lighthouse from  $p$  is N  $41.3^\circ$  E.  
The distance from  $q$  to  $h$  is 2.64 km.



- (i) Find the bearing of the lighthouse from  $q$ .
- (ii) The ship is sailing at a speed of 19 km/h. Find, correct to the nearest minute, the time taken to sail from  $p$  to  $q$ .

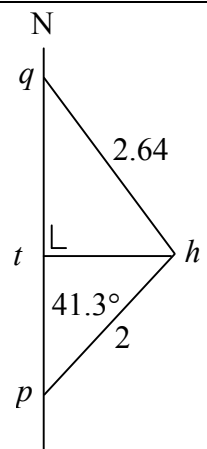
**(c)(i)**

**10 marks**

-

I  
 $\frac{|ph|}{\sin q} = \frac{|qh|}{\sin p}$   
 $\frac{2}{\sin q} = \frac{2.64}{\sin 41.3^\circ}$   
 $\Rightarrow \sin q = \frac{2 \sin 41.3^\circ}{2.64} = \frac{1.32}{2.64} = 0.5$   
 $\Rightarrow |\angle q| = 30^\circ$   
 Bearing S  $30^\circ$  E.

II  
 $\frac{|ht|}{2} = \sin 41.3^\circ$   
 $\Rightarrow |ht| = 2 \times 0.66 = 1.32$   
 $\Rightarrow \sin q = \frac{1.32}{2.64} = 0.5$   
 $\Rightarrow |\angle q| = 30^\circ$   
 Bearing S  $30^\circ$  E.



10 marks: Correct answer or  $|\angle q| = 30^\circ$  with work shown.

7 marks: A correct formula substituted with  $\sin q$  isolated or substitutes incorrectly into a correct formula and finishes accurately or finishes correctly with an error in a correct formula or finds  $|\angle thq|$  correctly and stops.

3 marks: Correct answer without work, or an incorrect relevant formula with substitution or displays ability to interpret question and begins to substitute or. has a relevant step e.g. draws [  $ht$  ].

0 marks: Nothing relevant written e.g. treats triangle  $phq$  as right-angled.

<b>(c)(ii)</b>	<b>10 marks</b>	-
<p>I:</p> $ \angle h  = 180^\circ - (41.3^\circ + 30^\circ) = 108.7^\circ$ $\frac{2}{\sin 30^\circ} = \frac{ pq }{\sin 108.7^\circ}$ $\Rightarrow  pq  = \frac{2 \sin 108.7^\circ}{\sin 30^\circ} = \frac{2(0.9472)}{0.5}$ $= 3.7888 \text{ km.}$ $\text{Time} = \frac{3.7888 \times 60}{19} = 11.96$ $= 12 \text{ minutes.}$	<p>II:</p> $ pq ^2 = 2.64^2 + 2^2$ $- 2 \times 2.64 \times 2 \times \cos 108.7^\circ$ $ pq ^2 = 10.9696 + 3.3856$ $= 14.3553$ $ pq  = 3.7888 \text{ km.}$ <p>Continue as in I</p>	<p>III:</p> <p>From II (c)(i)</p> $\frac{ qt }{2.64} = \cos 30^\circ$ $ qt  = 2.64 \times 0.866$ $= 2.2863.$ $\frac{ tp }{2} = \cos 41.3^\circ$ $ tp  = 1.5025$ $ pq  = 3.7888$ <p>Continue as in I.</p>

10 marks: Correct answer with work shown.

9 marks: Round-off omitted or incorrect.

7 marks: Calculates the distance and stops or makes an error in finding the distance but finishes correctly or substitutes incorrectly into a correct formula and finishes accurately or finishes correctly with an error in a correct formula.

3 marks: Correct answer without work, or has some relevant work for either distance or time or correctly identifies required quantity or a relevant step, e.g.  $108.7^\circ$  or  $2^2$  or  $|pq|^2 = 10.9696 - 3.3856 = 7.584 \Rightarrow |pq| = 2.75$  and stops or continues incorrectly.

0 marks: Nothing relevant written.



## QUESTION 6

<b>Part (a)</b>	<b>10 marks</b>	<b>Att 4</b>
<b>Part (b)</b>	<b>20 marks</b>	<b>Att 8</b>
<b>Part (c)</b>	<b>20 marks</b>	<b>Att 8</b>

**Part (a)** **10 (5, 5)marks** **Att (2, 2)**

- (i) Evaluate  $6!$
- (ii) Evaluate  $\binom{12}{3}$ .

**(a) (i)** **5 marks** **Att 2**

$$6! = 720 \quad \text{or} \quad 6.5.4.3.2.1 = 720 \quad \text{or} \quad 6.5.4.3.2 = 720 \quad \text{or} \quad {}^6P_6 = 720$$

**(a) (ii)** **5 marks** **Att 2**

$$\binom{12}{3} = \frac{12 \times 11 \times 10}{1 \times 2 \times 3} = 220 \quad \text{or} \quad \frac{12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 1 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1} = 220$$

- \* If parts of (a) are not identified, and if not obvious which part is being attempted, treat each part in order.
- \* Multiplication must be clearly indicated. So 6, 5, 4, 3, 2, 1 or 

6	5	4	3	2	1
---	---	---	---	---	---

 and stops merits Att 2.
- \* Any relevant integer, from the solutions, written down or used (and clearly separate from writing down the question) is Att 2, e.g. 1, 2, 3, 4, 5, 6 written singly or in combination (such as 6.5); but, for example, 30 with no work shown merits zero.

***In each part:***

Correct answer with or without work shown merits 5 marks.

Correct answer not fully worked out merits 4 marks.

An incorrect answer with relevant work merits 2 marks.

Otherwise 0 marks.

***Attempts (2 marks)***

- A1 One correct step (e.g. partial list) and stops, each part.
- A2 Writes any permutation or combination (but not factorial) symbol and stops, in part (i).
- A3 Writes any permutation or factorial (but not combination) symbol and stops, in part (ii).
- A4 Writes  ${}^6P_6$  and stops in (i). (Question asks *Evaluate*)
- A5 Writes  ${}^{12}P_3$  or  ${}^{12}P_9$  in (ii).
- A6 Writes  $\binom{12}{9}$  or  $\frac{12!}{3!9!}$  or  $\frac{{}^{12}P_3}{3!}$  and stops in (ii). (Question asks *Evaluate*)
- A7  $12 - 3$  and stops in (ii).
- A8 Relevant integer used.

**Part (b)****20 (5, 5, 5, 5) marks****Att (2, 2, 2, 2)**

Ten teams take part in a competition. The teams are divided into two groups. Teams A, B, C, D and E are in group 1 and teams U, V, X, Y, and Z are in group 2.

In the final, the winning team from group 1 plays the winning team from group 2.

Each team is equally likely to win its group.

- (i) How many different team pairings are possible for the final?
- (ii) What is the probability that team C plays team X in the final?
- (iii) What is the probability that team A plays in the final?
- (iv) What is the probability that team B does not play in the final?

**(b) (i)****5 marks****Att 2**

$5 \times 5 = 25$ or $5 + 5 + 5 + 5 + 5 = 25$ or 25	or	<table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <tr><td>AU</td><td>AV</td><td>AX</td><td>AY</td><td>AZ</td></tr> <tr><td>BU</td><td>BV</td><td>BX</td><td>BY</td><td>BZ</td></tr> <tr><td>CU</td><td>CV</td><td>CX</td><td>CY</td><td>CZ</td></tr> <tr><td>DU</td><td>DV</td><td>DX</td><td>DY</td><td>DZ</td></tr> <tr><td>EU</td><td>EV</td><td>EX</td><td>EY</td><td>EZ</td></tr> </table> <p style="text-align: center;">i.e. 25 team pairing</p>	AU	AV	AX	AY	AZ	BU	BV	BX	BY	BZ	CU	CV	CX	CY	CZ	DU	DV	DX	DY	DZ	EU	EV	EX	EY	EZ
AU	AV	AX	AY	AZ																							
BU	BV	BX	BY	BZ																							
CU	CV	CX	CY	CZ																							
DU	DV	DX	DY	DZ																							
EU	EV	EX	EY	EZ																							

\* Accept correct answer without work.

\* Any relevant integer written from solutions is Att 2 e.g. 2 and 5 are relevant,  $2 \times 5$  is relevant but 10 without work merits zero.

*Slips (-1)*

S1 Numerical errors to a maximum of 3.

*Attempts (2 marks)*

A1 Any relevant step [e.g. partial list] and stops.

A2 Any relevant integer, as above e.g. 2, 5.

A3 One or more elements from incomplete sample space.

A4 Any permutation, combination or factorial symbol for this part and stops.

*Worthless (0 marks)*

W1 Statement of Probability Theorem for this part and stops i.e. #E/#S.

**(b) (ii)****5 marks****Att 2**

$\frac{1}{25}$ or 0.04 or 4% or 1:25	5 marks
$\frac{1}{5} \times \frac{1}{5}$ or $\frac{1}{5^2}$ or $1 - \frac{24}{25}$	4 marks

\* Accept correct answer without work.

\* Accept additional work done [e.g. correct event clearly marked] on sample space in (i) that is for this part, otherwise attempt mark.

\* Accept candidate's sample space from part (i). Note that an incorrect sample space from that part could merit full marks in part (ii) if the answer is consistent with the work in part (i) but must be written as a fraction, otherwise attempt mark at most.

\* A sample space written for this part does not apply to part (i).

Attempts (2 marks)

- A1 Any relevant step and stops, e.g. pairings for this part.
- A2 Any relevant integer from solutions, i.e. 1, 2, 4, 5, 24, 25.
- A3 Inverted fraction,  $25/1$  or similar consistent with (b) (i) but see A4.
- A4 Any meaningful incorrect ratio in the form  $a/b$  or  $a:b$  but must be  $0 \leq a/b \leq 1$ , unless A3 is applied.
- A5 Sample space or partial sample space done for this part but successful outcome omitted.
- A6 Statement of Probability Theorem specifically for this part.

**(b) (iii)**

**5 marks**

**Att 2**

$\frac{1}{5}$ or $\frac{5}{25}$ or 20% or 0.2 or 1 : 5	5 marks
$1 - \frac{4}{5}$ or $1 - \frac{20}{25}$ or $\frac{5}{5^2}$	4 marks

- \* Accept correct answer without work.
- \* Accept correct sample space and correct outcomes clearly marked.
- \* Accept additional work done [e.g. correct event clearly marked] on sample space in (i) or (ii) that is for this part, otherwise attempt mark.

Attempts (2 marks)

- A1 Any relevant step and stops e.g. pairing for this part.
- A2 Any relevant integer from solutions, i.e. 1, 2, 4, 5, 20, 25.
- A3 Inverted fraction,  $5/1$  or similar consistent with earlier work but see A4.
- A4 Any incorrect ratio in the form  $a/b$  or  $a:b$  but  $0 \leq a/b \leq 1$  must apply unless A3 is applied.
- A5 Sample space or partial sample space done for this part but successful outcomes omitted.
- A6 Statement of Probability Theorem specifically for this part.

**(b) (iv)**

**5 marks**

**Att 2**

$\frac{4}{5}$ or $\frac{20}{25}$ or 0.8 or 80% or 4 : 5	5 marks
$1 - \frac{1}{5}$ or $1 - \frac{5}{25}$ or $\frac{20}{5^2}$	4 marks

- \* Accept correct answer without work
- \* Accept correct sample space and correct outcomes clearly marked
- \* Accept additional work done [e.g. correct event clearly marked ] on sample space in (i) or (ii) or (iii) that is for this part, otherwise attempt mark.
- \* Note that an incorrect sample space from parts (i), (ii) or (iii) could merit full marks in part (iv) if the answer is consistent with earlier work.

Attempts (2 marks)

- A1 Any relevant step and stops.
- A2 Any relevant integer from solutions, i.e. 1, 2, 4, 5, 8, 20, 25, 80.
- A3 Inverted fraction,  $25/20$ ,  $5/4$  or similar consistent with earlier work but see A4.
- A4 Any incorrect ratio in the form  $a/b$  or  $a:b$  but must be  $0 \leq a/b \leq 1$  unless A2 or A3 is applied.
- A5 Sample space or partial sample space done for this part but successful outcomes omitted.
- A6 Statement of Probability Theorem specifically for this part.

**Part (c)****20 (5, 5, 5, 5) marks****Att (2, 2, 2, 2)**

Seven horses run in a race.

All horses finish the race and no two horses finish the race at the same time.

- (i) In how many different orders can the seven horses finish the race?
- (ii) A person is asked to predict the correct order of the first three horses to finish the race. How many different such predictions can be made?
- (iii) A person is asked to predict, in any order, the first three horses to finish the race. How many different such predictions can be made?
- (iv) A person selects two of the seven horses at random. What is the probability that the selected horses are the first two horses to finish the race?

**(c) (i)****5 marks****Att 2**

5040

5 marks

 $7!$  or  $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$  or  $7 \times 6 \times 5 \times 4 \times 3 \times 2$  or  ${}^7P_7$ 

4 marks

\* Accept correct answer without work.

\* For the 4 marks, multiplication must be clearly indicated.

**Award** 5 marks for correct answer with or without work shown.  
 4 marks for correct answer not fully worked out.  
 2 marks for an incorrect answer with relevant work.  
 Otherwise 0 marks.

*Attempts (2 marks)*

A1 Any relevant step and stops.

A2 Any work with relevant integer 1, 2, 3, 4, 5, 6, 7 singly or in combination e.g. 5 or  $7 \times 6$  but 56 without work merits zero.

A3 Any permutation, combination or factorial symbol and stops.

**(c) (ii)****5 marks****Att 2**

210

5 marks

 ${}^7P_3$  or  $7 \times 6 \times 5$  or  $\frac{7!}{4!}$  or  $\frac{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{4 \times 3 \times 2 \times 1}$ 

4 marks

**(c) (iii)****5 marks****Att 2**

35

5 marks

 $\binom{7}{3}$  or  $\binom{7}{4}$  or  $\frac{7!}{3!4!}$  or  $\frac{{}^7P_3}{3!}$  or  $\frac{7 \times 6 \times 5}{1 \times 2 \times 3}$  or  $\frac{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 1 \times 4 \times 3 \times 2 \times 1}$ 

4 marks

\* Accept correct answer without work.

\* For the 4 marks, multiplication must be clearly indicated.

\* In (c)(iii), accept for 5 marks an answer which is 1/6 of the answer to (c)(ii).

**In each part award:** 5 marks for correct answer with or without work shown.  
 4 marks for correct answer not fully worked out.  
 2 marks for an incorrect answer with relevant work.  
 Otherwise 0 marks.

*Misreading (-1)*

M1 Misreads combination for permutation or vice versa, each time.

Attempts (2 marks)

A1 Any relevant step and stops.

A2 Any relevant integer 1, 2, 3, 4, 5, 6, 7 singly or in combination e.g. 4 or 2/6.

A3 Any permutation, combination or factorial symbol and stops.

(c) (iv)

5 marks

Att 2

Total outcomes: 5040

Outcomes of interest:  $\underline{A} \underline{B} \times 5! + \underline{B} \underline{A} \times 5!$  gives 240 outcomes

$$P(\text{success}) = \frac{240}{5040} \text{ or } \frac{1}{21}$$

or

$$\text{Possible pairs } \binom{7}{2} = 21$$

Each pair has equal chance of success

$$P(\text{success}) = \frac{1}{21}$$

or

Horses A, B, C, D, E, F, G.

42 possible outcomes,  
2 outcomes of interest

$$P(\text{success}) = \frac{2}{42} \text{ or } \frac{1}{21}$$

		second						
		A	B	C	D	E	F	G
winner	A		✓					
	B	✓						
	C							
	D							
	E							
	F							
	G							

or

$$P(\text{winner}) \times P(\text{second}) + P(\text{second}) \times P(\text{winner}) = \frac{1}{7} \times \frac{1}{6} + \frac{1}{7} \times \frac{1}{6} = \frac{2}{42} \text{ or } \frac{1}{21}$$

\* Work required for this part.

\* Accept for full marks 240 divided by the candidates answer in (c)(i).

Slips (-1)

S1 Correct answer with no work shown.

Attempts (2 marks)

A1 Any relevant step and stops.

A2 Any relevant integer from solutions, i.e. 1, 2, 3, 4, 5, 6, 7, 21, 42, 120, 240, 5040, singly or in combination e.g. 42 or 5/3.

A3 Correct inverted fraction e.g. 42/2 or similar.

A4 Any incorrect ratio in the form  $a/b$  or  $a:b$  but must be  $0 \leq a/b \leq 1$  unless A2 or A3 is applied.

A5 Sample space with no further work or partial sample space.

A6 Statement of Probability Theorem for this part.

A7 Any permutation, combination or factorial symbol and stops.

## QUESTION 7

<b>Part (a)</b>	<b>10 marks</b>	<b>Att 3</b>
<b>Part (b)</b>	<b>20 marks</b>	<b>Att 6</b>
<b>Part (c)</b>	<b>20 marks</b>	<b>Att 8</b>

**Part (a)** **10 marks** **Att 3**

Calculate the weighted mean of 10, 30 and 15, given that the weights are 3, 1 and 2, respectively.

**(a)** **10 marks** **Att 3**

$$\text{Weighted mean} = \frac{\sum xw}{\sum w} = \frac{10 \times 3 + 30 \times 1 + 15 \times 2}{3 + 1 + 2} = \frac{90}{6} \text{ or } 15$$

or

$x$	$w$	$xw$
10	3	30
30	1	30
15	2	30
	6	90

$$\Rightarrow \text{Weighted mean} = \frac{90}{6} \text{ or } 15$$

\* Accept correct answer without work i.e. uses calculator.

10 marks: Answer of 90/6 or 15.

7 marks: Answer of 90 and 6 without fraction  
or fraction written as 6/90  
or 90/? or ?/6.

4 marks: 90 or 6 or incorrect fraction with work.

3 marks: Some relevant step.

0 marks: Worthless work.

*Misreading (-1)*

M1 Obvious misreading with work shown e.g. 5 instead of 3.

*Attempts (3 marks)*

A1 Any relevant step e.g. arbitrary fraction.

A2 Weighted mean =  $\frac{\sum xw}{\sum w}$  and stops.

A3 A correct multiplication and stops.

A4 3 + 1 + 2 and stops.

**Part (b)****20 (10, 10) marks****Att (3, 3)**

There are fourteen questions in an examination.  
The table below shows the performance of the candidates.

Correct responses	0 – 2	3 – 5	6 – 8	9 – 11	12 – 14
Number of candidates	1	2	6	8	3

- (i) Using mid-interval values, calculate the mean number of correct responses.  
(ii) Calculate the standard deviation, correct to one decimal place.

**(b) (i)****10 marks****Att 3**

Mid-interval values 1, 4, 7, 10, 13 [3 marks]  
 Mean  $\bar{x} = \frac{1 \times 1 + 4 \times 2 + 7 \times 6 + 10 \times 8 + 13 \times 3}{1 + 2 + 6 + 8 + 3} = \frac{1 + 8 + 42 + 80 + 39}{20} = \frac{170}{20}$  or 8.5  
 [7 marks] [10 marks]

or

Interval	Mid-interval (x)	f	fx
0 – 2	1	1	1
3 – 5	4	2	8
6 – 8	7	6	42
9 – 11	10	8	80
12 – 14	13	3	39
		20	170

[7 marks]

Mean  $\bar{x} = \frac{\sum fx}{\sum f} = \frac{170}{20}$  or 8.5 [10 marks]

- \* Accept correct answer without work i.e. uses calculator.
- \* One table may be used for both parts of (b).
- \* All answers must be consistent with written mid-interval values and frequency values, otherwise incorrect answer without work merits zero.
- \* Award full marks for answer left in the form 170/20.

10 marks: Answer of 170/20 or 8.5.

7 marks: Answer of 170 and 20 without fraction  
or fraction written as 20/170  
or 170/? or ?/20.

4 marks: 170 or 20 or incorrect fraction with work. [Apply maximum of one blunder for numerator and one blunder for denominator].

3 marks Some relevant step.

0 marks Worthless work.

*Slips (-1)*

S1 Each numerical error to a maximum of 3.

S2 Each incorrect mid-interval value to a maximum of 3.

*Attempts (3 marks)*

A1 Some relevant step e.g. finds the median or modal class or draws a cumulative frequency curve or arbitrary fraction.

- A2 Weighted mean =  $\frac{\sum xf}{\sum f}$  and stops.  
 A3 A correct multiplication and stops.  
 A4  $1 + 2 + 6 + 8 + 3$  and stops.  
 A5 One or more correct mid-interval values and stops.  
 A6 Accept a reasonable estimate of  $8 \leq \bar{x} \leq 9$ .

**(b) (ii)** **10 marks** **Att 3**

Deviations  $1 - 8.5, 4 - 8.5, 7 - 8.5, 10 - 8.5, 13 - 8.5$  3 marks

$$\sigma = \sqrt{\frac{1(-7.5)^2 + 2(-4.5)^2 + 6(-1.5)^2 + 8(1.5)^2 + 3(4.5)^2}{1 + 2 + 6 + 8 + 3}}$$
 7 marks

$$= \sqrt{\frac{56.25 + 40.5 + 13.5 + 18 + 60.5}{20}} = \sqrt{\frac{189}{20}} = \sqrt{9.45} = 3.07 = 3.1$$
 10 marks

or

$x$	$f$	$fx$	$ x - \bar{x} $	$(x - \bar{x})^2$	$f(x - \bar{x})^2$	
1	1	1	7.5	56.25	56.25	
4	2	8	4.5	20.25	40.5	
7	6	42	1.5	2.25	13.5	
10	8	80	1.5	2.25	18.0	
13	3	39	4.5	20.25	60.75	
	20	170			189	7 marks

$$\sigma = \sqrt{\frac{\sum f(x - \bar{x})^2}{\sum f}} = \sqrt{\frac{189}{20}} = \sqrt{9.45} = 3.07 = 3.1$$
 10 marks

- \* Accept correct answer without work i.e. uses calculator.
- \* If one table is used for both parts, see table in part (i).
- \* Accept either positive or negative deviations.
- \* Accept candidate's values from (i) for working with in (ii).
- \* Where no marks have been given in part (i), the above table should be awarded 7 marks for part (i) and 7 marks for part (ii).

10 marks: Answer of 3.1.

9 marks: Answer of 3.07.

7 marks: Answer of 189 and 20 without fraction  
or fraction written as  $\frac{20}{189}$  or  $\frac{189}{?}$

4 marks: 189 or  $\sqrt{189}$  or incorrect fraction with work. [Apply maximum of one blunder for numerator and one blunder for denominator]  
or table without the bottom line totals.

3 marks Some relevant step 20 written without further work in this part.

0 marks Worthless work.

*Slips (-1)*

- S1 Each numerical error to a maximum of 3
- S2 Each incorrect deviation to a maximum of 3.
- S3 Incorrect or omitted round-off
- S4 Candidate uses sample standard deviation on calculator (Answer 3.153...)



Attempts (3 marks)

A1 Any relevant step, e.g. arbitrary fraction.

A2 Correct relevant formula and stops e.g.  $\sqrt{\frac{\sum fd^2}{\sum f}}$ .

A3 A correct multiplication and stops.

A4 Any correct deviation.

A5  $1 + 2 + 6 + 8 + 3$  and stops (if not already awarded in (b)(i)).

A6  $\sum f = 20$  stated in this part (if not already awarded in (b)(i)).

A7 Accept a reasonable estimate,  $2 \leq \sigma \leq 4$ .

**Part (c)**

**20 (5, 5, 5, 5) marks**

**Att (2, 2, 2, 2)**

A concert began at 8.00 p.m. The cumulative frequency table below gives the number of people in the concert hall at the times stated.

Time p.m.	7.10	7.20	7.30	7.40	7.50	8.00
Number of people	0	30	100	160	275	300

(i) Copy and complete the following frequency table to show the number of people who entered the hall during each time interval.

Time interval	7.10-7.20	7.20-7.30	7.30-7.40	7.40-7.50	7.50-8.00
Number of people					

(ii) In which interval does the median time of arrival lie?

(iii) In which time interval did the greatest number of people enter the concert hall?

(iv) What is the least number of people who could have been in the concert hall at 7.15 p.m.?

**(c)(i)**

**5 marks**

**Att 2**

Time interval	7.10 – 7.20	7.20 – 7.30	7.30 – 7.40	7.40 – 7.50	7.50 – 8.00
Number of people	30	70	60	115	25

Blunders (-3)

B1 Makes a cumulative frequency table from the given table:

Time interval	7.10 – 7.20	7.20 – 7.30	7.30 – 7.40	7.40 – 7.50	7.50 – 8.00
Number of people	30	130	290	565	865

Slips (-1)

S1 Each incorrect subtraction up to a maximum of 3. (Subsequent numbers may be correct)

S2 Each incorrect entry up to a maximum of 3.

Attempts (2 marks)

A1 One correct entry.

A2 Copies table.

**(c) (ii)**

**5 marks**

**Att 2**

7.30 – 7.40 interval

- \* Accept answers consistent with candidates table in (i).
- \* Draws cumulative frequency curve, gets  $x$  value corresponding to  $y = 150$  and states correct interval – accept for full marks.
- \* Accept “third interval”.

*Slips (-1)*

- S1 Gets correct  $x$ -value (7.38 p.m.) from ogive and stops.
- S2 Gets correct  $x$  value from ogive and then opts for incorrect time interval.
- S3 Gives a time between 7.30 and 7.40 without stating the interval.

*Attempts (2 marks)*

- A1 Draws cumulative frequency curve and stops.
- A2 States median relates to 150 people and stops.
- A3 Definition of median.

**(c) (iii)**

**5 marks**

**Att 2**

7.40 – 7.50 interval

*Blunders (-3)*

- B1 States 8 p.m. based on given cumulative frequency table.
- B2 States 7.40 p.m. and stops.
- B3 States 7.50 p.m. and stops.

*Misreadings (-1)*

- M1 Least number of people found = 25 (must be stated).

*Attempts (2 marks)*

- A1 Writes down 115 and stops.
- A2 Some attempt to read answer from ogive e.g. states that the steepest part of the ogive is from 7.40 to 7.50 p.m.

**(c) (iv)**

**5 marks**

**Att 2**

0

*Misreadings (-1)*

- M1 Greatest number of people = 30 (must be stated).

*Attempts (2 marks)*

- A1 Takes  $x = 7.15$  on ogive and finds corresponding value of  $y$  (18).
- A2 States 30 without work

## QUESTION 8

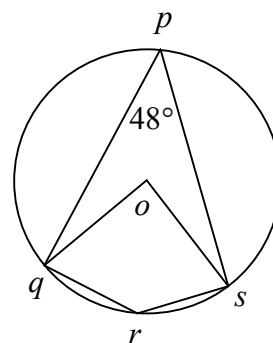
<b>Part (a)</b>	<b>10 marks</b>	<b>Att 4</b>
<b>Part (b)</b>	<b>20 marks</b>	<b>Att 7</b>
<b>Part (c)</b>	<b>20 marks</b>	<b>Att 8</b>

**Part (a)** **10 (5, 5) marks** **Att (2, 2)**

The points  $p, q, r$  and  $s$  lie on a circle, centre  $o$ .  
 $|\angle spq| = 48^\circ$ .

(i) Find  $|\angle soq|$ .

(ii) Find  $|\angle qrs|$ .



**(a) (i)** **5 marks** **Att 2**

$$|\angle soq| = 2|\angle spq| = 2(48^\circ) = 96^\circ$$

\* Accept  $264^\circ$ .

**(a) (ii)** **5 marks** **Att 2**

$$|\angle qrs| = 180^\circ - 48^\circ = 132^\circ$$

or

$$|\angle qrs| = \frac{1}{2}|\angle soq| = \frac{1}{2}(360^\circ - 96^\circ) = 132^\circ$$

- \* Accept answers clearly indicated on diagram.
- \* Accept correct answers without work.
- \* Angle notation or degree notation not required.

*Slips (-1)*

S1 Each numerical slip to a maximum of 3.

*Misreadings (-1)*

M1 Any obvious misreading that does not oversimplify or change the task.

*Attempts (2 marks)*

- A1 Sum of the four angles in quadrilateral  $pqrs$  is  $360^\circ$  and stops.
- A2 Sum of the four angles in quadrilateral  $qrs o$  is  $360^\circ$  and stops.
- A3 Joins  $po$  and indicates any correct equation e.g. exterior angles equal in measure to sum of interior opposite angles.
- A4 Shows  $|oq| = |os|$  and stops.
- A5 Any relevant step.

Prove that a line is a tangent to a circle at a point  $t$  of the circle if and only if it passes through  $t$  and is perpendicular to the line through  $t$  and the centre.

(b)

20 marks

Att 7

The two directions of the proof (“if” and “only if”) are as follows:

[1] If  $L$  passes through  $t$  and is perpendicular to  $ot$  then  $L$  is a tangent at  $t$ .

[2] If  $L$  is a tangent at  $t$  then  $L$  is perpendicular to  $ot$ .

[1]:  $[pt]$  is a diameter of circle  $K$ .

$L$  is a line such that  $pt \perp L$  at  $t$ .

To prove:  $L$  is a tangent to circle  $K$ .

Let  $q$  be any other point of  $L$ . Join  $oq$ .

In triangle  $otq$ ,  $|\angle otq| = 90^\circ$ .

Thus, triangle  $otq$  is right-angled at  $t$  and  $[oq]$  is the hypotenuse.

Hence,  $|oq| > |ot|$

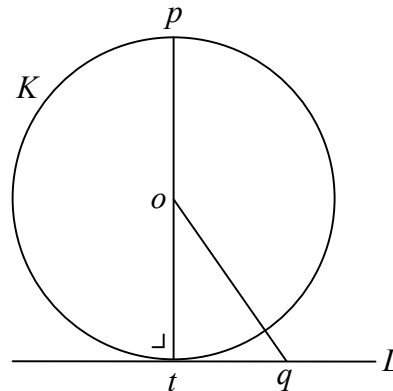
or  $|oq| > \text{radius}$

$\Rightarrow q$  is outside the circle.

Similarly, any other point of  $L$ , except  $t$ , is outside the circle.

Hence, the line  $L$  meets the circle at only one point,  $t$ .

Hence, the line  $L$  is a tangent.



Step 1. 7m

Step 2. 10 m

Step 3. 13 m

Step 4. 16 m

Step 5. 19 m

Step 6. 20 m

[2]:  $L$  is a tangent to  $K$  at  $t$ .

To prove:  $L \perp ot$ .

Suppose  $L$  not  $\perp ot$ .

Let  $s$  be a point of  $L$  such that  $os \perp L$ .

Since  $t \neq s$ , consider triangle  $ots$ .

$|\angle tso| = 90^\circ$

$\Rightarrow |\angle ots| = 90^\circ - |\angle sot| < 90^\circ$ .

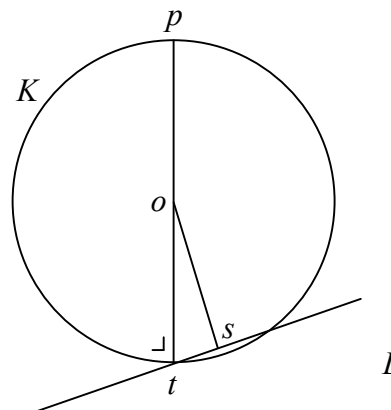
Hence,  $|os| < |ot| = \text{radius}$ .

Hence,  $s$  is a point inside the circle.

But this contradicts the fact that  $L$  is a tangent.

Hence, original supposition must be false.

Hence,  $L \perp ot$ .



Step 1. 7 m

Step 2. 10 m

Step 3. 13 m

Step 4. 16 m

Step 5. 19 m

Step 6. 20 m

\* Award full marks for a correct proof of either direction of the theorem (i.e. [1] or [2] above).

\* If candidate's work is not worthless, Att 7 at least must be awarded.

\* Clear diagram, without steps written (order of steps not clear), apply B2.

**Blunders (-3)**

B1 Incorrect step or part of step or step omitted.

B2 Steps in incorrect order, once.

**Attempts (7 marks)**

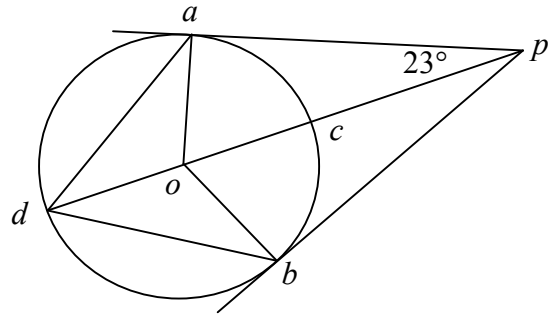
A1 Outline diagram with/without  $oq$  drawn. (Minimum required circle and tangent).

A2 Attempt at proof using special case.

A3 Memorised proof, without a diagram, if attempt can be reconciled with a diagram.

**Part (c)****20 (5, 5, 5, 5) marks****Att (2, 2, 2, 2)**

$pa$  and  $pb$  are tangents to a circle, centre  $o$ .  
 $po$  intersects the circle at  $c$  and  $d$ .  
 $|ao| = 5$ ,  $|pc| = 8$  and  $|\angle opa| = 23^\circ$ .



- (i) Find  $|pa|$ .
- (ii) Find  $|\angle aop|$ .
- (iii) Find  $|\angle adb|$ .
- (iv) Find  $|\angle dbo|$ .

**(c) (i)****5 marks****Att 2**

$$|pa|^2 + |ao|^2 = |po|^2 \Rightarrow |pa|^2 + 5^2 = 13^2 \Rightarrow |pa|^2 = 169 - 25 = 144 \Rightarrow |pa| = 12$$

- \* Accept use of Pythagorean triple 5, 12, 13.
- \* Accept trigonometric method.

*Blunders (-3)*

- B1 Incorrect step or part of step omitted.
- B2 Transposition error.
- B3 Mathematical blunder e.g.  $5^2 = 10$ .

*Slips (-1)*

- S1 Each numerical slip.

*Misreadings (-1)*

Any obvious misreading which does not oversimplify or change the task.

*Attempt (2 marks)*

- A1 Relevant step e.g. 13 or  $|\angle oap| = 90^\circ$

**(c) (ii)****5 marks****Att 2**

$$|\angle aop| + |\angle opa| = 90^\circ \Rightarrow |\angle aop| = 90^\circ - 23^\circ = 67^\circ$$

or

$$|\angle aop| = 180^\circ - (90^\circ + 23^\circ) = 67^\circ$$

- \* Accept answer written on a diagram
- \* Accept correct answer without work.

*Slips (-1)*

- S1 Numerical slips e.g.  $90 - 23 = 57$ .

*Attempt (2 marks)*

- A1 Sum of three angles in triangle is  $180^\circ$  and stops.
- A2 Marks  $\angle pao$  as right-angle for this part and stops, if not already awarded.
- A3 Some effort at getting result using trigonometry.

(c) (iii)

5 marks

Att 2

$$|\angle aop| = 90^\circ - 23^\circ = 67^\circ \Rightarrow |\angle aob| = 134^\circ.$$

$$|\angle adb| = \frac{1}{2}|\angle aob| = \frac{1}{2}(134^\circ) = 67^\circ.$$

or

$$|\angle adb| = |\angle adp| + |\angle pdb| = 2|\angle adp| = 2\left(\frac{1}{2}|\angle aop|\right) = 67^\circ$$

\* Accept answer written on a diagram.

\* Accept correct answer without work.

*Blunders (-3)*

B1  $|\angle adb| = \frac{1}{2}(67^\circ) = 33.5^\circ.$

*Attempts (2 marks)*

A1  $|\angle adp| = 23^\circ$

A2 Indicates congruent triangles.

(c) (iv)

5 marks

Att 2

$$|\angle dbo| = |\angle odb| = 0.5|\angle pob| = 0.5(67^\circ) = 33.5^\circ.$$

or

$$|\angle pob| = |\angle pdb| + |\angle dbo| = 2|\angle dbo| \Rightarrow 67^\circ = 2|\angle dbo| \Rightarrow |\angle dbo| = 33.5^\circ.$$

or

$$|\angle dbp| = 180^\circ - (23^\circ + 33.5^\circ) = 123.5^\circ \Rightarrow |\angle dbo| = 123.5^\circ - 90^\circ = 33.5^\circ$$

or

$$\text{In quadrilateral } dbpa \quad |\angle oad| + |\angle dbo| + 23^\circ + 23^\circ + 90^\circ + 90^\circ + 67^\circ = 360^\circ.$$

$$\text{Hence, } 2|\angle dbo| = 360^\circ - 293^\circ = 67^\circ \Rightarrow |\angle dbo| = 33.5^\circ$$

\* Accept answer written on a diagram.

\* Accept correct answer without work.

*Attempts (2 marks)*

A1 Indication of isosceles triangles e.g.  $|do| = |ob|$ .

A2 Correct relevant theorem e.g. exterior angle theorem.

A3 Some relevant step e.g.  $|\angle odb| = |\angle dbo|$ .

**Note:**

An incorrect answer in (ii) without work merits 0 marks.

But the same answer given for (iii) merits 5 marks and

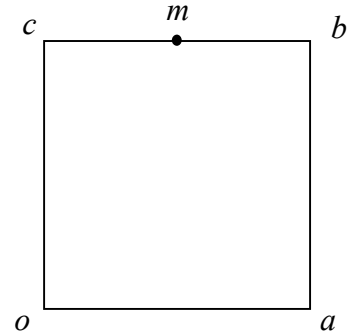
half the value of the answer given for (iv) merits 5 marks.

## QUESTION 9

<b>Part (a)</b>	<b>10 marks</b>	<b>Att 4</b>
<b>Part (b)</b>	<b>20 marks</b>	<b>Att 7</b>
<b>Part (c)</b>	<b>20 marks</b>	<b>Att 6</b>

**Part (a)** **10 (5, 5) marks** **Att (2, 2)**

$oabc$  is a square.  
 $m$  is the midpoint of  $[cb]$ .



- (i) Express  $\vec{b}$  in terms of  $\vec{a}$  and  $\vec{c}$ .  
 (ii) Express  $\vec{m}$  in terms of  $\vec{a}$  and  $\vec{c}$ .

**(a) (i)** **5 marks** **Att 2**

$$\vec{ob} = \vec{oc} + \vec{cb} = \vec{oc} + \vec{oa} = \vec{c} + \vec{a}.$$

or 
$$\vec{ob} = \vec{oa} + \vec{ab} = \vec{oa} + \vec{oc} = \vec{a} + \vec{c}.$$

- \* Accept correct answer without work shown in both parts.
- \* One blunder results in the attempt mark of 2, in both parts.
- \* Allow  $\vec{oa}$  for  $\vec{a}$  and  $\vec{oc}$  for  $\vec{c}$  in parts (i) and (ii).
- \* Accept without arrows

*Blunders (-3)*

- B1 Incorrect direction.
- B2 Error in using triangle or parallelogram law e.g.  $\vec{b} = \vec{a} - \vec{c}$ .
- B3 Does not simplify to  $\vec{a}$  and  $\vec{c}$ .

*Misreadings (-1)*

- M1 Any obvious misreading which does not oversimplify or change the task.

*Attempts (2 marks)*

- A1 Correct relevant step, e.g. relevant arrow added to given diagram.
- A2 Correct relevant application of vectors, e.g.  $\vec{ab} = \vec{c}$  or  $\vec{ab} = \vec{b} - \vec{a}$ .
- A3 Relevant statement e.g. "opposite sides of parallelogram are equal in measure".

*Worthless (0 marks)*

- W1 Diagram reproduced without modifications.

**(a) (ii)** **5 marks** **Att 2**

$$\vec{om} = \vec{oc} + \vec{cm} = \vec{oc} + \frac{1}{2}\vec{cb} = \vec{c} + \frac{1}{2}\vec{a}.$$

or 
$$\vec{cm} = \frac{1}{2}\vec{cb} \Rightarrow \vec{m} - \vec{c} = \frac{1}{2}\vec{a} \Rightarrow \vec{m} = \vec{c} + \frac{1}{2}\vec{a}.$$

*Blunders (-3)*

B1 Incorrect rule e.g.  $\vec{cm} = \vec{c} - \vec{m}$ .

B2 Error in using triangle or parallelogram law e.g.  $\vec{m} = \vec{c} - \frac{1}{2}\vec{a}$ .

B3 Does not simplify to  $\vec{a}$  and  $\vec{c}$ .

*Misreadings (-1)*

M1 Any obvious misreading which does not oversimplify or change the task.

*Attempts (2 marks)*

A1 Correct relevant step, e.g. relevant arrow added to given diagram.

A2 Correct relevant application of vectors, e.g.  $\vec{cb} = \vec{a}$ .

A3  $\vec{m} = \frac{1}{2}\vec{cb}$ .

*Worthless (0 marks)*

W1 Diagram reproduced without modifications.

W2  $\vec{mcb}$  or similar.

**Part (b)**

**20 (5, 5, 10) marks**

**Att (2, 2, 3)**

Let  $\vec{p} = 2\vec{i} + 3\vec{j}$  and  $\vec{q} = -5\vec{i} + 6\vec{j}$ .

(i) Express  $\vec{pq}$  in terms of  $\vec{i}$  and  $\vec{j}$ .

(ii) Express  $4\vec{p} + 2\vec{q}$  in terms of  $\vec{i}$  and  $\vec{j}$ .

(iii) Find the scalar  $k$  such that  $k\vec{p} - \vec{q} = 9\vec{i}$ .

**(b) (i)**

**5 marks**

**Att 2**

$$\begin{aligned}\vec{pq} &= \vec{q} - \vec{p} = -5\vec{i} + 6\vec{j} - (2\vec{i} + 3\vec{j}) \\ &= -5\vec{i} + 6\vec{j} - 2\vec{i} - 3\vec{j} = -7\vec{i} + 3\vec{j}.\end{aligned}$$

\* Accept correct answer without work shown in parts (i) and (ii).

\* One blunder results in the attempt mark of 2, in parts (i) and (ii).

*Blunders (-3)*

B1 Mixes up  $\vec{i}$ 's and  $\vec{j}$ 's.

B2  $\vec{pq} = \vec{p} - \vec{q}$  or  $\vec{pq} = \vec{q} + \vec{p}$ .

B3 Distributive law error e.g.  $-(2\vec{i} + 3\vec{j}) = -2\vec{i} + 3\vec{j}$ .

*Slips (-1)*

S1 Numerical slips to a maximum of 3.

S2 Stops at  $-5\vec{i} + 6\vec{j} - 2\vec{i} - 3\vec{j}$ .



*Misreadings (-1)*

M1 Writes  $\vec{qp}$  and continues correctly. ( Answer  $7\vec{i} - 3\vec{j}$  ).

*Attempts (2 marks)*

A1  $-7\vec{i}$  or  $3\vec{j}$  and stops.

A2 Plots one or more relevant vectors.

A3 Treats as  $\vec{p} \cdot \vec{q}$  with some relevant work.

*Worthless (0 marks)*

W1 Incorrect answer without work.

**(b) (ii)**

**5 marks**

**Att 2**

$$\begin{aligned} 4\vec{p} + 2\vec{q} &= 4(2\vec{i} + 3\vec{j}) + 2(-5\vec{i} + 6\vec{j}) = 8\vec{i} + 12\vec{j} - 10\vec{i} + 12\vec{j} \\ &= -2\vec{i} + 24\vec{j} . \end{aligned}$$

*Blunders (-3)*

B1 Mixes up  $\vec{i}$ 's and  $\vec{j}$ 's .

B2 Algebraic error e.g.  $-2\vec{i}^2 + 24\vec{j}^2$  , subject to S2.

B3 Distributive law error, similar to above.

*Slips (-1)*

S1 Numerical slips to a maximum of 3.

S2 Stops at  $8\vec{i} + 12\vec{j} - 10\vec{i} + 12\vec{j}$  .

*Misreadings (-1)*

M1 Writes  $4\vec{q} + 2\vec{p}$  and continues correctly. ( Answer  $-16\vec{i} + 30\vec{j}$  ).

*Attempts (2 marks)*

A1  $-2\vec{i}$  or  $24\vec{j}$  and stops.

A2  $8\vec{i}$  or  $12\vec{j}$  or  $-10\vec{i}$  or  $12\vec{j}$  and stops.

A3 Plots one or more relevant vectors.

A4 Some relevant step e.g.  $4(2\vec{i} + 3\vec{j})$  .

*Worthless (0 marks)*

W1 Incorrect answer without work, subject to attempts.

**(b) (iii)**

**10 marks**

**Att 3**

$$k \vec{p} - \vec{q} = 9 \vec{i}$$

$$\Rightarrow k(2 \vec{i} + 3 \vec{j}) - (-5 \vec{i} + 6 \vec{j}) = 9 \vec{i} + 0 \vec{j}$$

3 marks

$$\Rightarrow 2k \vec{i} + 3k \vec{j} + 5 \vec{i} - 6 \vec{j} = 9 \vec{i} + 0 \vec{j}$$

7 marks

$$\Rightarrow 2k + 5 = 9 \Rightarrow 2k = 4 \Rightarrow k = 2$$

10 marks

or  $\Rightarrow 3k - 6 = 0 \Rightarrow 3k = 6 \Rightarrow k = 2$

*Blunders (-3)*

B1 Mixes up  $\vec{i}$ 's and  $\vec{j}$ 's.

B2 Transposition error e.g.  $2k + 5 = 9 \Rightarrow 2k = 14 \Rightarrow k = 7$ .

B3 Distributive law error.

B4 Stops at  $2k \vec{i} + 3k \vec{j} + 5 \vec{i} - 6 \vec{j} = 9 \vec{i} + 0 \vec{j}$ .

*Slips (-1)*

S1 Numerical slips to a maximum of 3.

*Attempts (3 marks)*

A1  $k(2 \vec{i} + 3 \vec{j})$  or  $(-5 \vec{i} + 6 \vec{j})$  and stops.

A2 Some relevant step.

A3 Correct answer without work.

*Worthless (0 marks)*

W1 Incorrect answer without work.

W2  $k \vec{p} = \vec{p} - k$ .

**Part (c)**

**20 (10, 10) marks**

**Att (3, 3)**

Let  $\vec{x} = 5 \vec{i} + 4 \vec{j}$  and  $\vec{y} = 3 \vec{i} - 7 \vec{j}$ .

(i) Write  $\vec{x}^\perp$  in terms of  $\vec{i}$  and  $\vec{j}$  and show that  $|\vec{x}| = |\vec{x}^\perp|$ .

(ii) Calculate the dot product  $(\vec{x} + \vec{y}) \cdot (\vec{x} - \vec{y})$ .

**(c) (i)**

**10 marks**

**Att 3**

$$\vec{x} = 5 \vec{i} + 4 \vec{j} \Rightarrow \vec{x}^\perp = -4 \vec{i} + 5 \vec{j}$$

3 marks

$$|\vec{x}| = \sqrt{5^2 + 4^2} = \sqrt{25 + 16} = \sqrt{41}$$

7 marks

$$|\vec{x}^\perp| = \sqrt{(-4)^2 + 5^2} = \sqrt{16 + 25} = \sqrt{41}$$

10 marks

Thus,  $|\vec{x}| = |\vec{x}^\perp|$

*Blunders (-3)*

B1 Mixes up  $\vec{i}$ 's and  $\vec{j}$ 's.

B2 Sign error e.g.  $4 \vec{i} + 5 \vec{j}$  or  $4 \vec{i} - 5 \vec{j}$ .

- B3  $\vec{i}^2 \neq 1$  and/or  $\vec{j}^2 \neq 1$  .
- B4  $\vec{i} \cdot \vec{j} \neq 0$ .
- B5 Incorrect relevant formula e.g.  $\sqrt{a^2 - b^2}$  or  $\sqrt{(x_2 - x_1)^2 - (y_2 - y_1)^2}$  .
- B6 Mathematical error e.g.  $(-4)^2 = -16$  .
- B7 Leaves answer as  $\sqrt{5^2 + 4^2}$  and/or or =  $\sqrt{(-4)^2 + 5^2}$  .
- B8 Each step omitted.

*Slips (-1)*

S1 Numerical slips to a maximum of 3.

S2 No conclusion, if  $|\vec{x}| \neq |\vec{x}^\perp|$ .

*Attempts (3 marks)*

A1  $\vec{i}^2 = 1$  and/or  $\vec{j}^2 = 1$  and/or  $\vec{i} \cdot \vec{j} = 0$ .

A2 Correct modulus formula and stops.

A3 Some relevant step.

A4 No expression for  $\vec{x}^\perp$  is attempt at most.

**(c) (ii)**

**10 marks**

**Att 3**

$(\vec{x} + \vec{y}) \cdot (\vec{x} - \vec{y}) = (5\vec{i} + 4\vec{j} + 3\vec{i} - 7\vec{j}) \cdot (5\vec{i} + 4\vec{j} - 3\vec{i} + 7\vec{j})$	3 marks
$= (8\vec{i} - 3\vec{j}) \cdot (2\vec{i} + 11\vec{j})$	7 marks
$= 16 - 33$	9 marks
$= -17$	10 marks

*Blunders (-3)*

B1  $\vec{i}^2 \neq 1$  and/or  $\vec{j}^2 \neq 1$  .

B2  $\vec{i} \cdot \vec{j} \neq 0$ .

B3 Incorrect relevant formula e.g.  $\sqrt{a^2 - b^2}$  .

B4 Distribution error.

*Slips (-1)*

S1 Numerical slips to a maximum of 3.

*Attempts (3 marks)*

A1  $\vec{i}^2 = 1$  and/or  $\vec{j}^2 = 1$  and/or  $\vec{i} \cdot \vec{j} = 0$ .

A2 Correct modulus formula and stops.

A3 Some correct multiplication e.g.  $16\vec{i}^2$  .

A4 Some relevant step e.g.  $-5\vec{i} - 4\vec{j}$  or  $\vec{x}^2 - \vec{y}^2$  .

A5  $\vec{x} \cdot \vec{y}$  and finishes.

*Worthless (0 marks)*

$1.1 + 1. - 1 = 0$ .

## QUESTION 10

<b>Part (a)</b>	<b>10 marks</b>	<b>Att 3</b>
<b>Part (b)</b>	<b>20 marks</b>	<b>Att 6</b>
<b>Part (c)</b>	<b>20 marks</b>	<b>Att 6</b>

<b>Part (a)</b>	<b>10 marks</b>	<b>Att 3</b>
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Expand  $(1 - 2x)^4$  fully.

<b>(a)</b>	<b>10 marks</b>	<b>Att 3</b>
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$(1 - 2x)^4 = \binom{4}{0} + \binom{4}{1}(-2x) + \binom{4}{2}(-2x)^2 + \binom{4}{3}(-2x)^3 + \binom{4}{4}(-2x)^4$	4 marks
$= 1 + 4(-2x) + 6(4x^2) + 4(-8x^3) + 1(16x^4)$	7 marks
$= 1 - 8x + 24x^2 - 32x^3 + 16x^4$	10 marks

\* Accept long multiplication or Pascal's triangle and award 4 marks for 2 correct terms, 7 marks for 3 or 4 terms correct and 10 marks for 5 correct terms.

\* First line of solution box merits 4 marks.

\* Accept correct answer without work.

### *Blunders (-3)*

B1 Blunder in powers (once only).

B2 Blunder in working out binomial coefficients (once only).

B3 Puts powers of  $x$  as denominators e.g.  $\binom{4}{3}\left(\frac{-2x}{3}\right)$  or  $\frac{4}{3}(-2x)^3$ .

B4 Puts a + sign between coefficient and power of  $x$ .

B5 Expands  $(1 + 2x)^4$ .

### *Slips (-1)*

S1 Numerical slips to a maximum of 3.

### *Attempts (3 marks)*

A1 Any term written down correctly.

A2 Part of Pascal's triangle.

A3 Coefficients only.

A4 Any step towards getting a binomial coefficient e.g.  $\binom{4}{2}$ .

A5 Any correct step towards long multiplication.

A6 Effort at long multiplication subject to slips and blunders.

A7 Effort at Pascal's triangle.

### *Worthless (0 marks)*

W1 Writes  $4(1 - 2x)^3$  or  $4(1 - 2x)^3(-2)$ .

- (i) Find
- $S$
- , the sum to infinity of the geometric series

$$\frac{1}{3} + \frac{2}{9} + \frac{4}{27} + \dots$$

- (ii) The sum to infinity of another geometric series is also
- $S$
- .

The common ratio of this series is  $\frac{1}{3}$ .

Find the first term.

(b) (i)

10 marks

Att 3

$$\frac{1}{3} + \frac{2}{9} + \frac{4}{27} + \dots \Rightarrow a = \frac{1}{3} \quad \text{and} \quad r = \frac{\frac{2}{9}}{\frac{1}{3}} = \frac{2}{3}$$

3 marks

$$S = \frac{a}{1-r} = \frac{\frac{1}{3}}{1-\frac{2}{3}} = \frac{\frac{1}{3}}{\frac{1}{3}} = 1$$

7 marks

10 marks

\* Accept use of the limit of  $S_n$  as  $n$  tends to infinity.

*Blunders (-3)*B1 Incorrect  $a$ .B2 Incorrect  $r$ .B3 Incorrect relevant formula e.g.  $\frac{a}{1+r}$  giving answer  $\frac{1}{5}$ .

B4 Blunder in fractions.

*Slips (-1)*

S1 Numerical slips to a maximum of 3.

*Attempts (3 marks)*A1 Correct relevant formula and stops e.g.  $S_n = \frac{a(1-r^n)}{1-r}$ .A2 Some relevant step e.g. states the value for  $a$  or the value for  $r$ .A3 Adds 2 or more of the given terms e.g.  $S_2 = \frac{5}{9} = 0.5555$  or  $S_3 = \frac{19}{27} = 0.7037$ .

A4 One correct step in adding relevant fractions.

A5 Treats as arithmetic series with further work, e.g. identifies  $a$ .A6 Writes  $T_n = ar^{n-1}$  or  $\frac{1}{3}\left(\frac{2}{3}\right)^{n-1}$ .A7 Gives  $T_4 = 8/81$ .

A8 Correct answer without work.

*Worthless (0 marks)*

W1 Formula for arithmetic series and stops.

W2  $\frac{1}{3} + \frac{2}{9} + \frac{4}{27} = \frac{7}{39}$ .

(b) (ii)

10 marks

Att 3

$$S_{\infty} = \frac{a}{1-r} = 1 \quad \Rightarrow \quad \frac{a}{1-\frac{1}{3}} = 1 \quad \Rightarrow \quad \frac{a}{\frac{2}{3}} = 1 \Rightarrow a = \frac{2}{3}$$

3 marks

7 marks

10 marks

*Blunders (-3)*

B1 Incorrect  $r$ .

B2 Transposition error.

B3 Incorrect relevant formula e.g.  $\frac{a}{1+r} \Rightarrow a = \frac{4}{3}$  or  $S = \frac{1}{5}$  from (i) gives  $a = \frac{2}{15}$ , if not already penalised.

*Slips (-1)*

S1 Numerical slips to a maximum of 3.

*Attempts (3 marks)*

A1 Correct relevant formula and stops e.g.  $S_n = \frac{a(1-r^n)}{1-r}$ .

A2 Some relevant step e.g. states the value for  $r$ .

A3 Correct answer without work.

*Worthless (0 marks)*

W1 Incorrect answer without work

W2  $\frac{1}{3} \times \frac{1}{2}$  or  $\frac{1}{9}$ .

**Note:**  $a = \frac{1}{3}$  from (i) and  $r = \frac{1}{3}$  from (ii)  $\Rightarrow S_{\infty} = \frac{1}{2}$  [7 marks]

But  $S_{\infty} = 1 \Rightarrow a = \frac{2}{3}$ .

**Part (c)****20 (10, 10) marks****Att (3, 3)**

- (i) A machine costing €25 000 depreciates at the compound rate of 15% per annum. Find the value of the machine at the end of twelve years, correct to the nearest euro.
- (ii) A company invests €25 000 in machinery at the beginning of each year for twelve consecutive years. The machinery depreciates at the rate of 15% per annum compound depreciation. Using the formula for the sum of the first  $n$  terms of a geometric series, find the total value of the machinery at the end of the twelve years, correct to the nearest euro.

**(c) (i)****10 marks****Att 3**

$A = 25000(1 - 0.15)^{12}$	$= 25000(0.14224)$	$= 3556.09$	$= €3556.$
7 marks	9 marks	10 marks	

*Or: Calculation on a year by year basis:*

Year 1:	€25000	–	€3750
Year 2:	€21250	–	€3187.50
Year 3:	€18062.50	–	€2709.38
Year 4:	€15353.12	–	€2302.97
Year 5:	€13050.15	–	€1957.52
Year 6:	€11092.63	–	€1663.89
Year 7:	€9428.74	–	€1414.31
Year 8:	€8014.43	–	€1202.16
Year 9:	€6812.27	–	€1021.84
Year 10:	€5790.43	–	€868.56
Year 11:	€4921.87	–	€738.28
Year 12:	€4183.59	–	€627.54 = €3556.05 = €3556

\* Note:  $€25000(0.1422) = €3555$  - apply S3.

\* Accept long method of working from year to year.

*Blunders (-3)*B1 Incorrect  $r$ .

B2 Decimal error.

B3  $€25000(0.85)^{12}$  and stops.B4 Serious numerical error e.g.  $(0.85)^{12} = 10.2$ .B5 Sign error in formula e.g.  $A = P\left(1 + \frac{r}{100}\right)^n$  or  $1 - 0.85$  or  $1 + 0.15$ .

B6 Adds in long method.

*Slips (-1)*

S1 Numerical slips to a maximum of 3.

S2 Answer not rounded-off to nearest euro.

S3 Premature rounding that affects accuracy of final answer to a maximum of 3. (long method)

S4 Each year omitted in the long method, each time.

*Attempts (3 marks)*

A1 Mention of 0.15, 0.85, 15/100, 85/100.

A2 15% of €25000 = €3750 or 85% of €25000 = €21250 and stops.

A3 Correct answer without work.

*Worthless (0 marks)*W1  $€25000/15 = €1667$ .

(c) (ii)

10 marks

Att 3

$$\begin{aligned} S_{12} &= 25000(0.85)^{12} + 25000(0.85)^{11} + \dots + 25000(0.85) \\ &= 25000(0.85 + (0.85)^2 + (0.85)^3 + \dots + (0.85)^{12}) \\ &= 25000\left(\frac{0.85(1 - 0.85^{12})}{1 - 0.85}\right) \\ &= 25000\left(\frac{0.85(0.85776)}{0.15}\right) = 25000(4.86064) = 121516 \end{aligned}$$

*Blunders (-3)*

- B1 Incorrect  $a$ .
- B2 Incorrect  $r$ .
- B3 Decimal error.
- B4 Error in formula for sum.
- B5 Index error.
- B6 Serious mathematical error e.g.  $(0.85)^2 = 1.7$ .

*Slips (-1)*

- S1 Numerical slips to a maximum of 3.
- S2 Answer not rounded-off to nearest euro.
- S3 Premature rounding that affects accuracy of final answer to a maximum of 3.

*Attempts (3 marks)*

- A1 Mention of 0.15, 0.85, 15/100, 85/100.
- A2 15% of €25000 = €3750 or 85% of €25000 = €21250 and stops.
- A3 Correct answer without work.
- A4 Attempt at calculation on year-to-year basis, fully correct or otherwise.
- A5 Any relevant step.



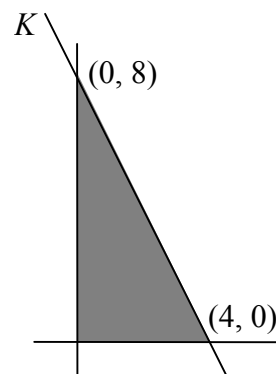
## QUESTION 11

<b>Part (a)</b>	<b>15 marks</b>	<b>Att 5</b>
<b>Part (b)</b>	<b>35 marks</b>	<b>Att 14</b>

**Part (a)** **15 (10, 5) marks** **Att (3, 2)**

The line  $K$  cuts the  $x$ -axis at  $(4, 0)$  and the  $y$ -axis at  $(0, 8)$ .

- (i) Find the equation of  $K$ .
- (ii) Write down the three inequalities that together define the region enclosed by  $K$ , the  $x$ -axis and the  $y$ -axis.



**(a) (i)** **10 marks** **Att 3**

Slope of $K$	$\frac{y_2 - y_1}{x_2 - x_1} = \frac{8 - 0}{0 - 4} = -2$	3 marks
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Equation of $K$ :	$y - y_1 = m(x - x_1)$	7 marks
	$y - 0 = -2(x - 4)$ or $y - 8 = -2(x - 0)$ or $2x + y - 8 = 0$ .	10 marks

or

Equation of $K$ :	$y = mx + c$	7 marks
	$0 = -2(4) + c \Rightarrow c = 8$ .	10 marks

- \* Accept  $y - 0 = -2(x - 4)$  without work.
- \* Apply scheme for Q2, Q3 where relevant.

### Blunders (-3)

B1 Incorrect relevant formula and continues.

B2 Mixes up  $x$ 's and  $y$ 's.

B3 Mathematical blunder, e.g.  $\frac{8 - 0}{0 - 4} = 2$ .

B4  $y - 0 = m(x - 4)$  where  $m$  is not equal to  $-2$  without work.

B5  $y - y_1 = -2(x - x_1)$  where  $(x_1, y_1)$  is not  $(0, 8)$  or  $(4, 0)$  without work.

B6 Transposing error. Penalise once.

### Slips (-1)

S1 Numerical slips to a maximum of 3.

### Attempts (3 marks)

A1 Any correct, relevant formula and stops.

**(a) (ii)**

**5 marks**

**Att 2**

$x \geq 0,$ 2 marks	$y \geq 0,$ 4 marks	$2x + y - 8 \leq 0$ or equivalent 5 marks
------------------------	------------------------	----------------------------------------------

- \* Accept correct inequalities without work.
- \* Accept  $x > 0$  etc.
- \* Accept inequality consistent with candidate's  $K$ .
- \* One inequality incorrect or not stated is a slip.

*Attempts ( 2 marks )*

- A1 One correct inequality written down and stops.
- A2 Substitutes any point and stops.
- A3  $x \leq 0$  or  $y \leq 0$  and stops, (without work).
- A4 Some correct work at simplifying  $K$ .
- A5 Incorrect or no conclusion e.g.  $2x + y - 8 = 0 \Rightarrow 2(0) + 0 - 8 = 0$ .
- A6 Mathematical error in testing a point ( e.g. sign error ).
- A7 Some relevant step e.g.  $x = 0$ .

*Slips (- 1 )*

- S1 Numerical slips to a maximum of 3.

*Worthless (0 marks)*

- W1 Writes equation of  $K$  and stops (without simplifying).
- W2 Draws the given diagram.

**Part (b)**

**35 (20, 10, 5) marks**

**Att 14 (8, 4, 2)**

A manufacturer of garden furniture produces plastic chairs and tables. Each chair requires 2 kg of raw material and each table requires 5 kg of raw material. In any working period the raw material used cannot exceed 800 kg.

Each chair requires 4 minutes of machine time and each table requires 4 minutes of machine time. The total machine time available in any working period is 1000 minutes.

- (i) Taking  $x$  as the number of chairs and  $y$  as the number of tables, write down two inequalities in  $x$  and  $y$  and illustrate these on graph paper.
- (ii) The manufacturer sells each chair for €20 and each table for €40.  
How many of each should be produced in each working period to maximise income?
- (iii) The manufacturer's costs for each chair are €17 and for each table are €34.70. Express the profit as a percentage of income, assuming the income has been maximised.

**(b) (i) Inequalities****10 (5, 5) marks****Att (2, 2)**

Raw materials:  $2x + 5y \leq 800$   
 Machine time:  $4x + 4y \leq 1000$

Also accept:

	Chairs	Tables	Maximum
Raw material	$2x$	$5y$	800
Machine time	$4x$	$4y$	1000

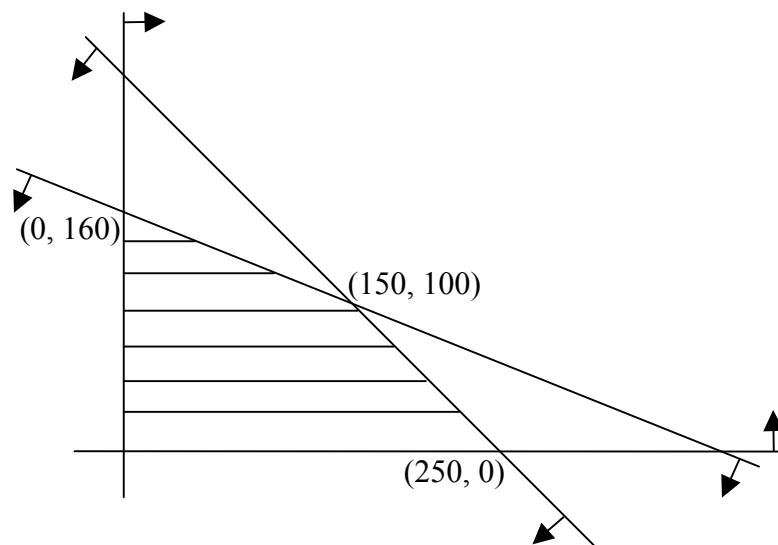
- \* Accept correct multiples / fractions of inequalities or different letters.
- \* Do not penalise here for incorrect or no inequality sign. Penalise in graph if used.
- \* Case :  $\frac{2}{4} \quad \frac{5}{4} \quad \frac{800}{1000}$  | Award 10 marks, but penalise in graph if linkup is incorrect.

*Blunders (-3)*

- B1 Mixes up  $x$ 's and  $y$ 's (once if consistent error).
- B2 Confuses rows and columns in table, e.g.  $2x + 4y \leq 800$  (once if consistent).
- B3 Misplaced decimal point, e.g.  $2x + 5y \leq 80$ , once.

*Attempts (2 marks for each inequality)*

- A1 Incomplete relevant data in table and stops (each inequality).
- A2 Any other correct inequality, e.g.  $x \geq 0$ ,  $y \geq 0$ , (each time).
- A3 Some variable  $\leq 800$  or  $\leq 1000$  (each time).
- A4  $2x$  and / or  $5y$  and stops (1 x Att 2).
- A5  $4x$  and / or  $4y$  and stops (1 x Att 2).

**(b) (i) Graph****10 (5, 5) marks****Att (2, 2)**

- \* Points or scales required.
- \* Each half-plane merits 5 marks, attempt 2 marks each.
- \* Half-planes required but no penalty for not indicating intersection if half-planes are indicated.
- \* If half-planes are indicated correctly, do not penalise for incorrect shading.
- \* Accept correct shading of intersection for half-planes but candidates may shade out areas that are not required and leave intersection blank.
- \* Correct shading over-rules arrows.
- \* Two lines drawn and **no shading**, only one of the following applies :
 

Case 1:	Two sets of arrows in expected direction	10 marks
Case 2:	Two sets of arrows in unexpected direction	10 marks
Case 3:	One set of arrows "correct" and the other "incorrect"	7 marks (5+Att 2)
Case 4:	One line with and the other without arrows	7 marks (5+Att 2)
Case 5:	No arrows	4 marks (Att 2, Att 2)

*Blunders (-3)*

- B1 No half-plane indicated ( each time ).
- B2 Blunder in plotting a line or calculations ( each line ).
- B3 Incorrect shading ( once ), e.g. one or both of the small triangles shaded.

*Attempts ( 2 marks each half-plane )*

- A1 Some relevant work towards a point on a line, i.e. 2m for each line attempted.
- A2 Draws axes or axes and one line ( 1 x Att 2m ).
- A3 Draws axes and two lines reasonably accurately (award Att 2 + Att 2 ).

**(b) (ii) Intersection of lines**

**5 marks**

**Att 2**

$$\begin{array}{rcl}
 2x + 5y = 800 & & 4x + 10y = 1600 \\
 4x + 4y = 1000 & \Rightarrow & 4x + 4y = 1000 \\
 \hline
 & & 6y = 600 \Rightarrow y = 100 \Rightarrow x = 150
 \end{array}$$

- \* Accept candidate's own equations from previous parts.
- \* If  $x$  is calculated, accept consistent value for  $y$  without further work and vice versa .

*Blunders (-3)*

- B1 Fails to multiply / divide both sides of equation(s) correctly when eliminating variable.
- B2 Sign error.
- B3  $x$  or  $y$  value only.
- B4 Transposing error.

*Slips (-1)*

- S1 Numerical slips to a maximum of 3.

*Attempts (2 marks)*

- A1 Correct or consistent answer without work or from a graph.  
[ Should get same values from graph as if they had been found algebraically e.g (151,100) on its own gets zero ]
- A2 Any relevant step towards solving equations.

*Worthless (0 marks)*

- W1 Incorrect answer without work and inconsistent with graph.

**(b) (ii) Income****5 marks****Att 2**

Step 1	Vertices	$20x + 40y$	Income
Step 2	( 0, 160)	0 + 6400	6400
Step 3	(150, 100)	3000 + 4000	7000
Step 4	(250, 0)	5000 + 0	5000
Step 5	150 chairs and 100 tables to maximise income.		

- \* Accept point of intersection from previous part.
- \* Information does not have to be in table form.
- \* Award 1 mark for each consistent step, subject to the attempt mark, using  $20x + 40y$ .
- \* Accept only vertices consistent with previously accepted work, not arbitrary ones.  
If (400,0) is tested and result is used to give max. income, apply (-1), otherwise ignore.
- \* Accept correct vertices or vertices from candidate's indicated area on non-simplified graph.
- \* Accept any correct multiple or fraction of  $20x + 40y$  here.
- \* If no marks have been awarded for intersection of lines and this point is written here award Att 2 for the previous part and also reward it here if the step is correct.
- \* Step 5 must be explicitly written to gain the final mark. Otherwise (-1).
- \* Testing only (150,100) to get 7000 merits Att 2 even if the candidate writes 150 chairs and 100 tables.
- \* No comparison means the attempt mark at most.

*Slips (-1)*

S1 Each arithmetic slip to a maximum of 3.

S2 Each step of the solution omitted, subject to the attempt mark [Step 1 may be implied].

*Attempts (2 marks)*A1 Any relevant work involving  $x$  or  $y$  and / or 20, 40 or similar.

A2 Any attempt at substituting coordinates into some expression.

A3 States 150 chairs and / or 100 tables with no other work.

**(b) (iii) Profit****5 marks****Att 2**

$$\begin{aligned} \text{Profit is } & 3x + 5.3y \\ & = 3(150) + 5.3(100) \text{ for 150 chairs and 100 tables} \\ & = 450 + 530 = \text{€}980. \end{aligned}$$

$$\text{Percentage profit is } \frac{980}{7000} \times 100 = 14\% .$$

- \* Accept candidates answer from previous part.

*Blunders (-3)*

B1 Mathematical error.

B2 Wrong denominator e.g. 6020 [ 7000 – 980 ].

B3 Inverts the final fraction e.g. 7000 / 980.

*Slips (-1)*

S1 Numerical slips to a maximum of 3.

*Attempt (2)*A1 Any correct step e.g.  $20 - 17 = 3$ .A2 Some work with  $3x$  and / or  $5.3y$ .

A3 Multiplies some fraction by 100.

*Worthless (0)*

W1 Simply writing down €17 or €34.70 and no other work.

## **BONUS MARKS FOR ANSWERING THROUGH IRISH**

Bonus marks are applied separately to each paper as follows:

If the mark achieved is less than 226, the bonus is 5% of the mark obtained, rounding *down*.  
(e.g. 198 marks  $\times$  5% = 9.9  $\Rightarrow$  bonus = 9 marks.)

If the mark awarded is 226 or above, the following table applies:

Marks obtained	Bonus
226 – 231	11
232 – 238	10
239 – 245	9
246 – 251	8
252 – 258	7
259 – 265	6
266 – 271	5
272 – 278	4
279 – 285	3
286 – 291	2
292 – 298	1
299 – 300	0