Coimisiún na Scrúduithe Stáit
State Examinations Commission

Scéim Mharcála	*Scrúduithe Ardteistiméireachta, 2004*
Matamaitic	*Ardleibhéal*

Marking Scheme	*Leaving Certificate Examination, 2004*
Mathematics	*Higher Level*

Contents

MARKING SCHEME

LEAVING CERTIFICATE EXAMINATION 2004

MATHEMATICS

HIGHER LEVEL

PAPER 1

GENERAL GUIDELINES FOR EXAMINERS - PAPER 1

1. Penalties of three types are applied to candidates' work as follows:
   - Blunders - mathematical errors/omissions (-3)
   - Slips - numerical errors (-1)
   - Misreadings (provided task is not oversimplified) (-1).

   Frequently occurring errors to which these penalties must be applied are listed in the scheme. They are labelled as B1, B2, B3,......, S1, S2, S3,...., M1, M2, etc. Note that these lists are not exhaustive.

2. When awarding attempt marks, e.g. Att(3), note that
   - any correct relevant step in a part of a question merits *at least* the attempt mark for that part
   - if deductions result in a mark which is lower than the attempt mark, then the attempt mark must be awarded
   - a mark between zero and the attempt mark is never awarded.

3. Worthless work is awarded zero marks. Some examples of such work are listed in the scheme and they are labelled as W1, W2,....etc.

4. The phrase “hit or miss” means that partial marks are not awarded – the candidate receives all of the relevant marks or none.

5. The *same* error in the *same* section of a question is penalised *once* only.

6. Special notes relating to the marking of a particular part of a question are indicated by an asterisk. These notes immediately follow the box containing the relevant solution.

7. Particular cases, verifications and answers derived from diagrams (unless requested) qualify for attempt marks at most.

8. A serious blunder, omission or misreading merits the ATTEMPT mark at most.

9. The phrase “and stops” means that no more work is shown by the candidate.

10. Accept the best of two or more attempts – even when attempts have been cancelled.

QUESTION 1

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 (10, 10) marks**	**Att (3, 3)**
Part (c)	**20 (5, 15) marks**	**Att (2, 5)**

Part (a) **10 marks** **Att 3**

1(a)

Express $\frac{1-\sqrt{3}}{1+\sqrt{3}}$ in the form $a\sqrt{3}-b$, where a and $b \in \mathbf{N}$

Part (a) **10 marks** **Att 3**

1(a)

$$\frac{1-\sqrt{3}}{1+\sqrt{3}}\cdot\frac{1-\sqrt{3}}{1-\sqrt{3}}=\frac{1-2\sqrt{3}+3}{(1)^2-(\sqrt{3})^2}=\frac{4-2\sqrt{3}}{-2}=\sqrt{3}-2$$

or

1(a)

$$\frac{1-\sqrt{3}}{1+\sqrt{3}} = a\sqrt{3}-b$$
$$1-\sqrt{3} = (1+\sqrt{3})(a\sqrt{3}-b)$$
$$1-\sqrt{3} = a\sqrt{3}-b+3a-b\sqrt{3}$$
$$1+(-1)\sqrt{3} = (3a-b)+(a-b)\sqrt{3}$$

$\Rightarrow$ $1 = 3a-b$ …………..(i) and $-1 = a-b$ ……………………(ii)

(i) : $3a-b=1$
(ii) : $\underline{a-b=-1}$
$2a = 2$
$a = 1$

(ii) $a-b=-1$
$1-b=-1$
$2=b$

Blunders (-3)
B1 indices.
B2 $(1+\sqrt{3}).(1-\sqrt{3})\neq -2$
B3 expansion $(1-\sqrt{3})^2$
B4 not in required form.
B5 not like to like.

Slips (-1)
S1 numerical.

Attempts
A1 no conjugate
A2 if over simplified (no surd)

Worthless
W1 calculator type solution.

Part (b)(i) **10 marks** **Att 3**

1(b)(i) Let $f(x)=x^3+kx^2-4x-12$, where k is a constant. Given that $x+3$ is a factor *f(x)*, find the value *k*.

Part (b)(i) **10 marks** **Att 3**

1(b)(i)

$$f(x)=x^3+kx^2-4x-12$$
$$(x+3) \text{ is factor } \Rightarrow f(-3)=0$$
$$f(-3)=(-3)^3+k(-3)^2-4(-3)-12=0$$
$$-27+9k+12-12=0$$
$$9k=27 \Rightarrow k=3$$

or

1(b)(i)

$$f(x)=(x^3+kx^2-4x-12) = (x+3)(x^2+ax-4)$$
$$x^3+kx^2-4x-12 = x^3+3x^2+ax^2+3ax-12-4x$$
$$x^3+kx^2-4x-12 = x^3+(3+a)x^2+(3a-4)x-12$$

Equating coefficients (i) $k = 3 + a$
(ii) $-4 = 3a - 4$

(ii) $-4 = 3a - 4$
$0 = 3a$
$\Rightarrow a=0$

(i) : $k = 3 + a$
$k = 3$

or

1(b)(i)

$$\begin{array}{r|l} & x^2 + (k-3)x + (5-3k) \\ \hline x+3 & x^3 + kx^2 - 4x - 12 \\ & x^3 + 3x^2 \\ \hline & (k-3)x^2 - 4x \\ & (k-3)x^2 + (3k-9)x \\ \hline & (5-3k)x - 12 \\ & (5-3k)x + (15-9k) \\ \hline & -12-(15-9k)=0 \end{array}$$

$$-12-15+9k=0$$
$$9k=27 \Rightarrow k = 3$$

Blunders (-3)
B1 deduction of root from factors.
B2 indices.
B3 not like to like.

Slips (-1)
S1 numerical.

S2 not changing sign when subtracting in division.

Worthless
W1 $f(x+3)$

Part (b)(ii) **10 marks** **Att 3**

1(b)(ii) Show that $\frac{3}{1+x^p}+\frac{3}{1+x^{-p}}$ simplifies to a constant.

Part (b)(ii) **10 marks** **Att 3**

1(b)(ii)

$$\frac{3}{1+x^p}+\frac{3}{1+x^{-p}}=\frac{3(1+x^{-p})+3(1+x^p)}{(1+x^p)(1+x^{-p})}$$
$$=\frac{3(1+x^{-p}+1+x^p)}{1+x^p+x^{-p}+x^0}$$
$$=\frac{3(2+x^{-p}+x^p)}{(2+x^{-p}+x^p)}$$
$$=3$$

or

1(b)(ii)

$$\frac{3}{1+x^p}+\frac{3}{1+x^{-p}}=\frac{3}{1+x^p}+\frac{3}{1+\frac{1}{x^p}}$$
$$=\frac{3}{1+x^p}+\frac{3x^p}{x^p+1}$$
$$=\frac{3(1+x^p)}{(1+x^p)}$$
$$=3$$

or

1(b)(ii)

Let $x^p=t \Rightarrow x^{-p}=\frac{1}{t}$

$$\frac{3}{1+x^p}+\frac{3}{1+x^{-p}}$$
$$=\frac{3}{1+t}+\frac{3}{1+\frac{1}{t}}$$
$$=\frac{3}{1+t}+\frac{3t}{t+1}$$
$$=\frac{3(1+t)}{(1+t)}$$
$$=3$$

Blunders (-3)
B1 indices.
B2 answer not simplified.
B3 $x^0\neq 1$

Slips (-1)
S1 numerical.

Worthless
W1 $x^{-p} = x^{p}$

Part (c)(i) **5 marks** **Att 2**

1(c)(i)	Show that $p^3 + q^3 - (p+q)^3 = -3pq(p+q)$

Part (c)(i) **5 marks** **Att 2**

1(c)(i)

$$
\begin{aligned}
& (p^3 + q^3) - (p+q)^3 \\
= \;& (p+q)(p^2 - pq + q^2) - (p+q)(p^2 + 2pq + q^2) \\
= \;& (p+q)[p^2 - pq + q^2 - p^2 - 2pq - q^2] \\
= \;& (p+q)(-3pq) \\
= \;& -3pq(p+q)
\end{aligned}
$$

or

1(c)(i)

$$
\begin{aligned}
& p^3 + q^3 - (p+q)^3 \\
= \;& p^3 + q^3 - (p^3 + 3p^2q + 3pq^2 + q^3) \\
= \;& p^3 + q^3 - p^3 - 3p^2q - 3pq^2 - q^3 \\
= \;& -3p^2q - 3pq^2 \\
= \;& -3pq(p+q)
\end{aligned}
$$

Blunders (-3)
B1 factors $(p^3 + q^3)$ once only.
B2 expansion $(p+q)^3$ once only.
B3 indices.
B4 expansion $(p+q)^2$ once only.
B5 value $\binom{n}{r}$ or no value $\binom{n}{r}$.
B6 root not verified, or root missing, provided one root found and verified.

Slips (-1)
S1 numerical.

Worthless
W1 numerical values.

Part (c)(ii) **15 marks** **Att 5**

1(c)(ii)	Hence, or otherwise, find, in terms of *a* and *b*, the three values of *x* for which $(a-x)^3+(b-x)^3-(a+b-2x)^3=0$

Part (c)(ii) **15 marks** **Att 5**

1(c)(ii) $(a-x)^3+(b-x)^3-(a+b-2x)^3=0$

Let $p=a-x$ and $q=b-x$

$\Rightarrow p+q = a+b-2x$

$\Rightarrow f(x) = -3(a-x)(b-x)(a+b-2x) = 0$

$\Rightarrow a-x=0$; $b-x=0$; $a+b-2x=0$

$a=x$; $b=x$; $a+b=2x$

$\frac{a+b}{2}=x$

or

1 (c)(ii) $f(x)=(a-x)^3+(b-x)^3-(a+b-2x)^3$

$f(a)=(a-a)^3+(b-a)^3-(a+b-2a)^3$

$=(0)^3+(b-a)^3-(b-a)^3$

$=0$

$\Rightarrow x=a$ root

$f(b)=(a-b)^3+(b-b)^3-(a+b-2b)^3$

$=(a-b)^3+(0)^3-(a-b)^3$

$=0$

$\Rightarrow x=b$ root

$$f\left(\frac{a+b}{2}\right)=\left[a-\frac{a+b}{2}\right]^3+\left[b-\frac{a+b}{2}\right]^3-\left[(a+b)-2\left(\frac{a+b}{2}\right)\right]^3$$

$$=\left[\frac{2a-a-b}{2}\right]^3+\left[\frac{2b-a-b}{2}\right]^3-[(a+b)-(a+b)]^3$$

$$=\left(\frac{a-b}{2}\right)^3+\left(\frac{b-a}{2}\right)^3-(0)^3$$

$$=\left(\frac{a-b}{2}\right)^3-\left(\frac{a-b}{2}\right)^3$$

$=0$

$\Rightarrow x=\frac{a+b}{2}$ root

* Accept expansion of $(a-x)^3,(b-x)^3$, and $(a+b-2x)^3$ etc.

Blunders (-3)

B1 deduction factors.

B2 deduction root from factor.

B3 incorrect $(p+q)$.

B4 expansion $(m-n)^3$.

B5 factors once only.

B6 root not verified or root missing, provided one root found and verified

Slips (-1)

S1 numerical.

Attempts

A1 one or more correct roots without work.

QUESTION 2

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 (10, 10) marks**	**Att (3 [or 2,2], 3)**
Part (c)	**20 (5, 5, 10) marks**	**Att (2, 2, 3)**

Part (a) **10 marks** **Att 3**

2(a) Solve, without using a calculator, the following simultaneous equations:

$$3x + y + z = 0$$
$$x - y + z = 2$$
$$2x - 3y - z = 9$$

Part (a) **10 marks** **Att 3**

2(a)

(i): $3x + y + z = 0$
(ii): $x - y + z = 2$
(iii): $2x - 3y - z = 9$

(i): $3x + y + z = 0$
(ii): $x - y + z = 2$
$2x + 2y = -2$ (iv)

(i) : $3x + y + z = 0$
(iii): $2x - 3y - z = 9$
$5x - 2y = 9$ (v)

(iv) $2x + 2y = -2$
(v) $5x - 2y = 9$
$7x = 7$
$x = 1$

(iv) $2x + 2y = -2$
$2(1) + 2y = -2$
$2y = -4$
$y = -2$

(ii) $x - y + z = 2$
$(1) - (-2) + z = 2$
$z = -1$

$\therefore\ x = 1 \qquad y = -2 \qquad z = -1$

Blunders (-3)

B1 multiplying one side of equation only.
B2 not finding 2nd unknown (having found 1st).
B3 not finding 3rd unknown (having found 1st and 2nd).

Slips (-1)

S1 numerical.

Worthless

W1 trial and error.

Part (b)(i) **10 marks** **Att 3**

[when treated as quadratic]

2(b)(i) Solve the inequality $\frac{x+1}{x-1} < 4$, where $x \in \mathbf{R}$ and $x \neq 1$

Part (b)(i) **10 marks** **Att 3**

2(b)(i)

$$\frac{x+1}{x-1} < 4$$

multiply across by $(x-1)^2 > 0$

$$(x+1)(x-1) < 4(x-1)^2$$

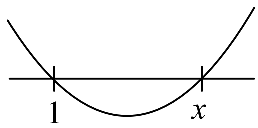
$$x^2 - 1 < 4x^2 - 8x + 4$$

$$0 < 3x^2 - 8x + 5$$

(*a*) Solve: $3x^2 - 8x + 5 = 0$

$$(3x-5)(x-1) = 0$$

$$x = \frac{5}{3} \text{ or } x = 1$$



$$f(x) > 0 \text{ when } \left\{x > \frac{5}{3}\right\} \cup \{x < 1\}$$

Blunders (-3)

B1 inequality sign.
B2 indices.
B3 expansion of $(x-1)^2$.
B4 factors once only.
B5 roots formula once only.
B6 deduction root from factor.
B7 range not stated.
B8 incorrect range.
B9 shape graph.

Slips (-1)

S1 numerical.

Attempts

A1 linear equation.

Worthless

W1 squares both sides.

Part (b)(i) **10 (5, 5)marks** **Att (2, 2)**
[when not treated as quadratic]

2(b)(i) Solve the inequality $\frac{x+1}{x-1} < 4$, where $x \in \mathbf{R}$ and $x \neq 1$

Part (b)(i) **10 (5, 5) marks** **Att (2, 2)**
(x – 1) < 0 5 marks **Att 2**
(x – 1) > 0 5 marks **Att 2**

2(b)(i) $\frac{x+1}{x-1} < 4 \quad \Leftrightarrow \quad \frac{x+1}{x-1} - 4 < 0$

$$\frac{(x+1)-4(x-1)}{(x-1)} < 0$$

$$\frac{x+1-4x+4}{x-1} < 0$$

$$\frac{5-3x}{x-1} < 0$$

(a) $(5-3x) > 0$ and $(x-1) < 0$

$5 > 3x$ $\qquad x < 1$

$\frac{5}{3} > x$ and $x < 1 \Rightarrow$ $\qquad x < 1$

(b) $(5-3x) < 0$ and $(x-1) > 0$

$5 < 3x$ $\qquad x > 1$

$\frac{5}{3} < x$ and $1 < x \Rightarrow$ $\qquad x > \frac{5}{3}$

So, answer is: $\{x < 1\} \cup \left\{x > \frac{5}{3}\right\}$

or

2(b)(i) $\frac{x+1}{x-1} < 4$

(a) $(x-1) > 0 \quad \Rightarrow x > 1$

$(x+1) < 4(x-1)$ since $(x-1) > 0$

$x+1 < 4x-4$

$5 < 3x$

$\frac{5}{3} < x$ and $x > 1 \quad \Rightarrow x > \frac{5}{3}$

(b) $(x-1) < 0 \quad \Rightarrow x < 1$

$(x+1) > 4(x-1)$

$x+1 > 4x-4$

$5 > 3x$

$\frac{5}{3} > x$ and $x < 1 \quad \Rightarrow x < 1$

So, answer is: $\{x < 1\} \cup \left\{x > \frac{5}{3}\right\}$

Blunders (-3)
B1 inequality sign.
B2 deduction of value.
B3 range not stated.
B4 incorrect range.

Slips (-1)
S1 numerical.

Part 2(b)(ii) **10 marks** **Att 3**

2 (b)(ii) The roots of $x^2 + px + q = 0$ are α and β, where p , $q \in R$,
Find the quadratic equation whose roots are $\alpha^2\beta$ and $\alpha\beta^2$

Part 2(b)(ii) **10 marks** **Att 3**

2(b)(ii)

$$x^2 + px + q = 0$$
$$x^2 - (-p)x + (q) = 0$$
$$x^2 - (\alpha + \beta)x + (\alpha\beta) = 0$$
$$\Rightarrow \alpha + \beta = -p \quad \text{and} \quad \alpha\beta = q$$

New roots : $\alpha^2\beta$ and $\beta^2\alpha$

New equation : $x^2 - (\alpha^2\beta + \alpha\beta^2)x + (\alpha^2\beta.\alpha\beta^2) = 0$

$$\alpha^2\beta + \beta^2\alpha = \alpha\beta(\alpha + \beta) = q(-p) = -pq$$
$$\alpha^3\beta^3 = (\alpha\beta)^3 = (q)^3 = q^3$$

$$\therefore \quad x^2 - (-pq)x + (q^3) = 0$$
$$x^2 + pqx + q^3 = 0$$

Blunders (-3)
B1 value of $(\alpha + \beta)$.
B2 value of $\alpha\beta$.
B3 indices.
B4 statement of quadratic equation.

Slips (-1)
S1 numerical.

Attempts
A1 not quadratic equation.

Part (c)(i)	**10 (5, 5)marks**	**Att (2, 2)**

2(c)(i) $f(x) = 2x + 1$, for $x \in R$

Show that there exists a real number k such that for all x

$f(x + f(x)) = kf(x)$

Part (c)(i)	$f(x + f(x))$ **5 marks**	**Att 2**
	k **5 marks**	**Att 2**

2(c)(i)

$$f(x) = 2x + 1$$
$$x + f(x) = x + (2x + 1) = 3x + 1$$
$$f[x + f(x)] = f(3x + 1) = 2(3x + 1) + 1$$
$$= 6x + 3$$

But $f[x + f(x)] = k[f(x)]$

$$\therefore \quad 6x + 3 = k(2x + 1)$$
$$3(2x + 1) = k(2x + 1)$$
$$\Rightarrow k = 3$$

Blunders (-3)

B1 $k = g(x)$

Slips (-1)

S1 numerical.

Attempts

A1 $f(x + f(x)) = f(x) + f(x)$

A2 particular values of *x*.

Note: A1 cannot lead to any further marks.

Part 2(c)(ii)	**10 marks**	**Att 3**

2(c)(ii) Show that for any real values of *a, b* and *h,* the quadratic equation $(x - a)(x - b) - h^2 = 0$ has real roots.

Part 2(c)(ii)	**10 marks**	**Att 3**

2(c)(ii)

$$(x - a)(x - b) - h^2 = 0$$
$$x^2 - ax - bx + ab - h^2 = 0$$
$$x^2 - (a + b)x + (ab - h^2) = 0$$

For real roots : $b^2 - 4ac \geq 0$

Here $b^2 - 4ac \Rightarrow [-(a + b)^2] - 4(1)(ab - h^2)$

$$= a^2 + 2ab + b^2 - 4ab + 4h^2$$
$$= a^2 - 2ab + b^2 + 4h^2$$
$$= (a - b)^2 + (2h)^2$$
$$\geq 0$$

$\Rightarrow$ real roots always

Blunders (-3)

B1 indices.
B2 incorrect value '*b*'.
B3 incorrect value '*a*'.
B4 incorrect value '*c*'.
B5 expansion $(a+b)^2$ once only.
B6 factors.
B7 inequality sign.

Slips (-1)

S1 numerical.

Attempts

A1 $b^2 - 4ac \geq 0$.

QUESTION 3

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 (10, 10) marks**	**Att (3, 3)**
Part (c)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, -, -)**

Part (a) **10 marks** **Att 3**

3(a) Find the real numbers p and q such that
$2(p+iq)+i(p-iq)=5+i$, where $i^2=-1$

Part (a) **10 marks** **Att 3**

3(a) $2(p+iq)+i(p-iq)=5+i$

$2p+2iq+pi-i^2q=5+i$

$(2p+q)+i(p+2q)=(5)+(1)i$

$\Rightarrow 2p+q=5\text{...........}(i)$ and $p+2q=1\text{...............}(ii)$

(i) $4p + 2q = 10$
(ii) $\underline{p + 2q = 1}$
$3p = 9$
$p = 3$

(ii) $p + 2q = 1$
$3 + 2q = 1$
$2q = -2$
$q = -1$

Blunders (-3)
B1 i
B2 indices.
B3 not real to real.
B4 not imaginary to imaginary.
B5 multiplying one side of equation only.
B6 not finding 2nd unknown (having found 1st)

Slips (-1)
S1 numerical.

Part (b)(i) **10 marks** **Att 3**

3(b)(i) $z_1=\cos\frac{4\pi}{3}+i\sin\frac{4\pi}{3}$ and $z_2=\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}$
Evaluate z_1z_2, giving your answer in the form $x+iy$

Part b(i) **10 marks** **Att 3**

3(b)(i) $z_1=\cos\frac{4\pi}{3}+i\sin\frac{4\pi}{3}$ and $z_2=\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}$

$\therefore\ z_1z_1=\cos\left(\frac{4\pi}{3}+\frac{\pi}{3}\right)+i\sin\left(\frac{4\pi}{3}+\frac{\pi}{3}\right)=\cos\frac{5\pi}{3}+i\sin\frac{5\pi}{3}$

$=\frac{1}{2}+i\left(-\frac{\sqrt{3}}{2}\right)=\frac{1}{2}-\frac{i\sqrt{3}}{2}$

or

3(b)(i) $z_1 z_2 = \left(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right)\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

$$= \cos\frac{4\pi}{3}\cos\frac{\pi}{3} + i\sin\frac{4\pi}{3}\cos\frac{\pi}{3} + i\cos\frac{4\pi}{3}\sin\frac{\pi}{3} + i^2\sin\frac{4\pi}{3}\sin\frac{\pi}{3}$$

$$= \left(\cos\frac{4\pi}{3}\cos\frac{\pi}{3} - \sin\frac{4\pi}{3}\sin\frac{\pi}{3}\right) + i\left(\sin\frac{4\pi}{3}\cos\frac{\pi}{3} + \cos\frac{4\pi}{3}\sin\frac{\pi}{3}\right)$$

$$= \cos\left(\frac{4\pi}{3} + \frac{\pi}{3}\right) + i\sin\left(\frac{4\pi}{3} + \frac{\pi}{3}\right)$$

$$= \cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}$$

$$= \left(\frac{1}{2}\right) + i\left(-\frac{\sqrt{3}}{2}\right) \qquad = \frac{1}{2} - \frac{i\sqrt{3}}{2}$$

Blunders (-3)
B1 argument once only.
B2 *i*.
B3 indices.
B4 trig formula.

Slips (-1)
S1 numerical.
S2 trig value.

Part (b)(ii) **10 marks** **Att 3**

3(b)(ii) $w_1 = a + ib$ and $w_2 = c + id$
prove that $\overline{(w_1 w_2)} = (\overline{w_1})(\overline{w_2})$, where $\overline{w}$ is complex conjugate of *w*.

Part (b)(ii) **10 marks** **Att 3**

3(b)(ii) $w_1 = a + ib \Rightarrow \overline{w_1} = a - ib$
$w_2 = c + id \Rightarrow \overline{w_2} = c - id$

$$\overline{w_1 w_2} = \overline{(a+ib)(c+id)}$$
$$= \overline{ac + ibc + iad + i^2 bd}$$
$$= \overline{(ac - bd) + i(bc + ad)}$$
$$= (ac - bd) - i(bc + ad) \text{............................(i)}$$

$$(\overline{w_1}).(\overline{w_2}) = (a - ib)(c - id)$$
$$= ac - ibc - iad + i^2 bd$$
$$= (ac - bd) - i(bc + ad) \text{......................(ii)}$$

$$\Rightarrow \overline{w_1 \cdot w_2} = (\overline{w_1})(\overline{w_2})$$

Blunders (-3)
B1 *i*
B2 conjugate.

Part (c)	**20(5, 5, 5, 5) marks**	**Att (2, 2, -, -)**

3(c) Let $A = \begin{pmatrix} 1 & -3 \\ -1 & 2 \end{pmatrix}$ and $P = \begin{pmatrix} 4 & 3 \\ -2 & -1 \end{pmatrix}$

(i) Evaluate $A^{-1}PA$ and hence $\left(A^{-1}PA\right)^{10}$

(ii) Use the fact that $\left(A^{-1}PA\right)^{10} = A^{-1}P^{10}A$ to evaluate P^{10}

Part (c) (i) Evaluate	**5 marks**	**Att 2**
Hence	**5 marks**	**Att 2**
(ii) P^{10}	**5 marks**	**hit/miss**
Evaluate	**5 marks**	**hit/miss**

3(c)(i) $A = \begin{pmatrix} 1 & -3 \\ -1 & 2 \end{pmatrix}$ $\quad P = \begin{pmatrix} 4 & 3 \\ -2 & -1 \end{pmatrix}$

$$A^{-1} = \frac{1}{2-3}\begin{pmatrix} 2 & 3 \\ 1 & 1 \end{pmatrix} = (-1)\begin{pmatrix} 2 & 3 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} -2 & -3 \\ -1 & -1 \end{pmatrix}$$

$$A^{-1}.P.A = \begin{pmatrix} -2 & -3 \\ -1 & -1 \end{pmatrix}\begin{pmatrix} 4 & 3 \\ -2 & -1 \end{pmatrix}\begin{pmatrix} 1 & -3 \\ -1 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} -2 & -3 \\ -1 & -1 \end{pmatrix}\begin{pmatrix} 1 & -6 \\ -1 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$$

$$\left(A^{-1}PA\right)^{10} = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}^{10} = \begin{pmatrix} 1 & 0 \\ 0 & 1024 \end{pmatrix}$$

3(c)(ii) $\left(A^{-1}PA\right)^{10} = \left(A^{-1}P^{10}A\right)$

$$A\left(A^{-1}PA\right)^{10}A^{-1} = A\left(A^{-1}P^{10}A\right)A^{-1}$$

$$A\left(A^{-1}PA\right)^{10}A^{-1} = P^{10}$$

$$\therefore \quad P^{10} = \begin{pmatrix} 1 & -3 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1024 \end{pmatrix}\begin{pmatrix} -2 & -3 \\ -1 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & -3 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} -2 & -3 \\ -1024 & -1024 \end{pmatrix}$$

$$= \begin{pmatrix} 3070 & 3069 \\ -2046 & -2045 \end{pmatrix}$$

Blunders (-3)

B1 value A^{-1} once only

B2 $A^{-1}.A \neq I$

B3 indices.

B4 incorrect order of multiplication.

Worthless

W1 P^{10} calculated by other means.

NOTE: $A^{-1}PA$ must be diagonal matrix for last 5 marks in (i); otherwise 0 marks.

QUESTION 4

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 (5, 10, 5) marks**	**Att (2, 3, 2)**
Part (c)	**20 (10, 10) marks**	**Att (3, 3)**

Part (a) **10 marks** **Att 3**

4(a) Show that $3\binom{n}{3} = n\binom{n-1}{2}$ for all natural numbers $n \geq 3$.

Part (a) **10 marks** **Att 3**

4(a) L.H.S. : $3\binom{n}{3} = \frac{3.n(n-1)(n-2)}{1.2.3} = \frac{n(n-1)(n-2)}{2}$

R.H.S.: $n\binom{n-1}{2} = n.\left[\frac{(n-1)(n-2)}{1.2}\right] = \frac{n(n-1)(n-2)}{2}$

$$\Rightarrow 3\binom{n}{3} = n\binom{n-1}{2}$$

or

4(a) L.H.S.: $3\binom{n}{3} = 3\left[\frac{n!}{3!(n-3)!}\right] = \frac{3}{6}\left[\frac{n(n-1)(n-2).(n-3)!}{(n-3)!}\right] = \frac{n(n-1)(n-2)}{2}$

R.H.S.: $n\binom{n-1}{2} = n\left[\frac{(n-1)!}{2![(n-1)-2]!}\right] = \frac{n(n-1)!}{2.(n-3)!}$

$$= \frac{n(n-1)(n-2)(n-3)!}{2(n-3)!} = \frac{n(n-1)(n-2)}{2}$$

$$\Rightarrow 3\binom{n}{3} = n\binom{n-1}{2}$$

Blunders (-3)

B1 definition of $\binom{n}{r}$.

B2 factorial.

Slips (-1)

S1 numerical.

Attempts

A1 correct with particular values.

Part (b) **20 (5, 10, 5)** **Att (2, 3, 2)**

4(b)(i)	Show that $\frac{2}{(2r-1)(2r+1)} = \frac{1}{2r-1} - \frac{1}{2r+1}, r \neq \pm\frac{1}{2}$
(ii)	Hence, find $\sum_{r=1}^{n} \frac{2}{(2r-1)(2r+1)}$
(iii)	Evaluate $\sum_{r=1}^{\infty} \frac{2}{(2r-1)(2r+1)}$

Part(b)(i)	**5 marks**	**Att 2**
(ii)	**10 marks**	**Att 3**
(iii)	**5 marks**	**Att 2**

4 (b)(i)

$$\frac{1}{2r-1} - \frac{1}{2r+1} = \frac{(2r+1)-(2r-1)}{(2r-1)(2r+1)}$$

$$= \frac{2r+1-2r+1}{(2r-1)(2r+1)}$$

$$= \frac{2}{(2r-1)(2r+1)}$$

or

4(b)(i) Let $\frac{2}{(2r-1)(2r+1)} = \frac{a}{2r-1} + \frac{b}{2r+1}$

Multiply across by $(2r-1)(2r+1)$:

$$2 = a(2r+1) + b(2r-1)$$

$$(0)r + (2) = 2ar + a + 2br - b$$

$$(0)r + (2) = (2a+2b)r + (a-b)$$

$\Rightarrow 2a + 2b = 0$

$a + b = 0$ ……….(i) and $a - b = 2$ ………………(ii)

(i): $a + b = 0$
(ii): $a - b = 2$
$2a = 2$
$a = 1 \quad \Rightarrow b = -1$

$$\Rightarrow \frac{2}{(2r-1)(2r+1)} = \frac{1}{2r-1} - \frac{1}{2r+1}$$

4(b)(ii) $\sum_{r=1}^{n} \frac{2}{(2r-1)(2r+1)}$

$$U_n = \frac{2}{(2n-1)(2n+1)} = \frac{1}{2n-1} - \frac{1}{2n+1}$$
$$U_{n-1} = \frac{2}{(2n-3)(2n-1)} = \frac{1}{2n-3} - \frac{1}{2n-1}$$
$$U_{n-2} = \frac{2}{(2n-5)(2n-3)} = \frac{1}{2n-5} - \frac{1}{2n-3}$$

……………………………………………………

……………………………………………………

$$U_3 = \frac{2}{5.7} = \frac{1}{5} - \frac{1}{7}$$
$$U_2 = \frac{2}{3.5} = \frac{1}{3} - \frac{1}{5}$$
$$U_1 = \frac{2}{1.3} = \frac{1}{1} - \frac{1}{3}$$
$$S_n = 1 - \frac{1}{2n+1}$$

4(b)(iii) $\sum_{r=1}^{\infty} \frac{2}{(2r-1)(2r+1)} = \lim_{n\to\infty}\left(1-\frac{1}{2n+1}\right) = 1.$

Blunders (-3)

B1 indices.
B2 cancellation must be shown or implied.
B3 not like to like.
B4 term or terms omitted
B5 S_r
B6 $r = \pm\frac{1}{2}$

Slips (-1)

S1 numerical.

NOTE: Must show 2 terms at start and 2 terms at finish.

Part (c)(i) **10 marks** **Att 3**

4 (c) (i)	The sequence $u_1, u_2, u_3, \ldots\ldots..$ is given by $u_{n+1} = \sqrt{4-(u_n)^2}$ and $u_1 = a > 0$. For what value of a will all of the terms of the sequence be equal to each other?

Part (c)(i) **10 marks** **Att 3**

4(c)(i) All terms equal $\Rightarrow$ *a, a, a, a, ...* That is, $u_1 = u_2 = u_3 = \ldots\ldots\ldots\ldots = a$

$$u_2 = \sqrt{4-u_1^{\,2}}$$
$$a = \sqrt{4-a^2}$$
$$a^2 = 4-a^2$$
$$2a^2 = 4$$
$$a^2 = 2$$
$$a = \pm\sqrt{2}$$

But $a > 0 \Rightarrow a = +\sqrt{2}$

Blunders (-3)

B1 indices.

B2 a must be > 0.

Slips (-1)

S1 numerical.

Attempts

A1 if $u_2 \neq a$.

Part (c)(ii) **10 marks** **Att 3**

4(c) (ii)	*p, q* and *r* are three numbers in arithmetic sequence. Prove that $p^2 + r^2 \geq 2q^2$

Part (c)(ii) **10 marks** **Att 3**

4(c)(ii) When *p, q, r* in arithmetic sequence, then $q = \frac{p+r}{2}$

$$p^2 + r^2 \geq 2q^2 \Leftrightarrow p^2 + r^2 - 2q^2 \geq 0.$$

Now, $p^2 + r^2 - 2q^2 = p^2 + r^2 - 2\left(\frac{p+r}{2}\right)^2$

$$= p^2 + r^2 - \frac{1}{2}\left(p^2 + 2pr + r^2\right)$$
$$= \frac{2p^2 + 2r^2 - p^2 - 2pr - r^2}{2}$$
$$= \frac{1}{2}\left(p^2 - 2pr + r^2\right)$$
$$= \frac{1}{2}(p-r)^2 \geq 0$$

or

4(c)(ii)	Let $p, p + d, p + 2d$ be 3 terms in A.P. $\left[p = p : q = p + d : r = p + 2d\right]$ $p^2 + r^2 \geq 2q^2 \;\Leftrightarrow\; p^2 + r^2 - 2q^2 \geq 0$. Now, $p^2 + r^2 - 2q^2 = p^2 + (p + 2d)^2 - 2(p + d)^2$ $= p^2 + p^2 + 4pd + 4d^2 - 2(p^2 + 2pd + d^2)$ $= 2p^2 + 4pd + 4d^2 - 2p^2 - 4pd - 2d^2$ $= 2d^2$ ≥ 0.

Blunders (-3)
B1 definition AP
B2 value of q.
B3 inequality sign.
B4 expansion of $(a + b)^2$ once only.
B5 factors once only.
B6 incorrect deduction or no deduction.
B7 indices.

Attempts
A1 particular values verified correctly.
A2 answer not as perfect square.

Worthless
W1 geometric sequence.

QUESTION 5

Part (a)	**10 marks**	**Att 3**
Part (b)	**20(10, 10) marks**	**Att (3, 3)**
Part (c)	**20(5, 5, 10) marks**	**Att (2, 2, 3)**

Part (a) **10 marks** **Att 3**

5(a) Find the fifth term in the expansion of $\left(x^2 - \frac{1}{x}\right)^6$ and show that it is independent of x.

Part 5(a) **10 marks** **Att 3**

5(a)

$$\left[x^2 + \left(-\frac{1}{x}\right)\right]^6$$

$$U_5 = \binom{6}{4}\left(x^2\right)^{6-4}\left(-\frac{1}{x}\right)^4$$

$$= \binom{6}{2}\left(x^2\right)^2\left(\frac{1}{x^4}\right)$$

$$= \frac{6.5}{1.2}x^4.\frac{1}{x^4}$$

$$= 15$$

or

5(a) $$\left[x^2 + \left(-\frac{1}{x}\right)\right]^6 = \left(x^2\right)^6 + \binom{6}{1}\left(x^2\right)^5\left(-\frac{1}{x}\right)^1 + \binom{6}{2}\left(x^2\right)^4\left(-\frac{1}{x}\right)^2 + \binom{6}{3}\left(x^2\right)^3\left(-\frac{1}{x}\right)^3 + \binom{6}{4}\left(x^2\right)^2\left(-\frac{1}{x}\right)^4 + \ldots\ldots\ldots\ldots$$

$$U_5 = \binom{6}{4}\left(x^2\right)^2\left(-\frac{1}{x}\right)^4$$

$$= \binom{6}{2}x^4.\frac{1}{x^4}$$

$$= \frac{6.5}{1.2}.x^0$$

$$= 15$$

Blunders (-3)

B1 general term.

B2 errors binomial expansion once only.

B3 indices.

B4 error value $\binom{n}{r}$ or no value $\binom{n}{r}$.

B5 $x^0 \neq 1$.

Slips (-1)

S1 numerical.

Part (b)(i) **10 marks** **Att 3**

5(b)(i)	In a geometric series, the second term is 8 and the fifth term is 27. Find the first term and the common ratio.

Part (b)(i) **10 marks** **Att 3**

5(b)(i) $u_2 = ar = 8$ $\qquad u_5 = ar^4 = 27$

$$\frac{u_5}{u_2} = \frac{ar^4}{ar} = \frac{27}{8}$$

$$r^3 = \frac{27}{8} \Rightarrow r = \frac{3}{2}$$

$$a(r) = 8$$

$$a\left(\frac{3}{2}\right) = 8 \Rightarrow a = \frac{16}{3}.$$

First term = $\frac{16}{3}$, common ratio = $\frac{3}{2}$

Blunders (-3)

B1 definition of term of GP.

B2 indices.

B3 not finding 2nd unknown (having found 1st)

Slips (-1)

S1 numerical.

Worthless

W1 uses AP.

W2 trial and error

Part (b)(ii) **10 marks** **Att 3**

5(b)(ii)	Solve $\log_4 x - \log_4 (x-2) = \frac{1}{2}$

Part (b)(ii) **10 mark** **Att 3**

5(b)(ii) $\log_4 x - \log_4 (x-2) = \frac{1}{2}$

$$\log_4\left(\frac{x}{x-2}\right) = \frac{1}{2}$$

$$\frac{x}{x-2} = 4^{\frac{1}{2}} = 2$$

$$x = 2(x-2)$$

$$x = 2x - 4$$

$$4 = x$$

or

(b)(ii)	$\log_4 x - \log_4(x-2) = \frac{1}{2}$ $\log_4\left(\frac{x}{x-2}\right) = \log_4(2)$ $\Rightarrow \frac{x}{x-2} = 2$ $x = 2x - 4$ $4 = x$

Blunders (-3)
B1 indices
B2 logs

Slips (-1)
S1 numerical
S2 excess value

Worthless
W1 drops logs

Part (c) **20(5, 5, 10)marks** **Att (2, 2, 3)**

5(c) Prove by induction that $2^n \geq n^2, n \in N, n \geq 4$.

Part (c)	***P(4)***	**5 marks**	**Att 2**
	P(k)	**5 marks**	**Att 2**
	P(k + 1)	**10 marks**	**Att 3**

5(c) To prove $2^n \geq n^2, n \in N, n \geq 4$

Test $n = 4 : 2^4 = 16$

$4^2 = 16$ $\Rightarrow p(4)$ true

Assume true for $n = k$

$\Rightarrow 2^k \geq k^2$

To prove : $2^{k+1} \geq (k+1)^2$

Proof: $2^{k+1} = 2.2^k \geq 2k^2$

$= k^2 + k^2$

$= k^2 + k.k$

$\geq k^2 + 3k$ … since $k \geq 4 > 3$

$= k^2 + 2k + k$

$\geq k^2 + 2k + 1$ … since $k \geq 4 > 1$

$= (k+1)^2$

$\therefore$ *P(k* + 1) true whenever *P(k)* true.

Since *P*(4) true, then by induction *P(n)* true for any positive integer *n,* $(n \in N, n \geq 4)$

or

5(c) [Base $n = 4$ and $P(k)$ as above]

To prove : $2^{k+1} \geq (k+1)^2$

$2^{k+1} = 2.2^k \geq 2k^2$

$= k^2 + k^2$

$\geq k^2 + (2k+1)$

$= (k+1)^2$

justify assertion $k^2 \geq 2k+1$:

$k^2 - 2k = k(k-2)$

$\geq 4(4-2)$ (since $k \geq 4$)

≥ 1.

or

$k^2 - 2k - 1 = (k-1)^2 - 2$

$\geq (4-1)^2 - 2$ (since $k \geq 4$)

$= 9 - 2 \geq 0$.

$\therefore$ *P(k* + 1) true whenever *P(k)* true.

Since $P(4)$ true, then by induction *P(n)* true for any positive integer *n,* $(n \in N, n \geq 4)$

Blunders (-3)

B1 fails to prove case $n = 4$; (not sufficient to say "true for $n = 4$").

B2 indices.

B3 $n \neq 4$

B4 fails to justify (as appropriate to method): (i) $k^2 \geq 2k+1$

(ii) $\left(\frac{k+1}{k}\right)^2 \leq 2$

(iii) $1 + \frac{1}{k} \leq 1 + \frac{1}{4} < \sqrt{2}$, since $k \geq 4$

or similar

QUESTION 6

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 (10, 10) marks**	**Att (3, 3)**
Part (c)	**20 (5, 10, 5) marks**	**Att (2, 3, -)**

Part (a) **10 marks** **Att 3**

6(a) Differentiate $\frac{1}{2+5x}$ with respect to x

Part (a) **10 marks** **Att 3**

6(a)

$$y = \frac{1}{2+5x} = (2+5x)^{-1}$$

$$\frac{dy}{dx} = -1(2+5x)^{-2}.(5) = \frac{-5}{(2+5x)^2}$$

or

6(a)

$$y = \frac{1}{2+5x} = \frac{u}{v}$$

$$\frac{dy}{dx} = \frac{(2+5x)(0)-(1)(5)}{(2+5x)^2} = \frac{-5}{(2+5x)^2}$$

Blunders (-3)
B1 differentiation.
B2 indices.

Attempts
A1 error in differentiation formula.

Part (b)(i) **10 marks** **Att 3**

6(b)(i) Given $y = \tan^{-1} x$, find the value of $\frac{dy}{dx}$ at $x = \sqrt{2}$.

Part (b)(i) **10 marks** **Att 3**

6(b)(i)

$$y = \tan^{-1} x$$

$$\frac{dy}{dx} = \frac{1}{1+x^2}$$

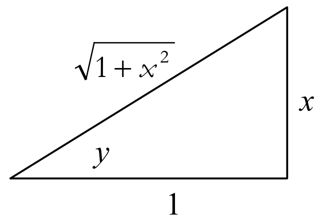
when $x = \sqrt{2}$, $\frac{dy}{dx} = \frac{1}{1+2} = \frac{1}{3}$

or

6(b)(i) $y = \tan^{-1} x$

$$\tan y = x$$
$$\sec^2 y.\frac{dy}{dx} = 1$$
$$\frac{dy}{dx} = \frac{1}{\sec^2 y} = \cos^2 y$$
$$\frac{dy}{dx} = \left(\frac{1}{\sqrt{1+x^2}}\right)^2 = \frac{1}{1+x^2}.$$

when $x = \sqrt{2}$, $\frac{dy}{dx} = \frac{1}{1+2} = \frac{1}{3}$



$$\tan y = x = \frac{x}{1}$$
$$\therefore \cos y = \frac{1}{\sqrt{1+x^2}}$$

Blunders (-3)
B1 differentiation.
B2 indices.
B3 definition of $\tan y$.
B4 definition of $\cos y$.
B5 no value of x.

Attempts
A1 error in differentiation formula.

Worthless
W1 integration.

Part (b)(ii) **10 marks** **Att 3**

6(b)(ii) Differentiate, from first principles, $\cos x$ with respect to x.

Part (b)(ii) **10 marks** **Att 3**

6(b)(ii)

$$f(x) = \cos x$$
$$f(x+h) = \cos(x+h)$$
$$f(x+h) - f(x) = \cos(x+h) - \cos x$$
$$\frac{f(x+h)-f(x)}{h} = \frac{-2\sin\left(\frac{2x+h}{2}\right)\sin\left(\frac{h}{2}\right)}{h}$$
$$\frac{f(x+h)-f(x)}{h} = -\sin\left(x+\frac{h}{2}\right)\frac{\sin\left(\frac{h}{2}\right)}{\left(\frac{h}{2}\right)}$$
$$\therefore \quad \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = -\sin x$$

or

6(b)(ii)	$y = \cos x$ $y + \Delta y = \cos(x + \Delta x)$ $\Delta y = \cos(x + \Delta x) - \cos x$ $\Delta y = -2\sin\left(x + \frac{\Delta x}{2}\right)\sin\left(\frac{\Delta x}{2}\right)$ $\frac{\Delta y}{\Delta x} = -\sin\left(x + \frac{\Delta x}{2}\right)\frac{\sin\left(\frac{\Delta x}{2}\right)}{\left(\frac{\Delta x}{2}\right)}$ $\therefore \quad \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = -\sin x$

Blunders (-3)

B1 trig formula.

B2 sum of angles.

B3 difference of angles

B4 not $\left(\frac{\sin\theta}{\theta}\right)$

B5 no limits shown or implied or no indication $\to 0$.

Worthless

W1 not 1st principles

Part (c)	**20(5, 10, 5) marks**	**Att (2, 3, -)**

6(c) Let $f(x) = x^3 + 6x^2 + 15x + 36, x \in \mathbf{R}$.

(i) Show that $f'(x)$ can be written in the form $3\left[(x+a)^2 + b\right]$, *a, b* $\in \mathbf{R}$ where $f'(x)$ is the first derivative of *f(x)*. .

(ii) Hence show that *f(x)* = 0 has only one real root.

Part (c) (i)	**5 marks**	**Att 2**
(ii) $f'(x) > 0$	**10 marks**	**Att 3**
***f(x)* increasing**	**5 marks**	**Hit or Miss**

6(c)	$f(x) = x^3 + 6x^2 + 15x + 36$
(i)	$f'(x) = 3x^2 + 12x + 15$ $= 3(x^2 + 4x + 5)$ $= 3\left[(x^2 + 4x + 4) + 1\right]$ $= 3\left[(x+2)^2 + 1\right]$
(ii)	$f'(x) = 3\left[(x+2)^2 + 1\right] > 0$ for all x $\Rightarrow$ *f(x)* is an increasing function $\Rightarrow$ curve cuts *x*-axis at only one point $\Rightarrow$ one real root.

or

6(c)(ii)	for max/min: $f'(x)=0$ $3x^2+12x+15=0$ $3(x^2+4x+5)=0$ $x^2+4x+5=0$ $x=\dfrac{-4\pm\sqrt{16-20}}{2}$ - not real values $\Rightarrow\ f'(x)\neq 0$ $\Rightarrow$ no turning points $\Rightarrow$ curve only cuts x-axis at only one point. $\Rightarrow$ one real root.

Blunders (-3)

B1 differentiation.
B2 completing square.
B3 not in required form.
B4 root formula once only.

Attempts

A1 particular values.
A2 $f'(x)=0$ giving real values.

QUESTION 7

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**
Part (c)	**20 (5, 5, 10) marks**	**Att (2, 2, 3)**

Part (a) **10 marks** **Att 3**

7(a) An object's distance from a fixed point is given by $s = 12 + 24t - 3t^2$, where s is in metres and t is in seconds.
Find the speed of the object when $t = 3$ seconds.

Part (a) **10 marks** **Att 3**

7(a) $s = 12 + 24t - 3t^2$

$$\frac{ds}{dt} = 24 - 6t$$

At $t = 3$: $\frac{ds}{dt} = 24 - 18 = 6$.

Blunders (-3)
B1 differentiation.
B2 gets acceleration.
B3 no value *t.*

Slips (-1)
S1 numerical.

Worthless
W1 no calculus.
W2 integration.

Part (b) **20(5, 5, 5, 5) marks** **Att (2, 2, 2, 2)**

7(b) The parametric equations of a curve are:
$x = 2\theta - \sin 2\theta$
$y = 1 - \cos 2\theta$, where $0 < \theta < \pi$.

(i) Find $\frac{dy}{dx}$

(ii) Show that the tangent to the curve at $\theta = \frac{\pi}{6}$ is perpendicular to the tangent at $\theta = \frac{2\pi}{3}$.

Part(b)(i)	$\frac{dx}{d\theta}$	**5 marks**	**Att 2**
	$\frac{dy}{d\theta}$	**5 marks**	**Att 2**
	$\frac{dy}{dx}$	**5 marks**	**Att 2**
(ii)		**5 marks**	**Att 2**

7(b)(i) $x = 2\theta - \sin 2\theta$ $\qquad y = 1 - \cos 2\theta$

$$\frac{dx}{d\theta} = 2 - 2\cos 2\theta \qquad \frac{dy}{d\theta} = 2\sin 2\theta$$

$$\frac{dy}{dx} = \frac{\left(\frac{dy}{d\theta}\right)}{\left(\frac{dx}{d\theta}\right)} = \frac{2\sin 2\theta}{2 - 2\cos 2\theta} = \frac{\sin 2\theta}{1 - \cos 2\theta}$$

(b) (ii) $m = \frac{dy}{dx} = \frac{\sin 2\theta}{1 - \cos 2\theta}$

$$\theta = \frac{\pi}{6}: \quad m_1 = \frac{\sin \frac{\pi}{3}}{1 - \cos \frac{\pi}{3}} = \frac{\frac{\sqrt{3}}{2}}{1 - \frac{1}{2}} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$$

$$\theta = \frac{2\pi}{3}: \quad m_2 = \frac{\sin \frac{4\pi}{3}}{1 - \cos \frac{4\pi}{3}} = \frac{-\frac{\sqrt{3}}{2}}{1 + \frac{1}{2}} = \frac{-\frac{\sqrt{3}}{2}}{\frac{3}{2}} = -\frac{1}{\sqrt{3}}$$

$$(m_1).(m_2) = \left(\sqrt{3}\right)\left(-\frac{1}{\sqrt{3}}\right) = -1$$

$\Rightarrow$ tangents are perpendicular

Blunders (-3)
B1 differentiation.
B2 indices.
B3 error in getting $\frac{dy}{dx}$
B4 trig formula
B5 incorrect deduction or no deduction from incorrect $(m_1)(m_2)$.
B6 calculator used for approximate values.

Slips (-1)
S1 trig value.
S2 numerical values.

Attempts
A1 error in differentiation formula.

Part 7(c)	20(5, 5, 10) marks	Att (2, 2, 3)

7(c) Given that $x = \dfrac{e^{2y}-1}{e^{2y}+1}$

(i) show that $e^{2y} = \dfrac{1+x}{1-x}$

(ii) show that $\dfrac{dy}{dx}$ can be expressed in the form $\dfrac{p}{1-x^q}$, $p, q \in \mathbf{N}$.

Part (c)(i)	5 marks	Att 2
(ii) $f'(x)$	5 marks	Att 2
value	10 marks	Att 3

7(c)(i)

$$x = \frac{e^{2y}-1}{e^{2y}+1}$$
$$x\left(e^{2y}+1\right) = e^{2y}-1$$
$$xe^{2y}+x = e^{2y}-1$$
$$1+x = e^{2y}-xe^{2y}$$
$$1+x = e^{2y}(1-x)$$
$$\frac{1+x}{1-x} = e^{2y}$$

or

7(c)(i)

$$\frac{1+x}{1-x} = \frac{1+\left(\frac{e^{2y}-1}{e^{2y}+1}\right)}{1-\left(\frac{e^{2y}-1}{e^{2y}+1}\right)}$$
$$= \frac{\left(e^{2y}+1\right)+\left(e^{2y}-1\right)}{\left(e^{2y}+1\right)-\left(e^{2y}-1\right)}$$
$$= \frac{2e^{2y}}{2}$$
$$= e^{2y}$$

7(c)(ii)

$$e^{2y} = \frac{1+x}{1-x}$$
$$2e^{2y}\frac{dy}{dx} = \frac{(1-x)(1)-(1+x)(-1)}{(1-x)^2}$$
$$2e^{2y}\frac{dy}{dx} = \frac{1-x+1+x}{(1-x)^2}$$
$$\frac{dy}{dx} = \frac{1}{2e^{2y}}\cdot\frac{2}{(1-x)^2}$$
$$= \frac{(1-x)}{(1+x)}\cdot\frac{1}{(1-x)^2}$$
$$= \frac{1}{(1+x)(1-x)}$$
$$= \frac{1}{1-x^2}$$

or

7(c)(ii)

$$e^{2y} = \frac{1+x}{1-x}$$

$$\ln\left(e^{2y}\right) = \ln\left(\frac{1+x}{1-x}\right)$$

$$2y\ln e = \ln(1+x) - \ln(1-x)$$

$$2y = \ln(1+x) - \ln(1-x)$$

$$2\frac{dy}{dx} = \frac{1}{1+x} - \frac{1}{1-x}(-1)$$

$$\frac{dy}{dx} = \frac{1}{2}\left[\frac{1}{1+x} + \frac{1}{1-x}\right]$$

$$= \frac{1}{2}\left[\frac{(1-x)+(1+x)}{(1+x)(1-x)}\right]$$

$$= \frac{1}{2}\left[\frac{2}{1-x^2}\right] = \frac{1}{1-x^2}$$

or

7(c)(ii)

$$x = \frac{e^{2y}-1}{e^{2y}+1}$$

$$\frac{dx}{dy} = \frac{\left(e^{2y}+1\right)\left(2e^{2y}\right) - \left(e^{2y}-1\right)\left(2e^{2y}\right)}{\left(e^{2y}+1\right)^2}$$

$$\frac{dy}{dx} = \frac{\left(e^{2y}+1\right)^2}{2e^{2y}\left[\left(e^{2y}+1\right) - \left(e^{2y}-1\right)\right]}$$

$$= \frac{\left(e^{2y}+1\right)^2}{4e^{2y}}$$

$$= \frac{1}{4e^{2y}}\left[e^{4y} + 2e^{2y} + 1\right]$$

$$= \frac{1}{4}\left[e^{2y} + 2 + \frac{1}{e^{2y}}\right]$$

$$= \frac{1}{4}\left[\frac{1+x}{1-x} + 2 + \frac{1-x}{1+x}\right]$$

$$= \frac{1}{4}\left[\frac{(1+x)^2 + 2(1+x)(1-x) + (1-x)^2}{(1-x)(1+x)}\right]$$

$$= \frac{1}{4\left(1-x^2\right)}\left[1 + 2x + x^2 + 2 - 2x^2 + 1 - 2x + x^2\right]$$

$$= \frac{1}{4\left(1-x^2\right)}\left[\frac{4}{1}\right] = \frac{1}{1-x^2}$$

Blunders (-3)

B1 cross multiplication.

B2 indices.

B3 logs.

B4 expansion $\left(e^{2y}+1\right)^2$ once only.

B5 expansion $(1+x)^2$ once only.

B6 expansion $(1-x)^2$ once only

Slips (-1)

S1 numerical.

S2 $\ln e \neq 1$

Attempts

A1 error in differentiation formula.

QUESTION 8

Part (a)	**10 (5, 5) marks**	**Att (2, 2)**
Part (b)	**20 (10, 10) marks**	**Att (3, 3)**
Part (c)	**20 (5, 5, 10) marks**	**Att (2, 2, 3)**

Part (a) **10 (5, 5)marks** **Att (2, 2)**

(a) Find **(i)** $\int \frac{1}{x^2} dx$ **(ii)** $\int \cos 6x dx$

Part(a) (i) **5 marks** **Att 2**
(ii) **5 marks** **Att 2**

8(a)(i) $\int \frac{1}{x^2} dx = \int x^{-2}.dx = \frac{x^{-1}}{-1} + c = \frac{-1}{x} + c$

8(a)(ii) $\int \cos 6x dx = \frac{\sin 6x}{6} + c$

* If c shown once $\Rightarrow$ no penalty

Blunders (-3)
B1 integration.
B2 no ‘c’ (Penalise 1st integration)
B3 indices.

Attempts
A1 only c correct.

Worthless
W1 differentiation instead of integration.

Part 8(b) **20 (10, 10)** **Att (3, 3)**

8(b)(i) Evaluate **(i)** $\int_3^6 \frac{dx}{\sqrt{36 - x^2}}$ **(ii)** $\int_0^{\frac{\pi}{3}} \sin x \cos^3 x dx$

Part (i) **10 marks** **Att 3**
Part(ii) **10 marks** **Att 3**

8(b)(i) $\int_3^6 \frac{dx}{\sqrt{36 - x^2}} = \int \frac{dx}{\sqrt{6^2 - x^2}} = \sin^{-1}\left(\frac{x}{6}\right)\Big]_3^6$

$= \sin^{-1}(1) - \sin^{-1}\left(\frac{1}{2}\right)$

$= \frac{\pi}{2} - \frac{\pi}{6}$

$= \frac{\pi}{3}$

8(b)(ii) $\int_0^{\frac{\pi}{3}} \sin x \cos^3 x dx$

$= \int (\cos^3 x)(\sin x dx)$

$u = \cos x$

$\frac{du}{dx} = -\sin x$

$du = -\sin x dx$

$-du = \sin x.dx$

$= \int u^3(-du)$

$= -\int u^3 du$

$= -\frac{u^4}{4}$

$= -\frac{1}{4}\left[(\cos x)^4\right]_0^{\frac{\pi}{3}} \quad = -\frac{1}{4}\left[\left(\cos\frac{\pi}{3}\right)^4 - (\cos 0)^4\right]$

$= -\frac{1}{4}\left[\left(\frac{1}{2}\right)^4 - (1)^4\right] \quad = -\frac{1}{4}\left[\frac{1}{16} - 1\right] \quad = -\frac{1}{4}\left(\frac{-15}{16}\right) \quad = \frac{15}{64}$

or

8(b)(ii) $\int_0^{\frac{\pi}{3}} \sin x \cos^3 x dx$

$= \int \sin x.\cos^2 x.\cos x dx$

$u = \sin x$

$\frac{du}{dx} = \cos x$

$du = \cos x.dx$

$= \int \sin x(1 - \sin^2 x).(\cos x dx)$

$= \int u(1 - u^2).du$

$= \int (u - u^3)du$

$u = \sin x$

$x = \frac{\pi}{3} \Rightarrow u = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

$x = 0 \Rightarrow u = \sin 0 = 0$

$= \left.\frac{u^2}{2} - \frac{u^4}{4}\right]_0^{\frac{\sqrt{3}}{2}}$

$= \frac{\left(\frac{\sqrt{3}}{2}\right)^2}{2} - \frac{\left(\frac{\sqrt{3}}{2}\right)^4}{4} \quad = \frac{3}{8} - \frac{9}{64} \quad = \frac{24-9}{64} \quad = \frac{15}{64}$

Blunders (-3)

B1 integration.

B2 indices.

B3 limits.

B4 no limits.

B5 incorrect order in applying limits.

B6 not calculating substituted limits.
B7 not changing limits.
B8 differentiation.

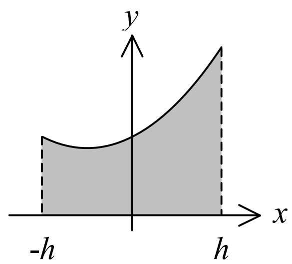
Slips (-1)
S1 numerical.
S2 trig value.

Worthless:
W1 differentiation instead of integration (except where other work merits attempt).

Note: Incorrect substitution and unable to finish yields attempt at most.
Note: (-3) is maximum deduction when evaluating limits.
Note: In Q8(b)(i) accept 60° for $\frac{\pi}{3}$ etc.

Part (c) **20(5, 5, 10) marks** **Att (2, 2, 3)**

8(c) The graph of the function $f(x) = ax^2 + bx + c$ from $x = -h$ to $x = h$ is shown in the diagram.



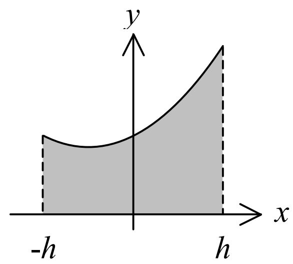
(i) Show that the area of the shaded region is

$$\frac{h}{3}\left[2ah^2 + 6c\right]$$

(ii) Given that $f(-h) = y_1, f(0) = y_2$ and $f(h) = y_3$, express the area of the shaded region in terms of y_1, y_2, y_3 and h.

Part (c)	**(i)**	**5 marks**	**Att 2**
	(ii) values of f(x)	**5 marks**	**Att 2**
	Express	**10 marks**	**Att 3**

8(c)(i) $A = \int_{-h}^{h} y\,dx = \int (ax^2 + bx + c)dx$



$$= \frac{ax^3}{3} + \frac{bx^2}{2} + cx\Big]_{(-h)}^{h}$$

$$= \left(\frac{ah^3}{3} + \frac{bh^2}{2} + ch\right) - \left(\frac{-ah^3}{3} + \frac{bh^2}{2} - ch\right)$$

$$= \frac{ah^3}{3} + \frac{bh^2}{2} + ch + \frac{ah^3}{3} - \frac{bh^2}{2} + ch$$

$$= \frac{2ah^3}{3} + 2ch$$

$$= \frac{h}{3}\left[2ah^2 + 6c\right]$$

8(c)(ii) $f(x) = ax^2 + bx + c$

$f(0) = 0 + 0 + c = y_2 \Rightarrow c = y_2$

$f(-h) = ah^2 - bh + c = y_1$

$f(h) \ = \ \underline{ah^2 + bh + c = y_3}$

$2ah^2 + 2c \quad = y_1 + y_3$

$$A = \frac{h}{3}\left[2ah^2 + 6c\right]$$

$$= \frac{h}{3}\left[\left(2ah^2 + 2c\right) + 4c\right]$$

$$= \frac{h}{3}\left[\left(y_1 + y_3\right) + 4y_2\right]$$

$$A = \frac{h}{3}\left[y_1 + 4y_2 + y_3\right]$$

Blunders (-3)
B1 indices
B2 integration.
B3 limits.
B4 no limits.
B5 incorrect order in applying limits.
B6 not calculating substituted limits.
B7 error with *f(x)*.
B8 uses $\pi \int y\,dx$

Slips (-1)
S1 numerical
S2 not in required form.

Attempts
A1 uses volume formula
A2 uses y^2 in formula.

Worthless
W1 differentiation instead of integration except where other work merits attempt.
W2 wrong area formula and no work.

Note: (-3) is maximum deduction when evaluating limits.

MARKING SCHEME

LEAVING CERTIFICATE EXAMINATION 2003

MATHEMATICS

HIGHER LEVEL

PAPER 2

GENERAL GUIDELINES FOR EXAMINERS - PAPER 2

1. Penalties of three types are applied to candidates' work as follows:
   - Blunders - mathematical errors/omissions (-3)
   - Slips - numerical errors (-1)
   - Misreadings (provided task is not oversimplified) (-1).

   Frequently occurring errors to which these penalties must be applied are listed in the scheme. They are labelled as B1, B2, B3,......, S1, S2, S3,...., M1, M2, etc. Note that these lists are not exhaustive.

2. When awarding attempt marks, e.g. Att(3), note that
   - any correct relevant step in a part of a question merits *at least* the attempt mark for that part
   - if deductions result in a mark which is lower than the attempt mark, then the attempt mark must be awarded
   - a mark between zero and the attempt mark is never awarded.

3. Worthless work is awarded zero marks. Some examples of such work are listed in the scheme and they are labelled as W1, W2,....etc.

4. The phrase "hit or miss" means that partial marks are not awarded – the candidate receives all of the relevant marks or none.

5. The *same* error in the *same* section of a question is penalised *once* only.

6. Special notes relating to the marking of a particular part of a question are indicated by an asterisk. These notes immediately follow the box containing the relevant solution.

7. Particular cases, verifications and answers derived from diagrams (unless requested) qualify for attempt marks at most.

8. A serious blunder, omission or misreading merits the ATTEMPT mark at most.

9. The phrase "and stops" means that no more work is shown by the candidate.

10. Accept the best of two or more attempts – even when attempts have been cancelled.

QUESTION 1

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**
Part (c)	**(5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**

Part (a) **10 marks** **Att 3**

1(a) A circle has centre (–1, 5) and passes through the point (1, 2).
Find the equation of the circle.

Equation of circle **5 marks** **Att 2**

1 (a)

$p(1,2)$, centre $q(-1,5)$. Radius $=|pq|=\sqrt{(1+1)^2+(2-5)^2}=\sqrt{13}$.

Equation of circle: $(x+1)^2+(y-5)^2=13$.

or

Circle: $x^2+y^2+2x-10y+c=0$. But (1, 2) ∈ Circle.

$\therefore\ 1+4+2-20+c=0 \Rightarrow c=13$.

Equation of circle: $x^2+y^2+2x-10y+13=0$.

Blunders (-3)
B1 Error in distance formula.
B2 Incorrect sign assigned to centre in equation of circle.

Slips (-1)
S1 Arithmetic error.

Attempts (3 marks)
A1 Radius length.
A2 Equation of circle without *c* or radius length evaluated.

Part (b) **20 (5, 5, 5, 5) marks** **Att (2, 2, 2, 2)**
Part (b) (i) **5 marks** **Att 2**

1 (b) (i) The point $a(5,2)$ is on the circle K: $x^2+y^2+px-2y+5=0$.
(i) Find the value of *p*.

Value of *p* **5 marks** **Att 2**

1 (b) (i)

$a(5,2) \in x^2+y^2+px-2y+5=0$

$\therefore 25+4+5p-4+5=0 \Rightarrow 5p=-30. \quad \therefore p=-6.$

Blunders (-3)
B1 Incorrect squaring.

Slips (-1)
S1 Arithmetic error.

Attempts (2 marks)
A1 Substitution of point and stops.

Part (b) (ii)	**15 marks (5, 5, 5)**	**Att (2, 2, 2)**

1 (b) (ii) The line $L: x - y - 1 = 0$ intersects the circle K.
Find the co-ordinates of the points of intersection.

Quadratic equation	**5 marks**	**Att 2**
Solving quadratic	**5 marks**	**Att 2**
Co-ordinates of intersection points	**5 marks**	**Att 2**

1 (b) (ii) $L: x - y - 1 = 0 \Rightarrow x = y + 1.$

$L \cap K : (y+1)^2 + y^2 - 6(y+1) - 2y + 5 = 0.$

$y^2 + 2y + 1 + y^2 - 6y - 6 - 2y + 5 = 0 \Rightarrow 2y^2 - 6y = 0$

$\therefore y^2 - 3y = 0 \Rightarrow y(y-3) = 0$

$\therefore y = 0$ or $y = 3.$

$y = 0 \Rightarrow x = 1$ and $y = 3 \Rightarrow x = 4.$

$\therefore$ Intersection points are (1, 0) and (4, 3).

Blunders (-3)

B1 Incorrect squaring.
B2 Incorrect factors.
B3 Error in quadratic formula.
B4 Failure to couple, e.g. x values without corresponding y values.

Slips (-1)

S1 Arithmetic error.

Attempts (2, 2, 2 marks)

A1 x in terms of y or y in terms of x.
A2 Quadratic not simplified.
A3Attempt at solving quadratic equation.

Part (c)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**
Part (c) (i)	**5 marks**	**Att 2**

1 (c) (i) The y-axis is a tangent to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$.
(i) Prove that $f^2 = c$.

Prove $f^2 = c$.	**5 marks**	**Att 2**

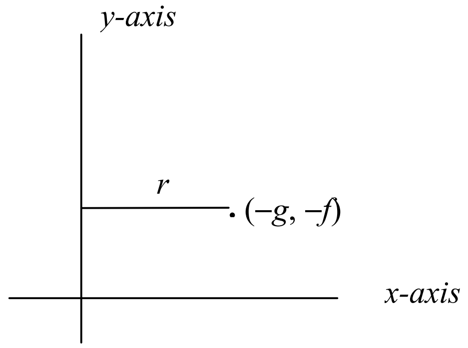
1 (c) (i)

$|-g| = r$

But $r^2 = g^2 + f^2 - c.$

$g^2 = g^2 + f^2 - c$

$\therefore f^2 = c.$



or

Perpendicular distance from $(-g, -f)$ to the line $x = 0$ equals radius.

$$\left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right| = \sqrt{g^2 + f^2 - c} \quad \text{where } a = 1,\ b = 0,\ c = 0,\ x_1 = -g,\ y_1 = -f.$$

$$\therefore \left|\frac{-g}{1}\right| = \sqrt{g^2 + f^2 - c} \Rightarrow g^2 = g^2 + f^2 - c. \quad \therefore f^2 = c.$$

Blunders (-3)

B1 Error in radius formula.

B2 Error in perpendicular distance formula or incorrect values assigned.

Slips (-1)

S1 Arithmetic error.

Attempts (2 marks)

A1 $r = |-g|$.

A2 Use of perpendicular distance formula.

Part (c) (ii) **15 marks (5, 5, 5)** **Att (2, 2, 2)**

1 (c) (ii) Find the equations of the circles that pass through the points (–3, 6) and (–6, 3) and have the *y*-axis as a tangent.

One equation in *f*, *g* and *c*	**5 marks**	**Att 2**
Three equations	**5 marks**	**Att 2**
Equations of circles	**5 marks**	**Att 2**

1 (c) (ii) Circle: $x^2 + y^2 + 2gx + 2fy + c = 0$.

$(-3, 6) \in \text{Circle} \Rightarrow 9 + 16 - 6g + 12f + c = 0$

$\therefore -6g + 12f + c = -45.$

$(-6, 3) \in \text{Circle} \Rightarrow 36 + 9 - 12g + 6f + c = 0$

$\therefore -12g + 6f + c = -45.$

$-12g + 24f + 2c = -90$

$-12g + 6f + c = -45$

$18f + c = -45 \Rightarrow c = -18f - 45.$

But *y*-axis a tangent $\therefore f^2 = c.$

$f^2 + 18f + 45 = 0 \Rightarrow (f + 3)(f + 15) = 0$

$\therefore f = -3$ or $f = -15.$

$f = -3 \Rightarrow c = 9$ and $f = -15 \Rightarrow c = 225.$

Substituting $f = -3$ and $c = 9$ into $-6g + 12f + c = -45$ gives $g = 3$.

Substituting $f = -15$ and $c = 225$ into $-6g + 12f + c = -45$ gives $g = 15$.

Required circles are: $x^2 + y^2 + 6x - 6y + 9 = 0$ and $x^2 + y^2 + 30x - 30y + 225 = 0$.

Blunders (-3)
B1 Incorrect factors.
B2 Error in quadratic formula.
B3 Correct values of *f, g* and *c* found and stops.

Slips (-1)
S1 Arithmetic error.

Attempts (2, 2, 2 marks)
A1 Two equations in *f, g* and *c*.
A2 Attempt at solving simultaneous equations.
A3 An equation in two variables.
A4 A correct value of either *f*, *g* or *c* found and stops.

QUESTION 2

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 (5, 10, 5) marks**	**Att (2, 3, 2)**
Part (c)	**20 (10, 10) marks**	**Att (3, 3)**

Part (a)	**10 marks**	**Att 3**

2 (a) $\vec{r} = 12\vec{i} - 35\vec{j}$. Find the unit vector in the direction of $\vec{r}$.

Unit vector	**5 marks**	**Att 2**

2 (a) Unit vector $= \dfrac{\vec{r}}{\left|\vec{r}\right|}$. $\quad \left|\vec{r}\right| = \left|12\vec{i} - 35\vec{j}\right| = \sqrt{144 + 1225} = \sqrt{1369} = 37.$

Unit vector $= \dfrac{12}{37}\vec{i} - \dfrac{35}{37}\vec{j}$.

Blunders (-3)
B1 Error in formula for norm of vector.
B2 Error in distance formula.

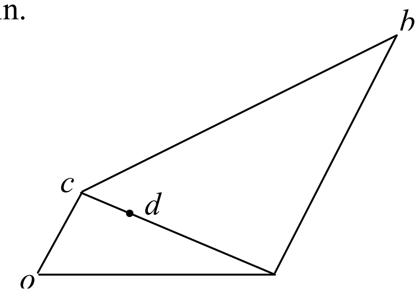
Slips (-1)
S1 Arithmetic error.

Attempts (3 marks)
A1 Norm of vector correct.

Part (b)	**20 (5, 10, 5) marks**	**Att (2, 3, 2)**
Part (b) (i)	**5 marks**	**Att 2**

2 (b) (i) *oabc* is a quadrilateral, where *o* is the origin.
$\overrightarrow{ad} = 3\overrightarrow{dc}$ and $\overrightarrow{ab} = 3\vec{c}$.

(i) Express $\vec{d}$ in terms of $\vec{a}$ and $\vec{c}$.



Express $\vec{d}$	**5 marks**	**Att 2**

2 (b) (i)

$$\vec{d} = \frac{\vec{a} + 3\vec{c}}{4} = \frac{1}{4}\vec{a} + \frac{3}{4}\vec{c}.$$

or

$\vec{d} = \vec{a} + \overrightarrow{ad} = \vec{a} + \frac{3}{4}\overrightarrow{ac}$ $= \vec{a} + \frac{3}{4}\left(\vec{c} - \vec{a}\right) = \frac{1}{4}\vec{a} + \frac{3}{4}\vec{c}$.

Blunders (-3)
B1 Error in ratio formula.
B2 $\overrightarrow{ac} = \vec{a} - \vec{c}$.
B3 Final answer not in terms of $\vec{a}$ and $\vec{c}$.

Slips (-1)
S1 Arithmetic error.

Attempts (2 marks)
A1 $\vec{d} = \vec{a} + \overrightarrow{ad}$.
A2 $\overrightarrow{ad} = \frac{3}{4}\overrightarrow{ac}$.

Part (b) (ii) **10 marks** **Att 3**

2 (b) (ii)	Express $\overrightarrow{db}$ in terms of $\vec{a}$ and $\vec{c}$.

Express $\overrightarrow{db}$ **10 marks** **Att 3**

2 (b) (ii)	$\overrightarrow{db} = \overrightarrow{da} + \overrightarrow{ab} = \frac{3}{4}\overrightarrow{ca} + 3\vec{c} = \frac{3}{4}\left(\vec{a} - \vec{c}\right) + 3\vec{c} = \frac{3}{4}\vec{a} + \frac{9}{4}\vec{c}$.

or

$\overrightarrow{db} = \vec{b} - \vec{d} = \vec{a} + \overrightarrow{ab} - \frac{1}{4}\vec{a} - \frac{3}{4}\vec{c} = \frac{3}{4}\vec{a} + \frac{9}{4}\vec{b}$.

Blunders (-3)
B1 $\overrightarrow{ca} = \vec{c} - \vec{a}$.
B2 Error in ratio formula.
B3 Final answer not in terms of $\vec{a}$ and $\vec{c}$.

Slips (-1)
S1 Arithmetic error.

Attempts (3 marks)
A1 $\overrightarrow{db} = \overrightarrow{da} + \overrightarrow{ab}$.
A2 $\overrightarrow{da} = \frac{3}{4}\overrightarrow{ca}$.
A3 $\overrightarrow{db} = \vec{b} - \vec{d}$.

Part (b) (iii) **5 marks** **Att 2**

2 (b) (iii) Show that *o*, *d* and *b* are collinear.

Show collinear **5 marks** **Att 2**

2 (b) (iii)

$$\vec{b} = \vec{a} + \vec{ab} = \vec{a} + 3\vec{c}$$

$$= 4\vec{od}. \qquad \therefore\ o, b \text{ and } d \text{ are collinear.}$$

or

$$\vec{db} = \frac{3}{4}\vec{a} + \frac{9}{4}\vec{c} = 3\left(\frac{1}{4}\vec{a} + \frac{3}{4}\vec{c}\right)$$

$$= 3\vec{d}. \qquad \therefore\ o, b \text{ and } d \text{ are collinear.}$$

or

$$\vec{ab} = 3\vec{c} \Rightarrow \vec{b} - \vec{a} = 3\vec{c}$$

$$\therefore \vec{b} = \vec{a} + 3\vec{c}. \text{ But } \vec{d} = \frac{1}{4}\left(\vec{a} + 3\vec{c}\right) \Rightarrow \vec{b} = 4\vec{d}. \quad \therefore o, b \text{ and } b \text{ collinear.}$$

Blunders (-3)

B1 $\vec{db}$ or $\vec{b}$ or $\vec{d}$ expressed in terms of $\vec{a}$ and $\vec{c}$ and stops.

Slips (-1)

S1 Arithmetic error.

Attempts (2 marks)

A1 $\vec{b} = \vec{a} + \vec{ab}$.

A2 $\vec{b} = 3\vec{c} + \vec{a}$.

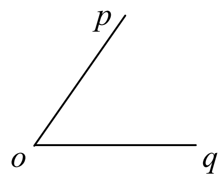
Part (c) **20 (10, 10)** **Att (3, 3)**

Part (c) (i) **10 marks** **Att 3**

2 (c) (i) *p* and *q* are points and *o* is the origin.

p, *q* and *o* are not collinear and $\left|\vec{p}\right| = \left|\vec{q}\right|$.

(i) Prove that $\vec{pq}$ is perpendicular to $\left(\vec{p} + \vec{q}\right)$.



Prove perpendicular **5 marks** **Att 3**

2 (c) (i)

$$\vec{pq}.\left(\vec{p} + \vec{q}\right) = \left(\vec{q} - \vec{p}\right)\left(\vec{q} + \vec{p}\right) = \left(\vec{q}\right)^2 - \left(\vec{p}\right)^2 = \left|\vec{q}\right|^2 - \left|\vec{p}\right|^2 = 0.$$

$$\therefore\ \vec{pq} \perp \left(\vec{p} + \vec{q}\right).$$

or

Let $\vec{p} = a\vec{i} + b\vec{j}$ and $\vec{q} = c\vec{i} + d\vec{j},\ a,b,c,d \in \mathbb{R}$.

$a^2 + b^2 = c^2 + d^2$ as $|\vec{p}| = |\vec{q}|$.

$$\vec{pq}.\left(\vec{p}+\vec{q}\right) = \left(\vec{q}-\vec{p}\right)\left(\vec{p}+\vec{q}\right) = \left[(c-a)\vec{i}+(d-b)\vec{j}\right].\left[(a+c)\vec{i}+(b+d)\vec{j}\right]$$

$$= (c-a)(a+c)+(d-b)(b+d) = c^2 - a^2 + d^2 - b^2$$

$$= (c^2 + d^2) - (a^2 + b^2)$$

$$= 0.$$

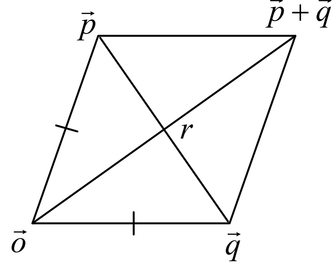
$$\therefore \vec{pq} \perp \left(\vec{p}+\vec{q}\right).$$

or

As $|\vec{p}| = |\vec{q}|$ then parallelogram is a rhombus.

$\therefore$ Diagonals are perpendicular.

$\therefore \vec{pq} \perp \left(\vec{p}+\vec{q}\right)$.



or

As $|\vec{p}| = |\vec{q}|$ triangle *oqr* and triangle *orp* and congruent.

$|\angle orq| = |\angle orp| = 90^0$.

$\therefore \vec{pq} \perp \left(\vec{p}+\vec{q}\right)$.

Blunders (-3)

B1 Incorrect vector multiplication.

B2 $\vec{pq} = \vec{p} - \vec{q}$.

B3 Error in formula for norm of vector.

B4 Stops at $|\vec{q}|^2 - |\vec{p}|^2$.

Slips (-1)

S1 Arithmetic error.

Attempts (3 marks)

A1 $\vec{p} = a\vec{i} + b\vec{j}$ and $\vec{q} = c\vec{i} + d\vec{j}$.

A2 Norm of vector correct.

A3 States congruent triangles or figure a rhombus.

A4 Stops at $c^2 - a^2 + d^2 - b^2$ and $a^2 + b^2 = c^2 + d^2$ not given.

Part (c) (ii) **10 marks** **Att 3**

2 (c) (ii)	Prove that $\vec{po}\,.\,\vec{pq} = \frac{1}{2}\left|\vec{pq}\right|^2$.

Prove **10 marks** **Att 3**

2 (c) (ii)

$$\vec{po}\,.\,\vec{pq} = -\vec{p}\left(\vec{q}-\vec{p}\right) = \left(\vec{p}\right)^2 - \vec{p}\,.\,\vec{q}$$

$$= \frac{1}{2}\left[2\left(\vec{p}\right)^2 - 2\,\vec{p}\,.\,\vec{q}\right] = \frac{1}{2}\left[\left(\vec{p}\right)^2 - 2\,\vec{p}\,.\,\vec{q} + \left(\vec{q}\right)^2\right], \text{ as } \left|\vec{p}\right| = \left|\vec{q}\right|.$$

$$= \frac{1}{2}\left[\left(\vec{q}-\vec{p}\right)^2\right] = \frac{1}{2}\left(\vec{pq}\right)^2 = \frac{1}{2}\left|\vec{pq}\right|^2, \text{ as } \vec{x}\,.\,\vec{x} = \left|\vec{x}\right|^2.$$

or

Let $\vec{p} = a\,\vec{i} + b\,\vec{j},\ \vec{q} = c\,\vec{i} + d\,\vec{j}$.

$$\left|\vec{p}\right| = \left|\vec{q}\right| \Rightarrow a^2 + b^2 = c^2 + d^2.$$

$$\vec{po}\,.\,\vec{pq} = \left(-a\,\vec{i} + b\,\vec{j}\right)\left[(c-a)\vec{i} + (d-b)\vec{j}\right]$$

$$= -a(c-a) - b(d-b) = -ac + a^2 - bd + b^2$$

$$= a^2 + b^2 - ac - bd.$$

But $\frac{1}{2}\left|\vec{pq}\right|^2 = \frac{1}{2}\left|(c-a)\vec{i} + (d-b)\vec{j}\right|^2 = \frac{1}{2}\left[(c-a)^2 + (d-b)^2\right]$

$$= \frac{1}{2}\left[c^2 - 2ac + a^2 + d^2 - 2bd + b^2\right]$$

$$= \frac{1}{2}\left[2a^2 + 2b^2 - 2ac - 2bd\right], \text{ as } a^2 + b^2 = c^2 + d^2.$$

$$= a^2 + b^2 - ac - bd.$$

$$\therefore\ \vec{po}\,.\,\vec{pq} = \frac{1}{2}\left|\vec{pq}\right|^2.$$

Blunders (-3)

B1 Incorrect vector multiplication.

B2 $\overrightarrow{pq} = \vec{p} - \vec{q}$

Slips (-1)

S1 Arithmetic error.

Attempts (3 marks)

A1 $\overrightarrow{pq}$ expressed in terms of $\vec{i}$ and $\vec{j}$

A2 $\frac{1}{2}\,|\,\overrightarrow{pq}\,|^2 = \frac{1}{2}(\vec{q} - \vec{p})^2$

A3 $\overrightarrow{po}.\overrightarrow{pq} = (\vec{p})^2 - \vec{p}.\vec{q}$.

QUESTION 3

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 (10, 10) marks**	**Att (3, 3)**
Part (c)	**20 marks (10, 10)**	**Att (3, 3)**

Part (a) **10 marks** **Att 3**

3 (a) $a(-1,4)$ and $b(9,-1)$ are two points and p is a point in $[ab]$.
Given that $|ap|:|pb| = 2:3$, find the co-ordinates of p.

Co-ordinates of *p* **10 marks** **Att 3**

3 (a) $p(x, y)$ where $x = \dfrac{mx_2 + nx_1}{m+n}$, $y = \dfrac{my_2 + ny_1}{m+n}$

$m = 2,\ n = 3,\ a(-1,\ 4) = (x_1,\ y_1)$ and $b(9,\ -1) = (x_2,\ y_2)$.

$\therefore\ x = \dfrac{18-3}{5} = 3$ and $y = \dfrac{-2+12}{5} = 2 \quad \Rightarrow \quad p(3,2)$.

or

a to *b*, *x* value up 10 units $\Rightarrow$ *a* to *p*, *x* value up 2/5(10) = 4
a to *b*, *y* value down 5 units $\Rightarrow$ *a* to *p*, *y* value down 2/5(−5) = −2
$\therefore$ *p* (3, 2).

Blunders (-3)
B1 Error in ratio formula.
B2 Error in translation.

Slips (-1)
S1 Arithmetic error.

Attempts (3 marks)
A1 Correct *x* or *y* value of point *p*.

Part (b) **20 (10, 10) marks** **Att (3, 3)**
Part (b) (i) **10 marks** **Att 3**

3 (b) (i) Calculate the perpendicular distance from the point (−1, −5) to the line $3x - 4y - 2 = 0$.

Calculate **10 marks** **Att 3**

3 (b) (i) Perpendicular distance $= \left|\dfrac{ax_1 + by_1 + c}{\sqrt{a^2+b^2}}\right| = \left|\dfrac{3(-1) - 4(-5) - 2}{\sqrt{9+16}}\right| = \left|\dfrac{15}{5}\right| = 3.$

Blunders (-3)
B1 Error in perpendicular distance formula.

Slips (-1)
S1 Arithmetic error.

Attempts (3 marks)

A1 Perpendicular distance with some substitution.

A2 Equation of line containing $(-1, -5)$ and perpendicular to $3x - 4y - 2 = 0$.

Part (b) (ii)	**10 marks**	**Att 3**

3 (b) (ii)	The point $(-1, -5)$ is equidistant from the lines $3x - 4y - 2 = 0$ and $3x - 4y + k = 0$, where $k \neq -2$. Find the value of *k*.

Value of *k*	**10 marks**	**Att 3**

3 (b) (ii)	$\left\lvert \frac{3(-1) - 4(-5) + k}{\sqrt{9+16}} \right\rvert = \left\lvert \frac{17+k}{5} \right\rvert = 3.$ $\therefore \lvert 17 + k \rvert = 15 \Rightarrow 17 + k = 15 \text{ or } 17 + k = -15$ $k \neq -2. \quad \therefore k = -32.$

Blunders (-3)

B1 Error in perpendicular distance formula.

B2 Incorrect application of absolute value.

Slips (-1)

S1 Arithmetic error.

Attempts (3 marks)

A1 Perpendicular distance to $3x - 4y + k = 0$.

A2 Work resulting in one *k* value of $k = -2$.

A3 Failure to use absolute value in distance formula.

Part (c)	**20 (5, 5, 10) marks**	**Att (2, 2, 3)**
Part (c) (i)	**10 (5, 5) marks**	**Att (2, 2)**

3 (c) (i)

f is the transformation $(x, y) \rightarrow (x', y')$, where $x' = 2x - y$ and $y' = x + y$.

L is the line $y = mx + c$.

K is the line through the origin that is perpendicular to L.

(i) Find the equation of $f(L)$ and the equation of $f(K)$.

Equation $f(L)$	**5 marks**	**Att 2**
Equation $f(K)$	**5 marks**	**Att 2**

3 (c) (i)

$$x' = 2x - y$$
$$\underline{y' = x + y}$$
$$x' + y' = 3x \Rightarrow x = \frac{1}{3}(x' + y')$$
$$y = y' - x = y' - \frac{1}{3}(x' + y') \Rightarrow y = \frac{1}{3}(-x' + 2y').$$
$$L: \ y = mx + c$$
$$\therefore f(L): \frac{1}{3}(-x' + 2y') = \frac{m}{3}(x' + y') + c$$
$$f(L): -x' + 2y' = mx' + my' + 3c$$
$$f(L): \ x'(m+1) + y'(m-2) + 3c = 0.$$

$$K: \ y = -\frac{1}{m}x$$
$$f(K): \frac{1}{3}(-x' + 2y') = -\frac{1}{m}.\frac{1}{3}(x' + y')$$
$$f(K): m(-x' + 2y') = -(x' + y')$$
$$\therefore f(K): x'(m-1) + y'(-2m-1) = 0.$$

Blunders (-3)

B1 Incorrect matrix.

B2 Incorrect matrix multiplication.

B3 Image line not in the form of $ax' + by' + c = 0$, apply once only.

Slips (-1)

S1 Arithmetic error.

Attempts (2, 2 marks)

A1 Expressing x or y in terms of primes in finding $f(L)$ or $f(K)$.

A2 Correct matrix for f in finding $f(L)$ or $f(K)$.

A3 Correct image point on $f(L)$.

A4 Correct image point on $f(K)$.

A5 Equation of line K.

Part (c) (ii) **10 marks** **Att 3**

3 (c) (ii)	Find the values of *m* for which $f(K) \perp f(L)$. Give your answer in surd form.

Values of *m* **10 marks** **Att 3**

3 (c) (ii)

$$\text{Slope } f(L) = \frac{m+1}{-m+2} \quad \text{and} \quad \text{Slope } f(K) = \frac{m-1}{2m+1}.$$

$$f(L) \perp f(K) \Rightarrow \frac{m+1}{-m+2} \cdot \frac{m-1}{2m+1} = -1$$

$$\therefore (m+1)(m-1) = (m-2)(2m+1)$$

$$m^2 - 1 = 2m^2 - 3m - 2 \Rightarrow m^2 - 3m - 1 = 0.$$

$$\therefore m = \frac{3 \pm \sqrt{9+4}}{2} = \frac{3 \pm \sqrt{13}}{2}.$$

Blunders (-3)
B1 Incorrect condition for perpendicularity.
B2 Error in quadratic formula.
B3 Error in determining slope, other than slip.

Slips (-1)
S1 Arithmetic error.

Attempts (3 marks)
A1 Slope of $f(L)$ or slope of $f(K)$.

QUESTION 4

Part (a)	**10 (5, 5) marks**	**Att (2, 2)**
Part (b)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**
Part (c)	**20 (10, 10) marks**	**Att (3, 3)**

Part (a) **10 marks (5, 5)** **Att (2, 2)**

4 (a) A is an acute angle such that $\tan A = \frac{8}{15}$.

Without evaluating A, find

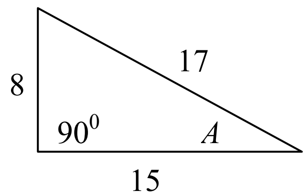
(i) $\cos A$

(ii) $\sin 2A$.

Find cos*A* **5 marks** **Att 2**

4 (a) (i)

$$\text{Tan } A = \frac{8}{15} \Rightarrow \cos A = \frac{15}{17}.$$



Find sin2*A* **5 marks** **Att 2**

4 (a) (ii)

$$\sin 2A = 2\sin A \cos A = 2\left(\frac{8}{15}\right)\left(\frac{15}{17}\right) = \frac{240}{289}.$$

or

$$\sin 2A = \frac{2\tan A}{1+\tan^2 A} = \frac{\frac{16}{15}}{1+\frac{64}{225}} = \frac{240}{289}.$$

Blunders (-3)

B1 Incorrect application of Pythagoras.

B2 Incorrect ratio of sides for cos *A*.

B3 Error in sin2*A* formula.

Slips (-1)

S1 Arithmetic error.

Attempts (2, 2 marks)

A1 Right angled triangle with $\angle A$, 8 and 15 correctly shown.

A2 Attempt at Pythagoras.

A3 Sin2*A* formula with some substitution.

Part (b)	20 (5, 5, 5, 5) marks	Att (2, 2, 2, 2)
Part (b) (i)	**10 marks (5, 5)**	**Att (2, 2)**

4 (b) (i)	Prove that $\cos 2A = \cos^2 A - \sin^2 A$. Deduce that $\cos 2A = 2\cos^2 A - 1$.

Prove Cos2*A*	5 marks	Att 2
Deduce Cos2*A*	**5 marks**	**Att 2**

4 (b) (i)

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

$\therefore \cos 2A = \cos A\cos A - \sin B\sin B = \cos^2 A - \sin^2 B.$

$\cos 2A = \cos^2 A - \sin^2 A = \cos^2 A - \left(1 - \cos^2 A\right)$

$= 2\cos^2 A - 1.$

Blunders (-3)

B1 Error in $\cos(A+B)$ formula.

B2 Error in $\sin^2 A$ conversion to $1-\cos^2 A$.

Slips (-1)

S1 Arithmetic error.

Attempts (2, 2 marks)

A1 Expansion of $\cos(A+B)$.

A2 $\sin^2 A = 1 - \cos^2 A$.

Part (b) (ii)	10 marks (5, 5)	Att (2, 2)

4 (b) (ii)	Hence, or otherwise, find the value of θ for which $2\cos\theta - 7\cos\left(\frac{\theta}{2}\right) = 0$, where $0^0 \le \theta \le 360^0$. Give your answer correct to the nearest degree.

Quadratic in Cos θ	5 marks	Att 2
Solve for $\theta/2$	**5 marks**	**Att 2**

4 (b) (ii)

$2\cos\theta - 7\cos\left(\frac{\theta}{2}\right) = 0$

$2\left[\cos^2\left(\frac{\theta}{2}\right) - 1\right] - 7\cos\left(\frac{\theta}{2}\right) = 0 \Rightarrow 4\cos^2\left(\frac{\theta}{2}\right) - 7\cos\left(\frac{\theta}{2}\right) - 2 = 0$

$\left[4\cos\left(\frac{\theta}{2}\right) + 1\right]\left[\cos\frac{\theta}{2} - 2\right] = 0$

$\therefore \cos\frac{\theta}{2} = -\frac{1}{4},\quad \cos\frac{\theta}{2} \ne 2$

$\frac{\theta}{2} = 104.47^0 \Rightarrow \theta = 208.94^0 = 209^0.$

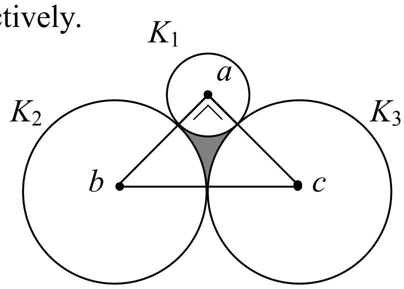
Blunders (-3)
B1 Incorrect substitution for $\cos\theta$.
B2 Error in factors.
B3 Error in quadratic formula.
B4 Incorrect solution.
B5 Correct solution for $\frac{\theta}{2}$ and stops.

Slips (-1)
S1 Arithmetic error.
S2 Solution not correct to nearest degree.

Attempts (2, 2 marks)
A1 $\cos\theta$ replaced by $\left(2\cos^2\frac{\theta}{2}-1\right)$.
A2 Correct factors.
A3 Solves quadratic and stops.

Part (c) **20 (10, 10) marks** **Att (3, 3)**
Part (c) (i) **10 marks** **Att 3**

4 (c) (i)
a, b and c are the centres of circles K_1, K_2 and K_3 respectively.
The three circles touch externally and $ab \perp ac$.
K_2 and K_3 each have radius $2\sqrt{2}$ cm.
(i) Find, in surd form, the length of the radius of K_1.



Radius of K_1 **10 marks** **Att 3**

4 (c) (i)

Let $|ab| = x, |ac| = x$

$|ab|^2 + |ac|^2 = |bc|^2 \Rightarrow 2x^2 = 32$

$x^2 = 16 \Rightarrow x = 4$

$\therefore$ radius $K_1 = 4 - 2\sqrt{2}$.

or

$|\angle abc| = 45^0$. $\therefore \cos 45^0 = \frac{|ab|}{|bc|} = \frac{|ab|}{4\sqrt{2}}$

$|ab| = 4\sqrt{2}\left(\frac{1}{\sqrt{2}}\right) = 4 \Rightarrow$ radius $K_1 = 4 - 2\sqrt{2}$.

Blunders (-3)
B1 Incorrect application of Pythagoras.
B2 Finds length of *ab* or *ac* and stops.

Slips (-1)
S1 Arithmetic error.

Attempts (3 marks)
A1 Length of *bc*.
A2 $\sin 45^0 = \dfrac{|ac|}{4\sqrt{2}}$ or $\cos 45^0 = \dfrac{|ab|}{4\sqrt{2}}$.
A3 Solution not in surd form.

Part (c) (ii)	**10 marks**	**Att 3**

4 (c) (ii)	Find the area of the shaded region in terms of π.

Area of shaded region	**10 marks**	**Att 3**

4 (c) (ii)

Area of shaded region = Area triangle $abc - K_1$ sector $- 2 \times K_2$ sector.

Area triangle $abc = \frac{1}{2}(4)(4) = 8$.

Area of smaller K_1 sector $= \frac{1}{2}r^2\theta = \frac{1}{2}\left(4-2\sqrt{2}\right)^2\frac{\pi}{2} = \frac{1}{4}\pi\left(16-16\sqrt{2}+8\right)$

$= \frac{1}{4}\pi\left(24-16\sqrt{2}\right) = 6\pi - 4\sqrt{2}\pi.$

$2 \times$ area of smaller K_2 sector $= \frac{1}{2}\left(2\sqrt{2}\right)^2\frac{\pi}{4}\times 2 = 2\pi$

$\therefore$ Area of shaded region $= 8 - 6\pi + 4\sqrt{2}\pi - 2\pi = 8 - 8\pi + 4\sqrt{2}\pi.$

Blunders (-3)
B1 Error in triangle area formula.
B2 Error in sector area formula.
B3 Finds area of triangle and sectors but fails to finish.

Slips (-1)
S1 Arithmetic error.

Attempts (3 marks)
A1 Area of triangle *abc*.
A2 Area of a sector.

QUESTION 5

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 (10, 10) marks**	**Att (3, 3)**
Part (c)	**20 (5, 5, 10) marks**	**Att (2, 2, 3)**

Part (a)	**10 marks**	**Att 3**

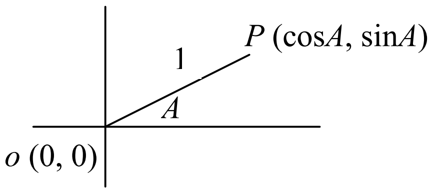
5 (a) Prove that $\cos^2 A + \sin^2 A = 1$, where $0^o \le A \le 90^o$.

Prove	**10 marks**	**Att 3**

5 (a)

$$|op|^2 = 1$$

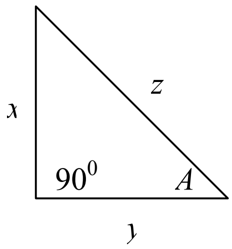
$$\therefore \cos^2 A + \sin^2 A = 1$$



or

$$\cos A = \frac{y}{z}, \quad \sin A = \frac{x}{z}$$

$$\cos^2 A + \sin^2 A = \frac{y^2 + x^2}{z^2} = \frac{z^2}{z^2} = 1$$



Blunders (-3)

B1 cos$A$ or sin$A$ expressed incorrectly as ratio of triangle sides.

B2 Error in distance formula.

Slips (-1)

S1 Arithmetic error.

Attempts (3 marks)

A1 Polar point $(\cos A, \sin A)$.

A2 cos$A$ or sin$A$ expressed as ratio of sides.

Part (b) **20 (10, 10) marks** **Att (3, 3)**

Part (b) (i) **10 marks** **Att 3**

5 (b) (i)	Show that $(\cos x + \sin x)^2 + (\cos x - \sin x)^2$ simplifies to a constant.

Show **10 marks** **Att 3**

5 (b) (i)	$(\cos x + \sin x)^2 + (\cos x - \sin x)^2$ $= \cos^2 x + 2\cos x \sin x + \sin^2 x + \cos^2 x - 2\cos x \sin x + \sin^2 x$ $= 2(\cos^2 x + \sin^2 x) = 2.$

Blunders (-3)

B1 Incorrect squaring.

B2 Stops at $2\sin^2 x + 2\cos^2 x$.

Slips (-1)

S1 Arithmetic error.

Attempts (3 marks)

A1 $(\cos x + \sin x)^2$ or $(\cos x - \sin x)^2$ squared correctly.

Part (b) (ii) **10 marks** **Att 3**

5 (b) (ii)	Express $1 - (\cos x - \sin x)^2$ in the form $a \sin bx$, where $a, b \in \mathbf{Z}$.

Express **10 marks** **Att 3**

5 (b) (ii)	$1 - (\cos x - \sin x)^2 = 1 - (\cos^2 x - 2\sin x \cos x + \sin^2 x)$ $= 1 - 1 + 2\sin x \cos x = 2\sin x \cos x$ $= \sin 2x.$

Blunders (-3)

B1 Incorrect squaring.

Slips (-1)

S1 Arithmetic error.

Attempts (3 marks)

A1 $1 - (\cos x - \sin x)^2$ with brackets removed.

A2 Replaces $\cos^2 x + \sin^2 x$ by 1.

A3 $1 - \sin^2 x = \cos^2 x$.

A4 $2\sin x \cos x$ replaced by $\sin 2x$.

Part (c) **20 (5, 5, 10) marks** **Att (2, 2, 3)**

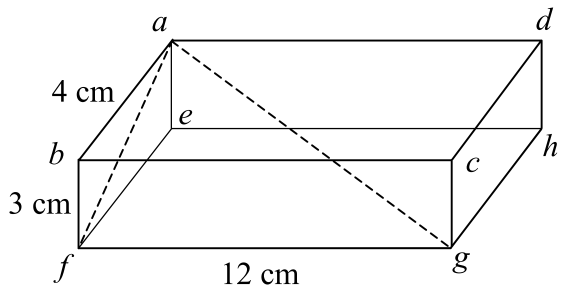
Part (c) (i) **5 marks** **Att 2**

5 (c) (i)

The diagram shows a rectangular box. Rectangle *abcd* is the top of the box and rectangle *efgh* is the base of the box.

$|ab| = 4$ cm, $|bf| = 3$ cm and $|fg| = 12$ cm.

(i) Find $|af|$.



Find $|af|$ **5 marks** **Att 2**

5 (c) (i)

$$|af|^2 = |ab|^2 + |bf|^2 = 16 + 9 = 25$$

$$\therefore |af| = 5.$$

Blunders (-3)

B1 Incorrect application of Pythagoras.

Slips (-1)

S1 Arithmetic error.

Attempts (2 marks)

A1 Application of Pythagoras.

Part (c) (ii) **5 marks** **Att 2**

5 (c) (ii) Find $|ag|$.

Find $|ag|$ **5 marks** **Att 2**

5 (c) (ii)

$$|ag|^2 = |af|^2 + |fg|^2 = 25 + 144 = 169$$

$$\therefore |ag| = 13.$$

Blunders (-3)

B1 Incorrect application of Pythagoras.

Slips (-1)

S1 Arithmetic error.

Attempts (2 marks)

A1 Application of Pythagoras.

Part (c) (iii)	**10 marks**	**Att 3**

5 (c) (iii)	Find the measure of the acute angle between $[ag]$ and $[df]$ Give your answer correct to the nearest degree.

Measure of acute angle	**10 marks**	**Att 3**

5 (c) (iii) Let *ag* and *fd* intersect at the point *r*.
As they bisect each other then $|ar| = 6.5$ and $|fr| = 6.5$.

$$\cos\angle arf = \frac{|ar|^2 + |fr|^2 - |af|^2}{2|ar|.|fr|} = \frac{(6.5)^2 + (6.5)^2 - 5^2}{2(6.5)(6.5)}$$

$$\cos\angle arf = \frac{59.5}{84.5} \Rightarrow |\angle aef| = 45.2^0 = 45^0.$$

Blunders (-3)
B1 Error in substitution into cosine formula.
B2 Incorrect evaluation of angle.
B3 Obtuse angle given as solution.

Slips (-1)
S1 Arithmetic error.
S2 Angle not correct to nearest degree.

Attempts (3 marks)
A1 Cosine formula with some substitution.

QUESTION 6

Part (a)	**10 (5, 5) marks**	**Att (- , 2)**
Part (b)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**
Part (c)	**20 (5, 5, 10) marks**	**Att (2, 2, 3)**

Part (a) **10 (5, 5) marks** **Att (- , 2)**

Part (a) (i) **5 marks** **Hit/Miss**

6 (a) (i) A committee of five is to be selected from six students and three teachers.
(i) How many different committees of five are possible?

Committees of five **5 marks** **Hit/Miss**

6 (a) (i) $^{9}C_{5} = 126.$

Part (a) (ii) **5 marks** **Att 2**

6 (a) (ii) How many of these possible committees have three students and two teachers?

Possible committees **5 marks** **Att 2**

6 (a) (ii) $^{6}C_{5} \times {}^{3}C_{2} = 20 \times 3 = 60.$

Blunders (-3)
B1 $^{6}C_{3} + {}^{3}C_{2}$.

Slips (-1)
S1 Arithmetic error.

Attempts (2 marks)
A1 $^{6}C_{3}$ or $^{3}C_{2}$.

Part (b)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**
Part (b) (i)	**15 (5, 5, 5) marks**	**Att (2, 2, 2)**

6 (b) (i) Solve the difference equation $3u_{n+2} - 2u_{n+1} - u_n = 0$, where $n \geq 0$, given that $u_0 = 3$ and $u_1 = 7$.

Characteristic roots	**5 marks**	**Att 2**
Simultaneous equations	**5 marks**	**Att 2**
Final solution	**5 marks**	**Att 2**

6 (b) (i)

$$3u_{n+2} - 2u_{n+1} - u_n = 0$$

$$3x^2 - 2x - 1 = 0$$

$$(3x+1)(x-1) = 0 \Rightarrow 3x+1 = 0 \text{ or } x - 1 = 0$$

$$\therefore x = -\frac{1}{3} \text{ or } x = 1.$$

$$u_n = l(\alpha)^n + k(\beta)^n \Rightarrow u^n = l(1)^n + k\left(-\frac{1}{3}\right)^n$$

$$u_0 = 3 \Rightarrow l + k = 3$$

$$u_1 = 7 \Rightarrow l - \frac{1}{3}k = 7$$

$$\frac{4}{3}k = -4 \Rightarrow k = -3 \text{ and } l = 6$$

$$u_n = 6(1)^n - 3\left(-\frac{1}{3}\right)^n = 6 - 3\left(-\frac{1}{3}\right)^n.$$

Blunders (-3)
B1 Error in characteristic equation.
B2 Error in factors or in quadratic formula.
B3 Incorrect use of initial conditions.

Slips (-1)
S1 Arithmetic error.

Attempts (2, 2, 2 marks)
A1 Correct characteristic equation.
A2 An equation in *k* and *l*.
A3 Correct value for *k* or *l*.

Part (b) (ii)	**5 marks**	**Att 2**

6 (b) (ii) Evaluate $\lim_{n\to\infty} u_n$.

Evaluate	**5 marks**	**Att 2**

6 (b) (ii)

$$\lim_{n\to\infty}\left[6 - 3\left(-\frac{1}{3}\right)^n\right] = 6 - 0 = 6.$$

Blunders (-3)

B1 $\lim_{n\to\infty}\left(-\frac{1}{3}\right)^n$ incorrect.

Slips (-1)

S1 Arithmetic error.

Attempts (2 marks)

A1 $\lim_{n\to\infty}\left(-\frac{1}{3}\right)^n$ correct.

Part (c)	**20 (5, 5, 10) marks**	**Att (2, 2, 3)**
Part (c) (i)	**5 marks**	**Att 2**

6 (c) (i)	Eight cards are numbered 1 to 8. The cards numbered 1 and 2 are red, the cards numbered 3 and 4 are blue, the cards numbered 5 and 6 are yellow and the cards numbered 7 and 8 are black. Four cards are selected at random from the eight cards. Find the probability that the four cards selected are: **(i)** all of different colours

All of different colour	**5 marks**	**Att 2**

6 (c) (i) P(all of different colour) $= \frac{{}^2C_1 \times {}^2C_1 \times {}^2C_1 \times {}^2C_1}{{}^8C_4} = \frac{16}{70} = \frac{8}{35}.$

or

$$P = \frac{2}{8} \times \frac{2}{7} \times \frac{2}{6} \times \frac{2}{5} \times 4! = \frac{8}{35}.$$

or

$$P = \frac{8}{8} \times \frac{6}{7} \times \frac{4}{6} \times \frac{2}{5} = \frac{8}{35}.$$

Blunders (-3)

B1 Incorrect number of possible outcomes.

B2 Addition of probabilities.

Slips (-1)

S1 Arithmetic error.

Attempts (2 marks)

A1 Correct number of possible outcomes.

A2 Correct number of favourable outcomes.

Part (c) (ii) **5 marks** **Att 2**

6 (c) (ii)	Find the probability that the four cards selected are: **(ii)** two odd-numbered cards and two even-numbered cards

Probability **5 marks** **Att 2**

6 (c) (ii) Probability $= \dfrac{{}^4C_2 \times {}^4C_2}{{}^8C_4} = \dfrac{36}{70} = \dfrac{18}{35}$.

or

Probability $= \dfrac{4}{8} \times \dfrac{3}{7} \times \dfrac{4}{6} \times \dfrac{3}{5} \times {}^4C_2 = \dfrac{18}{35}$.

Blunders (-3)
B1 Incorrect number of possible outcomes.
B2 Addition of probabilities.

Slips (-1)
S1 Arithmetic error.

Attempts (2 marks)
A1 Correct number of possible outcomes.
A2 Correct number of favourable outcomes.

Part (c) (iii) **10 marks** **Att 3**

6 (c) (iii)	Find the probability that the four cards selected are: **(iii)** all of different colours, two odd-numbered and two even-numbered.

Probability **10 marks** **Att 3**

6 (c) (iii) Probability $= \dfrac{{}^4C_2}{{}^8C_4} = \dfrac{6}{70} = \dfrac{3}{35}$.

or

{1, 3, 6, 8}, {1, 4, 5, 8}, {1, 4, 6, 7}, {2, 3, 5, 8}, {2, 3, 6, 7}, {2, 4, 5, 7}

Probability $= \dfrac{6}{70} = \dfrac{3}{35}$

or

Probability $= \dfrac{4}{8} \times \dfrac{3}{7} \times \dfrac{2}{6} \times \dfrac{1}{5} \times {}^4C_2 = \dfrac{3}{35}$.

Blunders (-3)
B1 Incorrect number of possible outcomes.
B2 Addition of probabilities.

Slips (-1)
S1 Arithmetic error.

Attempts (3 marks)
A1 Correct number of possible outcomes.
A2 Correct number of favourable outcomes.

QUESTION 7

Part (a)	**10 (5, 5) marks**	**Att(- , -)**
Part (b)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**
Part (c)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**

Part (a) **10 (5, 5) marks** **Att(- , -)**
Part (a) (i) **5 marks** **Hit/Miss**

7 (a) (i) At the Olympic Games, eight lanes are marked on the running track.
Each runner is allocated to a different lane. Find the number of ways in which the runners in a heat can be allocated to these lanes when there are

(i) eight runners in the heat

Eight runners in a heat **5 marks** **Hit/Miss**

7 (a) (i) $8! = 40320$ **or** ${}^{8}P_{8} = 40320.$

Part (a) (ii) **5 marks** **Hit/Miss**

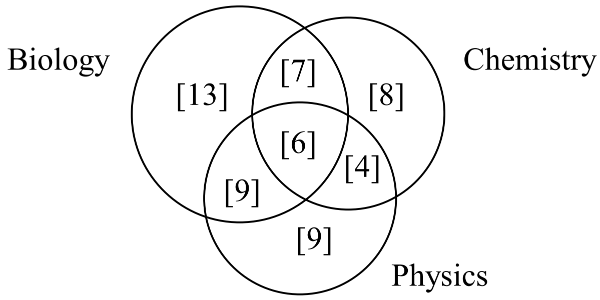
7 (a) (ii) **…**when there are
(ii) five runners in the heat and any five lanes may be used.

Five runners in a heat **5 marks** **Hit/Miss**

7 (a) (ii) ${}^{8}P_{5} = 6720.$

Part (b) **20 (5, 5, 5, 5)** **Att (2, 2, 2, 2)**
Part (b) (i) **5 marks** **Att 2**

7 (b) (i) In a class of 56 students, each studies at least one of the subjects Biology, Chemistry, Physics. The Venn diagram shows the numbers of students studying the various combinations of subjects.



(i) A student is picked at random from the whole class.
Find the probability that the student does not study Biology.

Probability **5 marks** **Att 2**

7 (b) (i) Probability = $\frac{21}{56} = \frac{3}{8}.$

or

Probability = 1−P(student does study Biology) $= 1 - \frac{35}{56} = \frac{21}{56} = \frac{3}{8}.$

Blunders (-3)
B1 Incorrect number of possible outcomes.

Slips (-1)
S1 Arithmetic error.

Attempts (2 marks)
A1 Correct number of possible outcomes.
A2 Correct number of favourable outcomes.

Part (b) (ii)	**5 marks**	**Att 2**

7 (b) (ii)	A student is picked at random from those who study at least two of the subjects. Find the probability that the student does not study Biology.

Probability	**5 marks**	**Att 2**

7 (b) (ii) Probability $= \frac{4}{26} = \frac{2}{13}$.

or

Probability = 1–P (student does study Biology) $= 1 - \frac{22}{26} = \frac{4}{26} = \frac{2}{13}$.

Blunders (-3)
B1 Incorrect number of possible outcomes.

Slips (-1)
S1 Arithmetic error.

Attempts (2 marks)
A1 Correct number of possible outcomes.
A2 Correct number of favourable outcomes.

Part (b) (iii)	**5 marks**	**Att 2**

7 (b) (iii)	Two students are picked at random from the whole class. Find the probability that they both study Physics.

Probability	**5 marks**	**Att 2**

7 (b) (iii) Probability $= \frac{{}^{28}C_2}{{}^{56}C_2} = \frac{27}{110}$.

or

Probability $= \frac{28}{56} \times \frac{27}{55} = \frac{27}{110}$.

Blunders (-3)
B1 Incorrect number of possible outcomes.

Slips (-1)
S1 Arithmetic error.

Attempts (2 marks)

A1 Correct number of possible outcomes.

A2 Correct number of favourable outcomes.

Part (b) (iv) **5 marks** **Att 2**

7 (b) (iv)	Two students are picked at random from those who study Chemistry. Find the probability that exactly one of them studies Biology.

Probability **5 marks** **Att 2**

(b) (iv) Probability $= \frac{^{13}C_1 \times ^{12}C_1}{^{25}C_2} = \frac{13}{25}.$

or

Probability $= \frac{13}{25} \times \frac{12}{24} \times 2 = \frac{13}{25}.$

Blunders (-3)

B1 Incorrect number of possible outcomes.

Slips (-1)

S1 Arithmetic error.

Attempts (2 marks)

A1 Correct number of possible outcomes.

A2 Correct number of favourable outcomes.

Part (c) **20 marks (5, 5, 5, 5)** **Att (2, 2, 2, 2)**

Part (c) (i) **5 marks** **Att 2**

7 (c) (i)	The mean of the real numbers p, q and r is $\bar{x}$ and the standard deviation is σ. **(i)** Show that the mean of the four numbers p, q, r and $\bar{x}$ is also $\bar{x}$.

Show mean **5 marks** **Att 2**

7 (c) (i) $\bar{x} = \frac{p+q+r}{3} \Rightarrow p+q+r = 3\bar{x}.$

$$\frac{p+q+r+\bar{x}}{4} = \frac{3\bar{x}+\bar{x}}{4} = \bar{x}.$$

Blunders (-3)

B1 Error in mean.

Slips (-1)

S1 Arithmetic error.

Attempts (2 marks)

A1 Correct mean of p, q and r.

A2 Correct mean of p, q, r and $\bar{x}$ in terms of p, q, r and $\bar{x}$.

Part (c) (ii)	15 marks	Att (2, 2, 2)

7 (c) (ii) The standard deviation of p, q, r and $\overline{x}$ is k.
Show that $k : \sigma = \sqrt{3} : 2$.

Standard deviation σ found	5 marks	Att 2
Standard deviation *k* found	5 marks	Att 2
Finish	5 marks	Att 2

7 (c) (ii)

$$\sigma = \sqrt{\frac{(p-\overline{x})^2 + (q-\overline{x})^2 + (r-\overline{x})^2}{3}} = \frac{1}{\sqrt{3}}\sqrt{(p-\overline{x})^2 + (q-\overline{x})^2 + (r-\overline{x})^2}$$

$$k = \sqrt{\frac{(p-\overline{x})^2 + (q-\overline{x})^2 + (r-\overline{x})^2 + (\overline{x}-\overline{x})^2}{4}} = \frac{1}{2}\sqrt{(p-\overline{x})^2 + (q-\overline{x})^2 + (r-\overline{x})^2}$$

$$\therefore\ k : \sigma = \frac{1}{2} : \frac{1}{\sqrt{3}} = \sqrt{3} : 2.$$

or

$$(\text{Standard deviation})^2 = \left(\frac{1}{n}\sum x^2\right) - (\overline{x})^2$$

$$\sigma = \sqrt{\frac{1}{3}\left(p^2 + q^2 + r^2\right) - (\overline{x})^2} = \frac{1}{\sqrt{3}}\sqrt{\left(p^2 + q^2 + r^2 - 3(\overline{x})^2\right)}$$

$$k = \sqrt{\frac{1}{4}\left(p^2 + q^2 + r^2 + (x)^2\right) - (\overline{x})^2} = \frac{1}{2}\sqrt{\left(p^2 + q^2 + r^2 - 3(\overline{x})^2\right)}$$

$$\therefore\ k : \sigma = \frac{1}{2} : \frac{1}{\sqrt{3}} = \sqrt{3} : 2.$$

Blunders (-3)
B1 Error in standard deviation.
B2 Any incorrect step in calculating σ or *k*.

Slips (-1)
S1 Arithmetic error.

Attempts (2, 2, 2 marks)
A1 Any correct deviation in calculating σ or *k*.
A2 Final ratio expressed but not simplified to $\sqrt{3} : 2$.
A3 Standard Deviation $= \sqrt{\left(\frac{1}{n}\sum x^2\right) - (\overline{x})^2}$.

QUESTION 8

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**
Part (c)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**

Part (a) **10 marks** **Att 3**

8 (a) Use integration by parts to find $\int x \sin x dx$.

Integration by parts **10 marks** **Att 3**

8 (a) $\int x\sin x dx = \int u dv = uv - \int v du.$

$u = x \Rightarrow du = dx. \quad dv = \sin x dx \Rightarrow v = \int \sin x dx = -\cos x.$

$\therefore \int x\sin x dx = -x\cos x + \int \cos x dx = -x\cos x + \sin x + \text{constant}.$

Blunders (-3)
B1 Incorrect differentiation or integration.
B2 Constant of integration omitted.
B3 Incorrect 'parts' formula.

Slips (-1)
S1 Arithmetic error.

Attempts (3 marks)
A1 Correct assigning to parts formula.
A2 Correct differentiation or integration.

Part(b) **20 marks (5, 5, 5, 5)** **Att (2, 2, 2, 2)**
Part (b) (i) **5 marks** **Att 2**

8 (b) (i) $f(x) = f(0) + \frac{f'(0)x}{1!} + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} + \ldots$ is the Maclaurin series.

(i) Derive the first five terms of the Maclaurin series for e^x.

Maclaurin series for e^x **5 marks** **Att 2**

8 (b) (i)
$f(x) = e^x \Rightarrow f(0) = 1$

$f'(x) = e^x \Rightarrow f'(0) = 1$

$f''(x) = e^x \Rightarrow f''(0) = 1$

$f'''(x) = e^x \Rightarrow f'''(0) = 1$

$f^{(iv)}(x) = e^x \Rightarrow f^{(iv)}(0) = 1$

$\therefore f(x) = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots$

Blunders (-3)
B1 Incorrect differentiation.
B2 Incorrect evaluation of $f^{(n)}(0)$.
B3 Each term not derived.

Slips (-1)
S1 Arithmetic error.

Attempts (2 marks)
A1 Correct expansion of e^x given but not derived.
A2 $f(0)$ correct.
A3 A correct differentiation.
A4 Any one correct term.

Part (b) (ii) **5 marks** **Att 2**

8 (b) (ii) Hence write down the first five terms of the Maclaurin series for e^{-x} and deduce the first three non-zero terms of the series for $\frac{e^x + e^{-x}}{2}$.

Hence and deduce **5 marks** **Att 2**

8 (b) (ii)

$$e^{-x} = 1 - \frac{x}{1!} + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!}$$

$$\therefore \frac{1}{2}\left(e^x + e^{-x}\right) = \frac{1}{2}\left(2 + \frac{2x^2}{2!} + \frac{2x^4}{4!}\right) = 1 + \frac{x^2}{2!} + \frac{x^4}{4!}$$

Blunders (-3)
B1 Incorrect sign in term due to incorrect squaring, cubing etc.
B2 Terms of $\frac{1}{2}\left(e^x + e^{-x}\right)$ not simplified to final answer.

Slips (-1)
S1 Arithmetic error.

Attempts (2 marks)
A1 Power series of e^{-x}.

Part (b) (iii) **10 (5, 5) marks** **Att (2, 2)**

8 (b) (iii) Write the general term of the series for $\frac{e^x + e^{-x}}{2}$ and use the Ratio Test to show that the series converges for all x.

General Term | **5 marks** | **Att 2**
Ratio Test | **5 marks** | **Att 2**

8 (b) (iii) $u_n = \dfrac{x^{2n-2}}{(2n-2)!} \quad \Rightarrow \quad u_{n+1} = \dfrac{x^{2n}}{(2n)!}$

$$\underset{n\to\infty}{\text{Limit}}\left|\frac{u_{n+1}}{u_n}\right| = \underset{n\to\infty}{\text{Limit}}\left|\frac{x^{2n}}{(2n)!} \times \frac{(2n-2)!}{x^{2n-2}}\right| = \underset{n\to\infty}{\text{Limit}}\left|\frac{x^2}{2n(2n-1)}\right| = |0|$$

$< 1 \quad \therefore$ Convergent.

Blunders (-3)

B1 Incorrect power in general term.
B2 Incorrect factorial expression.
B3 Error in u_{n+1}.
B4 Error in evaluating limit, other than slip.
B5 Evaluates limit as 0 and stops.

Slips (-1)

S1 Arithmetic error.

Attempts (2 marks)

A1 Power of x correct.
A2 Factorial correct.
A3 u_{n+1} correct.

Part (c) | **20 (5, 5, 5, 5) marks** | **Att (2, 2, 2, 2)**
Part (c) (i) | **5 marks** | **Att 2**

8 (c) (i) A solid cylinder has height h and radius r. The height of the cylinder, added to the circumference of its base, is equal to 3 metres.
(i) Express the volume of the cylinder in terms of r and π.

Express volume | **5 marks** | **Att 2**

8 (c) (i) $h + 2\pi r = 3 \quad \Rightarrow \quad h = 3 - 2\pi r$

Volume $= \pi r^2 h = \pi r^2 (3 - 2\pi r)$

$V = 3\pi r^2 - 2\ {}^2 r^3$.

Blunders (-3)

B1 Error in circumference formula.
B2 Error in volume formula.

Slips (-1)

S1 Arithmetic error.

Attempts (2 marks)

A1 $h + 2\pi r = 3$.

Part (c) (ii) **15 marks (5, 5, 5)** **Att (2, 2, 2)**

8 (c) (ii)	Find the maximum possible volume of the cylinder in terms of π.

Correct differentiation	**5 marks**	**Att 2**
Value of *r*	**5 marks**	**Att 2**
Maximum volume	**5 marks**	**Att 2**

8 (c) (ii)

$$V = 3\pi r^2 - 2\ \ {}^2 r^3$$

$$\frac{dV}{dr} = 6\pi r - 6\ \ {}^2 r^2$$

$$\text{For maximum, } \frac{dV}{dr} = 0$$

$$\therefore 6\pi r - 6\ \ {}^2 r^2 = 0 \quad \Rightarrow \quad r - \pi r^2 = 0$$

$$r(1 - \pi r) = 0 \quad \Rightarrow \quad r = \frac{1}{\ } \quad as \quad r \neq 0.$$

$$r = \frac{1}{\ } \quad \Rightarrow \quad V = \frac{3}{\ } - \frac{2}{\ } = \frac{1}{\ }.$$

$$\frac{d^2V}{dr^2} = 6\ \ - 12\ \ {}^2 r$$

$$\text{For } r = \frac{1}{\ }, \quad \frac{d^2V}{dr} = 6\ \ - 12\ \ = -6\pi < 0.$$

$$\therefore \text{ Maximum volume } = \frac{1}{\pi}\text{m}^3.$$

* $\frac{d^2v}{dr^2} < 0$ for $r = \frac{1}{\pi}$ not required.

Blunders (-3)
B1 Error in differentiation.
B2 Error in factors.
B3 Values of *r* and *h* found but maximum volume not evaluated.

Slips (-1)
S1 Arithmetic error.

Attempts (2, 2, 2 marks)
A1 Some part of differentiation correct.
A2 $\frac{dv}{dr} = 0$.
A3 Value of *h* found.

QUESTION 9

Part (a)	**10 marks**	**Hit/Miss**
Part (b)	**20 (5, 5, 10) marks**	**Att (2, 2, 3)**
Part (c)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**

Part (a) **10 marks** **Hit/Miss**

9 (a) z is a random variable with standard normal distribution.
Find the value of z_1 for which $P(z \le z_1) = 0.9370$.

Value of z_1 **10 marks** **Hit/Miss**

9 (a) $P(z \le z_1) = 0.9370$
$\therefore z_1 = 1.53.$

Part (b) **20 (5, 5, 10,) marks** **Att (2, 2, 3)**
Part (b) (i) **5 marks** **Att 2**

9 (b) (i) A child throws a ball at a group of three skittles. The probability that the ball will knock 0, 1, 2 or 3 of the skittles is given in the following probability distribution table:

x	0	1	2	3
$P(x)$	0.1	0.1	0.5	k

(i) Find the value of k.

Value of k **5 marks** **Att 2**

9 (b) (i) $\sum P(w) = 1 \Rightarrow 0.1 + 0.1 + 0.5 + k = 1. \quad \therefore k = 0.3.$

Slips (-1)
S1 Arithmetic error.

Attempts (2 marks)
A1 $\sum P(x) = 1.$
A2 $0.7 + k.$

Part (b)(ii) **5 marks** **Att 2**

9 (b) (ii) Find the mean of the distribution.

Find mean **5 marks** **Att 2**

9 (b) (ii) Mean = $\bar{x} = \sum_{x=0}^{3} xP(x) = 0 + 0.1 + 1 + 0.9 = 2.$

Blunders (-3)
B1 Incorrect formula for mean.

Slips (-1)

S1 Arithmetic error.

Attempts (2 marks)

A1 Any correct $x.P(x)$.

A2 $\dfrac{\sum x.P(x)}{\sum P(x)}$ with some substitution.

Part (b) (iii) **10 marks** **Att 3**

9 (b) (iii)	Find the standard deviation of the distribution, correct to two decimal places.

Find standard deviation **10 marks** **Att 3**

9 (b) (iii)

$$\sigma = \sqrt{\sum_{x=0}^{3} (x - \bar{x})^2 . P(w)}$$

$$\sigma = \sqrt{(0-2)^2.(0.1) + (1-2)^2.(0.1) + (2-2)^2.(0.5) + (3-2)^2.(0.3)}$$

$$\sigma = \sqrt{0.4 + 0.1 + 0 + 0.3} = \sqrt{0.8}$$

$$\sigma = 0.89.$$

or

$$\sigma^2 = \sum x^2 P(x) - (\bar{x})^2 = 0^2(0.1) + 1^2(0.1) + 2^2(0.5) + 3^2(0.3) - (2)^2$$

$$\sigma = \sqrt{0.1 + 2 + 2.7 - 4} = \sqrt{4.8 - 4} = \sqrt{0.8} = 0.89.$$

Blunders (-3)

B1 Incorrect deviation.

B2 Incorrect d^2.

B3 Stops at $\sqrt{0.8}$.

Slips (-1)

S1 Arithmetic error.

S2 Solution not correct to two decimal places.

Attempts (3 marks)

A1 Any correct deviation.

A2 Any $(x - \bar{x})^2 P(x)$ correct.

Part (c)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**

9 (c) Before local elections, a political party claimed that 30% of the voters supported it. In a random sample of 1500 voters, 400 said they would vote for that party.
Test the party's claim at the 5% level of significance.

Find $\bar{x}$	**5 marks**	**Att 2**
Find σ	**5 marks**	**Att 2**
Standard units	**5 marks**	**Att 2**
Conclusion	**5 marks**	**Att 2**

9 (c)

H_0: Claim is true.

$$n = 1500. \quad p = \frac{30}{100} = \frac{3}{10} \Rightarrow q = \frac{7}{10}.$$

$$\bar{x} = np = 1500\left(\frac{3}{10}\right) = 450.$$

$$\sigma = \sqrt{npq} = \sqrt{1500\left(\frac{3}{10}\right)\left(\frac{7}{10}\right)} = \sqrt{315}.$$

$$z = \frac{x - \bar{x}}{\sigma} = \frac{400 - 450}{\sqrt{315}} = \frac{-50}{\sqrt{315}} = -2.817.$$

$$z = -2.817 < -1.96$$

$\therefore$ Do not accept $H_0 \Rightarrow$ Party's claim not justified.

Blunders (-3)

B1 Incorrect value of p or of q.
B2 Incorrect formula for mean.
B3 Incorrect formula for σ.
B4 Error in standard units.
B5 Uses one tailed test.
B6 Incorrect conclusion.

Slips (-1)

S1 Arithmetic error.

Attempts (2, 2, 2, 2 marks)

A1 Correct value of p or of q.
A2 Correct formula for mean.
A3 Correct formula for σ.
A4 Correct expression for standard units.
A5 States two tailed test.

QUESTION 10

Part (a)	**30 (5, 5, 5, 10, 5) marks**	**Att (2, 2, 2, 3, 2)**
Part (b)	**20 (10, 10) marks**	**Att (3, 3)**

Part (a) **30 (5, 5, 5, 10, 5) marks** **Att (2, 2, 2, 3, 2)**

Part (a) (i) **5 marks** **Att 2**

10 (a) (i) The binary operation $*$ is defined by $a * b = a + b - ab$, where $a, b \in \mathbf{R} \setminus \{1\}$.
(i) Find the identity element.

Find identity element **5 marks** **Att 2**

10 (a) (i) Let e be the identity element.

$e * b = b$

$e * b = e + b - eb$

$\therefore e + b - eb = b$

$e(1 - b) = 0 \;\Rightarrow\; e = 0,\; (b \neq 1).$

Blunders (-3)
B1 $e * b$ incorrect.
B2 $e - eb = 0$ and stops.

Slips (-1)
S1 Arithmetic error.

Attempts (2 marks)
A1 $e * b = b$.
A2 $e * b = e + b - eb$.

Part (a) (ii) **5 marks** **Att 2**

10 (a) (ii) Calculate 3^{-1}, the inverse of 3.

Calculate 3^{-1} **5 marks** **Att 2**

10 (a) (ii) $3 * 3^{-1} = 3 + 3^{-1} - 3\left(3^{-1}\right) = 0$

$2\left(3^{-1}\right) = 3 \;\Rightarrow\; 3^{-1} = \frac{3}{2}.$

Blunders (-3)
B1 $3 * 3^{-1}$ incorrect.

Slips (-1)
S1 Arithmetic error.

Attempts (2 marks)
A1 $3 * 3^{-1}$ correct.
A2 $3 * 3^{-1} = 0$.

Part (a) (iii)	**5 marks**	**Att 2**

10 (a) (iii) Find x^{-1} in terms of x.

Find x^{-1}	**5 marks**	**Att 2**

10 (b) (iii) $x * x^{-1} = x + x^{-1} - x(x^{-1}) = 0$

$$x^{-1}(x-1) = x$$

$$\therefore\ x^{-1} = \frac{x}{x-1}.$$

Blunders (-3)
B1 $x * x^{-1}$ incorrect.

Slips (-1)
S1 Arithmetic error.

Attempts (2 marks)
A1 $x * x^{-1}$ correct.
A2 $x * x^{-1} = 0$.

Part (a) (iv)	**10 marks**	**Att 3**

10 (a) (iv) Show that $(a * b) * c = a * (b * c)$.

Show	**10 marks**	**Att 3**

10 (a) (iv)

$$(a*b)*c = (a+b-ab)*c = a+b+c-ab-ac-bc+abc.$$

$$a*(b*c) = a*(b+c-bc) = a+b+c-bc-ab-ac+abc.$$

$$\therefore (a*b)*c = a*(b*c).$$

Blunders (-3)
B1 Error in applying operation.

Slips (-1)
S1 Arithmetic error.

Attempts (3 marks)
A1 One correct operation other than $a*b$.
A2 $(a*b)*c$ correct.
A3 $a*(b*c)$ correct.

Part (a) (v) | **5 marks** | **Att 2**

10 (a) (v)	Show that $a * b \neq 1$, for all $a, b \in \mathbf{R} \setminus \{1\}$.

Show that $a*b \neq 1$ | **5 marks** | **Att 2**

10 (a) (v)	Let $a * b = 1$. $\therefore\ a + b - ab = 1 \ \Rightarrow\ a - 1 - b(a-1) = 0$ $(a-1)(1-b) = 0 \ \Rightarrow\ a = 1$ or $b = 1$. But $a \neq 1$ or $b \neq 1$ as $a, b \in R \setminus \{1\}$. $\therefore a * b \neq 1$.

Blunders (-3)
B1 Incorrect factors.
B2 Does not conclude $a \neq 1$ or $b \neq 1$.

Slips (-1)
S1 Arithmetic error.

Attempts (2 marks)
A1 Takes $a * b = 1$.
A2 $a + b - ab = 1$.
A3 Correct factors.

Part (b) | **20 (10, 10) marks** | **Att (3, 3)**

10 (b)	Prove that if H and K are subgroups of G, then so also is $H \cap K$.

Closure | **10 marks** | **Att 3**
Inverses | **10 marks** | **Att 3**

10 (b)	Let $a, b \in H \cap K$ $\therefore a, b \in H$ and $a, b \in K$. $\therefore ab \in H$ and $ab \in K$ as H and K are closed. $\therefore ab \in H \cap K \ \Rightarrow\ H \cap K$ is closed. $a \in H \cap K$ $\therefore a \in H$ and $a \in K$ $\therefore a^{-1} \in H$ and $a^{-1} \in K$ as H and K are groups. $\therefore a^{-1} \in H \cap K$. $\therefore H \cap K$ is a subgroup of G.

or

$a, b \in H \cap K$

$\therefore a, b \in H$ and $a, b \in K$

$\Rightarrow a * b^{-1} \in H$ and $a * b^{-1} \in K$, as H and K are groups.

$\therefore a * b^{-1} \in H \cap K$.

$\therefore H \cap K$ is a subgroup of G.

Blunders (-3)

B1 $ab \in H$ or $ab \in K$ not established.

B2 $ab \in H \cap K$ not given.

B3 $a^{-1} \in H \cap K$ not given.

Slips (-1)

S1 Arithmetic error.

Attempts (3, 3 marks)

A1 $a, b \in H \cap K$.

A2 $ab \in H$ or $ab \in K$.

A3 $a^{-1} \in H$ or $a^{-1} \in K$.

QUESTION 11

Part (a)	**20 marks (10, 10)**	**Att (3, 3)**
Part (b)	**30 (5, 10, 5, 5, 5) marks**	**Att (2, 3, 2, 2, 2)**

Part (a) **20 marks (10, 10)** **Att (3, 3)**

Part (a) (i) **10 marks** **Att 3**

11 (a) (i) f is the transformation $\begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} x' \\ y' \end{pmatrix}$ where $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 3 & -4 \\ 4 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 2 \\ -1 \end{pmatrix}$.

o is the point $(0, 0)$, p is the point $(1, 0)$ and q is the point $(0, 1)$.

(i) Find o', p' and q', the images of o, p and q, respectively under f.

Find image points **10 marks** **Att 3**

11 (a) (i)

$$\begin{pmatrix} 3 & -4 \\ 4 & 3 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}. \quad \therefore o' = (2, -1).$$

$$\begin{pmatrix} 3 & -4 \\ 4 & 3 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} + \begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 5 \\ 3 \end{pmatrix}. \quad \therefore p' = (5, 3).$$

$$\begin{pmatrix} 3 & -4 \\ 4 & 3 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} + \begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{pmatrix} -4 \\ 3 \end{pmatrix} + \begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{pmatrix} -2 \\ 2 \end{pmatrix}. \quad \therefore q' = (-2, 2).$$

Blunders (-3)

B1 Incorrect image point or only two correct image points given.

B2 Incorrect matrix multiplication, other than slip.

Slips (-1)

S1 Arithmetic error.

Attempts (3 marks)

A1 A correct image point.

Part (a) (ii) **10 marks** **Att 3**

11 (a) (ii) Verify that $|\angle p'o'q'| = 90^0$.

Verify **10 marks** **Att 3**

11 (a) (ii)

Slope $o'p' = \frac{3+1}{5-2} = \frac{4}{3}$.

Slope $o'q' = \frac{2+1}{-2-2} = -\frac{3}{4}$.

$\frac{4}{3} \times -\frac{3}{4} = -1 \Rightarrow |\angle p'o'q'| = 90^0$.

Blunders (-3)

B1 Error in slope formula.

B2 Incorrect condition for perpendicularity.

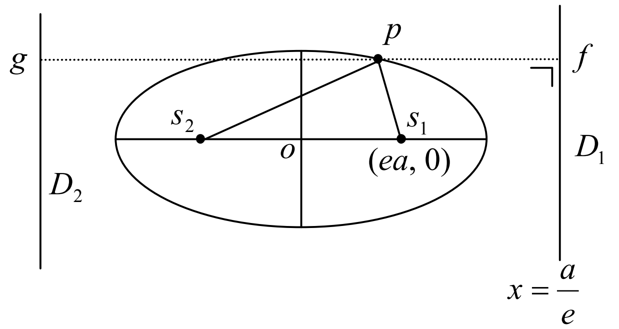
B3 Stops after finding slopes.

Slips (-1)
S1 Arithmetic error.

Attempts (2 marks)
A1 Slope op' or slope oq'.

Part (b) **30 (5, 10, 5, 5, 5) marks** **Att (2, 3, 2, 2, 2)**
Part (b) (i) **15 marks (5, 10)** **Att (2, 3)**

11 (b) (i)



The diagram shows an ellipse with eccentricity e, centred at the origin.
One focus is the point $s_1\,(ea, 0)$ and the other focus is s_2.

$x = \frac{a}{e}$ is the equation of the directrix D_1. p is any point on the ellipse.

Noting that $|ps_1| = e|pf|$, prove that $|ps_1| + |ps_2| = 2a$.

$|ps_2| = e|pg|$. **5 marks** **Att 2**
Finish **10 marks** **Att 3**

11 (b) (i) Draw $pg \perp D_2$.

$$|ps_1| + |ps_2| = e|pf| + |pg|$$
$$= e\big[|pf| + |pg|\big] = e|gf|$$
$$= e\left(2 \times \frac{a}{e}\right)$$
$$= 2a.$$

Blunders (-3)
B1 Stops at $|ps_1| + |ps_2| = e|gf|$.

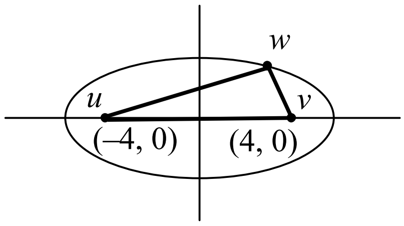
Slips (-1)
S1 Arithmetic error.

Attempts (2, 3 marks)
A1 Line pg shown.
A2 equation of D_2.
A3 $|gf| = \frac{2a}{e}$.

Part (b) (ii)	**15 marks (5, 5, 5)**	**Att (2, 2, 2)**

11 (b) (ii)

u (–4, 0) and v (4, 0) are two points.
w is a point such that the perimeter of triangle uvw has length 18.
The locus of w is an ellipse. Find its equation.

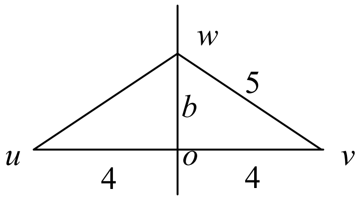


Value of *a*	**5 marks**	**Att 2**
Value of *b*	**5 marks**	**Att 2**
Equation of ellipse	**5 marks**	**Att 2**

11 (b) (ii) $|uv| = 8 \;\Rightarrow\; |uw| + |vw| = 10.$

But $|uw| + |vw| = 2a$. Proven in part (b) (i).

$\therefore 2a = 10 \;\Rightarrow\; a = 5.$



When w is on vertical axis $|vw| = |uw| = 5$.

Triangle ovw is right-angled with sides 5 and 4.

By Pythagoras, $|ow| = 3 \Rightarrow b = 3$.

$\therefore$ a $= 5$ and $b = 3 \;\Rightarrow\;$ Ellipse : $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1 \;\Rightarrow\; \dfrac{x^2}{25} + \dfrac{y^2}{9} = 1.$

Blunders (-3)

B1 $|uv|$ incorrect.

B2 $|uw| + |vw|$ incorrect.

B3 Incorrect application of Pythagoras.

B4 Equation of ellipse in central form.

Slips (-1)

S1 Arithmetic error.

Attempts (2, 2, 2 marks)

A1 $|uv|$ correct.

A2 $|uw| + |vw| = 2a.$

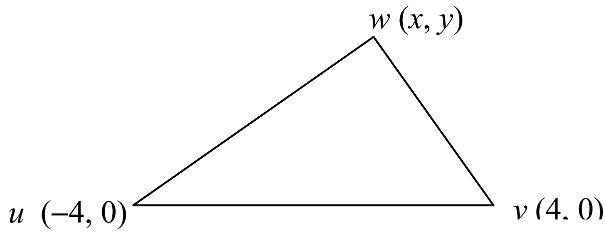
A3 $|wv| = 5.$

A4 Value of a or of b substituted into ellipse equation.

or

$\sqrt{(x-4)^2+y^2}+\sqrt{(x+4)^2+y^2}=10$	**5 marks**	**Att 2**
Correct work to 34–x^2–y^2	**5 marks**	**Att 2**
$36x^2+100y^2=900$	**5 marks**	**Att 2**

11 (b) (ii)



$|uv|+|vw|+|wu|=18.$ But $|uv|=8.$

$\therefore |vw|+|wu|=10.$

$\sqrt{(x-4)^2+y^2}+\sqrt{(x+4)^2+y^2}=10$

$x^2-8x+16+y^2+2\sqrt{(x-4)^2+y^2}.\sqrt{(x+4)^2+y^2}+x^2+8x+16+y^2=100.$

$\sqrt{(x-4)^2+y^2}.\sqrt{(x+4)^2+y^2}=34-x^2-y^2.$

$\left[(x-4)^2+y^2\right]\left[(x+4)^2+y^2\right]=\left(34-x^2-y^2\right)^2$

$(x-16)^2+y^4+y^2(x-4)^2+y^2(x+4)^2=1156+x^4+y^4-68x^2-68y^2+2x^2y^2.$

$\therefore x^4-32x^2+256+y^4+x^2y^2-8xy^2+16y^2+x^2y^2+8xy^2+16y^2$

$\quad =x^4+y^4-68x^2-68y^2+2x^2y^2+1156.$

$$36x^2+100y^2=900 \quad \Rightarrow \quad \frac{36x^2}{900}+\frac{100y^2}{900}=1$$

$$\therefore \text{ Ellipse}: \frac{x^2}{25}+\frac{y^2}{9}=1$$

Blunders (-3)
B1 Incorrect squaring.

Slips (-1)
S1 Arithmetic error.

Attempts (2, 2, 2 marks)
A1 $|uv|=8.$
A2 Some correct squaring.