Coimisiún na Scrúduithe Stáit
State Examinations Commission

LEAVING CERTIFICATE EXAMINATION, 2005

MATHEMATICS – ORDINARY LEVEL

PAPER 2 (300 marks)

MONDAY, 13 JUNE – MORNING, 9:30 to 12:00

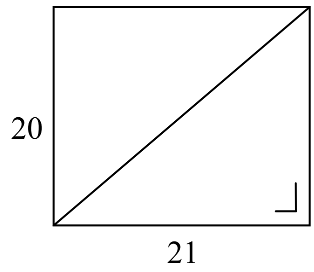
Attempt **FIVE** questions from **Section A** and **ONE** question from **Section B**.
Each question carries 50 marks.

WARNING: Marks will be lost if all necessary work is not clearly shown.

Answers should include the appropriate units of measurement, where relevant.

SECTION A
Attempt FIVE questions from this section.

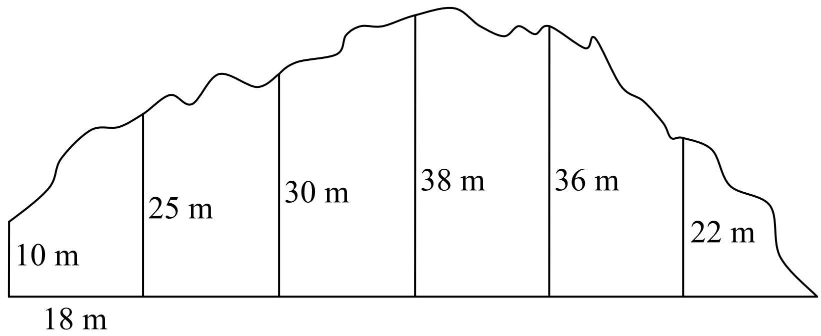
1. **(a)** A rectangle has length 21 cm and width 20 cm.



(i) Find the area of the rectangle.

(ii) Find the length of the diagonal.

(b) The sketch shows a lake bounded on one side by a straight dam.



At equal intervals of 18 m along the dam, perpendicular measurements are made to the opposite bank, as shown on the sketch.

(i) Use Simpson's Rule to estimate the area of the lake.

(ii) If the lake contains 15 000 m^3 of water, calculate the average depth of water in the lake, correct to the nearest metre.

(c) A steel-works buys steel in the form of solid cylindrical rods of radius 10 centimetres and length 30 metres.

The steel rods are melted to produce solid spherical ball-bearings. No steel is wasted in the process.

(i) Find the volume of steel in one cylindrical rod, in terms of π.

(ii) The radius of a ball-bearing is 2 centimetres.
How many such ball-bearings are made from one steel rod?

(iii) Ball-bearings of a different size are also produced.
One steel rod makes 225 000 of these new ball-bearings.
Find the radius of the new ball-bearings.

2. **(a)** Find the distance between the two points (3, 4) and (15, 9).

(b) L is the line $3x-4y+12=0$.
L intersects the x-axis at a and the y-axis at b.

(i) Find the co-ordinates of a and the co-ordinates of b.

(ii) K is the line that passes through b and is perpendicular to L.
Show L and K on a co-ordinate diagram.

(iii) Find the equation of K.

(iv) The point $(2t, 3t)$ is on the line K. Find the value of t.

(c) The lines $2x-y+3=0$ and $4x-y+k=0$ intersect at a point.

(i) Find, in terms of k, the co-ordinates of the point of intersection of the lines.

(ii) For what value of k is the point of intersection on the y-axis?

3. **(a)** The circle C has equation $x^2+y^2 = 49$.

(i) Write down the centre and the radius of C.

(ii) Verify that the point $(5, -5)$ lies outside the circle C.

(b) The line $y=10-2x$ intersects the circle $x^2+y^2=40$ at the points a and b.

(i) Find the co-ordinates of a and the co-ordinates of b.

(ii) Show the line, the circle and the points of intersection on a co-ordinate diagram.

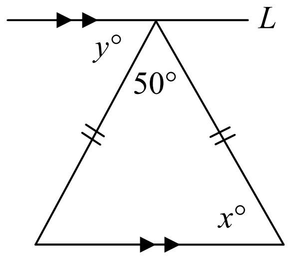
(c) The circle K has equation $(x+4)^2+(y-3)^2=36$.

(i) Write down the co-ordinates of the centre of K.

(ii) The point (2, 3) is one end-point of a diameter of K.
Find the co-ordinates of the other end-point.

(iii) The point $(-4, y)$ is on the circle K. Find the two values of y.

4. **(a)** In the diagram, the line L is parallel to the base of the isosceles triangle.



(i) Find x.

(ii) Find y.

(b) Prove that a line which is parallel to one side-line of a triangle, and cuts a second side, will cut the third side in the same proportion as the second.

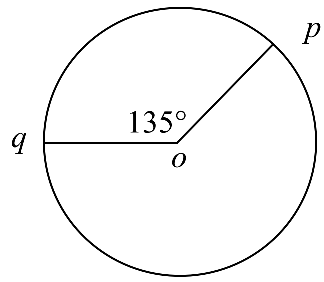
(c) **(i)** Draw a square $opqr$ with sides 8 cm.

(ii) Draw the image of this square under the enlargement with centre o and scale factor 0.25.

(iii) Calculate the area of this image square.

(iv) Under another enlargement the area of the image of the square $opqr$ is $100\ \text{cm}^2$. What is the scale factor of this enlargement?

5. **(a)** A circle has centre o and radius 14 cm.

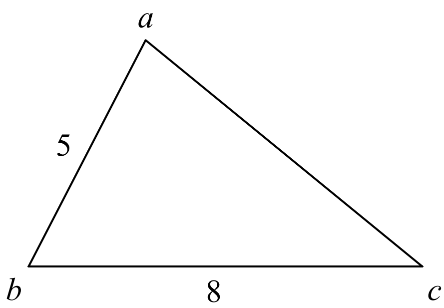


p and q are two points on the circle and $|\angle qop| = 135°$.

Find the length of the shorter arc pq.

Take $\pi = \dfrac{22}{7}$.

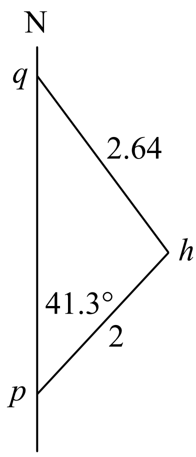
(b) In the triangle abc, $|ab| = 5$ cm and $|bc| = 8$ cm. The area of the triangle is $16.58\ \text{cm}^2$.



(i) Find $|\angle abc|$, correct to the nearest degree.

(ii) Find $|ac|$, correct to the nearest centimetre.

(c) A lighthouse, h, is observed from a ship sailing a straight course due North. The distance from p to h is 2 km and the bearing of the lighthouse from p is N 41.3° E. The distance from q to h is 2.64 km.



(i) Find the bearing of the lighthouse from q.

(ii) The ship is sailing at a speed of 19 km/h. Find, correct to the nearest minute, the time taken to sail from p to q.

6. **(a)** **(i)** Evaluate 6!

(ii) Evaluate $\binom{12}{3}$.

(b) Ten teams take part in a competition. The teams are divided into two groups. Teams A, B, C, D and E are in group 1 and teams U, V, X, Y and Z are in group 2.
In the final, the winning team from group 1 plays the winning team from group 2.
Each team is equally likely to win its group.

(i) How many different team pairings are possible for the final?

(ii) What is the probability that team C plays team X in the final?

(iii) What is the probability that team A plays in the final?

(iv) What is the probability that team B does not play in the final?

(c) Seven horses run in a race.
All horses finish the race and no two horses finish the race at the same time.

(i) In how many different orders can the seven horses finish the race?

(ii) A person is asked to predict the correct order of the first three horses to finish the race. How many different such predictions can be made?

(iii) A person is asked to predict, in any order, the first three horses to finish the race. How many different such predictions can be made?

(iv) A person selects two of the seven horses at random. What is the probability that the selected horses are the first two horses to finish the race?

7. **(a)** Calculate the weighted mean of 10, 30 and 15, given that the weights are 3, 1 and 2, respectively.

(b) There are fourteen questions in an examination.
The table below shows the performance of the candidates.

Correct responses	0 – 2	3 – 5	6 – 8	9 – 11	12 – 14
Number of candidates	1	2	6	8	3

(i) Using mid-interval values, calculate the mean number of correct responses.

(ii) Calculate the standard deviation, correct to one decimal place.

(c) A concert began at 8.00 p.m. The cumulative frequency table below gives the number of people in the concert hall at the times stated.

Time p.m.	7.10	7.20	7.30	7.40	7.50	8.00
Number of people	0	30	100	160	275	300

(i) Copy and complete the following frequency table to show the number of people who entered the hall during each time interval.

Time interval	7.10 – 7.20	7.20 – 7.30	7.30 – 7.40	7.40 – 7.50	7.50 – 8.00
Number of people					

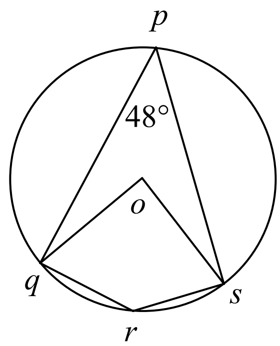
(ii) In which interval does the median time of arrival lie?

(iii) In which time interval did the greatest number of people enter the concert hall?

(iv) What is the least number of people who could have been in the concert hall at 7.15 p.m?

SECTION B
Attempt ONE question from this section.

8. **(a)** The points p, q, r and s lie on a circle, centre o.
$|\angle spq| = 48°$.

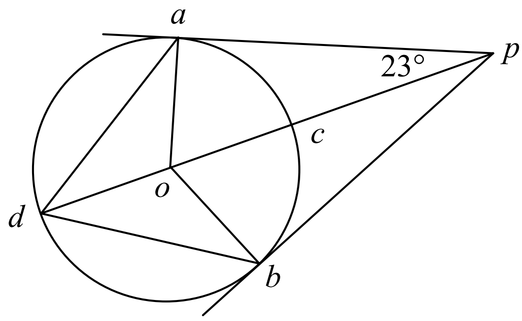


(i) Find $|\angle soq|$.

(ii) Find $|\angle qrs|$.

(b) Prove that a line is a tangent to a circle at a point t of the circle if and only if it passes through t and is perpendicular to the line through t and the centre.

(c) pa and pb are tangents to a circle, centre o.
po intersects the circle at c and d.
$|ao| = 5$, $|pc| = 8$ and $|\angle opa| = 23°$.



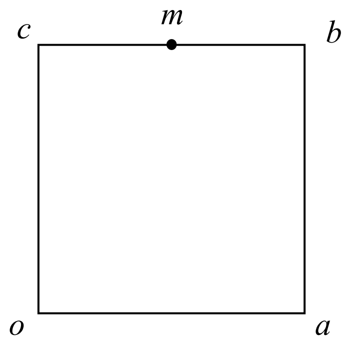
(i) Find $|pa|$.

(ii) Find $|\angle aop|$.

(iii) Find $|\angle adb|$.

(iv) Find $|\angle dbo|$.

9. **(a)** $oabc$ is a square.
m is the midpoint of $[cb]$.



(i) Express $\vec{b}$ in terms of $\vec{a}$ and $\vec{c}$.

(ii) Express $\vec{m}$ in terms of $\vec{a}$ and $\vec{c}$.

(b) Let $\vec{p} = 2\vec{i} + 3\vec{j}$ and $\vec{q} = -5\vec{i} + 6\vec{j}$.

(i) Express $\vec{pq}$ in terms of $\vec{i}$ and $\vec{j}$.

(ii) Express $4\vec{p} + 2\vec{q}$ in terms of $\vec{i}$ and $\vec{j}$.

(iii) Find the scalar k such that $k\vec{p} - \vec{q} = 9\vec{i}$.

(c) Let $\vec{x} = 5\vec{i} + 4\vec{j}$ and $\vec{y} = 3\vec{i} - 7\vec{j}$.

(i) Write $\vec{x}^{\perp}$ in terms of $\vec{i}$ and $\vec{j}$ and show that $|\vec{x}| = |\vec{x}^{\perp}|$.

(ii) Calculate the dot product $(\vec{x} + \vec{y}).(\vec{x} - \vec{y})$.

10. **(a)** Expand $(1-2x)^4$ fully.

(b) **(i)** Find S, the sum to infinity of the geometric series

$$\frac{1}{3} + \frac{2}{9} + \frac{4}{27} + \ldots$$

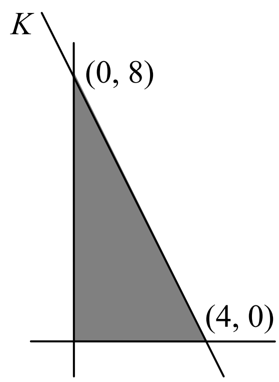
(ii) The sum to infinity of another geometric series is also S.
The common ratio of this series is $\frac{1}{3}$.
Find the first term.

(c) **(i)** A machine costing €25 000 depreciates at the compound rate of 15% per annum. Find the value of the machine at the end of twelve years, correct to the nearest euro.

(ii) A company invests €25 000 in machinery at the beginning of each year for twelve consecutive years. The machinery depreciates at the rate of 15% per annum compound depreciation.

Using the formula for the sum of the first n terms of a geometric series, find the total value of the machinery at the end of the twelve years, correct to the nearest euro.

11. **(a)** The line K cuts the x-axis at (4, 0) and the y-axis at (0, 8).



(i) Find the equation of K.

(ii) Write down the three inequalities that together define the region enclosed by K, the x-axis and the y-axis.

(b) A manufacturer of garden furniture produces plastic chairs and tables. Each chair requires 2 kg of raw material and each table requires 5 kg of raw material. In any working period the raw material used cannot exceed 800 kg.

Each chair requires 4 minutes of machine time and each table requires 4 minutes of machine time. The total machine time available in any working period is 1000 minutes.

(i) Taking x as the number of chairs and y as the number of tables, write down two inequalities in x and y and illustrate these on graph paper.

(ii) The manufacturer sells each chair for €20 and each table for €40.
How many of each should be produced in each working period to maximise income?

(iii) The manufacturer's costs for each chair are €17 and for each table are €34.70.
Express the profit as a percentage of income, assuming the income has been maximised.

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