



**Coimisiún na Scrúduithe Stáit**  
**State Examinations Commission**

*Scéim Mharcála*

*Scrúduithe Ardteistiméireachta, 2003*

*Matamaitic*

*Ardleibhéal*

*Marking Scheme*

*Leaving Certificate Examination, 2003*

*Mathematics*

*Higher Level*

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# MARKING SCHEME

## LEAVING CERTIFICATE EXAMINATION 2003

### MATHEMATICS

### HIGHER LEVEL

### PAPER 1

#### GENERAL GUIDELINES FOR EXAMINERS - PAPER 1

1. Penalties of three types are applied to candidates' work as follows:

- Blunders - mathematical errors/omissions (-3)
- Slips - numerical errors (-1)
- Misreadings (provided task is not oversimplified) (-1).

Frequently occurring errors to which these penalties must be applied are listed in the scheme. They are labelled as B1, B2, B3,....., S1, S2, S3,....., M1, M2, etc. Note that these lists are not exhaustive.

2. When awarding attempt marks, e.g. Att(3), note that
- any correct relevant step in a part of a question merits *at least* the attempt mark for that part
  - if deductions result in a mark which is lower than the attempt mark, then the attempt mark must be awarded
  - a mark between zero and the attempt mark is never awarded.
3. Worthless work is awarded zero marks. Some examples of such work are listed in the scheme and they are labelled as W1, W2,.....etc.
4. The *same* error in the *same* section of a question is penalised *once* only.
5. Special notes relating to the marking of a particular part of a question are indicated by an asterisk. These notes immediately follow the box containing the relevant solution.
6. Particular cases, verifications and answers derived from diagrams (unless requested) qualify for attempt marks at most.
7. A serious blunder, omission or misreading merits the ATTEMPT mark at most.
8. The phrase “and stops” means that no more work is shown by the candidate.
9. Accept the best of two or more attempts – even when attempts have been cancelled.

## QUESTION 1

<b>Part (a)</b>	<b>10 marks</b>	<b>Att 3</b>
<b>Part (b)</b>	<b>20 (10, 5, 5) marks</b>	<b>Att (3, 2, 2)</b>
<b>Part (c)</b>	<b>20 (10, 10) marks</b>	<b>Att (3, 3)</b>

**Part (a)** **10 marks** **Att 3**

**1(a)**

Express the following as a single fraction in its simplest form:

$$\frac{6y}{x(x+4y)} - \frac{3}{2x}$$

**Part (a)** **10 marks** **Att 3**

**1(a)**

$$\begin{aligned}\frac{6y}{x(x+4y)} - \frac{3}{2x} &= \frac{2(6y) - 3(x+4y)}{2x(x+4y)} \\ &= \frac{12y - 3x - 12y}{2x(x+4y)} \\ &= \frac{-3x}{2x(x+4y)} \\ &= \frac{-3}{2(x+4y)}\end{aligned}$$

*Blunders (-3)*

B1 indices.

B2 incorrect cancellation.

B3 answer not in simplest form.

*Slips (-1)*

S1 numerical.

*Worthless*

W1 values for  $x$  and  $y$ .

**Part (b)(i)**

**10 marks**

**Att 3**

**1(b)(i)**  $f(x) = ax^2 + bx + c$  where  $a, b, c \in \mathbf{R}$ .

Given that  $k$  is a real number such that  $f(k) = 0$ , prove that  $x - k$  is a factor of  $f(x)$

**Part (b)(i)**

**10 marks**

**Att 3**

**1(b)(i)**

$$f(x) = ax^2 + bx + c$$

$$f(k) = ak^2 + bk + c$$

$$f(x) - f(k) = a(x^2 - k^2) + b(x - k)$$

$$f(k) = 0 \Rightarrow f(x) = a(x - k)(x + k) + b(x - k)$$

$$= (x - k)[a(x + k) + b]$$

$\Rightarrow (x - k)$  is factor

or

**1(b)(i)**

$$\begin{array}{r} ax + (ak + b) \\ x - k \overline{) ax^2 + bx + c} \end{array}$$

$$\frac{ax^2 - akx}{(ak + b)x + c}$$

$$\frac{(ak + b)x - k(ak + b)}{(ak + b)x - k(ak + b)}$$

$$\frac{c + k(ak + b)}{c + k(ak + b)} = \frac{ak^2 + bk + c}{ak^2 + bk + c} = f(k)$$

But  $f(k) = 0$

$\Rightarrow (x - k)$  is factor

\* Possible to divide by  $(x - k)$  for full marks.

*Blunders (-3)*

B1 indices.

B2 factors.

B3  $f(k) = 0$ .

*Slips (-1)*

S1 numerical.

S2 not changing sign when subtracting in division

*Attempts*

A1 remainder  $\neq 0$  in division.

A2  $f(x) = ax^3 + bx^2 + cx + d$  finished correctly.

**1(b)(ii)** Show that  $2x - \sqrt{3}$  is a factor of  $4x^2 - 2(1 + \sqrt{3})x + \sqrt{3}$  and find the other factor.

**Show**

**5 marks**

**Att 2**

**Find**

**5 marks**

**Att 2**

**1(b)(ii)**

By inspection factors are

$$(2x - \sqrt{3}) \text{ and } (2x - 1)$$

$$\begin{aligned} \Rightarrow (2x - \sqrt{3})(2x - 1) &= 4x^2 - 2\sqrt{3}x - 2x + \sqrt{3} \\ &= 4x^2 - 2(1 + \sqrt{3})x + \sqrt{3} \end{aligned}$$

or

**1(b)(ii)**

$$\begin{array}{r} 2x - \sqrt{3} \overline{) 4x^2 - 2x - 2\sqrt{3}x + \sqrt{3}} \\ \underline{4x^2 \quad - 2\sqrt{3}x} \phantom{+ \sqrt{3}} \\ -2x + \sqrt{3} \\ \underline{-2x + \sqrt{3}} \\ 0 \end{array}$$

or

**1(b)(ii)**

If  $(2x - \sqrt{3})$  is a factor, then  $f\left(\frac{\sqrt{3}}{2}\right) = 0$

$$f(x): 4x^2 - 2x - 2\sqrt{3}x + \sqrt{3}$$

$$\begin{aligned} f\left(\frac{\sqrt{3}}{2}\right) &= 4\left(\frac{\sqrt{3}}{2}\right)^2 - 2\left(\frac{\sqrt{3}}{2}\right) - 2\sqrt{3}\left(\frac{\sqrt{3}}{2}\right) + \sqrt{3} \\ &= 3 - \sqrt{3} - 3 + \sqrt{3} \\ &= 0 \end{aligned}$$

$\Rightarrow (2x - \sqrt{3})$  is factor

$\Rightarrow (2x - 1)$  is other factor

*Blunders (-3)*

B1 indices.

B2 factors.

B3 deduction root from factor.

*Slips (-1)*

S1 numerical.

S2 not changing sign when subtracting in division.

**Part (c)**

**20 (10, 10) marks**

**Att (3, 3)**

**1(c)**

The real roots of  $x^2 + 10x + c = 0$  differ by  $2p$  where  $c, p \in \mathbf{R}$  and  $p > 0$ .

- (i) Show that  $p^2 = 25 - c$
- (ii) Given that one root is greater than zero and the other root is less than zero, find the range of possible values of  $p$ .

**(i) Show**

**10 marks**

**Att 3**

**(ii) Range**

**10 marks**

**Att 3**

**1(c)(i)**

$$x^2 + 10x + c = 0$$

$$x = \frac{-10 \pm \sqrt{100 - 4c}}{2(1)}$$

$$= \frac{-10 \pm 2\sqrt{25 - c}}{2}$$

$$\Rightarrow \text{roots are } = -5 \pm \sqrt{25 - c}$$

$$\text{roots: } -5 + \sqrt{25 - c} \text{ and } -5 - \sqrt{25 - c}$$

$$(-5 + \sqrt{25 - c}) - (-5 - \sqrt{25 - c}) = 2p$$

$$2\sqrt{25 - c} = 2p$$

$$p = \sqrt{25 - c}$$

$$p^2 = 25 - c$$

**1(c)(ii)**

$$\alpha = -5 + \sqrt{25-c} : \beta = -5 - \sqrt{25-c}$$

$$\underline{p > 0}$$

$$\alpha > \beta \Rightarrow \alpha > 0 \text{ and } \beta < 0$$

$$\alpha > 0 \Rightarrow -5 + \sqrt{25-c} > 0$$

$$-5 + p > 0$$

$$p > 5$$

$$\Rightarrow p > 5 \text{ (since } p > 0)$$

$$\beta < 0 \Rightarrow -5 - \sqrt{25-c} < 0$$

$$-5 - p < 0$$

$$-5 < p$$

or

**1(c)(i)**

Let  $\alpha$  and  $\beta$  be two roots and let  $\alpha > \beta \Rightarrow \alpha > 0$  and  $\beta < 0$

$$x^2 + 10x + c = 0$$

$$x^2 - (-10)x + c = 0$$

$$x^2 - (\alpha + \beta)x + (\alpha\beta) = 0$$

$$\Rightarrow \alpha + \beta = -10 \dots \dots \dots (i)$$

$$\alpha\beta = c \dots \dots \dots (ii)$$

$$\alpha - \beta = 2p \dots \dots \dots (iii)$$

$$(i) \quad \alpha + \beta = -10$$

$$(i) \quad \alpha + \beta = -10$$

$$(iii) \quad \alpha - \beta = 2p$$

$$(p-5) + \beta = -10$$

$$\underline{2\alpha} = 2p - 10$$

$$\alpha = p - 5$$

$$\beta = -5 - p$$

$$\beta = -p - 5$$

$$(iii) \quad \alpha\beta = c$$

$$(p-5)(-p-5) = c$$

$$-p^2 + 5p - 5p + 25 = c$$

$$25 - p^2 = c$$

**Q1(c)(ii)**

$$\alpha > 0$$

$$\beta < 0$$

$$p - 5 > 0$$

$$-p - 5 < 0$$

$$p > 5$$

$$-5 < p$$

$$(\Rightarrow p > 5, \text{ since } p > 0)$$

**or**

$$\alpha > 0 \text{ and } \beta < 0 \Rightarrow \alpha\beta < 0$$

$$c < 0$$

$$25 - p^2 < 0$$

$$25 < p^2$$

$$\Rightarrow p > 5, \text{ since } p > 0$$



*Blunders (-3)*

- B1 indices.
- B2 root formula once only.
- B3 mixing up roots once only.
- B4 inequality sign.
- B5 incorrect deduction from work or no deduction.
- B6 statement of quadratic equation once only.
- B7 incorrect  $(\alpha + \beta)$ .
- B8 incorrect  $\alpha\beta$ .
- B9 incorrect difference.

*Slips (-1)*

- S1 numerical.

## QUESTION 2

<b>Part (a)</b>	<b>15 (10, 5) marks</b>	<b>Att (3, 2)</b>
<b>Part (b)</b>	<b>20 (10, 10) marks</b>	<b>Att (3, 3)</b>
<b>Part (c)</b>	<b>15 (10, 5) marks</b>	<b>Att (3, 2)</b>

**Part (a)** **15 (10, 5) marks** **Att (3, 2)**

**2(a)**

Solve the simultaneous equations:

$$3x - y = 8$$

$$x^2 + y^2 = 10$$

**Part (a) Quadratic** **10 marks** **Att 3**  
**Finish** **5 marks** **Att 2**

**2(a)**

(i)  $3x - y = 8 \Rightarrow y = 3x - 8$

(ii)  $x^2 + y^2 = 10$

$$x^2 + (3x - 8)^2 = 10$$

$$x^2 + 9x^2 - 48x + 64 - 10 = 0$$

$$10x^2 - 48x + 54 = 0$$

$$5x^2 - 24x + 27 = 0$$

$$(5x - 9)(x - 3) = 0$$

$$\Rightarrow 5x - 9 = 0 \quad \text{or} \quad x - 3 = 0$$

$$5x = 9 \qquad \qquad \qquad x = 3$$

$$x = \frac{9}{5}$$

$$y = 3x - 8$$

$$x = \frac{9}{5} : \quad y = 3\left(\frac{9}{5}\right) - 8 = \frac{27 - 40}{5} = \frac{-13}{5} \Rightarrow \left(\frac{9}{5}, -\frac{13}{5}\right)$$

$$x = 3 \quad y = 3(3) - 8 = 9 - 8 = 1 \Rightarrow (3, 1)$$

*Blunders (-3)*

- B1 indices.
- B2 expansion of  $(3x - 8)^2$  once only.
- B3 factors once only.
- B4 deduction value from factor.
- B5 not getting 2<sup>nd</sup> value(having got 1<sup>st</sup>).
- B6 root formula once only.

*Slips (-1)*

- S1 numerical.

*Attempts*

- A1 not quadratic.

*Worthless*

- W1 trial and error.

Part (b)(i)

10 marks

Att 3

2(b)(i) Solve for  $x$ :

$$|4x + 7| < 1$$

Part (b)(i)

10 marks

Att 3

2(b)(i)

$$|4x + 7| < 1$$

$$\Rightarrow -1 < (4x + 7) < 1$$

$$-1 < 4x + 7$$

$$-8 < 4x$$

$$-2 < x$$

$$4x + 7 < 1$$

$$4x < -6$$

$$x < -\frac{6}{4}$$

$$x < -\frac{3}{2}$$

$$\text{Solution: } -2 < x < -\frac{3}{2}$$

Or

2(b)(i)

$$|4x + 7| < 1$$

$$(4x + 7)^2 < (1)^2$$

$$16x^2 + 56x + 49 < 1$$

$$16x^2 + 56x + 48 < 0$$

$$2x^2 + 7x + 6 < 0$$

(i)  $+2x^2$

$\Rightarrow$



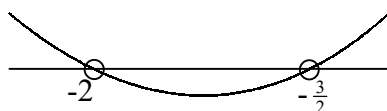
(ii) Solve  $2x^2 + 7x + 6 = 0$

$$(2x + 3)(x + 2) = 0$$

$$2x + 3 = 0 \quad \text{or} \quad x + 2 = 0$$

$$x = -\frac{3}{2} \quad \quad x = -2$$

(iii)



$$f(x) < 0 \quad \text{when} \quad -2 < x < -\frac{3}{2}$$

*Blunders (-3)*

- B1 upper limit.
- B2 lower limit.
- B3 expansion of  $(4x + 7)^2$  once only.
- B4 inequality sign, eg, changes to  $(=)$  , once only.
- B5 indices.
- B6 factors once only.
- B7 root formula, once only.
- B8 deduction root from factor.
- B9 incorrect range.
- B10 answer not stated.

*Slips (-1)*

- S1 numerical.
- S2  $\leq$ .

*Attempts*

- A1 ignore absolute value.

**Part (b)(ii)**

**10 marks**

**Att 3**

**2 (b)(ii)** Given that  $x^2 - ax - 3$  is a factor of  $x^3 - 5x^2 + bx + 9$  where  $a, b \in R$ , find the value of  $a$  and the value of  $b$  .

**Part (b)(ii)**

**10 marks**

**Att 3**

**2(b)(ii)**

$$\begin{array}{r} x + (a - 5) \\ x^2 - ax - 3 \overline{) x^3 - 5x^2 + bx + 9} \\ \underline{x^3 - ax^2 - 3x} \phantom{+ 9} \\ (a - 5)x^2 + (b + 3)x + 9 \\ \underline{(a - 5)x^2 - a(a - 5)x - 3(a - 5)} \\ (b + 3)x + (a^2 - 5a)x + 9 + 3a - 15 = 0 \end{array}$$

$$(b + 3 + a^2 - 5a)x + (3a - 6) = (0)x + (0)$$

$$\Rightarrow b + 3 + a^2 - 5a = 0 \dots\dots\dots(i) \quad \text{and} \quad 3a - 6 = 0 \dots\dots\dots(ii)$$

$$\begin{array}{ll} (i) \quad b + 3 + 4 - 10 = 0 & 3a = 6 \\ & a = 2 \\ & b = 3 \end{array}$$

or

**2 (b)(ii)** Let  $(x + p)$  be other factor

$$\therefore (x^2 - ax - 3)(x + p) = x^3 - 5x^2 + bx + 9$$

$$x^3 - (a - p)x^2 + (-3 - ap)x + (-3p) = x^3 - 5x^2 + bx + 9$$

Equating like to like

(i)  $a - p = 5$

(ii)  $-3 - ap = b$

(iii)  $-3p = 9$

(iii)  $p = -3$

(i)  $a - p = 5$

$$a - (-3) = 5$$

$$a = 2$$

(ii)  $-3 - ap = b$

$$-3 - 2(-3) = b$$

$$-3 + 6 = b \Rightarrow b = 3$$

or

**2(b)(ii)** Since  $(x^2 - ax - 3)$  is a factor of  $x^3 - 5x^2 + bx + 9$   
other factors is  $(x - 3)$  by inspection.

$$\Rightarrow f(3) = 0$$

$$f(x) = x^3 - 5x^2 + bx + 9$$

$$f(3) = (3)^3 - 5(3)^2 + b(3) + 9 = 0$$

$$27 - 45 + 3b + 9 = 0$$

$$3b = 9$$

$$b = 3$$

$$x^2 - ax - 3 \overline{) x^3 - 5x^2 + 3x + 9}$$

$$\underline{x^3 - ax^2 - 3x}$$

$$ax^2 - 5x^2 + 6x + 9$$

$$(a - 5)x^2 + 6x + 9$$

$$\underline{(a - 5)x^2 - a(a - 5)x - 3(a - 5)}$$

$$[x + (a - 5)] = [x + (-3)]$$

$$\Rightarrow (a - 5) = -3 \Rightarrow a = 2$$

or

**2 (b)(ii)**

$(x - 3)$  other factor by inspection.

$$(x - 3)(x^2 - ax - 3) = x^3 - 5x^2 + bx + 9$$

$$x^3 - (a + 3)x^2 + (3a - 3)x + 9 = x^3 - 5x^2 + bx + 9$$

Equating like to like:

(i)  $a + 3 = 5$

$$a = 2$$

(ii)  $3a - 3 = b$

$$3(2) - 3 = b$$

$$3 = b$$

*Blunders (-3)*

- B1 indices.
- B2 not like to like.
- B3 not getting  $2^{\text{nd}}$  value, having got  $1^{\text{st}}$ .
- B4 deduction root from factor.

*Slips (-1)*

- S1 numerical.
- S2 not changing sign when subtracting in division.

*Attempts*

- A1 remainder  $\neq 0$  in division.
- A2 any attempt at division.

**Part (c)(i)**

**10 marks**

**Att 3**

**2(c)(i)** Solve for  $y$  :

$$2^{2y+1} - 5(2^y) + 2 = 0$$

**Part (c)(i)**

**10 marks**

**Att 3**

**2(c)(i)**

$$2^{2y+1} - 5(2^y) + 2 = 0$$

Let  $x = 2^y$

$$2^{2y+1} = 2^{2y} \cdot 2^1 = 2(2^y)^2 = 2x^2$$

$$2x^2 - 5x + 2 = 0$$

$$(2x - 1)(x - 2) = 0$$

$$\Rightarrow 2x - 1 = 0 \quad \text{or} \quad x - 2 = 0$$

$$x = \frac{1}{2} \quad \text{or} \quad x = 2$$

$$x = \frac{1}{2} : \quad x = 2^y = 2^{-1} \quad \Rightarrow \quad y = -1$$

$$x = 2 : \quad x = 2^y = 2^1 \quad \Rightarrow \quad y = 1$$

*Blunders (-3)*

- B1 indices.
- B2 factors once only.
- B3 root formula once only.
- B4 if  $x$  used for  $2^y$ , not getting  $y$  once only.
- B5 deduction value from factor.

*Slips (-1)*

- S1 numerical.

**Part (c)(ii)****5 marks****Att 2****2(c)(ii)**Given that  $x = \alpha$  and  $x = \beta$  are the solutions of the quadratic equation $2k^2x^2 + 2ktx + t^2 - 3k^2 = 0$  where  $k, t \in \mathbb{R}$  and  $k \neq 0$ , show that  $\alpha^2 + \beta^2$  is independent of  $k$  and  $t$ .**Part (c)(ii)****5 marks****Att 2****2(c)(ii)**

$$2k^2x^2 + 2ktx + t^2 - 3k^2 = 0$$

$$x^2 - \left(-\frac{t}{k}\right)x + \left(\frac{t^2 - 3k^2}{2k^2}\right) = 0$$

$$x^2 - (\alpha + \beta)x + (\alpha\beta) = 0$$

$$\Rightarrow \alpha + \beta = -\frac{t}{k} \quad \text{and} \quad \alpha\beta = \frac{t^2 - 3k^2}{2k^2} = \left(\frac{t^2}{2k^2} - \frac{3}{2}\right)$$

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2(\alpha\beta)$$

$$= \left(\frac{-t}{k}\right)^2 - 2\left(\frac{t^2}{2k^2} - \frac{3}{2}\right)$$

$$= \frac{t^2}{k^2} - \frac{t^2}{k^2} + 3$$

$$= 3$$

or

**2(c)(ii)**

$$(2k^2)x^2 + (2kt)x + (t^2 - 3k^2) = 0$$

$$\text{Roots: } x = \frac{-2kt \pm \sqrt{4k^2t^2 - 4(2k^2)(t^2 - 3k^2)}}{2(2k^2)}$$

$$= \frac{-2kt \pm \sqrt{4k^2[6k^2 - t^2]}}{4k^2}$$

$$= \frac{-(2k)t \pm (2k)\sqrt{6k^2 - t^2}}{4k^2} = \frac{-t \pm \sqrt{6k^2 - t^2}}{2k}$$

$$\Rightarrow \alpha = \frac{-t + \sqrt{6k^2 - t^2}}{2k} \quad \text{and} \quad \beta = \frac{-t - \sqrt{6k^2 - t^2}}{2k}$$

$$\alpha^2 + \beta^2 = \frac{1}{4k^2} \left[ \left( t^2 - 2t\sqrt{6k^2 - t^2} + 6k^2 - t^2 \right) + \left( t^2 + 2t\sqrt{6k^2 - t^2} + 6k^2 - t^2 \right) \right]$$

$$= \frac{1}{4k^2} [2t^2 + 6k^2 - 2t^2 + 6k^2]$$

$$= \frac{12k^2}{4k^2}$$

$$= 3$$

*Blunders (-3)*

- B1 indices.
- B2 statement of quadratic equation once only.
- B3 expansion of  $(\alpha + \beta)^2$  once only.
- B4 root formula once only.
- B5 incorrect  $(\alpha + \beta)$
- B6  $\alpha\beta$ .

*Slips (-1)*

- S1 numerical.

*Attempts*

- A1  $t$  and/or  $k$  in answer.
- A2 substitute  $x = \alpha$  and/or  $x = \beta$



### QUESTION 3

<b>Part (a)</b>	<b>10 (5, 5) marks</b>	<b>Att (2, 2)</b>
<b>Part (b)</b>	<b>20 (5, 5, 10) marks</b>	<b>Att (2, 2, 3)</b>
<b>Part (c)</b>	<b>20 (15, 5) marks</b>	<b>Att (5, 2)</b>

<b>Part (a)</b>	<b>10 (5, 5) marks</b>	<b>Att (2, 2)</b>
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**3(a)**

$$\text{Evaluate } (1 \quad -2) \begin{pmatrix} 3 & 0 \\ -5 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

<b>Part (a) 1<sup>st</sup> Multiplication</b>	<b>5 marks</b>	<b>Att 2</b>
<b>2<sup>nd</sup> Multiplication</b>	<b>5 marks</b>	<b>Att 2</b>

**3(a)**

$$\begin{aligned} & (1 \quad -2) \begin{pmatrix} 3 & 0 \\ -5 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ -2 \end{pmatrix} \\ & = (1 \quad -2) \begin{pmatrix} 3 \\ -7 \end{pmatrix} \quad \text{or} \quad (13 \quad -2) \begin{pmatrix} 1 \\ -2 \end{pmatrix} \\ & = (17) \quad \quad \quad = (17) \end{aligned}$$

*Slips (-1)*

S1 each incorrect element.

S2 numerical.

\* Note: cannot get 2<sup>nd</sup> 5 marks if 1<sup>st</sup> multiplication not correct “dimension”

<b>Part (b)(i)</b>	<b>10(5, 5) mark</b>	<b>Att (2, 2)</b>
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**3(b)(i)**

Given that  $z = 2 - i$ , calculate  $|z^2 - z + 3|$  where  $i^2 = -1$

<b>Part (b)(i) Complex Number</b>	<b>5 marks</b>	<b>Att 2</b>
<b>Modulus</b>	<b>5 marks</b>	<b>Att 2</b>

**3(b)(i)**

$$z = 2 - i$$

$$z^2 = (2 - i)^2 = 4 - 4i + i^2 = 3 - 4i$$

$$z^2 - z + 3 = (3 - 4i) - (2 - i) + 3$$

$$= 4 - 3i$$

$$|z^2 - z + 3| = |4 - 3i| = \sqrt{4^2 + (-3)^2} = \sqrt{25} = 5$$

*Blunders (-3)*

B1  $i$

B2  $(2-i)^2$  once only.

B3 modulus.

*Slips (-1)*

S1 numerical.

**Part (b)(ii)**

**10 marks**

**Att 3**

**3(b)(ii)**  $k$  is a real number such that  $\frac{-1+i\sqrt{3}}{-4\sqrt{3}-4i} = ki$ .

Find  $k$ .

**Part (b)(ii)**

**10 marks**

**Att**

**3(b)(ii)**

$$\begin{aligned} \frac{-1+i\sqrt{3}}{-4\sqrt{3}-4i} \cdot \frac{-4\sqrt{3}+4i}{-4\sqrt{3}+4i} &= \frac{4\sqrt{3}-12i-4i+4\sqrt{3}i^2}{(48)+(16)} \\ &= \frac{-16i}{64} \\ &= -\left(\frac{1}{4}\right)i \end{aligned}$$

or

**3(b)(ii)**

$$\begin{aligned} \frac{-1+i\sqrt{3}}{-4\sqrt{3}-4i} &= ki \\ \rightarrow -1+i\sqrt{3} &= (-4k\sqrt{3})i + 4k \\ \Rightarrow (-1) + (\sqrt{3})i &= (4k) + (-4k\sqrt{3})i \end{aligned}$$

Equating like to like:

$$-1 = 4k \quad \text{or} \quad -4k\sqrt{3} = \sqrt{3}$$

$$-\frac{1}{4} = k$$

$$k = -\frac{1}{4}$$

*Blunders (-3)*

B1  $i$

B2 indices.

B3 cross multiplication.

B4 not like to like.

*Slips (-1)*

S1 numerical.

*Attempts*

A1 not using correct conjugate.

Part (c)

20(15, 5) marks

Att (5, 2)

3(c)  $1, \omega, \omega^2$  are the three roots of the equation  $z^3 - 1 = 0$ .

(i) Prove that  $1 + \omega + \omega^2 = 0$

(ii) Hence, find the value of  $(1 - \omega - \omega^2)^5$

Part (c) (i) Prove  
(ii) Value

15 marks  
5 marks

Att 5  
Att 2

3(c)(i)

$$z^3 - 1 = 0$$

$$z^3 = 1$$

$$\begin{aligned} z^3 &= r(\cos \theta + i \sin \theta) \\ &= 1(\cos 2n\pi + i \sin 2n\pi) \end{aligned}$$

$$z = (\cos 2n\pi + i \sin 2n\pi)^{\frac{1}{3}}$$

$$z = \cos \frac{2n\pi}{3} + i \sin \frac{2n\pi}{3}$$

$$n = 0 : Z_0 = \cos(0) + i \sin(0) = 1$$

$$n = 1 : Z_1 = \cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3} = -\frac{1}{2} + i \left( \frac{\sqrt{3}}{2} \right) = \omega$$

$$n = 2 : Z_2 = \cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3} = -\frac{1}{2} - i \left( \frac{\sqrt{3}}{2} \right) = \omega^2$$

$$\begin{aligned} 1 + \omega + \omega^2 &= 1 + \left[ -\frac{1}{2} + i \frac{\sqrt{3}}{2} \right] + \left[ -\frac{1}{2} - i \frac{\sqrt{3}}{2} \right] \\ &= 0 \end{aligned}$$

$$\begin{array}{|c} 1 \\ \hline |z^3| = 1 \\ \theta = 2n\pi \end{array}$$

or

3(c)(i)

$$z^3 - 1 = 0$$

$$z^3 - (0)z^2 + (0)z - 1 = 0$$

$$z^3 - (\sum \alpha)z^2 + (\sum \alpha\beta)z - (\alpha\beta\gamma) = 0$$

$$\Rightarrow \sum \alpha = 1 + \omega + \omega^2 = 0$$

or

**3(c)(i)**

$$z^3 - 1 = 0$$

$$(z-1)(z^2 + z + 1) = 0$$

$$z = 1 \quad \text{or} \quad z = \frac{-1 \pm \sqrt{1-4}}{2} = \frac{-1 \pm \sqrt{-3}}{2} = \frac{-1 \pm i\sqrt{3}}{2}$$

$$\begin{aligned} 1 + \omega + \omega^2 &= 1 + \left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) + \left(-\frac{1}{2} - i\frac{\sqrt{3}}{2}\right) \\ &= 0 \end{aligned}$$

or

**3(c)(i)**

$$z^3 - 1 = 0$$

$$(z-1)(z^2 + z + 1) = 0$$

$$z-1 = 0 \quad \text{or} \quad z^2 + z + 1 = 0$$

$$z = 1$$

$$f(z) = z^2 + z + 1$$

Since  $\omega$  is root,

$$f(\omega) = (\omega)^2 + (\omega) + 1 = 0$$

$$\Rightarrow 1 + \omega + \omega^2 = 0$$

**3(c)(ii)**

$$1 + \omega + \omega^2 = 0$$

$$1 = -\omega - \omega^2$$

$$1 = -(\omega + \omega^2)$$

$$\left[1 - \omega - \omega^2\right]^5 = \left[1 - (\omega + \omega^2)\right]^5$$

$$= [1+1]^5$$

$$= 32$$

*Blunders (-3)*

- B1 modulus.
- B2 argument.
- B3 formula De Moivre once only.
- B4 application De Moivre.
- B5 indices.
- B6 polar formula once only.
- B7  $i$
- B8 factors once only.
- B9 roots formula once only.
- B10 sum of roots.

*Slips (-1)*

- S1 numerical.
- S2 trig value.
- S3  $1 = \omega + \omega^2$

*Attempts*

- A1 no general solution when applying De Moivre.

## QUESTION 4

<b>Part (a)</b>	<b>10 marks</b>	<b>Att 3</b>
<b>Part (b)</b>	<b>20 (10, 10) marks</b>	<b>Att (3, 3)</b>
<b>Part (c)</b>	<b>20 (10, 10) marks</b>	<b>Att (3, 3)</b>

**Part (a)** **10 marks** **Att 3**

**4(a)**

Express the recurring decimal  $0.252525\dots$  in the form  $\frac{p}{q}$  where  $p, q \in \mathbf{N}$  and  $q \neq 0$

**Part (a)** **10 marks** **Att 3**

**4(a)(i)**

$$0.\dot{2}\dot{5} = 0.252525\dots$$

$$= \frac{25}{100} + \frac{25}{10000} + \frac{25}{1000000} + \dots$$

Geometric Series to  $\infty$ :  $a = \frac{25}{100}$      $r = \frac{1}{100}$

$$S_{\infty} = \frac{a}{1-R} = \frac{\frac{25}{100}}{1 - \frac{1}{100}} = \frac{\left(\frac{25}{100}\right)}{\left(\frac{99}{100}\right)} = \frac{25}{99}$$

or

**4(a)(i)**

Let  $x = 0.\dot{2}\dot{5}$

$$\begin{array}{r} x = 0.252525\dots \\ 100x = 25.252525\dots \\ \hline 99x = 25 \end{array}$$

$$x = \frac{25}{99}$$

\* Accept  $\frac{25}{99}$  for full marks and no work shown.

*Blunders (-3)*

B1 infinity formula once only.

B2  $a$ .

B3  $R$ .

*Slips (-1)*

S1 numerical.

Part (b)

20 (10, 10)

Att (3, 3)

4 (b) In an arithmetic series, the sum of the second term and the fifth term is 18. The sixth term is greater than the third term by 9.

(i) Find the first term and the common difference.

(ii) What is the smallest value of  $n$  such that  $S_n > 600$  where  $S_n$  is the sum of the first  $n$  terms of the series?

Part(b)(i)

10 marks

Att 3

(ii)

10 marks

Att 3

4 (b)(i)

Arithmetic series:  $u_2 + u_5 = 18 \Rightarrow (a + d) + (a + 4d) = 18$

$$2a + 5d = 18 \dots\dots\dots(i)$$

$$u_6 - u_3 = 9$$

$$(a + 5d) - (a + 2d) = 9$$

$$3d = 9$$

$$d = 3 \dots\dots\dots(ii)$$

(i)  $2a + 5d = 18$

$$2a + 15 = 18$$

$$a = \frac{3}{2}$$

$$a = \frac{3}{2}$$

$$d = 3$$

4(b)(ii)

Let  $S_n > 600$

$$\frac{n}{2}[2a + (n-1)d] > 600$$

$$\frac{n}{2}[3 + 3n - 3] > 600$$

$$3n^2 > 1200$$

$$n^2 > 400$$

$$n > 20$$

$$n = 21$$

$$(n > 0)$$



*Blunders (-3)*

- B1 term.
- B2 reversing terms in second condition.
- B3 formula AP once only.
- B4 incorrect 'a' in formula.
- B5 incorrect 'd' in formula.
- B6 inequality sign.
- B7 indices.
- B8 incorrect deduction or no deduction.
- B9 not getting 2<sup>nd</sup> value (having got 1<sup>st</sup> value)

*Worthless*

- W1 GP

**Part (c)(i)**

**10 marks**

**Att 3**

**4 (c)**

(i)  $u_1, u_2, u_3, u_4, u_5, \dots$  is a sequence where  $u_1 = 2$  and  $u_{n+1} = (-1)^n u_n + 3$

Evaluate  $u_2, u_3, u_4, u_5$  and  $u_{10}$ .

**Part (c)(i)**

**10 marks**

**Att 3**

**4(c)(i)**

$$u_{n+1} = (-1)^n u_n + 3 \quad : \quad u_1 = 2$$

$$u_2 = (-1)^1 (2) + 3 = 1$$

$$u_3 = (-1)^2 (1) + 3 = 4$$

$$u_4 = (-1)^3 (4) + 3 = -1$$

$$u_5 = (-1)^4 (-1) + 3 = 2$$

$$\Rightarrow 2, 1, 4, -1, 2, 1, 4, -1, 2, 1, \dots$$

↓

$u_{10}$

$$u_{10} = 1$$

*Blunders (-3)*

- B1 indices.
- B2 term omitted.
- B3 deduction  $u_{10}$  or no  $u_{10}$ .
- B4 formula once only.

*Slips (-1)*

- S1 numerical.



Part (c)(ii)

10 marks

Att 3

4(c) (ii)  $a, b, c, d$  are the first, second, third and fourth terms of a geometric sequence, respectively.

Prove that  $a^2 - b^2 - c^2 + d^2 \geq 0$

Part (c)(ii)

10 marks

Att 3

4(c)(ii)

4 terms in G.P.  $\Rightarrow a, b, c, d \Rightarrow a, ar, ar^2, ar^3$

$$\begin{aligned} a^2 - b^2 - c^2 + d^2 &= a^2 - (ar)^2 - (ar^2)^2 + (ar^3)^2 \\ &= a^2 - a^2r^2 - a^2r^4 + a^2r^6 \\ &= a^2[1 - r^2 - r^4 + r^6] \\ &= a^2[(1 - r^2) - r^4(1 - r^2)] \\ &= a^2(1 - r^2)[1 - r^4] \\ &= a^2(1 - r^2)(1 - r^2)(1 + r^2) \\ &= a^2(1 - r^2)^2(1 + r^2) \\ &\geq 0 \end{aligned}$$

or

4(c)(ii)

$$\begin{aligned} a, b, c, d \text{ in GP} &\Rightarrow \frac{b}{a} = \frac{c}{b} = \frac{d}{c} = r \\ &\Rightarrow \frac{b}{a} = \frac{d}{c} \Rightarrow d = \frac{bc}{a} \end{aligned}$$

$$\begin{aligned} &a^2 - b^2 - c^2 + d^2 \\ &= a^2 - b^2 - c^2 + \frac{b^2c^2}{a^2} \\ &= \frac{a^4 - a^2b^2 - a^2c^2 + b^2c^2}{a^2} \\ &= \frac{a^2(a^2 - c^2) - b^2(a^2 - c^2)}{a^2} \\ &= \frac{(a^2 - c^2)(a^2 - b^2)}{a^2} \end{aligned}$$

$$\begin{aligned} \text{If GP is increasing } a < c &\Rightarrow (a^2 - c^2) < 0 \\ |a| < |b| &\Rightarrow (a^2 - b^2) < 0 \\ &\Rightarrow \frac{(a^2 - c^2)(a^2 - b^2)}{a^2} \geq 0 \end{aligned}$$

$$\begin{aligned} \text{If GP is decreasing } a > c &\Rightarrow (a^2 - c^2) > 0 \\ |a| > |b| &\Rightarrow (a^2 - b^2) > 0 \\ &\Rightarrow \frac{(a^2 - c^2)(a^2 - b^2)}{a^2} \geq 0 \end{aligned}$$

*Blunders (-3)*

- B1 definition GP
- B2 indices.
- B3 factors once only.
- B4 incorrect deduction or no deduction.
- B5 inequality sign.

*Attempts*

- A1 particular values verified.

*Worthless*

- W1 AP

## QUESTION 5

<b>Part (a)</b>	<b>10 (5, 5)marks</b>	<b>Att (2, 2)</b>
<b>Part (b)</b>	<b>20(5, 5, 10) marks</b>	<b>Att (2, 2, 3)</b>
<b>Part (c)</b>	<b>20(5, 5, 5, 5) marks</b>	<b>Att (2, 2, 2, 2)</b>

**Part (a)** **10(5, 5,) marks** **Att 2**

**5(a)** Solve for  $x$ :

$$x = \sqrt{7x - 6} + 2$$

**Quadratic** **5 marks** **Att 2**  
**Finish** **5 marks** **Att 2**

**5(a)(i)**

$$\begin{aligned}
 x - 2 &= \sqrt{7x - 6} \\
 (x - 2)^2 &= 7x - 6 \\
 x^2 - 4x + 4 &= 7x - 6 \\
 x^2 - 11x + 10 &= 0 \\
 (x - 1)(x - 10) &= 0 \\
 \Rightarrow x - 1 = 0 & \quad \text{or} \quad x - 10 = 0 \\
 x = 1 & \quad \text{or} \quad x = 10
 \end{aligned}$$

Test  $x = 1$       LHS = -1      RHS =  $\sqrt{7x - 6} = \sqrt{1} = 1$   
LHS  $\neq$  RHS

$x = 10$       LHS = 8      RHS =  $\sqrt{70 - 6} = \sqrt{64} = 8$   
LHS = RHS

Solution:  $x = 10$

*Blunders (-3)*

- B1 indices.
- B2 factors once only.
- B3 root formula once only.
- B4 deductions values from factors or no values.

*Slips (-1)*

- S1 numerical.
- S2 extra value.

*Attempts*

- A1  $x = 10$  and no other work merits 2 marks.

Part (b)

20(5, 5, 10) marks

Att (2, 2, 3)

5(b)

Use induction to prove that 8 is a factor of  $7^{2n+1} + 1$  for any positive integer  $n$ .

P(1)

5 marks

Att 2

Assume

5 marks

Att 2

$P(k+1)$

10 marks

Att 3

5(b)

$$\begin{aligned} \text{Let } n = 1 \quad 7^{2n+1} + 1 &= 7^3 + 1 = 344 \div 8 = 43 \\ &\Rightarrow \text{true for } n = 1 \end{aligned}$$

Assume true for  $n = k \Rightarrow (7^{2k+1} + 1)$  is divisible by 8.

To prove  $[7^{2(k+1)+1} + 1]$  is divisible by 8

i.e. that  $(7^{2k+3} + 1)$  is divisible by 8

[ using difference between  $P(k)$  and  $P(k+1)$  ]

$$(7^{2k+3} + 1) - (7^{2k+1} + 1) = 7^{2k+1} \cdot 7^2 - 7^{2k+1}$$

$$= 7^{2k+1}(49 - 1)$$

$$= 7^{2k}(48),$$

which is divisible by 8

$\therefore P(k+1)$  true whenever  $P(k)$  is true

Since  $P(1)$  true, then, by induction,  $P(n)$  true for any positive integer  $n$ .

or

5(b)

$$\begin{aligned} \text{Let } n = 1 \quad 7^{2n+1} + 1 &= 7^3 + 1 = 344 \div 8 = 43 \\ &\Rightarrow \text{true for } n = 1 \end{aligned}$$

Assume true for  $n = k: \Rightarrow (7^{2k+1} + 1)$  is divisible by 8

$$\therefore 7^{2k+1} + 1 = 8A, \text{ where } A \text{ is an integer.}$$

$$\therefore 7^{2k+1} = 8A - 1$$

To prove:  $(7^{2(k+1)+1} + 1)$  is divisible by 8:

$$\begin{aligned} 7^{2k+3} + 1 &= 7^2 \cdot 7^{2k+1} + 1 \\ &= 49[8A - 1] + 1 \\ &= 49(8A) - 49 + 1 \\ &= 49(8A) - 48 \\ &= 8[49A - 6] \end{aligned}$$

which is divisible by 8

$\therefore P(k+1)$  true whenever  $P(k)$  is true

Since  $P(1)$  true, then, by induction,  $P(n)$  true for any positive integer  $n$ .

or

**5(b)**

$$\text{Let } n = 1 \quad 7^{2n+1} + 1 = 7^3 + 1 = 344 \div 8 = 43 \\ \Rightarrow \text{ true for } n = 1$$

$$P(k+1) = (7^{2k+3} + 1) = 7^2 \cdot 7^{2k+1} + 1 \\ = 49(7^{2k+1}) + 1 \\ = 48(7^{2k+1}) + (7^{2k+1} + 1) \\ = 48(7^{2k+1}) + P(k),$$

which is divisible by 8 since  $P(k)$  divisible by 8

$\therefore P(k+1)$  true whenever  $P(k)$  is true

Since  $P(1)$  true, then, by induction,  $P(n)$  true for any positive integer  $n$ .

*Blunders (-3)*

B1 indices.

B2  $n \neq 1$  (accept  $n = 0$ )

\* Note must prove  $n = 1$  (not sufficient to state  $P(n)$  true when  $n = 1$ ).

**Part (c)**

**20(5, 5, 5, 5)marks**

**Att (2, 2, 2, 2)**

**5(c)** Consider the binomial expansion of  $\left(ax + \frac{1}{bx}\right)^8$ , where  $a$  and  $b$  are non-zero real numbers.

(i) Write down the general term.

(ii) Given that the coefficient of  $x^2$  is equal to the coefficient of  $x^4$ , show that  $ab = 2$ .

**General Term**

**5 marks**

**Att 2**

**$x^2$  Term**

**5 marks**

**Att 2**

**$x^4$  Term**

**5 marks**

**Att 2**

**Show**

**5 marks**

**Att 2**

**5(c)**  $\left(ax + \frac{1}{bx}\right)^8$

(i)  $u_{R+1} = \binom{8}{R} (ax)^{8-R} \left(\frac{1}{bx}\right)^R$

(ii) Let  $u_{R+1}$  be term with  $x^2$

$$\binom{8}{R} (ax)^{8-R} (bx)^{-R} = kx^2$$

$$k[x^{8-R} \cdot x^{-R}] = kx^2$$

$$x^{8-2R} = x^2$$

$$\Rightarrow 8 - 2R = 2$$

$$6 = 2R$$

$$3 = R$$

$$U_4 = \binom{8}{3} (ax)^5 \left(\frac{1}{bx}\right)^3$$

$$= \frac{8 \cdot 7 \cdot 6}{1 \cdot 2 \cdot 3} (a^5 x^5) \frac{1}{b^3 x^3} = \left(\frac{56a^5}{b^3}\right) x^2$$

Now let  $U_{R+1}$  be term with  $x^4$

$$x^{8-2R} = x^4$$

$$\Rightarrow 8 - 2R = 4$$

$$4 = 2R$$

$$2 = R$$

$$U_3 = \binom{8}{2} (ax)^6 \left(\frac{1}{bx}\right)^2$$

$$= \frac{8 \cdot 7}{1 \cdot 2} \cdot a^6 x^6 \frac{1}{b^2 x^2} = \left(\frac{28a^6}{b^2}\right) x^4$$

Equating coefficients:

$$\frac{56a^5}{b^3} = \frac{28a^6}{b^2}$$

$$56a^5 b^2 = 28a^6 b^3$$

$$ab = 2$$

\* Accept multiplication or Binomial expansion simplified.

*Blunders (-3)*

B1 general term.

B2 error binomial expansion once only.

B3  $\binom{n}{r}$  or no  $\binom{n}{r}$ .

B4 indices.

B5 not like to like.

*Slips (-1)*

S1 numerical.

*Attempts*

A1 correct trial and error.

## QUESTION 6

<b>Part (a)</b>	<b>10 marks</b>	<b>Att 3</b>
<b>Part (b)</b>	<b>20 (5, 5, 5, 5) marks</b>	<b>Att (2, 2, 2, 2)</b>
<b>Part (c)</b>	<b>20 (5, 5, 10) marks</b>	<b>Att (2, 2, 3)</b>

**Part (a)** **10 marks** **Att 3**

**6(a)**

Differentiate  $\sqrt{1+4x}$  with respect to  $x$

**Part (a)** **10 marks** **Att 3**

**6(a)**

$$y = \sqrt{1+4x} = (1+4x)^{\frac{1}{2}}$$

$$\frac{dy}{dx} = \frac{1}{2}(1+4x)^{-\frac{1}{2}} \cdot (4) = \frac{2}{(1+4x)^{\frac{1}{2}}} = \frac{2}{\sqrt{1+4x}}$$

*Blunders (-3)*

B1 differentiation.

B2 indices.

*Attempts*

A1 error in differentiation formula.

**Part (b)** **20 (5, 5, 5, 5) marks** **Att (2, 2, 2, 2)**

**6(b)** Show that the equation  $x^3 - 4x - 2 = 0$  has a root between 2 and 3.

Taking  $x_1 = 2$  as the first approximation to this root, use the Newton-Raphson method to find  $x_3$ , the third approximation. Give your answer correct to two decimal places.

**Show** **5 marks** **Att 2**

$f'(x)$  **5 marks** **Att 2**

$x_2$  **5 marks** **Att 2**

$x_3$  **5 marks** **Att 2**

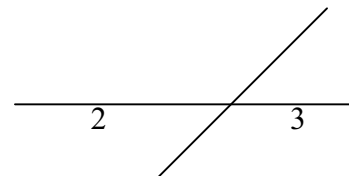
**6(b)**

$$f(x) = x^3 - 4x - 2$$

$$f(2) = 8 - 8 - 2 < 0$$

$$f(3) = 27 - 12 - 2 > 0$$

$\Rightarrow$  root between 2 and 3  $\Rightarrow$  root = 2.abc



$$f(x) = x^3 - 4x - 2$$

$$f'(x) = 3x^2 - 4$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$x_n$	$f(x) = x^3 - 4x - 2$	$f'(x) = 3x^2 - 4$	$x_{n+1}$
2	-2	8	$2 - \frac{(-2)}{8} = 2.25$
2.25	0.3906	11.1875	$2.25 - 0.0349 = 2.2151$

$$x_3 = 2.2151 = 2.22$$

*Blunders (-3)*

- B1 Newton-Raphson formula once only.
- B2 differentiation.
- B3 indices.
- B4  $x_1 \neq 2$ , once only.
- B5 inequality sign.
- B6 deduction from work or no deduction (root).
- B7 incorrect value in table (unless an obvious slip)

*Slips (-1)*

- S1 numerical.
- S2 answer not to 2 decimal places.

*Worthless*

- W1 incorrect answer and no work.

\* Accept  $x_3 = 2.22$  by calculation.

**Part (c)**

**20(5, 5, 10) marks**

**Att (2, 2, 3)**

**6(c)** The function  $f(x) = \frac{1}{1-x}$  is defined for  $x \in \mathbf{R} \setminus \{1\}$ .

- (i) Prove that the graph of  $f$  has no turning points and no points of inflection.
- (ii) Write down a reason that justifies the statement: “ $f$  is increasing at every value of  $x \in \mathbf{R} \setminus \{1\}$ ”.
- (iii) Given that  $y = x + k$  is a tangent to the graph of  $f$  where  $k$  is a real number, find the two possible values of  $k$ .



**Turning points & points of inflection**      **5 marks**  
**Reason**      **5 marks**  
**Values of  $k$**       **10 marks**

**Att 2**  
**Att 2**  
**Att 3**

**6(c)(i)**       $f(x) = \frac{1}{(1-x)} = (1-x)^{-1}$   
 $f'(x) = (-1)(1-x)^{-2}(-1) = (1-x)^{-2}$   
 $f''(x) = -2(1-x)^{-3}(-1) = 2(1-x)^{-2} = \frac{2}{(1-x)^3}$   
 $f'(x) = \frac{1}{(1-x)^2} \neq 0 \Rightarrow$  no turning points  
 $f''(x) = \frac{2}{(1-x)^3} \neq 0 \Rightarrow$  no points of inflection

**(ii)**       $f'(x) = \frac{1}{(1-x)^2} > 0$  for all  $x \Rightarrow f(x)$  is an increasing function for all  $x$

**(iii)**       $y = x + k$  is tangent  
Slope  $m = f'(x) = 1 \quad \Rightarrow \quad \frac{1}{(1-x)^2} = 1$   
 $\Rightarrow (1-x)^2 = 1$   
 $1 - 2x + x^2 = 1$   
 $x^2 - 2x = 0$   
 $x(x-2) = 0$   
 $\Rightarrow x = 0 \quad \text{or} \quad x = 2$

$x = 0: \quad y = \frac{1}{1-x} = \frac{1}{1-0} = 1 \Rightarrow (0,1)$

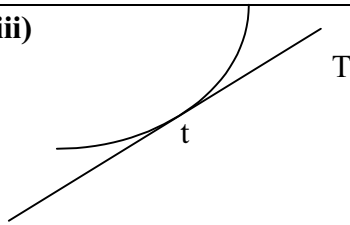
$x = 2: \quad y = \frac{1}{1-x} = \frac{1}{1-2} = -1 \Rightarrow (2,-1)$

$(0, 1) \in y = x + k$       or       $(y - y_1) = m(x - x_1)$   
 $1 = 0 + k$        $m = 1: (x_1, y_1) = (0,1)$   
 $\Rightarrow k = 1$        $(y - 1) = 1(x - 0)$   
 $y = x + 1 \Rightarrow k = 1$

$(2,-1) \in y = x + k$        $m = 1: (x_1, y_1) = (2,-1)$   
 $-1 = 2 + k$        $(y + 1) = 1(x - 2)$   
 $-3 = k$        $y = x - 3 \Rightarrow k = -3$

or

6(c)(iii)



$$\begin{aligned}T \cap \{\text{curve}\} &= \{t\} \\ \left\{y = \frac{1}{1-x}\right\} \cap \{y = x+k\} \\ \Rightarrow \frac{1}{1-x} &= x+k \\ 1 &= (x+k)(1-x) \\ 1 &= x - x^2 + k - kx \\ x^2 + kx - x + 1 - k &= 0 \\ x^2 + (k-1)x + (1-k) &= 0\end{aligned}$$

Since T is tangent to curve, quadratic has only one root.

$$\begin{aligned}\Rightarrow b^2 - 4ac &= 0 \\ (k-1)^2 - 4(1-k) &= 0 \\ k^2 - 2k + 1 - 4 + 4k &= 0 \\ k^2 + 2k - 3 &= 0 \\ (k+3)(k-1) &= 0 \\ k+3 = 0 \quad \text{or} \quad k-1 = 0 \\ k = -3 \quad \text{or} \quad k = 1\end{aligned}$$

*Blunders (-3)*

- B1 differentiation.
- B2 indices.
- B3  $f'(x) \neq 0$
- B4  $f''(x) \neq 0$
- B5 deduction.
- B6 factors once only.
- B7 root formula once only.
- B8 no  $y$  values.
- B9 equation straight line
- B10  $b^2 - 4ac \neq 0$ .
- B11 deduction roots from factors.
- B12 expansion of  $(k-1)^2$  once only.

*Slips (-1)*

- S1 numerical.

*Attempts*

- A1 error in differentiation formula.
- A2 graphical methods.

*Worthless*

- W1 integration.

## QUESTION 7

<b>Part (a)</b>	<b>10 (5, 10)marks</b>	<b>Att (2, 3)</b>
<b>Part (b)</b>	<b>20 (10, 10) marks</b>	<b>Att (3, 3)</b>
<b>Part (c)</b>	<b>15 (10, 5) marks</b>	<b>Att (3, 2)</b>

**Part (a)** **15 (5, 10) marks** **Att (2, 3)**

**7(a)**

Differentiate each of the following with respect to  $x$  :

(i)  $\cos^4 x$                       (ii)  $\sin^{-1} \frac{x}{5}$

(i) **5 marks** **Att 2**

(ii) **10 marks** **Att 3**

**7(a)(i)**

$$y = \cos^4 x = (\cos x)^4$$

$$\frac{dy}{dx} = 4(\cos x)^3 (-\sin x) = -4 \cos^3 x \sin x$$

**(ii)**

$$y = \sin^{-1} \left( \frac{x}{5} \right) \qquad a = 5$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{a^2 - x^2}} = \frac{1}{\sqrt{25 - x^2}}$$

*Blunders (-3)*

B1 differentiation.

B2 indices.

*Attempts*

A1 error in differentiation formula.

**Part (b)(i)**

**10 marks**

**Att 3**

**7(b)(i)** The parametric equations of a curve are:

$$x = \cos t + t \sin t$$

$$y = \sin t - t \cos t \text{ where } 0 < t < \frac{\pi}{2}.$$

Find  $\frac{dy}{dx}$  and write your answer in its simplest form.

**Part(b)(i)**

**10 marks**

**Att**

**7(b)(i)**

$$x = \cos t + t \sin t$$

$$\frac{dx}{dt} = (-\sin t) + [t(\cos t) + \sin t(1)]$$

$$= -\sin t + t \cos t + \sin t$$

$$= t \cos t$$

$$y = \sin t - t \cos t$$

$$\frac{dy}{dt} = \cos t - [t(-\sin t) + (\cos t)(1)]$$

$$= \cos t + t \sin t - \cos t$$

$$= t \sin t$$

$$\frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{t \sin t}{t \cos t} = \tan t$$

*Blunders (-3)*

B1 differentiation.

B2 indices.

B3  $\frac{dy}{dx}$

*Slips (-1)*

S1 not in simplest form.

*Attempts*

A1 error in differentiation formula.

Part (b)(ii)

10 marks

Att 3

7(b)(ii) Given that  $\frac{1}{x} + \frac{1}{y} = \frac{1}{6}$ , find the value of  $\frac{dy}{dx}$  at the point (2, -3)

Part(b)(ii)

10 marks

Att 3

7(b)(ii)

$$\frac{1}{x} + \frac{1}{y} = \frac{1}{6}$$

$$\frac{1}{y} = \frac{1}{6} - \frac{1}{x}$$

$$\frac{1}{y} = \frac{x-6}{6x}$$

$$y = \frac{6x}{x-6}$$

$$\frac{dy}{dx} = \frac{(x-6)(6) - (6x)(1)}{(x-6)^2} = \frac{6x-36-6x}{(x-6)^2} = \frac{-36}{(x-6)^2}$$

At (2, -3) :  $x = 2$ :  $\frac{dy}{dx} = \frac{-36}{(x-6)^2} = \frac{-36}{(2-6)^2} = \frac{-36}{16} = \frac{-9}{4}$

Or

7(b)(ii)

$$\frac{1}{x} + \frac{1}{y} = \frac{1}{6}$$

$$x^{-1} + y^{-1} = \frac{1}{6}$$

$$-1 \cdot x^{-2} + (-1) \cdot y^{-2} \cdot \frac{dy}{dx} = 0$$

$$-\frac{1}{x^2} - \frac{1}{y^2} \cdot \frac{dy}{dx} = 0$$

$$-\frac{1}{x^2} = \frac{1}{y^2} \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{-y^2}{x^2}$$

At (2, -3)  $x = 2$ ,  $y = -3$   $\frac{dy}{dx} = \frac{-(-3)^2}{(2)^2} = \frac{-9}{4}$

*Blunders (-3)*

B1 algebra.

B2 differentiation.

B3 substitution of (2, -3) or no substitution.

*Slips (-1)*

S1 numerical.

*Attempts*

A1 error in differentiation formula.

Part (c)(i)

10 marks

Att 3

7(c)(i) Given that  $y = \ln \frac{1+x^2}{1-x^2}$  for  $0 < x < 1$ , find  $\frac{dy}{dx}$  and write your answer in the form

$$\frac{kx}{1-x^k} \text{ where } k \in \mathbf{N}.$$

Part (c)(i)

10 marks

Att 3

7(c)(i)

$$\begin{aligned} y &= \ln \left( \frac{1+x^2}{1-x^2} \right) \\ y &= \ln(1+x^2) - \ln(1-x^2) \\ \frac{dy}{dx} &= \frac{1}{1+x^2} (2x) - \frac{1}{1-x^2} (-2x) \\ &= \frac{2x}{(1+x^2)} + \frac{2x}{(1-x^2)} \\ &= \frac{2x(1-x^2) + 2x(1+x^2)}{(1+x^2)(1-x^2)} \\ &= \frac{2x - 2x^3 + 2x + 2x^3}{1-x^4} = \frac{4x}{1-x^4} \end{aligned}$$

or

7(c)(i)

$$\begin{aligned} y &= \ln \left( \frac{1+x^2}{1-x^2} \right) \\ \frac{dy}{dx} &= \frac{1}{\left[ \frac{1+x^2}{1-x^2} \right]} \cdot \frac{(1-x^2)(2x) - (1+x^2)(-2x)}{(1-x^2)^2} \\ &= \frac{(1-x^2)}{(1+x^2)} \cdot \frac{2x - 2x^3 + 2x + 2x^3}{(1-x^2)^2} \\ &= \frac{4x}{(1+x^2)(1-x^2)} \\ &= \frac{4x}{1-x^4} \end{aligned}$$

*Blunders (-3)*

B1 differentiation.

B2 logs.

B3 indices.

B4 not in required form.

*Slips (-1)*

S1 numerical.

*Attempts*

A1 error in differentiation formula.

**Part (c)(ii)****5 marks****Att 2**

**(c)(ii)** Given that  $f(\theta) = \sin(\theta + \pi)\cos(\theta - \pi)$ , find the derivative of  $f(\theta)$  and express it in the form  $\cos n\theta$ , where  $n \in \mathbb{Z}$

**Part (c)(ii)****5 marks****Att 2****7(c)(ii)**

$$\begin{aligned}
 f(\theta) &= \sin(\theta + \pi)\cos(\theta - \pi) \\
 f'(\theta) &= \sin(\theta + \pi)[- \sin(\theta - \pi)] + \cos(\theta - \pi)[\cos(\theta + \pi)] \\
 &= \cos(\theta + \pi)\cos(\theta - \pi) - \sin(\theta + \pi)\sin(\theta - \pi) \\
 &= \cos A.\cos B - \sin A.\sin B \\
 &= \cos[(\theta + \pi) + (\theta - \pi)] \\
 &= \cos 2\theta
 \end{aligned}$$

or

**7(c)(ii)**

$$\begin{aligned}
 f(\theta) &= \sin(\theta + \pi)\cos(\theta - \pi) \\
 &= \frac{1}{2}[2\sin(\theta + \pi)\cos(\theta - \pi)] \\
 &= \frac{1}{2}[\sin[(\theta + \pi) + (\theta - \pi)] + \sin[(\theta + \pi) - (\theta - \pi)]] \\
 f(\theta) &= \frac{1}{2}[\sin 2\theta + \sin 2\pi] \\
 f'(\theta) &= \frac{1}{2}[2\cos 2\theta] = \cos 2\theta
 \end{aligned}$$

or

**7(c)(ii)**

$$f(\theta) = \sin(\theta + \pi)\cos(\theta - \pi)$$

$$\begin{aligned}
 \sin(\theta + \pi) &= \sin \theta \cos \pi + \cos \theta \sin \pi \\
 &= -\sin \theta + 0 \\
 &= -\sin \theta \\
 \cos(\theta - \pi) &= \cos \theta \cos \pi + \sin \theta \sin \pi \\
 &= -\cos \theta + 0 \\
 &= -\cos \theta
 \end{aligned}$$

$$\begin{aligned}
 \cos \pi &= -1 \\
 \sin \pi &= 0
 \end{aligned}$$

$$f(\theta) = \sin(\theta + \pi)\cos(\theta - \pi) = (-\sin \theta)(-\cos \theta)$$

$$f(\theta) = \sin \theta \cos \theta = \frac{1}{2}[\sin 2\theta]$$

$$f'(\theta) = \frac{1}{2}[2\cos 2\theta] = \cos 2\theta$$

*Blunders (-3)*

B1 trig formula.

B2 differentiation.

B3 not in required form.

*Slips (-1)*

S1 numerical.

S2 trig value.

*Attempts*

A1 error in differentiation formula.

## QUESTION 8

<b>Part (a)</b>	<b>10 (5, 5)marks</b>	<b>Att (2, 2)</b>
<b>Part (b)</b>	<b>20 (10, 10)</b>	<b>Att (3, 3)</b>
<b>Part (c)</b>	<b>20 (10, 10) marks</b>	<b>Att (3, 3)</b>

**Part (a)** **10 (5, 5)marks** **Att (2, 2)**

**(a)**

Find **(i)**  $\int (x^3 + 2) dx$  **(ii)**  $\int e^{7x} dx$

**Part(a) (i)** **5 marks** **Att 2**  
**(ii)** **5 marks** **Att 2**

**8(a)** **(i)**  $\int (x^3 + 2) dx = \frac{x^4}{4} + 2x + c$

**(ii)**  $\int e^{7x} dx = \frac{e^{7x}}{7} + c$

\* If  $c$  shown once  $\Rightarrow$  no penalty

*Blunders (-3)*

B1 integration.

B2 no ' $c$ ' once only. (Penalise 1<sup>st</sup> integration)

B3 indices.

*Attempts*

A1 only  $c$  correct.

*Worthless*

W1 differentiation instead of integration.



Part (b)

20 (10, 10)

Att (3, 3)

**8(b)(i)** Evaluate  $\int_0^1 \frac{2x}{\sqrt{1+x^2}} dx$

**(ii)** By letting  $u = \sin x$ , evaluate  $\int_0^{\frac{\pi}{2}} \cos x \sin^6 x dx$

Part (i)

10 marks

Att 3

Part(ii)

10 marks

Att 3

<b>8(b)(i)</b>	$\int_0^1 \frac{2x}{\sqrt{1+x^2}} dx$ $= \int \frac{dw}{w^{\frac{1}{2}}}$ $= \int w^{-\frac{1}{2}} \cdot dw$ $= \frac{w^{\frac{1}{2}}}{\frac{1}{2}}$ $= 2 \cdot w^{\frac{1}{2}} \Big _1^2$ $= 2[\sqrt{2} - \sqrt{1}] = 2(\sqrt{2} - 1)$	<p>Let <math>w = 1 + x^2</math></p> $\frac{dw}{dx} = 2x$ $dw = 2x \cdot dx$ <p><math>x = 1 : w = 1 + x^2 = 1 + 1 = 2</math></p> <p><math>x = 0 : w = 1 + x^2 = 1 + 0 = 1</math></p>
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or

**8(b)(i)**

$$2w^{\frac{1}{2}} = 2(1+x^2)^{\frac{1}{2}} \Big|_0^1$$

$$= 2\left[(2)^{\frac{1}{2}} - (1)^{\frac{1}{2}}\right]$$

$$= 2[\sqrt{2} - 1]$$

<b>8(b)(ii)</b>	$\int_0^{\frac{\pi}{2}} \sin^6 x \cdot (\cos x dx)$ $= \int u^6 \cdot du$ $= \frac{u^7}{7} \Big _0^1$ $= \frac{1}{7}[1 - 0]$ $= \frac{1}{7}$	<p><math>u = \sin x</math></p> $\frac{du}{dx} = \cos x$ $du = \cos x \cdot dx$ <p><math>x = \frac{\pi}{2} : u = \sin \frac{\pi}{2} = 1</math></p> <p><math>x = 0 : u = \sin 0 = 0</math></p>
-----------------	--	--

or

**8(b)(ii)**

$$\begin{aligned} \dots \frac{1}{7} [u^7] &= \frac{1}{7} [(\sin x)^7]_{\frac{\pi}{2}}^0 \\ &= \frac{1}{7} \left[ \left( \sin \frac{\pi}{2} \right)^7 - (\sin 0)^7 \right] \\ &= \frac{1}{7} [1^7 - 0^7] \\ &= \frac{1}{7} (1) = \frac{1}{7} \end{aligned}$$

*Blunders (-3)*

- B1 integration.
- B2 indices.
- B3 limits.
- B4 no limits.
- B5 incorrect order in applying limits.
- B6 not calculating substituted limits.
- B7 not changing limits.
- B8 differentiation.

*Slips (-1)*

- S1 numerical.
- S2 trig value.

*Worthless:*

- W1 differentiation instead of integration except where other work merits attempt.

Note: Incorrect substitution and unable to finish yields attempt at most.

Note: (-3) is maximum deduction when evaluating limits

**Part (c)(i)**

**10 marks**

**Att 3**

**8(c)(i)** Show that  $\int_a^{2a} \sin 2x dx = \sin 3a \sin a$ .

**Part (c)(i)**

**10 marks**

**Att 3**

**8(c)(i)**

$$\begin{aligned} \int_a^{2a} \sin 2x dx &= \left[ \frac{-\cos 2x}{2} \right]_a^{2a} \\ &= -\frac{1}{2} [\cos 2(2a) - \cos 2(a)] \\ &= -\frac{1}{2} [\cos 4a - \cos 2a] \\ &= -\frac{1}{2} [-2 \sin 3a \sin a] \\ &= \sin 3a \sin a \end{aligned}$$

or

$$\begin{aligned} \mathbf{8(c)(i)} \quad \int_a^{2a} \sin 2x dx &= \int 2 \sin x (\cos x dx) & u &= \sin x \\ &= \int 2u \cdot du & \frac{du}{dx} &= \cos x \\ &= u^2 & du &= \cos x \cdot dx \\ &= \sin^2 x \\ &= \frac{1}{2} [1 - \cos 2x]_a^{2a} = \frac{1}{2} [\cos 2a - \cos 4a] = \sin 3a \cdot \sin a \end{aligned}$$

*Blunders (-3)*

- B1 integration.
- B2 limits.
- B3 no limits.
- B4 incorrect order in applying limits.
- B5 trig formula.
- B6 not calculating substituted limits.

*Slips (-1)*

- S1 numerical

*Worthless*

- W1 differentiation instead of integration except where other work merits attempts.

Note: (-3) is maximum deduction in evaluation of limits.

Note: Incorrect substitution and unable to finish merits attempt at most.

**Part (c)(ii)**

**10 marks**

**Att 3**

**(c)(ii)** Use integration methods to show that the volume of a sphere with radius  $r$  is  $\frac{4}{3}\pi r^3$

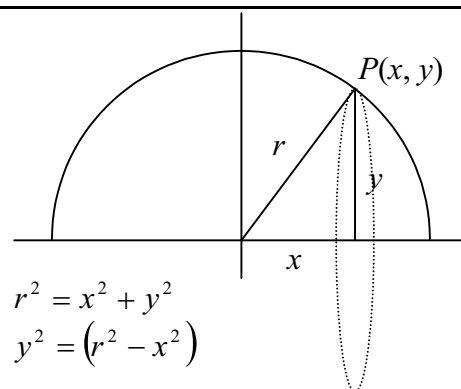
**Part (c)(ii)**

**10 marks**

**Att 3**

**8(c)(ii)**

$$\begin{aligned} V &= \int_{-r}^r \pi y^2 dx \\ &= \pi(2) \int_0^r y^2 dx \\ &= 2\pi \int (r^2 - x^2) dx \\ &= 2\pi \left[ r^2 x - \frac{x^3}{3} \right]_0^r \\ &= 2\pi \left[ r^3 - \frac{r^3}{3} \right] \\ &= 2\pi \left( \frac{2r^3}{3} \right) = \frac{4\pi r^3}{3} \end{aligned}$$



*Blunders (-3)*

- B1 equation circle.
- B2 volume formula.(must be quadratic).
- B3 limits.
- B4 no limits.
- B5 incorrect order in applying limits.
- B6 indices.

*Slips (-1)*

- S1 numerical

*Attempts*

- A1 uses  $v = \pi y$

*Worthless*

- W1 differentiation instead of integration.
- W2  $\int 4\pi R^2 dR$

Note: (-3) is maximum deduction in evaluation of limits.

# MARKING SCHEME

## LEAVING CERTIFICATE EXAMINATION 2003

### MATHEMATICS

### HIGHER LEVEL

### PAPER 2

#### GENERAL GUIDELINES FOR EXAMINERS - PAPER 2

1. Penalties of three types are applied to candidates' work as follows:

- Blunders - mathematical errors/omissions (-3)
- Slips - numerical errors (-1)
- Misreadings (provided task is not oversimplified) (-1).

Frequently occurring errors to which these penalties must be applied are listed in the scheme. They are labelled as B1, B2, B3,....., S1, S2, S3,....., M1, M2, etc. Note that these lists are not exhaustive.

2. When awarding attempt marks, e.g. Att(3), note that
  - any correct relevant step in a part of a question merits *at least* the attempt mark for that part
  - if deductions result in a mark which is lower than the attempt mark, then the attempt mark must be awarded
  - a mark between zero and the attempt mark is never awarded.
3. Worthless work is awarded zero marks. Some examples of such work are listed in the scheme and they are labelled as W1, W2,.....etc.
4. The *same* error in the *same* section of a question is penalised *once* only.
5. Special notes relating to the marking of a particular part of a question are indicated by an asterisk. These notes immediately follow the box containing the relevant solution.
6. Particular cases, verifications and answers derived from diagrams (unless requested) qualify for attempt marks at most.
7. A serious blunder, omission or misreading merits the ATTEMPT mark at most.
8. The phrase “and stops” means that no more work is shown by the candidate.
9. Accept the best of two or more attempts – even when attempts have been cancelled.

## QUESTION 1

<b>Part (a)</b>	<b>10 (5, 5) marks</b>	<b>Att (2, 2)</b>
<b>Part (b)</b>	<b>20 (5, 5, 5, 5) marks</b>	<b>Att (2, 2, 2, -)</b>
<b>Part (c)</b>	<b>20 (10,10) marks</b>	<b>Att (3, 3)</b>

**Part (a)** **10 (5, 5) marks** **Att (2, 2)**

**Substitutes into  $x^2$  and  $y^2$**

**or Cartesian point on circle found**

**or  $t^2(t)$  in terms of  $x$  and in terms of  $y$**  **5 marks** **Att 2**

**Finish** **5 marks** **Att 2**

**1(a)**

For all values of  $t \in \mathbf{R}$ , the point  $\left(\frac{3-3t^2}{1+t^2}, \frac{6t}{1+t^2}\right)$  lies on the circle

$$x^2 + y^2 = r^2.$$

Find  $r$ , the radius of the circle.

**Radius of circle** **10 (5, 5) marks** **Att (2, 2)**

**1 (a)**

$$t = 0 \Rightarrow (3, 0) \in x^2 + y^2 = r^2 \Rightarrow 9 = r^2. \quad \therefore r = 3.$$

**or**

$$\begin{aligned} \left(\frac{3-3t^2}{1+t^2}\right)^2 + \left(\frac{6t}{1+t^2}\right)^2 &= \frac{9-18t^2+9t^4+36t^2}{(1+t^2)^2} = \frac{9t^4+18t^2+9}{(1+t^2)^2} \\ &= \frac{9(t^4+2t^2+1)}{(t^2+1)^2} = \frac{9(t^2+1)^2}{(t^2+1)^2} = 9. \end{aligned}$$

$$\therefore r^2 = 9 \Rightarrow r = 3.$$

**or**

$$x = \frac{3-3t^2}{1+t^2} \Rightarrow x + xt^2 = 3 - 3t^2 \Rightarrow t^2(x+3) = 3-x$$

$$\therefore t^2 = \frac{3-x}{3+x}.$$

$$\text{But } y^2 = \frac{36t^2}{(1+t^2)^2} = \frac{36\left(\frac{3-x}{3+x}\right)}{\left(1+\frac{3-x}{3+x}\right)^2} = \frac{36(3-x)(3+x)}{36}.$$

$$\therefore y^2 = 9 - x^2 \Rightarrow x^2 + y^2 = 9. \quad \therefore r = 3.$$

\* Accept  $r^2 = 9$ .

*Blunders (-3)*

B1 incorrect squaring (apply once if same type of error).

B2 incorrect factorisation.

*Slips (-1)*

S1 arithmetic error.

Attempts ( 2, 2 marks)

- A1 substitutes for x only or substitutes for y only.
- A2 fails to eliminate t.
- A3 assigns value for t.
- A4 attempt at simplification

**Part (b)** **20 (5, 5, 5, 5) marks** **Att (2, 2, 2, -)**

**Part (b) (i)** **15 (5, 5, 5) marks** **Att (2, 2, 2)**

**Centre & radius  $C_1$**  **5 marks** **Att 2**

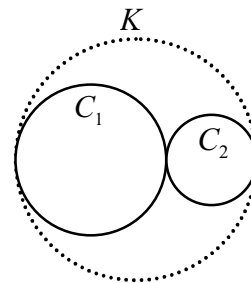
**Centre & radius  $C_2$**  **5 marks** **Att 2**

**Final conclusion** **5 marks** **Att 2**

**1 (b) (i)**

$C_1: x^2 + y^2 + 2x - 2y - 23 = 0$  and  
 $C_2: x^2 + y^2 - 14x - 2y + 41 = 0$  are two circles.

**(i)** Prove that  $C_1$  and  $C_2$  touch externally.



**$C_1$  and  $C_2$  touch externally** **15 (5, 5, 5) marks** **Att (2, 2, 2)**

**1 (b) (i)**

Centre  $C_1 = c_1(-1, 1)$ , radius  $= \sqrt{1+1+23} = 5$ .  $\therefore r_1 = 5$ .

Centre  $C_2 = c_2(7, 1)$ , radius  $= \sqrt{49+1-41} = 3$ .  $\therefore r_2 = 3$ .

$|c_1c_2| = \sqrt{64+0} = 8$ .

As  $|c_1c_2| = r_1 + r_2$ , the circles touch externally.

Blunders (-3)

- B1 incorrect sign in centre point or incorrect  $g, f$ .
- B2 error in radius length formula.
- B3 error in distance formula.
- B4 fails to conclude.

Slips (-1)

- S1 arithmetic error.

Attempts ( 2, 2, 2 marks)

- A1 a correct centre
- A2 a correct radius.
- A3 sum of radii.

Part (b) (ii)

5 marks

Hit/Miss

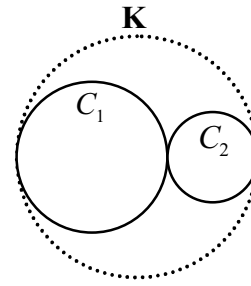
1 (b) (ii)

$K$  is a third circle.

Both  $C_1$  and  $C_2$  touch  $K$  internally.

The centres of the three circles lie in a straight line.

Find the equation of  $K$ .

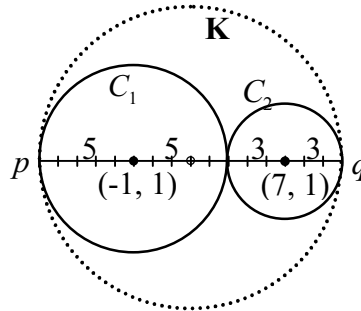


Equation of circle  $K$

5 marks

Hit/Miss

1 (b) (ii)



Diameter of  $K = 10 + 6 = 16 \Rightarrow$  radius = 8.

$p$  is 5 units to the left of  $(-1, 1) \Rightarrow p (-6, 1)$

$q$  is 3 units to the right of  $(7, 1) \Rightarrow q (10, 1)$

Midpoint of  $[pq] =$  centre of circle  $K = (2, 1)$ .

$\therefore$  Equation of circle  $K : (x - 2)^2 + (y - 1)^2 = 64$ .

\* Award full marks to candidates who indicate some understanding of the criteria for touching circles. For example:

(i) states  $d = R - r$ ,  $d = R + r$  or similar

(ii) finds any one of the relevant quantities, e.g. centre and/or radius of one or more of the three circles involved.

\* **Note:** work presented in part (b)(i) may be of merit for part (b)(ii).



Part (c)  
Part (c) (i)

20 (10, 10) marks  
10 marks

Att (3, 3)  
Att 3

1 (c) (i)

The line  $ax + by = 0$  is a tangent to the circle  $x^2 + y^2 - 12x + 6y + 9 = 0$   
where  $a, b \in \mathbf{R}$  and  $b \neq 0$ .

(i) Show that  $\frac{a}{b} = -\frac{3}{4}$ .

Show

10 marks

Att 3

1 (c) (i)

Centre of circle =  $(6, -3)$  and radius =  $\sqrt{36 + 9 - 9} = 6$ .

$\perp$  distance from  $(6, -3)$  to tangent  $ax + by = 0$  equals radius 6.

$$\therefore \left| \frac{6a - 3b}{\sqrt{a^2 + b^2}} \right| = 6 \Rightarrow \left| \frac{2a - b}{\sqrt{a^2 + b^2}} \right| = 2.$$

$$(2a - b)^2 = 4a^2 + 4b^2$$

$$4a^2 - 4ab + b^2 = 4a^2 + 4b^2 \Rightarrow 4ab = -3b^2 \Rightarrow 4a = -3b$$

$$\therefore \frac{a}{b} = -\frac{3}{4}.$$

or

$$y = -\frac{ax}{b} \Rightarrow x^2 + \frac{a^2 x^2}{b^2} - 12x - \frac{6ax}{b} + 9 = 0$$

$$\therefore x^2 \left( 1 + \frac{a^2}{b^2} \right) + x \left( -12 - \frac{6a}{b} \right) + 9 = 0.$$

$$\text{Line a tangent, so perfect square} \Rightarrow \left( -12 - \frac{6a}{b} \right)^2 - 4 \left( 1 + \frac{a^2}{b^2} \right) (9) = 0$$

$$144b^2 + 144ab + 36a^2 - 36b^2 - 36a^2 = 0$$

$$108b^2 + 144ab = 0 \Rightarrow 3b + 4a = 0. \quad \therefore \frac{a}{b} = -\frac{3}{4}.$$

*Blunders (-3)*

- B1 error in perpendicular distance formula.
- B2 error in centre point.
- B3 error in radius distance formula.
- B4 error in squaring.
- B5 error in quadratic formula.

*Slips (-1)*

- S1 arithmetic error.

*Attempts (3 marks)*

- A1 uses perpendicular distance and fails to finish.
- A2 quadratic in  $x$  or  $y$  and fails to finish

Part (c) (ii)

10 marks

Att 3

1 (c) (ii)

(ii) Hence, or otherwise, find the co-ordinates of the point of contact.

Point of contact

10 marks

Att 3

1(c) (ii)

$$ax + by = 0 \Rightarrow y = \frac{3}{4}x \text{ as } \frac{a}{b} = -\frac{3}{4}.$$

$$y = \frac{3}{4}x \cap x^2 + y^2 - 12x + 6y + 9 = 0$$

$$\text{gives } x^2 + \frac{9x^2}{16} - 12x + \frac{9}{2}x + 9 = 0$$

$$25x^2 - 120x + 144 = 0$$

$$(5x - 12)(5x - 12) = 0 \Rightarrow 5x - 12 = 0$$

$$\therefore x = \frac{12}{5} \text{ and hence } y = \frac{9}{5}. \text{ Point of contact is } \left(\frac{12}{5}, \frac{9}{5}\right).$$

*Blunders (-3)*

B1 error in factors or quadratic formula..

B2 finds  $x$  value but fails to give  $y$  value.

*Slips (-1)*

S1 arithmetic error.

*Attempts (3 marks)*

A1 quadratic in  $x$  or  $y$  and fails to finish.

A2 slope or equation of normal.

## QUESTION 2

<b>Part (a)</b>	<b>10 marks</b>	<b>Att 3</b>
<b>Part (b)</b>	<b>20 (10,10) marks</b>	<b>Att (3, 3)</b>
<b>Part (c)</b>	<b>20 (10, 5, 5) marks</b>	<b>Att (3, 2, 2)</b>

**Part (a)** **10 marks** **Att 3**

**2 (a)**

$oabc$  is a parallelogram where  $o$  is the origin,  $\vec{a} = 3\vec{i} - \vec{j}$  and  $\vec{b} = 4\vec{i} + 3\vec{j}$ .

Express  $\vec{c}$  in terms of  $\vec{i}$  and  $\vec{j}$ .

**Express  $\vec{c}$**  **10 marks** **Att 3**

**2 (a)**

$$\vec{c} = \vec{ab} = \vec{b} - \vec{a} = 4\vec{i} + 3\vec{j} - 3\vec{i} + \vec{j} \Rightarrow \vec{c} = \vec{i} + 4\vec{j}.$$

or

$$\vec{b} = \vec{a} + \vec{c} \Rightarrow \vec{c} = \vec{b} - \vec{a} = \vec{i} + 4\vec{j}.$$

*Blunders (-3)*

B1  $\vec{ab} = \vec{a} - \vec{b}$ .

B2 incorrect translation.

B3 error in translation.

*Slips (-1)*

S1 arithmetic error.

*Attempts (3 marks)*

A1 diagram with correct order of points  $oabc$ .

A2  $\vec{oc} = \vec{ab}$  and stops.

**Part (b)** **20 (10, 10) marks** **Att (3, 3)**

**Part (b) (i)** **10 marks** **Att 3**

**2 (b) (i)**

$\vec{p} = 2\vec{i} + \vec{j}$ ,  $\vec{q} = 3\vec{i} + k\vec{j}$ ,  $\vec{r} = 3\vec{i} + t\vec{j}$  where  $k, t \in \mathbf{R}$  and  $o$  is the origin.

(i) Given that  $\vec{p} \perp \vec{q}$ , calculate the value of  $k$ .

**Calculate value of  $k$**  **10 marks** **Att 3**

**2 (b) (i)**

$$\vec{p} \perp \vec{q} \Rightarrow \vec{p} \cdot \vec{q} = 0.$$

$$\therefore (2\vec{i} + \vec{j})(3\vec{i} + k\vec{j}) = 0 \Rightarrow 6 + k = 0. \quad k = -6.$$

*Blunders (-3)*

B1  $\vec{p} \cdot \vec{q} = -1$ .

B2 incorrect vector multiplication.

B3 incorrect formula.

*Slips (-1)*

S1 arithmetic error.

*Attempts (3 marks)*

A1  $\vec{p} \cdot \vec{q} = 0$ .

A2 a vector perpendicular to  $\vec{p}$  or  $\vec{q}$ .

**Part (b) (ii)**

**10 marks**

**Att 3**

**2 (b) (ii)**

**(ii)** Given that  $|\angle por| = 45^\circ$ , calculate the two possible values of  $t$ .

**Values of  $t$**

**10 marks**

**Att 3**

**2 (b) (ii)**

$$\cos 45^\circ = \frac{\vec{p} \cdot \vec{q}}{|\vec{p}| |\vec{q}|} \Rightarrow \frac{1}{\sqrt{2}} = \frac{(2\vec{i} + \vec{j})(3\vec{i} + t\vec{j})}{|2\vec{i} + \vec{j}| |3\vec{i} + t\vec{j}|}$$

$$\frac{1}{\sqrt{2}} = \frac{6+t}{\sqrt{5}\sqrt{9+t^2}} \Rightarrow 5(9+t^2) = 2(6+t)^2$$

$$\therefore 45 + 5t^2 = 72 + 24t + 2t^2$$

$$3t^2 - 24t - 27 = 0 \Rightarrow t^2 - 8t - 9 = 0 \Rightarrow (t-9)(t+1) = 0$$

$$\therefore t = 9 \text{ or } t = -1.$$

*Blunders (-3)*

B1 incorrect formula for norm of vector.

B2 incorrect vector multiplication.

B3 incorrect squaring.

B4 incorrect factors.

B5 error in slope formula.

B6 sign error in  $\tan \theta$  formula.

B7 only one value of  $t$  found.

B8  $\vec{op} \cdot \vec{ro}$ .

*Slips (-1)*

S1 arithmetic error.

*Attempts (3 marks)*

A1 scalar product with substitution.

A2 norm of vector found.

A3  $\vec{p} \cdot \vec{r}$  correct.

A4 one correct slope.

Part (c)

20 (10, 5, 5)

Att (3, 2, 2)

Part (c) (i)

10 marks

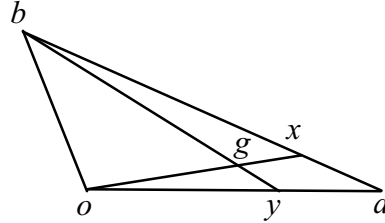
Att 3

2 (c) (i)

$oab$  is a triangle where  $o$  is the origin.

(i)  $x$  is a point on  $[ab]$  such that  $|ax| : |xb| = 1 : 3$ .

Express  $\vec{x}$  in terms of  $\vec{a}$  and  $\vec{b}$ .



Express  $\vec{x}$

10 marks

Att 3

2 (c) (i)

$$\vec{x} = \frac{3}{4}\vec{a} + \frac{1}{4}\vec{b} \quad \text{or} \quad \vec{x} = \vec{a} + \frac{1}{4}\vec{ab} = \vec{a} + \frac{1}{4}(\vec{b} - \vec{a}) \Rightarrow \vec{x} = \frac{3}{4}\vec{a} + \frac{1}{4}\vec{b}.$$

*Blunders (-3)*

B1 error in ratio formula.

B2  $\vec{ab} = \vec{a} - \vec{b}$ .

*Slips (-1)*

S1 arithmetic error.

*Attempts (3 marks)*

A1  $\vec{x} = \vec{a} + \frac{1}{4}\vec{ab}$ .

Part (c) (ii)

5 marks

Att 2

2 (c) (ii)

(ii)  $y$  is a point on  $[oa]$  such that  $|oy| : |ya| = 2 : 1$ .

Express  $\vec{by}$  in terms of  $\vec{a}$  and  $\vec{b}$ .

Express  $\vec{by}$

5 marks

Att 2

2 (c) (ii)

$$\vec{by} = \vec{y} - \vec{b} = \frac{2}{3}\vec{a} - \vec{b} \quad \text{or} \quad \vec{by} = \vec{bo} + \vec{oy} = -\vec{b} + \frac{2}{3}\vec{a}.$$

*Blunders (-3)*

B1 error in ratio formula.

B2  $\vec{by} = \vec{b} - \vec{y}$ .

Slips (-1)

S1 arithmetic error.

Attempts ( 2 marks)

A1  $\vec{by} = \vec{ba} + \vec{ay}$ .

**Part (c) (iii)**

**5 marks**

**Att 2**

**2 (c) (iii)**

(iii)  $[ox]$  and  $[by]$  intersect at  $g$ .

Given that  $\vec{g} = m\vec{x}$  and  $\vec{bg} = n\vec{by}$  where  $m, n \in \mathbf{R}$ ,  
find the value of  $m$  and the value of  $n$ .

**Find value of  $m$  and of  $n$**

**5 marks**

**Att 2**

**2 (c) (iii)**

$$\vec{g} = m\vec{x} \Rightarrow \vec{g} = \frac{3m}{4}\vec{a} + \frac{m}{4}\vec{b}.$$

$$\vec{bg} = n\vec{by} \Rightarrow \vec{g} - \vec{b} = \frac{2n}{3}\vec{a} - n\vec{b}$$

$$\therefore \vec{g} = \frac{2n}{3}\vec{a} + \vec{b}(1-n).$$

$$\therefore \frac{3m}{4} = \frac{2n}{3} \text{ and } \frac{m}{4} = 1-n.$$

$$\frac{2n}{3} = 3(1-n) \quad \therefore n = \frac{9}{11}. \quad \text{But } m = 4(1-n) \Rightarrow m = \frac{8}{11}.$$

Blunders (-3)

B1 error in solving other than slip.

B2 finds  $m$  but fails to give  $n$  value.

Slips (-1)

S1 arithmetic error.

Attempts ( 2 marks)

A1  $\vec{g}$  in terms of  $\vec{a}$  and  $\vec{b}$  and scalar  $m$  or  $\vec{g}$  in terms of  $\vec{a}$  and  $\vec{b}$  and scalar  $n$ .

A2 equates coefficients.

A3 two equations in  $m$  and  $n$  and stops.

### QUESTION 3

<b>Part (a)</b>	<b>20 marks</b>	<b>Att 6</b>
<b>Part (b)</b>	<b>30 (5, 5, 5, 5, 5, 5) marks</b>	<b>Att (2, 2, 2, 2, 2, 2)</b>

<b>Part (a)</b>	<b>20 marks</b>	<b>Att 6</b>
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**3 (a)**  $f$  is the transformation  $(x, y) \rightarrow (x', y')$  where  $x' = x + y$  and  $y' = x - y$ .  
 $L$  is the line  $4x - 2y - 1 = 0$ .  
 Find the equation of  $f(L)$ , the image of  $L$  under  $f$ .

<b>Find equation of <math>f(L)</math></b>	<b>20 marks</b>	<b>Att 6</b>
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**3 (a)**

$$x' = x + y$$

$$y' = x - y \Rightarrow x' + y' = 2x \quad \therefore x = \frac{1}{2}(x' + y')$$

$$\text{But } y = x' - x = x' - \frac{1}{2}(x' + y') \Rightarrow y = \frac{1}{2}(x' - y')$$

$$f(L): 2(x' + y') - (x' - y') = 1$$

$$f(L): x' + 3y' - 1 = 0.$$

**or**

$$(0, -\frac{1}{2}), (\frac{1}{4}, 0) \in L \quad ; \quad f(0, -\frac{1}{2}) = (-\frac{1}{2}, \frac{1}{2}) \text{ and } f(\frac{1}{4}, 0) = (\frac{1}{4}, \frac{1}{4}).$$

$$\text{Slope of } f(L) = \frac{\frac{1}{2} - \frac{1}{4}}{-\frac{1}{2} - \frac{1}{4}} = -\frac{1}{3}.$$

$$f(L): y' - \frac{1}{2} = \frac{-1}{3}(x' + \frac{1}{2}) \Rightarrow 3y' - \frac{3}{2} = -x' - \frac{1}{2}$$

$$\therefore f(L): x' + 3y' - 1 = 0.$$

*Blunders (-3)*

- B1 incorrect matrix.
- B2 error in slope formula.
- B3 image line not in the form of  $ax' + by' + c = 0$ .
- B4 incorrect matrix multiplication.

*Slips (-1)*

- S1 arithmetic error.

*Attempts ( 6 marks)*

- A1 expressing  $x$  or  $y$  in terms of primes.
- A2 correct matrix for  $f$ .
- A3 a correct image point on  $f(L)$ .

<b>Part (b)</b>	<b>30 (5, 5, 5, 5, 5, 5) marks</b>	<b>Att (2, 2, 2, 2, 2, 2)</b>
<b>Part (b) (i)</b>	<b>10 (5, 5) marks</b>	<b>Att (2, 2)</b>
<b>Point on <math>M</math></b>	<b>5 marks</b>	<b>Att 2</b>
<b>Equation of <math>M</math></b>	<b>5 marks</b>	<b>Att 2</b>

**3 (b) (i)**

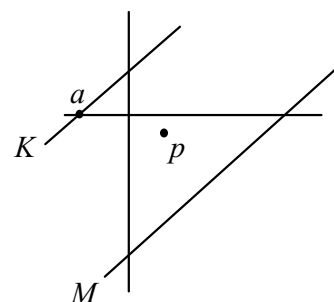
$K$  is the line  $3x - 4y + 9 = 0$ .

The point  $a(-3, 0)$  is on  $K$ .

The line  $M$  is parallel to  $K$ .

The point  $p(2, -1)$  is midway between  $K$  and  $M$ .

(i) Find the equation of  $M$ .



<b>Equation of <math>M</math></b>	<b>10 (5, 5) marks</b>	<b>Att (2, 2)</b>
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**3 (b) (i)**

$p$  mid-point of  $[aq]$  where  $q \in M$ .

$a(-3, 0) \rightarrow p(2, -1) \rightarrow q(7, -2)$  and  $q \in M$ .

Equation of  $M : 3x - 4y = k$  as  $M$  parallel to  $K$ .

$(7, -2) \in M \Rightarrow 21 + 8 = k. \therefore k = 29$ .

Equation of line  $M : 3x - 4y = 29$ .

*Blunders (-3)*

B1 error in translation or central symmetry.

B2 error in mid-point formula.

B3 error in slope of  $M$  other than slip.

B4 error in equation line formula.

*Slips (-1)*

S1 arithmetic error.

*Attempts (2, 2 marks)*

A1 one component of point on  $M$  correct.

A2 slope of  $M$ .

<b>Part (b) (ii)</b>	<b>5 marks</b>	<b>Att 2</b>
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**3 (b) (ii)** Calculate the distance between  $K$  and  $M$ .

<b>Distance between <math>K</math> and <math>M</math></b>	<b>5 marks</b>	<b>Att 2</b>
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**3 (b) (ii)**

$\perp$  distance from point  $a(-3, 0)$  to the line  $3x - 4y - 29 = 0$

$$\frac{|-9 + 0 - 29|}{\sqrt{9 + 16}} = \frac{|-38|}{5} = \frac{38}{5}$$



*Blunders (-3)*

B1 error in perpendicular distance formula.

*Slips (-1)*

S1 arithmetic error.

*Attempts ( 2 marks)*

A1 perpendicular distance formula with some substitution.

**Part (b) (iii)**

**5 marks**

**Att 2**

**3 (b) (iii)**

**(iii)** Calculate the measure of the acute angle between  $ap$  and  $K$ .  
Give your answer correct to the nearest degree.

**Measure of acute angle**

**5 marks**

**Att 2**

**3 (b) (iii)**

$$a(-3, 0), p(2, -1). \text{ Slope } ap = \frac{0+1}{-3-2} = -\frac{1}{5}.$$

$$K : 3x - 4y + 9 = 0. \text{ Slope } K = \frac{3}{4}.$$

$$\tan \theta = \pm \frac{\frac{3}{4} + \frac{1}{5}}{1 - \frac{3}{20}} = \pm \frac{19}{17}. \text{ For acute angle, } \theta = \tan^{-1}\left(\frac{19}{17}\right) = 48^\circ.$$

*Blunders (-3)*

B1 error in slope formula.

B2 sign error in  $\tan \theta$  formula.

B3 error in  $\tan^{-1}\left(\frac{19}{17}\right)$  calculation.

*Slips (-1)*

S1 arithmetic error.

S2 angle not correct to nearest degree.

*Attempts ( 2 marks)*

A1 one correct slope.

A2  $\tan \theta$  formula with some substitution.

Part (b) (iv)

10 (5, 5) marks

Att (2, 2)

$|ab|$  or  $|ab|^2$  in terms of  $x$  and  $y$

5 marks

Att 2

Finish

5 marks

Att 2

3 (b) (iv)

- (iv)  $b(x, y)$  is a point on  $K$  such that  $|ab| = 15$  and  $x > 0$ .  
Find the value of  $x$  and the value of  $y$ .

Value of  $x$  and value of  $y$

10 (5, 5) marks

Att (2, 2)

3 (b) (iv)

$$a(-3, 0), b(x, y) \text{ and } |ab| = 15.$$

$$(x+3)^2 + (y-0)^2 = 225.$$

$$(x, y) \in 3x - 4y + 9 = 0 \Rightarrow y = \frac{1}{4}(3x + 9).$$

$$\therefore (x+3)^2 + \frac{1}{16}(3x+9)^2 = 225$$

$$16x^2 + 96x + 144 + 9x^2 + 54x + 81 = 225$$

$$25x^2 + 150x - 3375 = 0 \Rightarrow x^2 + 6x - 135 = 0$$

$$(x-9)(x+15) = 0 \Rightarrow x = 9. \quad x \neq -15 \text{ as } x > 0.$$

$$\therefore x = 9 \text{ and } y = 9. \quad \therefore b(9, 9).$$

*Blunders (-3)*

B1 incorrect squaring.

B2 incorrect factors.

B3  $x$  value found and stops.

*Slips (-1)*

S1 arithmetic error.

*Attempts (2, 2 marks)*

A1 expression for  $|ab| = 15$ .

A2  $K$ : expressed  $y$  in terms of  $x$  or  $x$  in terms of  $y$ .

A3 quadratic in  $x$  or in  $y$ .

## QUESTION 4

<b>Part (a)</b>	<b>10 marks</b>	<b>Att 3</b>
<b>Part (b)</b>	<b>25 (10, 15) marks</b>	<b>Att (3, 5)</b>
<b>Part (c)</b>	<b>15 (10, 5) marks</b>	<b>Att (3, 2)</b>

**Part (a)** **10 marks** **Att 3**

**4 (a)**            The circumference of a circle is  $30\pi$  cm.  
                       The area of a sector of the circle is  $75 \text{ cm}^2$ .  
                       Find, in radians, the angle in this sector.

**Measure of angle in sector** **10 marks** **Att 3**

**4 (a)**

Circumference =  $2\pi r = 30\pi \Rightarrow r = 15\text{cm}$ .  
 Area of sector =  $\frac{1}{2}r^2\theta = \frac{1}{2}(15)^2\theta = 75$ .  
 $\therefore \theta = \frac{2}{3}$  radians.

*Blunders (-3)*

- B1 error in circumference formula.
- B2 error in area sector formula.

*Slips (-1)*

- S1 arithmetic error.

*Attempts (3 marks)*

- A1 correct value for  $r$ .

<b>Part (b)</b>	<b>25 (10, 15) marks</b>	<b>Att (3, 5)</b>
<b>Express as product</b>	<b>10 marks</b>	<b>Att 3</b>
<b>Solutions for <math>x</math></b>	<b>15 marks</b>	<b>Att 5</b>

**4 (b)**            Find all the solutions of the equation

$\sin 2x + \sin x = 0$

in the domain  $0^\circ \leq x \leq 360^\circ$ .

**Find solutions** **25 (10, 15)** **Att (3, 5)**

**4 (b)**

$\sin 2x + \sin x = 0$

$2 \sin \frac{3x}{2} \cos \frac{x}{2} = 0 \Rightarrow \sin \frac{3x}{2} = 0$  or  $\cos \frac{x}{2} = 0$ .

$\frac{3x}{2} = 0^\circ, 180^\circ, 360^\circ, 540^\circ$  or  $\frac{x}{2} = 90^\circ$ .

$x = 0^\circ, 120^\circ, 240^\circ, 360^\circ$  or  $x = 180^\circ$ .

Solution =  $\{0^\circ, 120^\circ, 180^\circ, 240^\circ, 360^\circ\}$  in the domain  $0^\circ \leq x \leq 360^\circ$ .

**or**

$$\sin 2x + \sin x = 0$$

$$2\sin x \cos x + \sin x = 0$$

$$\sin x(2\cos x + 1) = 0$$

$$\sin x = 0 \quad \text{or} \quad \cos x = -\frac{1}{2}.$$

$$x = 0^\circ, 180^\circ, 360^\circ \quad \text{or} \quad x = 120^\circ, 240^\circ$$

$$x = \{0^\circ, 120^\circ, 240^\circ, 180^\circ, 360^\circ\}.$$

\* ignore solutions outside of range, correct or otherwise.

*Blunders (-3)*

B1 error in  $\sin A + \sin B$  formula.

B2 error in  $\sin 2A$  formula.

B3 missing solution or incorrect solution.

*Slips (-1)*

S1 arithmetic error.

*Attempts (3, 5 marks)*

A1 expresses  $\sin 2x = 2\sin x \cos x$  and stops.

A2 one correct solution.

A3 solutions for  $x$  not given.

**Part (c)**

**15 (10, 5) marks**

**Att (3, 2)**

**Part (c) (i)**

**10 marks**

**Att 3**

**4 (c) (i)**

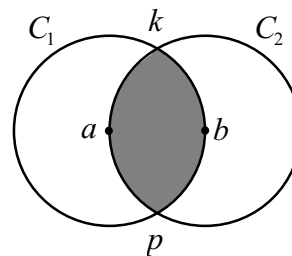
$C_1$  is a circle with centre  $a$  and radius  $r$ .

$C_2$  is a circle with centre  $b$  and radius  $r$ .

$C_1$  and  $C_2$  intersect at  $k$  and  $p$ .

$a \in C_2$ .

$b \in C_1$ .



**(i)** Find, in radians, the measure of angle  $k a p$ .

**Measure of angle  $kap$**

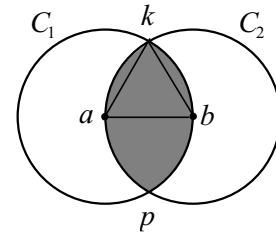
**10 marks**

**Att 3**

**4 (c) (i)**

Triangle  $kab$  is equilateral with sides of length  $r$ .

$$\therefore |\angle kab| = \frac{\pi}{3} \Rightarrow |\angle kap| = \frac{2\pi}{3}$$



\* accept angle in degrees. Penalise in part (c) (ii).

*Blunders (-3)*

B1 an incorrect length in one triangle side used.

B2 error in cosine rule formula.

B3  $|\angle kab| = 60^\circ$  and  $|\angle kap|$  not given.

*Slips (-1)*

S1 arithmetic error.

*Attempts ( 3 marks)*

A1 draws triangle showing length of side.

**Part (c) (ii)**

**5 marks**

**Att 2**

**4 (c) (ii)**

Calculate the area of the shaded region.

Give your answer in terms of  $r$  and  $\pi$ .

**Area of shaded region 5 marks**

**Att 2**

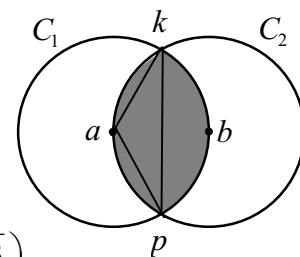
**4 (c) (ii)**

$$\text{Area triangle } kap = \frac{1}{2}r^2 \sin \frac{2\pi}{3} = \frac{1}{2}r^2 \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4}r^2.$$

$$\text{area of sector } apk \text{ in } C_1 = \frac{1}{2}r^2\theta = \frac{1}{2}r^2 \frac{2\pi}{3} = \frac{\pi}{3}r^2.$$

$$\text{Area of shaded region } kpb \text{ in } C_1 = \frac{\pi}{3}r^2 - \frac{\sqrt{3}}{4}r^2 = r^2 \left( \frac{\pi}{3} - \frac{\sqrt{3}}{4} \right)$$

$$\therefore \text{Area of required shaded region} = 2r^2 \left( \frac{\pi}{3} - \frac{\sqrt{3}}{4} \right).$$



*Blunders (-3)*

B1 error in triangle area formula.

B2 error in sector area formula.

B3 finds area of triangle and of sector but fails to finish.

*Slips (-1)*

S1 arithmetic error.

*Attempts ( 2 marks)*

A1 triangle area formula with some substitution.

A2 sector area formula with some substitution.

## QUESTION 5

<b>Part (a)</b>	<b>10 marks</b>	<b>Att 3</b>
<b>Part (b)</b>	<b>25 (15, 10) marks</b>	<b>Att (5, 3)</b>
<b>Part (c)</b>	<b>15 (10, 5) marks</b>	<b>Att (3, 2)</b>

**Part (a)** **10 marks** **Att 3**

**5 (a)** Find the value of  $\sin 15^\circ$  in surd form.

**Value of  $\sin 15^\circ$**  **10 marks** **Att 3**

**5 (a)**

$$\begin{aligned} \sin 15^\circ &= \sin(45^\circ - 30^\circ) \\ &= \sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ \\ &= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \cdot \frac{1}{2} = \frac{\sqrt{3} - 1}{2\sqrt{2}}. \end{aligned}$$

*Blunders (-3)*

- B1 error in  $\sin(A - B)$  formula.  
 B2  $\sin 45^\circ$  etc incorrect.

*Slips (-1)*

- S1 arithmetic error.

*Attempts (3 marks)*

- A1  $\sin 15^\circ = \sin(45^\circ - 30^\circ)$  or  $\sin 15^\circ = \sin(60^\circ - 45^\circ)$ .  
 A2  $\sin 45^\circ$  etc not in surd form.

*Worthless (0)*

- W1  $\sin 15^\circ$  evaluated from calculator.

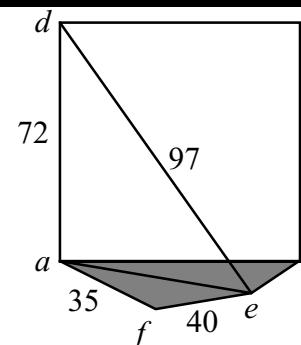
**Part (b)** **25 (15, 10) marks** **Att (5, 3)**

**Part (b) (i)** **15 marks** **Att 5**

**5 (b) (i)**  $a, f$  and  $e$  are points on horizontal ground.  
 $d$  is a point on a vertical wall directly above  $a$ .

$|ad| = 72$  m,  $|de| = 97$  m,  
 $|af| = 35$  m and  $|fe| = 40$  m.

**(i)** Calculate  $|ae|$ .



**Calculate** |  $ae$  |

**15 marks**

**Att 5**

**5 (b) (i)**

$$|ae|^2 = 97^2 - 72^2 = 4225$$

$$|ae| = \sqrt{4225} = 65.$$

*Blunders (-3)*

- B1 incorrect squaring.
- B2 incorrect application of Pythagoras.
- B3 error in square root.

*Slips (-1)*

- S1 arithmetic error.

*Attempts ( 5 marks)*

- A1 application of Pythagoras.

**Part (b) (ii)**

**10 marks**

**Att 3**

**5 (b) (ii)** Hence, calculate  $|\angle afe|$ .

**Calculate** |  $\angle afe$  |

**10 marks**

**Att 3**

**5 (b) (ii)**

$$\cos \angle afe = \frac{35^2 + 40^2 - 65^2}{2(35)(40)} = \frac{-1400}{2800} = -\frac{1}{2}$$

$$|\angle afe| = 120^\circ.$$

*Blunders (-3)*

- B1 error in cosine rule formula.
- B2 error in substitution into cosine rule formula.
- B3 incorrect squaring.
- B4 incorrect evaluation of angle.

*Slips (-1)*

- S1 arithmetic error.

*Attempts ( 3 marks)*

- A1 cosine formula with some substitution.

Part (c)

15 (10, 5) marks

Att (3, 2)

Part (c) (i)

10 marks

Att 3

5 (c) (i) Using the identity  $\cos(A - B) = \cos A \cos B + \sin A \sin B$ , or otherwise, prove:  $\sin(A + B) = \sin A \cos B + \cos A \sin B$ .

Prove  $\sin(A+B)$

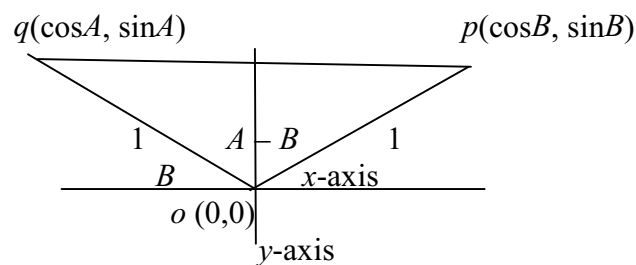
10 marks

Att 3

5 (c) (i)

$$\begin{aligned} \sin(A + B) &= \cos(90^\circ - (A + B)) \\ &= \cos((90^\circ - A) - B) \\ &= \cos(90^\circ - A) \cos B + \sin(90^\circ - A) \sin B \\ &= \sin A \cos B + \cos A \sin B. \end{aligned}$$

or



$$\text{Area triangle } opq = \frac{1}{2}(1)(1)\sin(A-B)$$

$$\text{Area triangle } opq = \frac{1}{2}|\sin A \cos B - \cos A \sin B|.$$

$$\therefore \sin(A-B) = \sin A \cos B - \cos A \sin B$$

$$\begin{aligned} \text{Replacing } B \text{ by } -B \Rightarrow \sin(A+B) &= \sin A \cos(-B) - \cos A \sin(-B) \\ &= \sin A \cos B + \cos A \sin B. \end{aligned}$$

Blunders (-3)

B1 incorrect expansion of  $\cos[(90^\circ - A) - B]$

Slips (-1)

S1 arithmetic error.

Attempts (3 marks)

A1  $\sin(A + B) = \cos[90^\circ - (A + B)]$

A2 area of triangle  $opq$ .



Part (c) (ii)

5 marks

Att 2

5 (c) (ii) Prove:  $\sin(A + B)\sin(A - B) = (\sin A + \sin B)(\sin A - \sin B)$ .

Prove

5 marks

Att 2

5 (c) (ii)

$$\begin{aligned}\sin(A + B)\sin(A - B) &= [\sin A \cos B + \cos A \sin B][\sin A \cos B - \cos A \sin B] \\ &= \sin^2 A \cos^2 B - \cos^2 A \sin^2 B \\ &= \sin^2 A(1 - \sin^2 B) - (1 - \sin^2 A)\sin^2 B \\ &= \sin^2 A - \sin^2 A \sin^2 B - \sin^2 B + \sin^2 A \sin^2 B \\ &= \sin^2 A - \sin^2 B \\ &= (\sin A + \sin B)(\sin A - \sin B).\end{aligned}$$

or

$$\begin{aligned}\sin(A + B)\sin(A - B) &= \frac{1}{2}(\cos 2B - \cos 2A) = \frac{1}{2}(1 - 2\sin^2 B - 1 + 2\sin^2 A) \\ &= \sin^2 A - \sin^2 B = (\sin A + \sin B)(\sin A - \sin B).\end{aligned}$$

*Blunders (-3)*

B1 error in expansion of  $\sin(A + B)$  or  $\sin(A - B)$ .

B2 error in  $\sin(A + B)$ .  $\sin(A - B)$  formula.

*Slips (-1)*

S1 arithmetic error.

*Attempts (2 marks)*

A1 correct expansion of  $\sin(A - B)$ .

A2 correct application of  $\sin(A + B)$ .  $\sin(A - B)$  formula.

## QUESTION 6

<b>Part (a)</b>	<b>10 (5, 5) marks</b>	<b>Att (2, 2)</b>
<b>Part (b)</b>	<b>20 (5, 5, 5, 5) marks</b>	<b>Att (2, 2, 2, 2)</b>
<b>Part (c)</b>	<b>20 (5, 5, 5, 5) marks</b>	<b>Att (2, 2, 2, 2)</b>

<b>Part (a)</b>	<b>10 (5, 5) marks</b>	<b>Att (2, 2)</b>
<b>Part (a) (i)</b>	<b>5 marks</b>	<b>Att 2</b>

**6 (a) (i)**

Eight people, including Kieran and Anne, are available to form a committee. Five people must be chosen for the committee.

- (i) In how many ways can the committee be formed if both Kieran and Anne must be chosen?

<b>Part (a) (i)</b>	<b>5 marks</b>	<b>Att 2</b>
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**6 (a) (i)** Kieran and Anne chosen.  $\therefore$  Select 3 from 6.  
Solution =  ${}^6C_3 = 20$ .

*Blunders (-3)*

B1  ${}^6C_5$ .

*Slips (-1)*

S1 arithmetic error.

*Attempts (2 marks)*

A1  ${}^8C_3$ .

<b>Part (a) (ii)</b>	<b>5 marks</b>	<b>Att 2</b>
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**6 (a) (ii)** In how many ways can the committee be formed if neither Kieran nor Anne can be chosen?

<b>Part (a) (ii)</b>	<b>5 marks</b>	<b>Att 2</b>
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**6 (a) (ii)** Both Kieran and Anne not chosen.  $\therefore$  Select 5 from 6.  
Solution =  ${}^6C_5 = 6$ .

*Blunders (-3)*

B1  ${}^6C_3$ .

*Slips (-1)*

S1 arithmetic error.

*Attempts (2 marks)*

A1  ${}^8C_5$ .

<b>Part (b)</b>	<b>20 (5, 5, 5, 5) marks</b>	<b>Att (2, 2, 2, 2)</b>
<b>Part (b) (i)</b>	<b>15 (5, 5, 5) marks</b>	<b>Att (2, 2, 2)</b>
<b>Characteristic roots</b>	<b>5 marks</b>	<b>Att 2</b>
<b>Simultaneous equations</b>	<b>5 marks</b>	<b>Att 2</b>
<b>Final solution</b>	<b>5 marks</b>	<b>Att 2</b>

**6 (b) (i)** Solve the difference equation  $u_{n+2} - 4u_{n+1} + 3u_n = 0$ , where  $n \geq 0$ , given that  $u_0 = -2$  and  $u_1 = 4$ .

**Solve** **15 (5, 5, 5) marks** **Att (2, 2, 2)**

**6 (b) (i)**

$$u_{n+2} - 4u_{n+1} + 3u_n = 0$$

$$x^2 - 4x + 3 = 0$$

$$(x-3)(x-1) = 0 \Rightarrow x-3 = 0 \text{ or } x-1 = 0$$

$$x = 3 \text{ or } x = 1.$$

$$u_n = l(\alpha)^n + k(\beta)^n = l(3)^n + k(1)^n.$$

$$u_n = l(3)^n + k$$

$$u_0 = -2 \therefore l + k = -2$$

$$u_1 = 4 \therefore 3l + k = 4 \Rightarrow -2l = -6$$

$$\therefore l = 3, k = -5.$$

$$u_n = 3(3)^n - 5.$$

*Blunders (-3)*

- B1 error in characteristic equation.
- B2 error in factors or in quadratic formula.
- B3 incorrect use of initial conditions.

*Slips (-1)*

- S1 arithmetic error.

*Attempts (2, 2, 2 marks)*

- A1 correct characteristic equation.
- A2 an equation in  $k$  and  $l$ .
- A3 correct value for  $k$  or  $l$ .

**Part (b) (ii)**

**5 marks**

**Att 2**

**6 (b) (ii)** Verify that the solution you have obtained in (i) satisfies the difference equation.

**Verify**

**5 marks**

**Att 2**

**6 (b) (ii)**

$$\begin{aligned} & u_{n+2} - 4u_{n+1} + 3u_n \\ &= 3(3)^{n+2} - 5 - 12(3)^{n+1} + 20 + 9(3)^n - 15 \\ &= 27(3)^n - 36(3)^n + 9(3)^n \\ &= 0. \end{aligned}$$

*Blunders (-3)*

B1  $u_{n+2}$  or  $u_{n+1}$  incorrect.

B2 error in indices.

*Slips (-1)*

S1 arithmetic error.

*Attempts (2 marks)*

A1 correct substitution into difference equation.

A2  $u_{n+2}$  or  $u_{n+1}$  correct.

**Part (c)**

**20 (5, 5, 5, 5) marks**

**Att (2, 2, 2, 2)**

**Part (c) (i)**

**5 marks**

**Att 2**

**6 (c) (c)** Ten discs, each marked with a different whole number from 1 to 10, are placed in a box. Three of the discs are drawn at random (without replacement) from the box.

**(i)** What is the probability that the disc with the number 7 is drawn?

**Probability disc with 7 is drawn**

**5 marks**

**Att 2**

**6 (c) (i)**

Number of possible outcomes =  ${}^{10}C_3 = 120$ .

Number of favourable outcomes (one 7 and any two others) =  ${}^9C_2 = 36$

$$\text{Probability} = \frac{36}{120} = \frac{3}{10}.$$

**or**

$$\text{Probability} = \frac{1}{10} \cdot \frac{9}{9} \cdot \frac{8}{8} \times 3 = \frac{3}{10}. \quad (\times 3 \text{ as the } 7 \text{ can be drawn on the first or second or third disc}).$$

*Blunders (-3)*

B1 favourable and possible outcomes given, but probability not expressed.

*Slips (-1)*

S1 arithmetic error.

Attempts ( 2 marks)

A1 correct number of possible outcomes.

A2 correct number of favourable outcomes.

A3 failure to allow for order, i.e. number 7 can be drawn first, second or third.

**Part (c) (ii)**

**5 marks**

**Att 2**

**6 (c) (ii)** What is the probability that the three numbers on the discs drawn are odd?

**Probability of three odd numbers**

**5 marks**

**Att 2**

**6 (c) (ii)**

Number of possible outcomes =  ${}^{10}C_3 = 120$ .

Number of favourable outcomes =  ${}^5C_3 = 10$

Probability =  $\frac{10}{120} = \frac{1}{12}$ .

**or**

$P(\text{three odd}) = P(\text{first odd}).P(\text{second odd}).P(\text{third odd}) = \frac{5}{10} \cdot \frac{4}{9} \cdot \frac{3}{8} = \frac{60}{720} = \frac{1}{12}$ .

Blunders (-3)

B1 favourable and possible outcomes given, but probability not expressed.

Slips (-1)

S1 arithmetic error.

Attempts ( 2 marks)

A1 correct number of possible outcomes.

A2 correct number of favourable outcomes.

**Part (c) (iii)**

**5 marks**

**Att 2**

**6 (c) (iii)** What is the probability that the product of the three numbers on the discs drawn is even?

**Probability product even**

**5 marks**

**Att 2**

**6 (c) (iii)**

For product even as least one of the numbered discs must be even.

At least one even = all possible outcomes – outcome of all three odd.

Possible outcomes =  ${}^{10}C_3 = 120$ .

Favourable outcomes =  ${}^{10}C_3 - {}^5C_3 = 120 - 10 = 110$ .

$\therefore$  Probability (three odd) =  $\frac{110}{120} = \frac{11}{12}$ .

**or**

$$\begin{aligned} P(\text{Three odd}) &= P(E, E, E) + P(E, E, O) + P(E, O, O) \\ &= \frac{5}{10} \cdot \frac{4}{9} \cdot \frac{3}{8} + \frac{5}{10} \cdot \frac{4}{9} \cdot \frac{5}{8} \times 3 + \frac{5}{10} \cdot \frac{5}{9} \cdot \frac{4}{8} \times 3 \\ &= \frac{60 + 300 + 300}{720} = \frac{660}{720} = \frac{11}{12}. \end{aligned}$$

*Blunders (-3)*

B1 favourable and possible outcomes given, but probability not expressed.

*Slips (-1)*

S1 arithmetic error.

*Attempts (2 marks)*

A1 correct number of possible outcomes.

A2 correct number of favourable outcomes.

A3 P(even, even, even) or P(even, even, odd) or P(even, odd, odd) calculated.

A4 solution =  $1 - P(\text{three odd})$  and stops.

**Part (c) (iv)**

**5 marks**

**Att 2**

**6 (c) (iv)** What is the probability that the smallest number on the discs drawn is 4?

**Probability smallest number is 4**

**5 marks**

**Att 2**

**6 (c) (iv)**

Choose 4 and any two discs numbered 5, 6, 7, 8, 9, 10.

Favourable outcomes =  ${}^6C_2 = 15$ .

Possible outcomes =  ${}^{10}C_3 = 120$ .

$$\text{Probability} = \frac{15}{120} = \frac{1}{8}.$$

**or**

$$\text{Probability} = \frac{1}{10} \cdot \frac{6}{9} \cdot \frac{5}{8} \times 3 = \frac{90}{720} = \frac{1}{8}.$$

*Blunders (-3)*

B1 favourable and possible outcomes given, but probability not expressed.

*Slips (-1)*

S1 arithmetic error.

*Attempts (2 marks)*

A1 correct number of possible outcomes.

A2 correct number of favourable outcomes.

A3 failure to allow for order, i.e. number 4 can be drawn first, second or third.

## QUESTION 7

<b>Part (a)</b>	<b>10 (5, 5) marks</b>	<b>Att (2, 2)</b>
<b>Part (b)</b>	<b>20 (5, 5, 5, 5) marks</b>	<b>Att (2, 2, 2, 2)</b>
<b>Part (c)</b>	<b>20 (5, 5, 10) marks</b>	<b>Att (2, 2, 3)</b>

<b>Part (a)</b>	<b>10 (5, 5) marks</b>	<b>Att (2, 2)</b>
<b>Part (a) (i)</b>	<b>5 marks</b>	<b>Att 2</b>

- 7 (a)** Five cars enter a car park. There are exactly five vacant spaces in the car park.
- (i)** In how many different ways can the five cars park in the vacant spaces?

<b>Part (a) (i)</b>	<b>5 marks</b>	<b>Att 2</b>
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- 7 (a) (i)**
- Solution =  $5 \times 4 \times 3 \times 2 \times 1 = 5! = 120$ .

*Blunders (-3)*

B1  $5^5$ .

*Slips (-1)*

S1 arithmetic error.

*Attempts (2 marks)*

A1 two cars parked in  $5 \times 4$  ways.

<b>Part (a) (ii)</b>	<b>5 marks</b>	<b>Att 2</b>
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- 7 (a) (ii)** Two of the cars leave the car park without parking.  
In how many different ways can the remaining three cars park in the five vacant spaces?

<b>Part (a) (ii)</b>	<b>5 marks</b>	<b>Att 2</b>
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- 7 (a) (ii)**
- Solution =  $5 \times 4 \times 3 = 60$ .

*Blunders (-3)*

B1  $5^3$ .

*Slips (-1)*

S1 arithmetic error.

*Attempts (2 marks)*

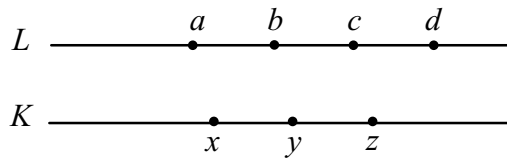
A1 two cars parked in  $5 \times 4$  ways.

Part (b)  
Part (b) (i)

20 (5, 5, 5, 5)  
5 marks

Att (2, 2, 2, 2)  
Att 2

7 (b) (i)



$L$  and  $K$  are distinct parallel lines.

$a, b, c$  and  $d$  are points on  $L$  such that  $|ab| = |bc| = |cd| = 1$  cm.

$x, y$  and  $z$  are points on  $K$  such that  $|xy| = |yz| = 1$  cm.

- (i) How many different triangles can be constructed using three of the named points as vertices?

Number of triangles

5 marks

Att 2

7 (b) (i)

Form triangles with one point from  $K$  and two points from  $L$   
or one point from  $L$  and two points from  $K$ .

$$\text{Solution} = {}^3C_1 \times {}^4C_2 + {}^4C_1 \times {}^3C_2 = 18 + 12 = 30.$$

Slips (-1)

S1 arithmetic error.

Attempts (2 marks)

$$\text{A1 } {}^3C_1 \times {}^4C_2 \text{ or } {}^4C_1 \times {}^3C_2$$

Part (b) (ii)

5 marks

Att 2

7 (b) (ii) How many different quadrilaterals can be constructed using four of the named points as vertices?

Quadrilaterals

5 marks

Att 2

7 (b) (ii)

Form quadrilateral with two points from  $L$  and two points from  $K$ .

$$\text{Solution} = {}^4C_2 \times {}^3C_2 = 6 \times 3 = 18.$$

Slips (-1)

S1 arithmetic error.

Attempts (2 marks)

$$\text{A1 } {}^4C_2 \text{ or } {}^3C_2 \text{ or } {}^4C_1 + {}^3C_2.$$



**Part (b) (iii)**

**5 marks**

**Att 2**

**7 (b) (iii)** How many different parallelograms can be constructed using four of the named points as vertices?

**Parallelograms**

**5 marks**

**Att 2**

**7 (b) (iii)**

$$[xy] \text{ with } [ab] \text{ or } [bc] \text{ or } [cd] = 1 \times 3 = 3$$

$$[yz] \text{ with } [ab] \text{ or } [bc] \text{ or } [cd] = 1 \times 3 = 3$$

$$[xz] \text{ with } [ac] \text{ or } [bd] = 1 \times 2 = 2$$

$$\therefore \text{Total} = 8.$$

*Blunders (-3)*

B1 one set of parallelograms omitted.

*Slips (-1)*

S1 arithmetic error.

*Attempts (2 marks)*

A1 one set of possible parallelograms correctly counted.

**Part (b) (iv)**

**5 marks**

**Att 2**

**7 (b) (iv)** If one quadrilateral is constructed at random, what is the probability that it is *not* a parallelogram?

**Probability**

**5 marks**

**Att 2**

**7 (b) (iv)**

$$\text{Number of quadrilaterals} = 18. \quad \text{Solution from part (b) (ii).}$$

$$\text{Number of parallelograms} = 8 \quad \text{Solution from part (b) (iii).}$$

$$\therefore \text{Number of quadrilaterals not parallelograms} = 18 - 8 = 10.$$

$$\text{Probability} = \frac{10}{18} = \frac{5}{9}.$$

*Blunders (-3)*

B1 possible and favourable outcomes found but probability not given.

*Slips (-1)*

S1 arithmetic error.

*Attempts (2 marks)*

A1 correct number of favourable outcomes.

A2 correct number of possible outcomes.

**Part (c)**

**20 marks (5, 5, 10)**

**Att (2, 2, 3)**

**Part (c) (i)**

**5 marks**

**Att 2**

**7 (c)** The mean of the real numbers  $a$  and  $b$  is  $\bar{x}$ .  
The standard deviation is  $\sigma$ .

**(i)** Express  $\sigma$  in terms of  $a$ ,  $b$  and  $\bar{x}$ .

**Express  $\sigma$**

**5 marks**

**Att 2**

**7 (c) (i)**

$$\sigma = \sqrt{\frac{(a - \bar{x})^2 + (b - \bar{x})^2}{2}}.$$

*Blunders (-3)*

B1 error in standard deviation formula.

*Slips (-1)*

S1 arithmetic error.

*Attempts (2 marks)*

A1 any correct deviation.

**Part (c) (ii)**

**5 marks**

**Att 2**

**7 (c) (ii)** Hence, express  $\sigma$  in terms of  $a$  and  $b$  only.

**Express  $\sigma$**

**5 marks**

**Att 2**

**7 (c) (ii)**

$$\begin{aligned}\bar{x} &= \frac{a+b}{2}. \\ \sigma &= \sqrt{\frac{\left(a - \frac{a+b}{2}\right)^2 + \left(b - \frac{a+b}{2}\right)^2}{2}} = \sqrt{\frac{(a-b)^2 + (b-a)^2}{8}} \\ &= \sqrt{\frac{2a^2 - 4ab + 2b^2}{8}} = \sqrt{\frac{a^2 - 2ab + b^2}{4}} = \sqrt{\frac{(a-b)^2}{4}} \\ \sigma &= \frac{a-b}{2}.\end{aligned}$$

*Blunders (-3)*

B1 error in mean.

*Slips (-1)*

S1 arithmetic error.

*Attempts (2 marks)*

A1 correct mean.

Part (c) (iii)

10 marks

Att 3

7 (c) (iii) Show that  $\bar{x}^2 - \sigma^2 = ab$ .

Show

10 marks

Att 3

7 (c) (iii)

$$\bar{x}^2 - \sigma^2 = \left(\frac{a+b}{4}\right)^2 - \left(\frac{a-b}{4}\right)^2 = \frac{a^2 + 2ab + b^2 - a^2 + 2ab - b^2}{4}$$

$$\bar{x}^2 - \sigma^2 = \frac{4ab}{4} = ab.$$

*Blunders (-3)*

B1 incorrect squaring.

*Slips (-1)*

S1 arithmetic error.

*Attempts (3 marks)*

A1  $\bar{x}^2 - \sigma^2$  expressed in terms of  $a$  and of  $b$ .

## QUESTION 8

<b>Part (a)</b>	<b>10 marks</b>	<b>Att 3</b>
<b>Part (b)</b>	<b>25 (15, 5, 5) marks</b>	<b>Att (5, 2, 2)</b>
<b>Part (c)</b>	<b>15 (5, 5, 5) marks</b>	<b>Att (2, 2, 2)</b>

**Part (a)** **10 marks** **Att 3**

**8 (a)** Use integration by parts to find  $\int xe^{-5x} dx$ .

**Integration** **10 marks** **Att 3**

**8 (a)**

$$\int xe^{-5x} dx = \int u dv = uv - \int v du.$$
$$u = x \Rightarrow du = dx. \quad dv = e^{-5x} dx \Rightarrow v = \int e^{-5x} dx = -\frac{1}{5} e^{-5x}.$$
$$\therefore \int xe^{-5x} dx = -\frac{1}{5} xe^{-5x} + \frac{1}{5} \int e^{-5x} dx = -\frac{1}{5} xe^{-5x} - \frac{1}{25} e^{-5x} + \text{constant}.$$

*Blunders (-3)*

- B1 incorrect differentiation or integration.
- B2 constant of integration omitted.
- B3 incorrect 'parts' formula.

*Slips (-1)*

- S1 arithmetic error.

*Attempts (3 marks)*

- A1 correct assigning to parts formula.
- A2 correct differentiation or integration.

**Part (b)** **25 marks (15, 5, 5)** **Att (5, 2, 2)**  
**Part (b) (i)** **15 marks** **Att 5**

**8 (b) (i)**

$$f(x) = f(0) + \frac{f'(0)x}{1!} + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} + \dots \text{ is the Maclaurin series.}$$

**(i)** Derive the Maclaurin series for  $f(x) = \log_e(1+x)$  up to and including the term containing  $x^4$ .

**(i) Maclaurin series****15 marks****Att 5****8 (b) (i)**

$$f(x) = \log_e(1+x) \quad \Rightarrow \quad f(0) = 0$$

$$f'(x) = \frac{1}{1+x} = (1+x)^{-1} \quad \Rightarrow \quad f'(0) = 1$$

$$f''(x) = -1(1+x)^{-2} \quad \Rightarrow \quad f''(0) = -1$$

$$f'''(x) = 2(1+x)^{-3} \quad \Rightarrow \quad f'''(0) = 2$$

$$f^{(4)}(x) = -6(1+x)^{-4} \quad \Rightarrow \quad f^{(4)}(0) = -6.$$

$$\therefore f(x) = 0 + x - \frac{x^2}{2!} + \frac{2x^3}{3!} - \frac{6x^4}{4!} + \dots$$

$$\therefore f(x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$$

*Blunders (-3)*

B1 incorrect differentiation.

B2 incorrect evaluation of  $f^n(0)$ .

B3 each term not derived.

*Slips (-1)*

S1 arithmetic error.

*Attempts (5 marks)*A1  $f(0)$  correct.

A2 a correct differentiation.

A3 any one correct term.

**Part (b) (ii)****10 marks (5, 5)****Att (2, 2)****General term****5 marks****Att 2****Show****5 marks****Att 2**

**8 (b) (ii)** Write down the general term and use the Ratio Test to show that the series converges for  $-1 < x < 1$ .

**Part (b) (ii)****10 marks (5, 5)****Att (2, 2)****8 (b) (ii)**

$$u_n = \frac{(-1)^{n+1} x^n}{n} \quad \Rightarrow \quad u_{n+1} = \frac{(-1)^{n+2} x^{n+1}}{n+1}.$$

$$\text{Limit}_{n \rightarrow \infty} \left| \frac{u_{n+1}}{u_n} \right| = \text{Limit}_{n \rightarrow \infty} \left| \frac{(-1)^{n+2} x^{n+1}}{n+1} \times \frac{n}{(-1)^n x^n} \right|$$

$$\text{Limit}_{n \rightarrow \infty} \left| \frac{(-1)xn}{n+1} \right| = \text{Limit}_{n \rightarrow \infty} \left| \frac{x}{1 + \frac{1}{n}} \right| = |x|.$$

$$\text{Convergent for } |x| < 1 \quad \Rightarrow \quad -1 < x < 1.$$

*Blunders (-3)*

- B1 (-1) omitted from general term.
- B2 an incorrect power in general term.
- B3 error in  $u_{n+1}$ .
- B4 error in evaluating limit other than slip.
- B5 evaluates limit as  $|x|$  and stops.

*Slips (-1)*

- S1 arithmetic error.

*Attempts ( 2, 2 marks)*

- A1 any part of  $u_n$  correct.
- A2  $u_{n+1}$  correct.
- A3 states ratio test correctly.

**Part (c)**

**15 (5, 5, 5) marks**

**Att (2, 2, 2)**

**Part (c) (i)**

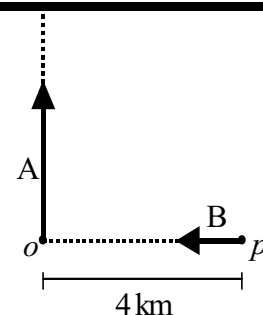
**5 marks**

**Att 2**

**8 (c)** The point  $p$  is 4 km due east of the point  $o$ .

At noon, A leaves  $o$  and travels north at a steady speed of 12 km/h. At the same time, B leaves  $p$  and travels towards  $o$  at a steady speed of 6 km/h.

- (i) Write down expressions in  $x$  for the distances that A and B will each have travelled at  $x$  minutes after noon.



**(i) Expressions in  $x$**

**5 marks**

**Att 2**

**8 (c) (i)**

$A$  travels at 12km/h and  $B$  travels at 12km/h.

In  $x$  minutes,  $A$  travels a distance of  $x \times \frac{12}{60} = \frac{x}{5}$  km.

In  $x$  minutes,  $B$  travels a distance of  $x \times \frac{6}{60} = \frac{x}{10}$  km.

*Blunders (-3)*

- B1 distance travelled by A given as  $12x$  and/or distance travelled by B given as  $6x$ .

*Slips (-1)*

- S1 arithmetic error.

*Attempts ( 2 marks)*

- A1 correct distance travelled by A given or correct distance travelled by B given.

Part (c) (ii)

5 marks

Att 2

8 (c) (ii) Find an expression in  $x$  for the distance that B will be from A at  $x$  minutes after noon.

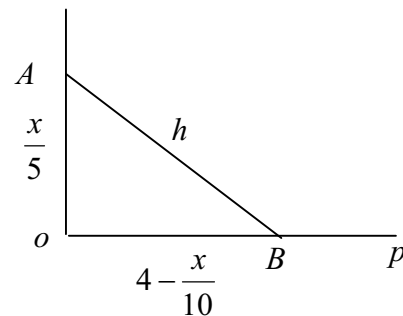
Distance from A to B

5 marks

Att 2

8 (c) (ii)

$$h = \sqrt{\left(\frac{x}{5}\right)^2 + \left(4 - \frac{x}{10}\right)^2}$$



Blunders (-3)

B1 error in distance from B to A.

Slips (-1)

S1 arithmetic error.

Attempts (2 marks)

A1 correct distance from o to B.

Part (c) (iii)

5 marks

Att 2

8 (c) (iii) At how many minutes after noon will B be closest to A?

How many minutes

5 marks

Att 2

8 (c) (iii)

$$h = \left(\frac{x^2}{25} + \left(4 - \frac{x}{10}\right)^2\right)^{\frac{1}{2}}$$

$$\frac{dh}{dx} = \frac{1}{2} \left(\frac{x^2}{25} + \left(4 - \frac{x}{10}\right)^2\right)^{-\frac{1}{2}} \cdot \left(\frac{2x}{25} + 2\left(4 - \frac{x}{10}\right)\left(-\frac{1}{10}\right)\right)$$

$$\text{For minimum } \frac{dh}{dx} = 0$$

$$\therefore \frac{2x}{25} - \frac{1}{5} \left(4 - \frac{x}{10}\right) = 0 \Rightarrow 2x - 20 + \frac{x}{2} = 0$$

$$5x = 40 \Rightarrow x = 8 \text{ minutes.}$$

$$\text{Note: } \frac{d^2h}{dx^2} > 0 \text{ for } x = 8 \Rightarrow \text{minimum.}$$

**or**

$$\begin{aligned}\frac{x^2}{25} + 16 - \frac{4x}{5} + \frac{x^2}{100} &= \frac{x^2}{20} - \frac{4x}{5} + 16 \\ &= \frac{1}{20}(x^2 - 16x + 320) = \frac{1}{20}(x^2 - 16x + 64 + 256) \\ &= \frac{1}{20}((x-8)^2 + 256.)\end{aligned}$$

Minimum value for  $x - 8 = 0$ .

Minimum at  $x = 8$  minutes.

\*  $\frac{d^2x}{dh^2} > 0$  for  $x = 8$  not required.

*Blunders (-3)*

B1 error in differentiation.

*Slips (-1)*

S1 arithmetic error.

*Attempts (2 marks)*

A1 equates  $dh/dx = 0$  and stops.

A2 some correct differentiation and stops.

A3 if previous error does not allow solution.



## QUESTION 9

<b>Part (a)</b>	<b>10 marks</b>	<b>Att 3</b>
<b>Part (b)</b>	<b>20 (5, 5, 5, 5) marks</b>	<b>Att (2, 2, 2, 2)</b>
<b>Part (c)</b>	<b>20 (5, 5, 5, 5) marks</b>	<b>Att (2, 2, 2, 2)</b>

**Part (a)** **10 marks** **Att 3**

**9 (a)**  $z$  is a random variable with standard normal distribution.  
Calculate  $P(-2.13 < z \leq 1.46)$ .

**Calculate** **10 marks** **Att 3**

**9 (a)**

$$\begin{aligned} & P(-2.13 < z \leq 1.46) \\ &= P(z \leq 1.46) - P(z \leq -2.13) \\ &= P(z \leq 1.46) - [1 - P(z < 2.13)] \\ &= 0.9279 - (1 - 0.9834) \\ &= 0.9113. \end{aligned}$$

*Blunders (-3)*

- B1 incorrect area defined.  
B2 incorrect reading from tables.

*Slips (-1)*

- S1 arithmetic error.

*Attempts (3 marks)*

- A1  $P(z \leq 1.46)$  found.  
A2  $P(z < 2.13)$  found.

**Part (b)** **20 (5, 5, 5, 5) marks** **Att (2, 2, 2, 2)**  
**Part (b) (i)** **5 marks** **Att 2**

**9 (b)** Whenever Anne's mobile phone rings, the probability that she answers the call is  $\frac{3}{4}$ .  
A friend phones Anne six times.  
**(i)** What is the probability that she misses all the calls?

**P(misses all calls)** **5 marks** **Att 2**

**9 (b) (i)**

$$\begin{aligned} P(\text{Anne answers}) &= \frac{3}{4}. & P(\text{Anne does not answer}) &= \frac{1}{4}. \\ P(\text{Anne misses all calls}) &= \left(\frac{1}{4}\right)^6 = \frac{1}{2096}. \end{aligned}$$

*Blunders (-3)*

- B1 incorrect value for  $q$ .  
B2 error in binomial.

*Slips (-1)*

S1 arithmetic error.

*Attempts ( 2 marks)*

A1  $q = \frac{1}{4}$ .

**Part (b)(ii)**

**5 marks**

**Att 2**

**9 (b) (ii)** What is the probability that she misses the first two calls and answers the others?

**Probability**

**5 marks**

**Att 2**

**9 (b) (ii)**

$$\text{Probability} = \left(\frac{1}{4}\right)^2 \left(\frac{3}{4}\right)^4 = \frac{81}{4096}.$$

*Blunders (-3)*

B1 powers of probability interchanged.

*Slips (-1)*

S1 arithmetic error.

*Attempts ( 2 marks)*

A1  $(\frac{1}{4})^2$  or  $(\frac{3}{4})^4$  in solution

**Part (b) (iii)**

**5 marks**

**Att 2**

**9 (b) (iii)** What is the probability that she answers exactly one of the calls?

**Probability**

**5 marks**

**Att 2**

**9 (b) (iii)**

$$\text{Probability} = {}^6C_1 \left(\frac{1}{4}\right)^5 \left(\frac{3}{4}\right)^1 = \frac{9}{2048}.$$

*Blunders (-3)*

B1 binomial coefficient omitted.

B2  $p$  and  $q$  interchanged in solution.

*Slips (-1)*

S1 arithmetic error.

*Attempts ( 2 marks)*

A1 a correct substitution into binomial.

**Part (b) (iv)**

**5 marks**

**Att 2**

**9 (b) (iv)** What is the probability that she answers at least two of the calls?

**Probability**

**5 marks**

**Att 2**

**9 (b) (iv)**

$$P(\text{At least two calls answered}) = 1 - [P(\text{one answered}) + P(\text{none answered})].$$

$$= 1 - \left[ {}^6C_1 \left(\frac{1}{4}\right)^5 \left(\frac{3}{4}\right)^1 + \left(\frac{1}{4}\right)^6 \right].$$

$$= 1 - \frac{18}{4096} - \frac{1}{4096}$$

$$= \frac{4077}{4096}.$$

*Blunders (-3)*

B1 omits one essential probability.

*Slips (-1)*

S1 arithmetic error.

*Attempts (2 marks)*

A1  $P = 1 - [P(\text{one answered}) + P(\text{none answered})]$ .

**Part (c)**

**20 (5, 5, 5, 5) marks**

**Att (2, 2, 2, 2)**

**Find  $\bar{x}$**

**5 marks**

**Att 2**

**Find  $\sigma$**

**5 marks**

**Att 2**

**Correct confidence interval**

**5 marks**

**Att 2**

**Final solution**

**5 marks**

**Att 2**

**9 (c)** In a newspaper advertisement, a driving school claimed that 80% of its clients passed their driving test on their first attempt.

1000 people who had attended the school and who had taken the test for the first time were randomly selected.

Find, at the 5% level of significance, the interval in which the number who passed should lie in order that the claim made in the advertisement be accepted.

9 (c)

 $H_0$  : Claim is true.

$$p = \frac{80}{100} = 0.8 \Rightarrow q = 0.2. \quad n = 1000.$$

$$\bar{x} = np = 1000(0.8) = 800.$$

$$\sigma = \sqrt{npq} = \sqrt{160} = 12.65.$$

$$\begin{aligned} 5\% \text{ confidence interval} &= \bar{x} \pm 1.96\sigma \\ &= 800 \pm 1.96(12.65) \\ &= 800 \pm 24.79 \\ &= [776, 824] \end{aligned}$$

$\therefore$  Accept claim if number lies between 776 and 824 inclusive.

*Blunders (-3)*

- B1 incorrect value of  $p$  or  $q$
- B2 incorrect formula for mean.
- B3 incorrect formula for  $\sigma$ .
- B4 error in confidence given.
- B5 final answer not simplified.

*Slips (-1)*

- S1 arithmetic error.

*Attempts (2, 2, 2, 2 marks)*

- A1 correct value of  $p$  or of  $q$ .
- A2 correct formula for mean.
- A3 correct formula for  $\sigma$ .
- A4 error in confidence interval.
- A5 substitutes into confidence and stops.

## QUESTION 10

<b>Part (a)</b>	<b>10 (5, 5) marks</b>	<b>Att (2, 2)</b>
<b>Part (b)</b>	<b>20 (5, 5, 10) marks</b>	<b>Att (2, 2, 3)</b>
<b>Part (c)</b>	<b>20 (10, 10) marks</b>	<b>Att (3, 3)</b>

<b>Part (a)</b>	<b>10 (5, 5) marks</b>	<b>Att (2, 2)</b>
<b>Show <math>A^2 = B</math></b>	<b>5 marks</b>	<b>Att 2</b>
<b>Finish</b>	<b>5 marks</b>	<b>Att 2</b>

**10 (a)**

$$A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}, \quad C = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \quad D = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

$M = \{A, B, C, D\}$  is a cyclic group under matrix multiplication.

Verify that  $A$  is a generator of  $M$ .

<b>Verify <math>A</math> is a generator of <math>M</math></b>	<b>10 (5, 5) marks</b>	<b>Att (2, 2)</b>
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**10 (a)**

$$A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \quad \therefore \quad A^2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} = B.$$

$$A^3 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = C.$$

$$A^4 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = D.$$

$\therefore A$  is a generator.

*Blunders (-3)*

B1 incorrect matrix multiplication.

B2  $A^3$  or  $A^4$  not evaluated.

*Slips (-1)*

S1 arithmetic error.

*Attempts (2, 2 marks)*

A1 attempt at evaluating  $A^2$ .

A2 attempt at evaluating  $A^3$  or  $A^4$ .

Part (b)

20 (5, 5, 10) marks

Att (2, 2, 3)

Part (b) (i)

10 (5, 5) marks

Att (2, 2)

Find one composition

5 marks

Att 2

Find other composition and finish

5 marks

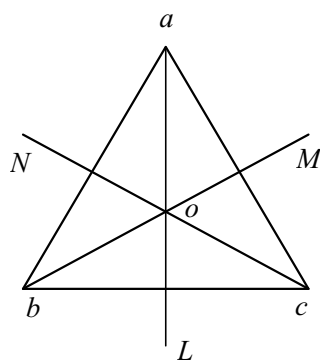
Att 2

10 (b) (i)

$abc$  is an equilateral triangle.

$L, M$  and  $N$ , the perpendicular bisectors of the sides, intersect at  $o$ .

$D_3 = \{I_\pi, R_{120^\circ}, R_{240^\circ}, S_L, S_M, S_N\}$ , under composition, is the symmetry group of  $abc$ .



(i) Investigate whether  $S_L \circ S_M = S_M \circ S_L$ .

Investigate

10 (5, 5) marks

Att (2, 2)

10 (b) (i)

$$S_L \circ S_M = R_{120^\circ} \quad \text{and} \quad S_M \circ S_L = R_{240^\circ}$$

$$\therefore S_L \circ S_M \neq S_M \circ S_L.$$

Blunders (-3)

B1  $S_L \circ S_M$  evaluated as incorrect rotation.

B2  $S_M \circ S_L$  evaluated as incorrect rotation.

Slips (-1)

S1 arithmetic error.

Attempts (2, 2 marks)

A1  $S_L \circ S_M = R_{2\alpha}$ .

A2  $S_M \circ S_L = R_{2\beta}$ .

**Part (b) (ii)**

**10 marks**

**Att 3**

**10 (b) (ii)** Write down the centralizer of  $R_{120^\circ}$ .

**Centralizer of  $R_{120^\circ}$**

**10 marks**

**Att 3**

**10 (b) (ii)**

Elements that commute with  $R_{120^\circ}$

$$C(R_{120^\circ}) = \{I_\pi, R_{120^\circ}, R_{240^\circ}\}.$$

*Blunders (-3)*

B1 gives exactly two elements, both correct.

*Slips (-1)*

S1 arithmetic error.

*Attempts (3 marks)*

A1 given exactly one element, which is correct.

**Part (c)**

**20 (10, 10)**

**Att (3, 3)**

$f(R_{120^\circ}^{-1}) = f(R_{240^\circ})$  or  $[f(R_{120^\circ})]^{-1}$  **10 marks**

**Att 3**

**Final conclusion**

**10 marks**

**Att 3**

**10 (c)**

$$P = \left\{ \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix} \right\}$$

$P$  is a group under composition of permutations.

$D_3$  is the symmetry group of the equilateral triangle  $abc$ , as described in (b).

$f: D_3 \rightarrow P$  is an isomorphism where  $f(R_{120^\circ}) = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$ .

Find  $f(R_{120^\circ}^{-1})$ , justifying your answer.

**Find  $f(R_{120^\circ}^{-1})$**

**20 marks (10, 10)**

**Att (3, 3)**

**10 (c)**

$$f(R_{120^\circ}^{-1}) = f(R_{240^\circ}) = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}.$$

**or**

$$f(R_{120^\circ}^{-1}) = [f(R_{120^\circ})]^{-1} = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}.$$

*Blunders (-3)*

B1 incorrect inverse permutation, but with one column correct and two columns incorrect (given that only 1, 2 and 3 are used).

*Slips (-1)*

S1 arithmetic error.

*Attempts (3, 3 marks)*

A1  $R_{120^\circ}^{-1} = R_{240^\circ}$ .

A2 incorrect inverse permutation with all columns incorrect but uses 1, 2 and 3 in 'solution'.



## QUESTION 11

<b>Part (a)</b>	<b>10 marks</b>	<b>Att 3</b>
<b>Part (b)</b>	<b>20 (5, 5, 5, 5) marks</b>	<b>Att (2, 2, 2, 2)</b>
<b>Part (c)</b>	<b>20 (10, 5, 5) marks</b>	<b>Att (3, 2, 2)</b>

**Part (a)** **10 marks** **Att 3**

**11 (a)**

The polar of a point  $p$  with respect to the circle  $x^2 + y^2 = 9$  is  $2x - 5y = 27$ .

Given that the polar of the point  $(x_1, y_1)$  with respect to the circle  $x^2 + y^2 = r^2$  is  $xx_1 + yy_1 = r^2$ , find the coordinates of  $p$ .

**Find coordinates of  $p$**  **10 marks** **Att 3**

**11 (a)**

$x^2 + y^2 = 9$ . The polar of  $p$  is  $2x - 5y = 27$ .

$2x - 5y = 27 \Rightarrow \frac{2}{3}x - \frac{5}{3}y = 9$ . Line is in the form of  $xx_1 + yy_1 = r^2$ .

$\therefore p$  is  $\left(\frac{2}{3}, -\frac{5}{3}\right)$ .

*Blunders (-3)*

B1 error in  $dy/dx$ .

*Slips (-1)*

S1 arithmetic error.

*Attempts (3 marks)*

A1 line expressed in the form of  $xx' + yy' = r^2$ .

A2 finds  $dy/dx$ .

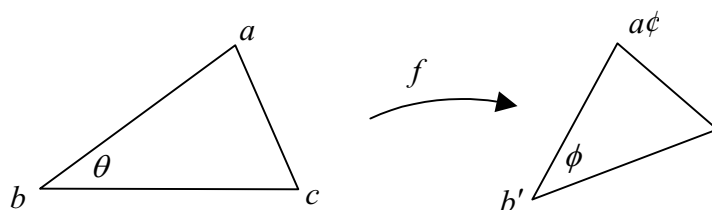
A3 solves between line and circle.

<b>Part (b)</b>	<b>20 (5, 5, 5, 5) marks</b>	<b>Att (2, 2, 2, 2)</b>
<b>Cos <math>\theta</math></b>	<b>5 marks</b>	<b>Att 2</b>
<b>Cos <math>\phi</math></b>	<b>5 marks</b>	<b>Att 2</b>
<b><math> p'q'  = k pq </math></b>	<b>5 marks</b>	<b>Att 2</b>
<b>Finish</b>	<b>5 marks</b>	<b>Att 2</b>

**11 (b)**  $f$  is a similarity transformation.  
 $f$  maps the angle  $\theta$  onto the angle  $\phi$ .  
 Prove that  $\theta$  and  $\phi$  are equal in measure.

**Prove** **20 (5, 5, 5, 5) marks** **Att (2, 2, 2, 2)**

**11 (b)**



$$\cos \theta = \frac{|ab|^2 + |bc|^2 - |ac|^2}{2|ab||bc|}$$

$$\cos \phi = \frac{|a'b'|^2 + |b'c'|^2 - |a'c'|^2}{2|a'b'||b'c'|}$$

$f$  is a similarity transformation.

$\therefore |p'q'| = k|pq|$  where  $k$  is a scalar.

$$\therefore \cos \phi = \frac{k^2|ab|^2 + k^2|bc|^2 - k^2|ac|^2}{2k^2|ab||bc|}$$

$$\cos \phi = \frac{|ab|^2 + |bc|^2 - |ac|^2}{2|ab||bc|} = \cos \theta.$$

$$\cos \theta = \cos \phi \Rightarrow |\angle \theta| = |\angle \phi| \text{ as } 0^\circ \leq \theta \leq 180^\circ.$$

*Blunders (-3)*

- B1 error in cosine formula.
- B2  $\cos \phi$  not simplified fully.
- B3 error in definition of similarity transformation.

*Slips (-1)*

- S1 arithmetic error.

*Attempts (2, 2, 2, 2 marks)*

- A1 diagram of value.
- A2 fails to square  $k$ .
- A3  $k^2$  incorrectly cancelled.

Part (c)

20 (10, 5, 5) marks

Att (3, 2, 2)

Part (c) (i)

10 marks

Att 3

11 (c) (i)  $[cd]$  is a diameter of the ellipse  $\frac{x^2}{100} + \frac{y^2}{25} = 1$  where  $c$  is the point  $\left(5\sqrt{2}, \frac{5}{2}\sqrt{2}\right)$ .

Find the equation of the tangent to the ellipse at  $c$ .

Equation of tangent

10 marks

Att 3

11 (c) (i)

$$\frac{x^2}{100} + \frac{y^2}{25} = 1, \quad \text{point } c \left(5\sqrt{2}, \frac{5}{2}\sqrt{2}\right).$$

$$\text{Tangent at } (x_1, y_1) \text{ is } \frac{xx_1}{100} + \frac{yy_1}{25} = 1$$

$$\backslash \text{ tangent at } c \left(5\sqrt{2}, \frac{5}{2}\sqrt{2}\right) \text{ is } \frac{5\sqrt{2}x}{100} + \frac{5\sqrt{2}y}{25} = 1$$

$$\backslash \frac{10x}{100} + \frac{10y}{50} = \sqrt{2} \quad \text{P} \quad \frac{x}{10} + \frac{y}{5} = \sqrt{2}.$$

*Blunders (-3)*

B1 error in tangent formula.

*Slips (-1)*

S1 arithmetic error.

*Attempts (3 marks)*

A1 formula for equation of tangent.

A2 finds  $dy/dx$ .

Part (c) (ii)

10 (5, 5) marks

Att (2, 2)

Equation of  $st$

5 marks

Att 2

Coordinates of  $s$  and  $t$

5 marks

Att 2

11 (c) (ii) The diameter  $[st]$  is conjugate to the diameter  $[cd]$ .  
Find the equation of  $st$  and the coordinates of the points  $s$  and  $t$ .

Part (c) (ii)

10 (5, 5) marks

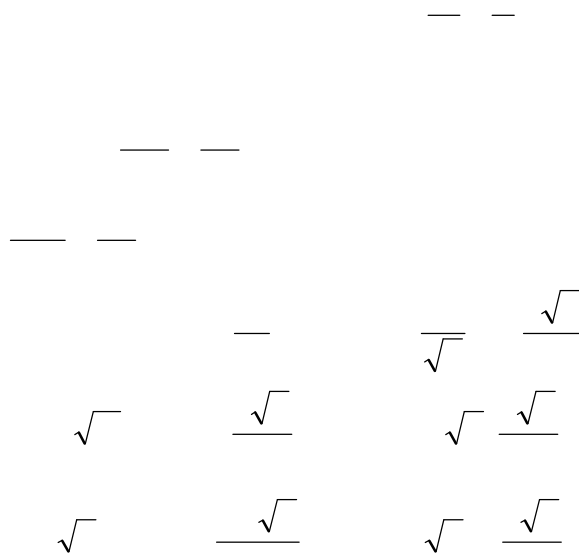
Att (2, 2)

11 (c) (ii)

$[st]$  and  $[cd]$  are conjugate diameters of ellipse  $E$ .

$\therefore$  Tangent at  $c$  is parallel to  $st$ .

Equation of  $st$  is  $\frac{x}{10} + \frac{y}{5} = 1$ .



*Blunders (-3)*

- B1 error in slope.
- B2 error in line equation formula.
- B3 incorrect squaring.
- B4 finds the coordinates of one point.

*Slips (-1)*

- S1 arithmetic error.

*Attempts ( 2, 2 marks)*

- A1 recognises that tangent at  $c$  is parallel to  $st$ .
- A2 correct slope of  $st$ .
- A3 equation in  $x^2$  or  $y^2$  from solving.