MARKING SCHEME
JUNIOR CERTIFICATE EXAMINATION 2007
MATHEMATICS - HIGHER LEVEL - PAPER 2

GENERAL GUIDELINES FOR EXAMINERS

1. Penalties of three types are applied to candidates' work as follows:
   - Blunders - mathematical errors/omissions (-3)
   - Slips- numerical errors (-1)
   - Misreadings (provided task is not oversimplified) (-1).

   ✍ means that work relevant to correct answer must be shown for full marks

   Frequently occurring errors to which these penalties must be applied are listed in the scheme. They are labelled: B1, B2, B3,…, S1, S2,…, M1, M2,…etc. These lists are not exhaustive.

2. When awarding attempt marks, e.g. Att(3), note that
   - any *correct, relevant* step in a part of a question merits at least the attempt mark for that part
   - if deductions result in a mark which is lower than the attempt mark, then the attempt mark must be awarded
   - a mark between zero and the attempt mark is never awarded.

3. Worthless work is awarded zero marks. Some examples of such work are listed in the scheme and they are labelled as W1, W2,…etc.

4. The phrase "hit or miss" means that partial marks are not awarded – the candidate receives all of the relevant marks or none.

5. The phrase "and stops" means that no more work is shown by the candidate.

6. Special notes relating to the marking of a particular part of a question are indicated by an asterisk. These notes immediately follow the box containing the relevant solution.

7. The sample solutions for each question are not intended to be exhaustive lists – there may be other correct solutions.

8. Unless otherwise indicated in the scheme, accept the best of two or more attempts – even when attempts have been cancelled.

9. The *same* error in the *same* section of a question is penalised *once* only.

10. Particular cases, verifications and answers derived from diagrams (unless requested) qualify for attempt marks at most.

11. A serious blunder, omission or misreading results in the attempt mark at most.

12. Do not penalise the use of a comma for a decimal point, e.g. €5.50 may be written as €5,50.

QUESTION 1

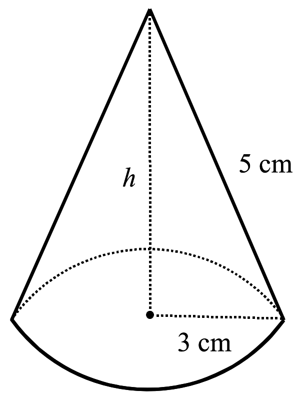
Part (a)	**10(5,5) marks**	**Att (2,2)**
Part (b)	**20(5,5,10)marks**	**Att (2,2,3)**
Part (c)	**20(5,10,5) marks**	**Att (2,3,2)**

Part (a) **10 (5,5) marks** **Att (2,2)**

(a) A cone has a base radius of 3 cm and a slant height of 5 cm.

(i) ✍ Find h, the perpendicular height of the cone.

(ii) ✍ Find the volume of the cone in terms of π.



(a)(i) **5 marks** **Att 2**

$$5^2 = 3^2 + h^2 \Rightarrow h^2 = 16 \Rightarrow h = \sqrt{16} \text{ cm or } 4 \text{ cm}$$

Blunders (-3)
B1 Correct answer without work (✍)
B2 Pythagoras incorrect
B3 Incorrect squaring
B4 $h^2 = 16$ and stops

Sli ps (-1)
S1 Arithmetic slips to a max of 3

Attempts (2 marks)
A1 Pythagoras indicated
A2 Diagram drawn with right angle indicated

(a)(ii) **5 marks** **Att 2**

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi 3^2 4 = 12\pi \text{ cms}^3$$

Blunders (-3)
B1 Correct answer without work (✍)
B2 Incorrect substitution into correct formula
B3 Incorrect relevant volume formula and continues

Slips (-1)
S1 Arithmetic slips to a max of 3
S2 Answer not expressed in terms of π

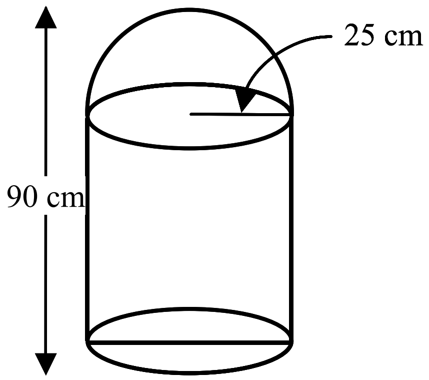
Attempts (2 marks)
A1 Correct formula with some substitution

Worthless (0)
W1 πrl with or without substitution

Part (b) **20 (5,5,10) marks** **Att(2,2,3)**

A hot water container is in the shape of a hemisphere on top of a cylinder as shown. The hemisphere has a radius of 25 cm and the container has a height of 90 cm.

✍ Find the internal volume of the container in litres, giving your answer correct to the nearest litre.



(b) Cylinder **5 marks** **Att 2**

Volume of cylinder = $\pi r^2 h = \pi 25^2\, 65$ cm^3

Blunders (-3)
B1 Incorrect substitution into correct formula
B2 Incorrect h
B3 Incorrect relevant volume formula

Attempts (2 marks)
A1 Correct formula with some substitution
A2 Correct h indicated

Worthless (0)
W1 Area formula

Hemisphere **5 marks** **Att 2**

$$\text{Volume of hemisphere} = \frac{2}{3}\pi r^3 = \frac{2}{3}\pi 25^3 \text{ cms}^3$$

Blunders (-3)

B1 Incorrect substitution into correct formula

B2 Incorrect relevant volume formula

Attempts (2 marks)

A1 Correct formula without substitution

Worthless (0)

W1 Area formula

Total volume **10 marks** **Att3**

$$\pi 25^2 65 + \frac{2}{3}\pi 25^3$$

$$= 40625\pi + 10416 \cdot 6667\pi$$

$$= 51041 \cdot 66667\pi = 160352 \cdot 125 \text{ cms}^3$$

Total Volume = 160 litres

Blunders (-3)

B1 Correct answer without work (✍)

B2 Volume of container expressed as difference of both parts

B3 Answer not expressed in litres

B4 Using a value of π which affects accuracy of answer

B5 Early rounding off which affects accuracy of answer

B6 Incorrect squaring and/or cubing

Slips (-1)

S1 Arithmetic slips to a max of 3

S2 Not rounding to nearest litre

Attempts (3 marks)

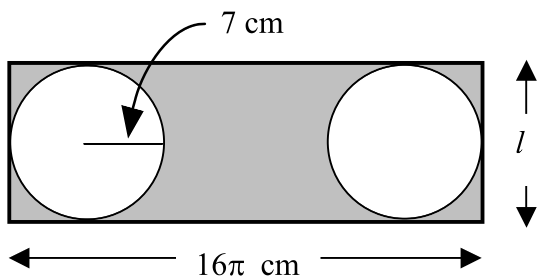
A1 Effort at calculating volume of either hemisphere or cylinder

A2 Indication of conversion to litres

Part (c) **20 (5,10,5) marks** **Att(2,3,2)**

A rectangular piece of metal has a width of 16π cm.

Two circular pieces, each of radius 7 cm, are cut from the rectangular piece, as shown.



(i) Find the length, *l*, of the rectangular piece of metal.

(ii) ✍ Calculate the area of the metal not used (i.e. the shaded section), giving your answer in terms of π.

(iii) ✍ Express the area of the metal not used as a percentage of the total area.

(c) (i) **5 marks** **Att 2**

$l = 14$ cm

Blunders (-3)

B1 Length l =7 cm

B2 Length l = 14π cm

Attempts (2 marks)

A1 Length l = 7π cm

Worthless (0)

W1 Length = 16π or 8π

(c) (ii) **10 marks** **Att 3**

Area of rectangle = 16π .14 = 224π cm²

Area of discs = $2\pi r^2 = 2\pi 7^2$ = 98π cm²

Unused = 224π - 98π = 126 π cm²

Blunders (-3)

B1 Correct answer without work (✍)

B2 Incorrect substitution into correct formula

B3 Incorrect r

B4 Value of l inconsistent with (c)(i)

B5 Incorrect relevant area formula

B6 Area of one disc (rather than two)

Slips(-1)
S1 Arithmetic slips to a max of 3
S2 Answer not in terms of π

Attempts (3 marks)
A1 Correct formula with some substitution
A2 Area of rectangle indicated
A3 Area of both discs indicated
A4 Perimeter of rectangle and area of one or both discs and stops
A5 Circumference of one or both discs and area of rectangle and stops

Worthless (0)
W1 Perimeter of rectangle and circumference of discs

(c) (iii) **5 marks** **Att 2**

Percentage unused $= \frac{126\pi}{224\pi}.100 = 56.25\%$

Blunders (-3)
B1 Correct answer without work (✍)
B2 Ratio not simplified
B3 Ratio inverted
B4 Decimal error
B5 Early rounding off which affects accuracy of answer
B6 Ratio not converted to percentage

Slips (-1)
S1 Arithmetic slips to a max of 3

Attempts (2 marks)
A1 Unused area expressed as a ratio
A2 Any use of 100

QUESTION 2

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 (10,10)marks**	**Att (3,3)**
Part (c)	**20 (5,5,5,5) marks**	**Att (2,2,2,2)**

Part (a) **10 marks** **Att 3**

p (2 , 4) and q (-1 , 1) are two points.

q is the midpoint of [pr].

✍ Find the co-ordinates of r.

(a) **10 marks** **Att 3**

$p \to q \to r$

$p(2,4) \to q(-1,1) \to r(-1-3, 1-3) = (-4,-2)$

$r = (-4, -2)$

or q midpoint pr where $r(x,y)$

$\left(\frac{2+x}{2}, \frac{4+y}{2}\right) = (-1,1)$

$2 + x = -2 \quad 4 + y = 2$

$x = -4 \quad y = -2$

Blunders (-3)

B1 Correct answer without work (✍)
B2 Substitutes for r correctly but point not found (translation method)
B3 Takes r as midpoint of pq
B4 Incorrect midpoint formula and continues
B5 Mixes both x and y in substitution
B6 Finds one co-ordinate only

Slips (-1)

S1 Arithmetic slips to a max of 3

Misreading (-1)

M1 Takes p as midpoint of [qr]

Attempts (3 marks)

A1 Writes midpoint formula without or with some substitution
A2 Graphical solution correct

Part (b) **20(10,10) marks** **Att (3,3)**

(0, 6) and (4, -2) are two points on the line M.

(i) ✍ Find the slope of M.

(ii) ✍ Find the equation of the line N through (4, -2), which is perpendicular to M.

Give your answer in the form $ax + by + c = 0$, where a, b and $c \in \mathbf{Z}$.

(b) (i) **10 marks** **Att 3**

(i) Slope of $M = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-2-6}{4-0} = \dfrac{-8}{4}$ or -2

Blunders (-3)
B1 Correct answer without work (✍)
B2 Incorrect slope formula and continues
B3 Mixes both x and y in substitution
B4 Substitutes correctly but slope not found

Slips (-1)
S1 Arithmetic slips to a max of 3

Attempts (3 marks)
A1 Writes slope formula with or without some substitution
A2 Effort at difference of y's and/or difference of x's

(b) (ii) **10 marks** **Att 3**

Slope of $N = \dfrac{1}{2}$ or $\dfrac{4}{8}$

Equation of N: $y - y_1 = m(x - x_1)$

$y - -2 = ½ (x - 4)$

$y + 2 = ½ (x - 4)$

$2y + 4 = x - 4$

$x - 2y - 8 = 0.$

Blunders (-3)
B1 Correct answer without work (✍)
B2 Incorrect relevant formula and continues
B3 Switches both x and y in substitution
B4 Substitutes correctly for x and y but incorrect slope
B5 $y + 2 = \dfrac{1}{2}(x - 4)$ and stops

Slips (-1)
S1 Arithmetic slips to a max of 3
S2 a, b, c in integer form but not written as $ax + by + c = 0$

Attempts (3marks)
A1 Correct line formula with or without some substitution
A2 Indicates product of perpendicular slopes equals -1

Part (c) **20(5,5,5,5) marks** **Att (2,2,2,2)**

L is the line $x - 2y + 2 = 0$ and K is the line $x + 2y - 6 = 0$.

(i) ✍ Find the coordinates of u, the point of intersection of L and K.

(ii) ✍ L cuts the y-axis at the point v. Find the coordinates of v.

(iii) ✍ Show that w (0, 3) is on the line K.

(iv) ✍ Show that $|uw| = |uv|$.

(c)(i) **5 marks** **Att 2**

$$x - 2y + 2 = 0$$
$$\underline{x + 2y - 6 = 0}$$
$$2x \quad -4 = 0$$
$$\Rightarrow \quad x = 2 \qquad 2 - 2y + 2 = 0 \Rightarrow y = 2 \qquad u(2,2)$$

* Accept (2,2) $\in$ L and (2,2) $\in$ K shown in each case

Blunders (-3)
B1 Correct answer without work (✍)
B2 Transposition error
B3 No substitution for second value

Slips (-1)
S1 Arithmetic slips to a max of 3

Attempts (2marks)
A1 Any correct step and stops
A2 Effort at graphical solution e.g. lets $x = 0$ and/or $y = 0$

(c)(ii) **5 marks** **Att 2**

$x = 0 \Rightarrow 0\text{-}2y + 2 = 0 \quad \Rightarrow y = 1$

$v = (0,1)$

Blunders (-3)

B1 Correct answer without work (✍)

B2 Takes $y = 0$ and finds x

B3 Transposition error

Slips (-1)

S1 Arithmetic slips to a max of 3

Misreading (-1)

M1 Takes K instead of L

Attempts (2marks)

A1 Graphical solution correct

(c)(iii) **5 marks** **Att 2**

$0 + 2(3) - 6 = 6 - 6 = 0 \Rightarrow w$ (0,3) on K

Blunders (-3)

B1 Correct answer without work (✍)

B2 Mixes x and y in substitution

Slips (-1)

S1 Arithmetic slips to a max of 3

Attempts (2 marks)

A1 Graphical solution correct

A2 Any effort at substitution

Worthless (0)

W1 Graphical solution incorrect

(c)(iv) **5 marks** **Att 2**

Formula $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$|uw| = \sqrt{(2-0)^2 + (2-3)^2} = \sqrt{2^2 + (-1)^2} = \sqrt{4+1}$ or $\sqrt{5}$

$|uv| = \sqrt{(2-0)^2 + (2-1)^2} = \sqrt{2^2 + 1^2} = \sqrt{4+1}$ or $\sqrt{5}$

$|uw| = |uv|$

Blunders (-3)

B1 Correct answer without work (✍)
B2 Incorrect relevant formula and continues
B3 Switches both x and y in substitution
B4 Substitutes correctly for x and y in each case but does not simplify
B5 $(-1)^2 \neq 1$

Slips (-1)

S1 $|uw| \neq |uv|$ without a conclusion from work
S2 Arithmetic slips to a max of 3

Attempts(2 marks)

A1 Correct formula with or without some substitution
A2 Incorrect relevant formula with some correct substitution
A3 Effort at translation

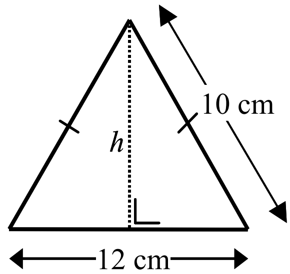
QUESTION 3

Part (a)	**10 marks**	**Att 3**
Part (b)	**25 (15,10) marks**	**Att (5,3)**
Part (c)	**15 (5,5,5) marks**	**Att (2,2,2)**

Part (a) **10 marks** **Att 3**

The isosceles triangle shown in the diagram, has a base of length 12 cm and the other two sides are each 10 cm in length.

✍ Find h, the perpendicular height of the triangle.



(a) **10 marks** **Att 3**

$$10^2 = h^2 + 6^2 \Rightarrow h^2 = 64 \Rightarrow h = \sqrt{64} \text{ or } 8 \text{ cm}$$

Blunders (-3)

B1 Correct answer without work (✍)
B2 Pythagoras incorrect
B3 Incorrect squaring
B4 $h^2 = 64$ and stops

Slips (-1)

S1 Arithmetic slips to a max of 3

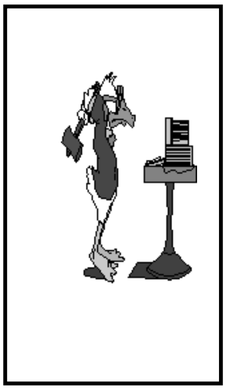
Attempts (3 marks)

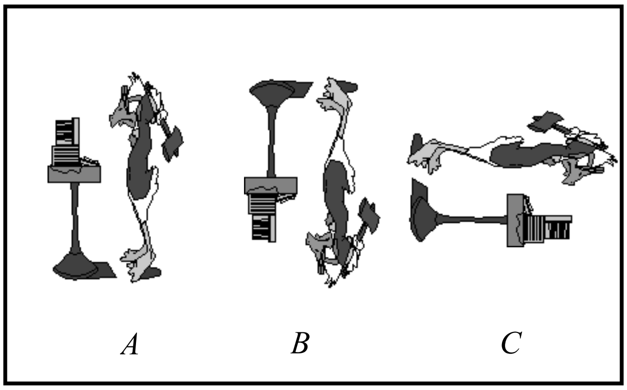
A1 Pythagoras indicated
A2 Reference to 3, 4, 5 and stops
A3 Using 12 in any version of Pythagoras
A4 States $\frac{12}{2} = 6$

Part (b) **25 (15,10) marks** **Att (5,3)**

(i) ✍ Prove that if two sides of a triangle are equal in measure, then the angles opposite these sides are equal in measure.

(ii) Each of the three figures labelled *A*, *B* and *C* shown below in the box on the right is the image of the figure shown in the box on the left under a transformation. For each of *A*, *B* and *C*, state what the transformation is (translation, central symmetry, axial symmetry or rotation) and in the case of a rotation, state the angle.

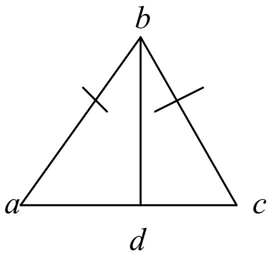




(b)(i) **15 marks** **Att 5**

Given: Triangle abc with $|ab| = |bc|$
To prove : $|\angle bac| = |\angle bca|$
Construction: Construct $[bd]$, where d is the midpoint of $[ac]$ step1
Proof: Taking Δabd and Δbcd, $|ab| = |bc|$ (given)
$|bd| = |bd|$ (same line segment) step2
$|da| = |dc|$ (d midpoint $[ac]$ step3
Δabd and Δbcd congruent by SSS step4
$\Rightarrow |\angle bac| = |\angle bca|$ step5

or Construction: From b drop a perpendicular to$[ac]$ intersecting $[ac]$ at d step1
Proof: Taking Δabd and Δbcd, $|ab| = |bc|$ (given)
$|bd| = |bd|$ (same line segment) step2
$|\angle bda| = |\angle bdc|$ (both right angles) step3
Δabd and Δbcd congruent by RHS step4
$\Rightarrow |\angle bac| = |\angle bca|$ step5



or Construction: Let the bisector of $\angle abc$ meet $[ac]$ at d step1
Proof: Taking Δabd and Δbcd , $|ab| = |bc|$ (given)
$|bd| = |bd|$ (same line segment) step2
$|\angle abd| = |\angle cbd|$ (Construction) step3
Δabd and Δbcd congruent by SAS step 4
$\Rightarrow |\angle bac| = |\angle bca|$ step 5

* Some steps may be indicated on the diagram

Blunders(-3)
B1 Each step incorrect or omitted
B2 Each step incomplete

Attempts (5marks)
A1 Diagram with triangle drawn and equal sides indicated

Worthless(0)
W1 Wrong Theorem
W2 Triangle and nothing else

(b)(ii) **10 marks** **Att 3**

A	Axial Symmetry
B	Central Symmetry or Rotation 180°
C	Rotation 90°(clockwise) or 270° (anti-clockwise)

* Accept angle of rotation without reference to clockwise or anticlockwise
* One correct transformation 4 marks
* Two correct transformations 7 marks
* Three correct transformations 10 marks

Slips (-1)
S1 No angle or incorrect angle of rotation

Attempts (3 marks)
A1 Any attempt at drawing the original figure under one of the given transformations

Part (c) **15 (5,5,5) marks** **Att (2,2,2)**

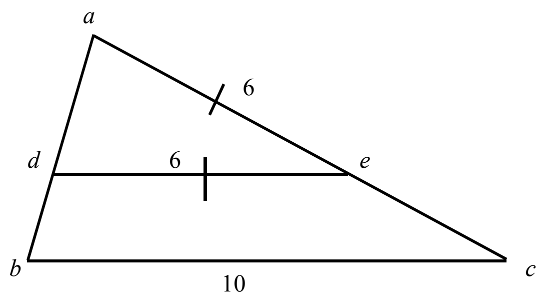
In the triangle abc, $bc \parallel de$, $|ae| = |de| = 6$ and $|bd| = ½\,|ce|$.

$|bc| = 10$.

(i) ✍ Find $|ce|$.

(ii) ✍ Find $|ad|$.

(iii) ✍ Find $|ab|$.



(c)(i) **5 marks** **Att2**

$$\frac{6}{|ac|} = \frac{6}{10} \Rightarrow |ac| = 10 \Rightarrow |ce| = 4$$

Blunders(-3)

B1 Correct answer without work (✍)

B2 $\frac{6}{|ec|} = \frac{6}{10}$

B3 $\frac{|ac|}{6} = \frac{6}{10}$ or equivalent

B4 Transposition error

Slips (-1)

S1 Arithmetic slips to a max of 3

Attempts (2marks)

A1 $|ac| = 10$ and stops

A2 Any effort at a relevant ratio

Worthless (0)

W1 $|ce| = 10$ without work shown

(c) (ii) **5 marks** **Att 2**

$$\frac{|ad|}{2} = \frac{6}{4} \quad \Rightarrow |ad| = 3$$

Blunders(-3)

B1 Correct answer without work (✍)

B2 $|db| \neq ½ |ec|$

B3 $\frac{|ad|}{2} = \frac{6}{10}$ or equivalent

B4 $\frac{|ad|}{2} = \frac{4}{6}$ or equivalent

B5 Transposition error

Slips (-1)

S1 Arithmetic slips to a max of 3

Attempts (2marks)

A1 Some effort at ratio and stops

A2 $|db| = 2$ and stops

Worthless (0)

W1 $|ad| = 6$ without work

(c) (iii) **5 marks** **Att 2**

$$|ab| = |ad| + |db| = 3 + 2 \text{ or } 5$$

Blunders (-3)

B1 Correct answer without work (✍)

B2 Inverted ratio

Slips (-1)

S1 Arithmetic slips to a max of 3

Misreadings (-1)

M1 Finding $|ac|$

Attempts (2 marks)

A1 Addition of more sides than required (work shown)

QUESTION 4

Part (a)	**10 (5,5) marks**	**Att (2,2)**
Part (b)	**20 marks**	**Att 7**
Part (c)	**20 (5,10,5) marks**	**Att (2,3,2)**

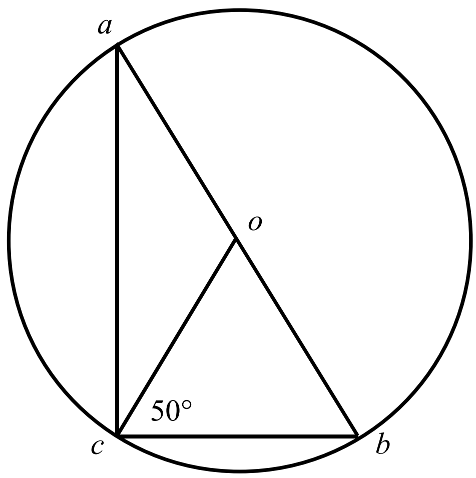
Part (a) **10 (5,5) marks** **Att (2,2)**

[*ab*] is the diameter of a circle of centre *o*.

$|\angle ocb| = 50°$.

(i) ✍ Find $|\angle boc|$.

(ii) ✍ Find $|\angle bac|$.



(a)(i) **5 marks** **Att 2**

$|oc| = |ob| \Rightarrow |\angle ocb| = |\angle obc| = 50° \Rightarrow |\angle boc| = 180° - (50° + 50°)$

$|\angle boc| = 80°$

* Accept work on diagram

Blunders(-3)

B1 Correct answer without work (✍)

B2 Sum of angles in triangle ≠ 180°

B3 Incorrectly indicates equal sides in isosceles triangle

Slips

S1 Arithmetic slips to a max of 3

Attempts(2 marks)

A1 $|oc| = |ob|$ indicated and stops

A2 Angle sum of triangle = 180°

Worthless(0)

W1 Assumes any angle in Δocb is a right angle and stops

(a)(ii) **5 marks** **Att 2**

$\lvert <aco \rvert = 40°$ since $\lvert <acb \rvert = 90°$ (angle in semi-circle) $\lvert ao \rvert = \lvert co \rvert$ (radii) $\Rightarrow \lvert \angle bac \rvert = 40°$

* Accept any correct approach

Blunders (-3)
B1 Correct answer without work (✍)
B2 Sum of angles in triangle ≠ 180°
B3 Incorrectly indicates equal sides in isosceles triangle
B4 $\lvert <acb \rvert \neq 90°$

Slips
S1 Arithmetic slips to a max of 3
Attempts (2 marks)
A1 Indicates sum of angles in a triangle equals 180°
A2 Identifies |*acb*| right angle
A3 States straight line angle = 180° and stops

Worthless (0)
W1 Assumes any angle in Δoac is a right angle

Part (b) **20 marks** **Att 7**

✍ Prove that the measure of the angle at the centre of the circle is twice the measure of the angle at the circumference, standing on the same arc.

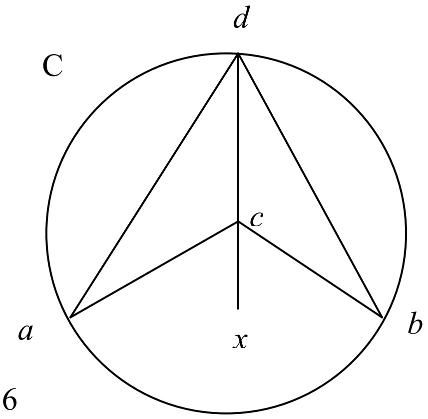
(b) **20 marks** **Att 7**

Given: Circle C, centre c, with points a,b,d on arc

Construction: Join ac, bc, ad, bd

Join dc and produce to x Step1

To prove: $|<acb| = 2|<adb|$



Proof:

$|ac| = |cd|$. both radii

⇨ $|<cad| = |<adc|$ angles in isosceles triangle step 2

But $|<acx| = |<cad| + |<adc|$

exterior = sum interior opposites step 3

⇨ $|<acx| = 2|<adc|$ step 4

Similarly $|<bcx| = 2|<bdc|$ step 5

⇨ $|<acx| + |<bcx| = 2|<adc| + 2|<bdc|$ step 6

⇨ $|<acb| = 2|<adb|$ step 7

* Some steps may be indicated on diagram

Blunders(-3)

B1 Each incorrect or omitted step

B2 Each step incomplete

B3 Theorem proven for angle in semicircle

Attempts (7marks)

A1 Diagram with angle at centre and/or angle at arc indicated

A2 Diagram with angle in a semicircle

Part (c) **20 (5,10,5) marks** **Att(2,3,2)**

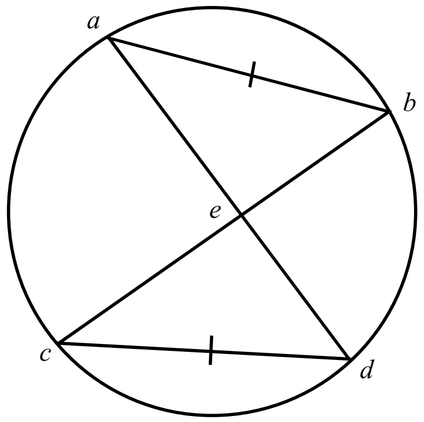
[*ab*] and [*cd*] are chords of the circle as shown and |*ab*| = |*cd*|.

The chords [*ad*] and [*bc*] intersect at the point *e*.

(i) State why $|\angle bad| = |\angle bcd|$.

(ii) ✍ Prove that the triangles *bae* and *dce* are congruent.

(iii) ✍ Prove |*ad*| = |*bc*|.



(c) (i) **5 marks** **Att 2**

Same arc

Attempts (2marks)

A1 Indicates angle at centre of circle

Worthless (0)

W1 Assumes *e* is centre of circle

W2 States that angles are alternate angles

(c) (ii) **10 marks** **Att 3**

$|\angle bad| = |\angle bcd|$ (given)

|*ab*| = |*cd*| given — step 1

|<abc| = |<adc| (on arc ac) or |<aeb| = |<ced| (vertically opposite) — step 2

Triangles *bae* and *dce* are congruent by ASA — step 3

* Some steps may be indicated on diagram drawn by candidate

Blunders (-3)

B1 Correct answer without work (✍)

B2 Each step incorrect or omitted

B3 Each step incomplete

Attempts (3marks)

A1 Diagram with triangles drawn and equal angles from (c)(i) indicated

Worthless (0)

W1 Diagram from examination paper either partially or totally drawn

W2 [*ab*] and [*cd*] parallel and stops

(c) (iii) **5 marks** **Att 2**

$\lvert ae \rvert = \lvert ce \rvert$ and $\lvert be \rvert = \lvert ed \rvert$ due to congruent triangles in (c)(ii) $\lvert ae \rvert + \lvert ed \rvert = \lvert ce \rvert + \lvert be \rvert$ $\lvert ad \rvert = \lvert bc \rvert$.

Blunders (-3)
B1 Correct answer without work (✍)
B2 $\lvert ae \rvert \neq \lvert ce \rvert$ and/or $\lvert be \rvert \neq \lvert ed \rvert$

Attempts (2 marks)
A1 Expression in terms of ratios
A2 $\lvert ad \rvert = \lvert ae \rvert + \lvert ed \rvert$ and stops

Worthless (0)
W1 Assuming either or both triangles are isosceles triangles
W2 Taking *e* as centre of circle

QUESTION 5

Part (a)	**10 marks**	**Att 3**
Part (b)	**20(5,10,5) marks**	**Att (2,3,2)**
Part (c)	**20(10,5,5) marks**	**Att (3,2,2)**

Part (a) **10 marks** **Att 3**

✍ If $\sin A = -\frac{1}{2}$, find the two values for the angle A, where $0° \leq A \leq 360°$.

(a) **10 marks** **Att 3**

$Sin\ A = -\frac{1}{2}$ in 3rd and 4th Quadrants

$A = 180° + 30°$ and $360° - 30° = 210°$ and $330°$.

Blunders (-3)

B1 Correct answer without work (✍)
B2 Second value of A not found
B3 Value(s) of A not in range $0° \leq A \leq 360°$
B4 Identifies incorrect quadrant(s)

Slips (-1)

S1 Arithmetic slips to a max of 3

Attempts (3 marks)

A1 Circle with all four quadrants indicated
A2 Some indication of the use of 30°
A3 Right angled triangle with A, -1, 2 indicated
A4 $Sine\ A = \frac{opp}{hyp}$

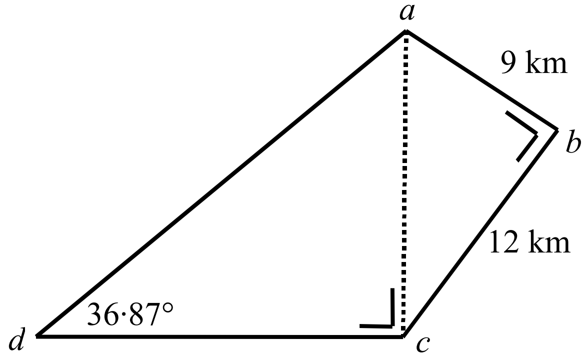
Worthless (0)

W1 Incorrect answer without work

Part (b) **20 (5,10,5)marks** **Att (2,3,2)**

In the diagram opposite, *abcd* represents the course taken in a triathlon. Competitors must swim the 9 km from *a* to *b*, then run the 12 km from *b* to *c* and cycle from *c* to *d* and back to *a*.

$|\angle adc| = 36{\cdot}87°$.



(i) ✍ Find the distance from *a* to *c*.

(ii) ✍ Find the distance from *c* to *d*, correct to the nearest km.

(iii) ✍ Find the total length of the course.

(b)(i) **5 marks** **Att 2**

$|ac|^2 = 9^2 + 12^2 = 81+144 = 225$

$|ac| = \sqrt{225}$ or 15 km

Blunders (-3)

B1 Correct answer without work (✍)

B2 Pythagoras incorrect

B3 Incorrect squaring

B4 $|ac|^2 = 225$ and stops

Slips (-1)

S1 Arithmetic slips to a max of 3

Attempts (2 marks)

A1 Pythagoras indicated

A2 Reference to 3,4,5 and stops

A3 Calculates $|\angle acb|$ or $|\angle cab|$ and stops

Worthless (0)

W1 Assigning a value to either <*bac* or <*bca*(with or without further work)

(b) (ii) **10 marks** **Att 3**

$$Tan\ 53.13° = \frac{|cd|}{15} \Rightarrow |cd| = 15\ Tan\ 53.13° \quad \text{or} \quad Tan 36.87° = \frac{15}{|cd|}$$

$$= 15(1.333) = 19.999\text{km} \qquad \Rightarrow |cd| = \frac{15}{Tan 36.87^0} = \frac{15}{0.75}$$

$$|cd| = 19.999$$

$$|cd| = 20\text{km to nearest km}$$

Blunders (-3)

B1 Correct answer without work (✍)
B2 Incorrect ratio for *Tan* function
B3 Error in cross multiplication
B4 Reads wrong page of tables or uses calculator in incorrect mode

Slips (-1)

S1 Arithmetic slips to a max of 3
S2 Slip reading tables (wrong column)
S3 Fails to distinguish between degrees and minutes and degrees in decimal form

Attempts (3marks)

A1 Indicates use of $|cd|$ in a ratio
A2 Indicates use of 15 or equivalent in a ratio
A3 $Tan\ A = \frac{opp}{adj}$ or $Tan 36.87°$ or $Tan\ 53.13°$ and stops

(b)(iii) **5 marks** **Att 2**

$$|ad|^2 = 20^2 + 15^2 = 400 + 225 = 625 \text{ km} \quad \textbf{or} \quad Sine 36.87° = \frac{15}{|ad|}$$

$$|ad| = 25 \text{ km} \qquad |ad| = \frac{15}{0.6} = 25 \text{ km}$$

Total =12 + 9 + 20 + 25 = 66km

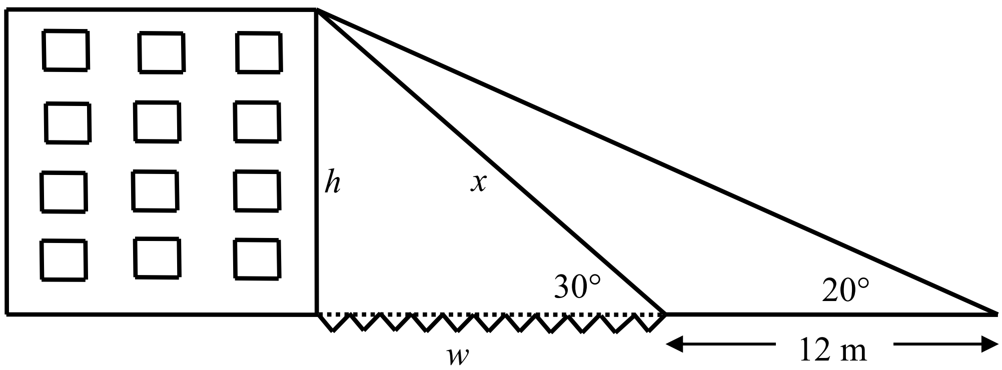
Blunders (-3)

B1 Correct answer without work (✍)
B2 Pythagoras incorrect
B3 Incorrect squaring
B4 $|ad|^2 = 625$ and stops
B5 Incorrect ratio for *Trig* function
B6 Error in cross multiplication
B7 Reads wrong page of tables or uses calculator in incorrect mode
B8 Incorrect ratio for Sine Rule

Slips (-1)
S1 Arithmetic slips to a max of 3
S2 Each side omitted in sum after calculation of $|ad|$
S3 $|ac|$ included in sum

Attempts (2 marks)
A1 Pythagoras indicated
A2 Reference to 3,4,5 and stops

Part (c) **20 (10,5,5) marks** **Att (3,2,2)**



The diagram shows an office block built on a river bank. From a point on the opposite river bank the angle of elevation of the top of the office block is 30°. From a point 12 m further back the angle of elevation is 20°.

(i) ✍ Find x, correct to 2 decimal places.

(ii) ✍ Find h, the height of the office block, correct to 2 decimal places.

(iii) ✍ Find w, the width of the river, correct to 2 decimal places.

(c) (i) **10 marks** **Att 3**

$$\frac{Sine 10^0}{12} = \frac{Sine 20^0}{x}$$

$$x = \frac{12\, Sine 20}{Sine 10^0}$$

$$= \frac{12(\cdot 342)}{\cdot 1736}$$

$=23\cdot6405$ $= 23\cdot64$m to 2 decimal places

Blunders (-3)
B1 Correct answer without work (✍)
B2 Incorrect ratio in use of Sine Rule
B3 Error in cross multiplication
B4 Reads wrong page of tables or uses calculator in incorrect mode
B5 Early rounding off which affects the answer

Slips (-1)
S1 Arithmetic slips to a max of 3
S2 Answer not to 2 decimal places

Attempts (3marks)
A1 Sine Rule with some substitution
A2 Identifies 150° or 10°

Worthless (0)
W1 Treats triangle as right angled

(c) (ii) **5 marks** **Att 2**

$$Sine30^\circ = \frac{h}{x} = \frac{h}{23 \cdot 64} \quad \Rightarrow \quad h = 23{\cdot}64\ Sine30^\circ = 23{\cdot}64({\cdot}5) = 11{\cdot}82\text{m}$$

Blunders (-3)
B1 Correct answer without work (✍)
B2 Incorrect ratio for Trig function
B3 Error in cross multiplication
B4 Reads wrong page of tables or uses calculator in incorrect mode
B5 Rounding early which affects answer

Slips (-1)
S1 Arithmetic slips to a max of 3
S2 Answer not to two decimal places

Attempts (2marks)
A1 Indicates use of h in a ratio
A2 Indicates use of 23.64 or equivalent in a ratio
A3 Tan 30° = $\frac{h}{w}$ or Tan 60° = $\frac{w}{h}$ or writes value of Tan 30° or Tan 60°

(c) (iii) **5 marks** **Att 2**

$$x^2 = h^2 + w^2 \Rightarrow 23{\cdot}64^2 = 11{\cdot}82^2 + w^2$$
$$w^2 = 558{\cdot}8496 - 139{\cdot}7124 = 419{\cdot}1372$$
$$w = 20{\cdot}47284 = 20{\cdot}47\text{m to 2 decimal places}$$

* This solution can be achieved in a variety of ways using trigonometric methods

Blunders (-3)
B1 Correct answer without work (✍)
B2 Pythagoras incorrect
B3 Incorrect squaring
B4 $w^2 = 419{\cdot}1372$ and stops
B5 Incorrect ratio for Trig function
B6 Error in cross multiplication
B7 Reads wrong page of tables or uses calculator in incorrect mode
B8 Incorrect ratio for Sine Rule
B9 Early rounding which affects answer

Slips (-1)
S1 Arithmetic slips to a max of 3
S2 Calculates w +12 and stops
S3 Answer not to two decimal places

Attempts (2 marks)
A1 Pythagoras indicated
A2 Indicates use of h and/or x values in a ratio

QUESTION 6

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 (10,5,5)marks**	**Att (3,2,2)**
Part (c)	**20 (5,5,10) marks**	**Att (2,2,3)**

Part (a) **10 marks** **Att 3**

In 4 games, a soccer player scored 1, x, 4 and 3 goals respectively.

The mean number of goals scored by the player per game was 2.

✍ Find the number of goals scored in the second game i.e. the value of x.

(a) **10 marks** **Att 3**

$$\frac{1+x+4+3}{4} = 2 \Rightarrow \quad 8 + x = 8 \Rightarrow \quad x = 0$$

Blunders (-3)

B1 Correct answer without work (✍)
B2 Incorrect denominator
B3 Error in transposition
B4 8x in numerator
B5 12x in numerator

Slips (-1)

S1 Arithmetic slips to a max of 3

Attempts (3 marks)

A1 Adds some or all of the numbers
A2 Indication of division by 4
A3 $\frac{1+x+4+3}{4}$ and stops

Part (b) **20 (10,5,5) marks** **Att (3,2,2)**

Over a period of one month, the owner of a factory recorded the number of days that each of his 50 employees was absent from work. The following table shows the results.

No. days absent	0	1	2	3	4	5
No. of employees	7	9	11	12	7	4

(i) ✍ Find the mean number of days the employees were absent.

(ii) ✍ Find the percentage of employees who were absent for more than the mean number of days.

(iii) Write down the mode.

(b) (i) **10 marks** **Att 3**

$$\frac{7(0)+9(1)+11(2)+12(3)+7(4)+4(5)}{7+9+11+12+7+4}$$

$$\frac{0+9+22+36+28+20}{50} \quad = \frac{115}{50} \quad = 2\cdot 3$$

Blunders (-3)
B1 Correct answer without work (✍)
B2 Transposition error
B3 Division by 6
B4 Division by 15 (sum of class intervals)
B5 Consistently adds interval value to frequency instead of multiplying
B6 $\frac{50}{115}$ and continues

Slips (-1)
S1 Arithmetic slips to a max of 3

Attempts (3 marks)
A1 One correct multiplication in numerator
A2 Indicates use of 50
A3 Sum of frequencies divided by 6

Worthless (0)
W1 $\frac{15}{6}$

(b) (ii) **5 marks** **Att 2**

Percentage greater than mean = $\dfrac{(12+7+4).100}{50}$ = 46%

* Accept candidates answer from b(i)

Blunders (-3)

B1 Correct answer without work (✍)
2 Divisor other than 50
B3 Not applying percentage to answer

Slips (-1)

S1 Arithmetic slips to a max of 3
S2 Omits one of the relevant values above mean

Attempts (2 marks)

A1 Indicates division by 50
A2 Adds two or more relevant values
A3 Indicates some use of 100

(b) (iii) **5 marks** **Att 2**

Mode = 3.

Blunders (-3)

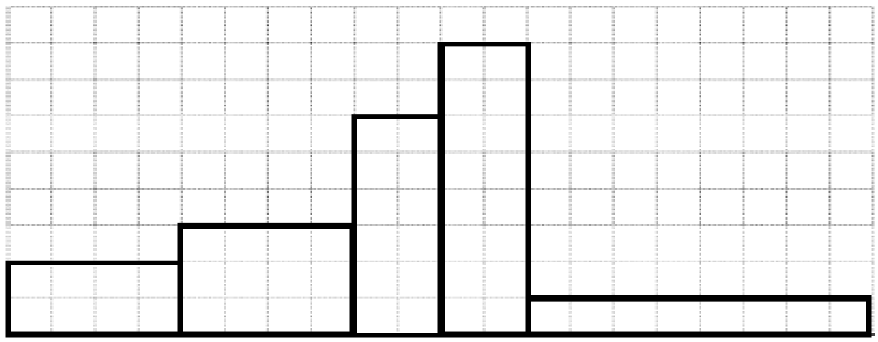
B1 Mode =12

Attempts (2 marks)

A1 Indication of division by 2
A2 Rearranges frequencies and finds the mode of these

Part (c) **20(5,5,10) marks** **Att (2,2,3)**

The distribution of the ages of people living in an apartment block is shown in the histogram below.



(i) Given that there are 10 people in the 0 – 10 age group, copy and complete the frequency table below.

Ages in years	0 – 10	10 – 20	20 – 25	25 – 30	30 – 50
No. of people	10				

[Note: 10 – 20 means 10 years or more but less than 20 years old, etc.]

(ii) Copy and complete the cumulative frequency table below.

Ages in years	< 10	< 20	< 25	< 30	< 50
No. of people					

(iii) ✍ Construct an ogive and use it to estimate the median age.

(c) (i) **5 marks** **Att 2**

(i)

Ages in years	0 – 10	10 – 20	20 – 25	25 – 30	30 – 50
No. of people	10	**15**	**15**	**20**	**10**

Blunders (-3)
B1 One box = 0·8 and continues
Slips (-1)
S1 Arithmetic slips to a max of 3
S2 Each incorrect entry
Attempts (2 marks)
A1 8, 12, 12, 16, 8 for frequencies
A2 One box = 1·25 and stops
A3 Work with base and stops
Worthless (0)
W1 Copies table and stops without making any further entries

(c) (ii) **5 marks** **Att 2**

Ages in years	< 10	< 20	< 25	< 30	< 50
No. of people	**10**	**25**	**40**	**60**	**70**

* Accept candidate's frequency table

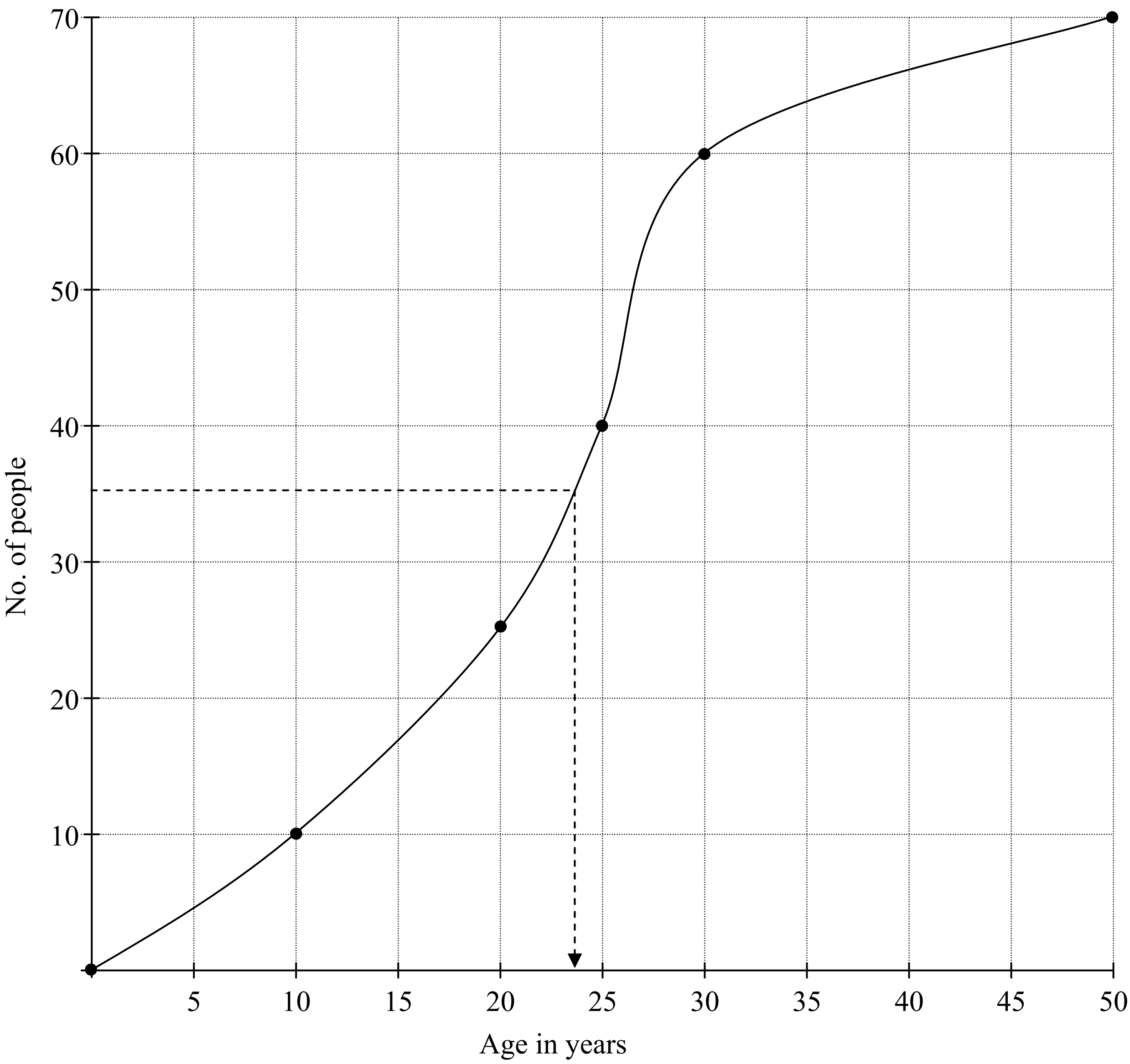
Blunders (-3)
B1 Subtracting frequencies instead of adding

Slips (-1)
S1 Arithmetic slips to a max of 3
S2 Each incorrect entry

Attempts (2 marks)
A1 Any one value filled correctly into table
A2 Any indication of addition of frequencies

Worthless (0)
W1 Copies table and stops
W2 Repeats (c) (i) table

(c) (iii) **10 marks** **Att 3**



Median = 23

* Accept median clearly marked on graph

* Accept median in the range 20-25 years

Blunders (-3)
B1 Incorrect scales
B2 Plots points but not joined
B3 Draws a ‘cumulative’ histogram
B4 Draws a ‘cumulative’ cumulative ogive
B5 Line drawn from incorrect starting point of correct axis for median
B6 Uses horizontal axis for starting point for median

Slips (-1)
S1 Each incorrect plot
S2 Each point omitted
S3 Work for median correct but outside tolerance

Attempts (3 marks)
A1 Correct scale on base axis