



**Coimisiún na Scrúduithe Stáit**  
*State Examinations Commission*

# **JUNIOR CERTIFICATE EXAMINATION**

**2012**

**MARKING SCHEMES**

**MATHEMATICS  
HIGHER LEVEL**





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# **JUNIOR CERTIFICATE EXAMINATION**

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**MARKING SCHEME**

**MATHEMATICS  
HIGHER LEVEL  
PAPER 1**

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**MATHEMATICS - HIGHER LEVEL - PAPER 1**

GENERAL GUIDELINES FOR EXAMINERS

1. Penalties of three types are applied to candidates' work as follows:

- Blunders - mathematical errors/omissions (-3)
- Slips- numerical errors (-1)
- Misreadings (provided task is not oversimplified) (-1).

Frequently occurring errors to which these penalties must be applied are listed in the scheme. They are labelled: B1, B2, B3,..., S1, S2,..., M1, M2,...etc. These lists are not exhaustive.

2. When awarding attempt marks, e.g. Att(3), note that
- any *correct, relevant* step in a part of a question merits at least the attempt mark for that part
  - if deductions result in a mark which is lower than the attempt mark, then the attempt mark must be awarded
  - a mark between zero and the attempt mark is never awarded.
3. Worthless work is awarded zero marks. Some examples of such work are listed in the scheme and they are labelled as W1, W2,...etc.
4. The phrase "hit or miss" means that partial marks are not awarded – the candidate receives all of the relevant marks or none.
5. The phrase "and stops" means that no more work is shown by the candidate.
6. Special notes relating to the marking of a particular part of a question are indicated by an asterisk. These notes immediately follow the box containing the relevant solution.
7. The sample solutions for each question are not intended to be exhaustive lists – there may be other correct solutions.
8. Unless otherwise indicated in the scheme, accept the best of two or more attempts – even when attempts have been cancelled.
9. The *same* error in the *same* section of a question is penalised *once* only.
10. Particular cases, verifications and answers derived from diagrams (unless requested) qualify for attempt marks at most.
11. A serious blunder, omission or misreading results in the attempt mark at most.
12. Do not penalise the use of a comma for a decimal point, e.g. €5.50 may be written as €5,50.

## BONUS MARKS FOR ANSWERING THROUGH IRISH

Bonus marks are applied separately to each paper as follows:

If the mark achieved is 225 or less, the bonus is 5% of the mark obtained, rounded **down**.  
(e.g. 198 marks  $\times$  5% = 9.9  $\Rightarrow$  bonus = 9 marks.)

If the mark awarded is above 225, the following table applies:

Bunmharc (Marks obtained)	Marc Bónais (Bonus Mark)	Bunmharc (Marks obtained)	Marc Bónais (Bonus Mark)
226	11	261 – 266	5
227 – 233	10	267 – 273	4
234 – 240	9	274 – 280	3
241 – 246	8	281 – 286	2
247 – 253	7	287 – 293	1
254 – 260	6	294 – 300	0

## QUESTION 1

<b>Part (a)</b>	<b>10 marks</b>	<b>Att (2,2)</b>
<b>Part (b)</b>	<b>20 marks</b>	<b>Att (3,3)</b>
<b>Part (c)</b>	<b>20 marks</b>	<b>Att (2,3,2)</b>

**Part (a)** **10(5,5) marks** **Att (2,2)**

- |   |
|---|
| <b>(a) (i)</b> List the divisors of 30.<br><b>(ii)</b> State which of these divisors are prime numbers. |
|---|

**(a) (i)** **5 marks** **Att 2**

Divisors of 30 → 1, 2, 3, 5, 6, 10, 15, 30.
---

\* Accept correct answer for full marks. No work required, no ✍

*Slips (-1)*

S1 Each incorrect or missing number to a maximum of -3, must have at least one correct

*Misreadings (-1)*

M1 Misreads 30, but continues correctly, provided oversimplification does not occur

*Attempts (2 marks)*

A1 Any correct divisor

A2 Any relevant step

*Worthless (0)*

W1 Incorrect answer, but note A1

W2 Multiples of 30

**(a) (ii)** **5 marks** **Att 2**

Prime divisors of 30 → 2, 3, 5.
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\* Answer may be dependent on part (i)

*Slips (-1)*

S1 Each incorrect or missing prime number to a maximum of -3, must have at least one correct

S2 Includes 1 as a prime number

*Attempts (2 marks)*

A1 Any correct prime divisor

A2 Shows some knowledge of a prime number

*Worthless (0)*

W1 Incorrect answer with no work of merit

**Part (b)****20 (10,10) marks****Att (3,3)**

- (i) €900 is invested for two years at 3% per annum compound interest.  
✍ Find the value of the investment at the end of the second year
- (ii) John has a gross weekly wage of €600.  
After tax his net weekly wage is €554.  
✍ Calculate his tax credits if he is taxed at the standard rate of 20%.

**(b) (i)****10 marks****Att 3**

<b>I</b>			
$P_1$	= €900	$I_1 = \frac{P \times R}{100} = \frac{900 \times 3}{100}$	<b>3m</b>
		$= €27$	<b>4m</b>
$P_2$	= €900 + €27 = €927	$I_2 = \frac{P \times R}{100} = \frac{927 \times 3}{100} = €27.81$	<b>7m</b>
$A_2$	= €927 + €27.81		<b>7m</b>
	= €954.81		<b>10m</b>
<b>II</b>			
	$A = P(1 + i)^t$		
	$A = €900 \left(1 + \frac{3}{100}\right)^2$ or $€900(1 + .03)^2$		<b>3m</b>
	$A = €900(1.03)^2$		<b>4m</b>
	$A = €900(1.0609)$		<b>7m</b>
	$A = €954.81$		<b>10m</b>

- \* Do not penalise for the omission of € symbol
- \* Ignore missing brackets if final answer is not affected
- \* Final answer of €954 is 4 marks

*Blunders (-3)*

- B1 Correct answer, no work shown ✍
- B2 3% of an incorrect number
- B3 Decimal error
- B4 Incorrect operation
- B5 Mathematical error
- B6 Error in squaring
- B7 Error in formula
- B8 Precedent error
- B9 Fails to calculate final step

*Slips (-1)*

- S1 Numerical error to a max of -3

*Misreadings (-1)*

- M1 Misreads a digit provided it doesn't oversimplify the question

*Attempts (3 marks)*

- A1 Relevant correct formula which is not in log tables
- A2 Finds 3% of some number other than €900 and stops
- A3 Divides by 100
- A4  $900 \times 3$
- A5  $\frac{P \times R \times T}{100} = 54$ , oversimplification
- A6 Identifies  $P = €900$  and/or  $R = 3\%$  or  $\cdot 03$
- A7  $3\% = \frac{3}{100}$  or  $\cdot 03$
- A8 Any correct substitution
- A9 Any relevant step

*Worthless (0)*

- W1 Incorrect answer no work shown
- W2  $F = P (1 + i)^t$  with no correct substitution



(b) (ii)

10 marks

Att 3

<b>I</b>			
Gross income	=	€600	
Net income	=	€554	..... Given in question
Tax paid	=	€600 – €554	3 m
	=	€46	4 m
Tax @20%	=	20% of €600	
	=	€120	7m
Tax credits	=	€120 – €46	
	=	€74	10m
<b>Steps are interchangeable:</b>			
Tax @20%	=	20% of €600	3m
	=	€120	4m
Tax paid	=	€600 – €554	4m
	=	€46	7 m
Tax credits	=	€120 – €46	7m
	=	€74	10m
<b>II</b>			
Tax @20%	=	20% of €600	3m
	=	€120	4m
	=	€600 – €120	
	=	€480	7m
	=	€554 – €480	
	=	€74	10m

*Blunders (-3)*

- B1 Correct answer, no work shown ✍
- B2 Decimal error
- B3 Incorrect operation
- B4 Inversion
- B5 Mathematical error
- B6 Finds 20% of an incorrect figure
- B7 Fails to calculate final step

*Slips (-1)*

- S1 Numerical error to a max of -3

*Misreadings (-1)*

- M1 Misreads a digit provided it doesn't oversimplify the question

*Attempts (3 marks)*

- A1 Finds 20% correctly of a figure other than €600 and stops
- A2  $20\% = \frac{20}{100}$  or equivalent
- A3 Divides by 100
- A4 Shows some knowledge of tax credits e.g. writes "Tax payable = total tax – tax credits"
- A5 Any relevant step

*Worthless (0)*

- W1 Incorrect answer no work shown
- W2  $600 \times 554$
- W3 No work of merit

Part (c)

20 (5,10,5) marks

Att (2,3,2)

- (i) ✍ By rounding to the nearest whole number, estimate the value of

$$\frac{3 \cdot 89 \times 7 \cdot 24 - \sqrt{8 \cdot 94}}{8 \cdot 52 - 3 \cdot 65}$$

- (ii) ✍ Evaluate  $\frac{3 \cdot 89 \times 7 \cdot 24 - \sqrt{8 \cdot 94}}{8 \cdot 52 - 3 \cdot 65}$ , correct to two decimal places.

- (iii) ✍ Simplify  $\sqrt{5}(\sqrt{2} + \sqrt{5}) - \sqrt{8}(\sqrt{2} - \sqrt{5})$  without the use of a calculator.  
Express your answer in the form  $a + b\sqrt{c}$ , where  $a, b, c \in \mathbb{N}$ .

(c) (i)

5 marks

Att 2

$$\begin{aligned} & \frac{4 \times 7 - \sqrt{9}}{9 - 4} && \mathbf{2m} \\ = & \frac{28 - 3}{5} && \mathbf{2m} \\ = & \frac{25}{5} && \mathbf{2m} \\ = & 5 && \mathbf{5m} \end{aligned}$$

*Blunders (-3)*

- B1 Correct answer, no work shown ✍
- B2 Rounds incorrectly, once if consistent
- B3 Incorrect operation
- B4 Inversion
- B5 Mathematical error
- B6 Precedent error
- B7 Square root error
- B8 Invalid cancellation
- B9 Sign error
- B10 Fails to calculate final step, stops at  $\frac{25}{5}$

*Slips (-1)*

- S1 Numerical errors to a max of -3

*Misreadings (-1)*

- M1 Misreads a digit provided it doesn't oversimplify the question

*Attempts (2 marks)*

- A1 Some correct rounding
- A2 5·17 with work

*Worthless (0)*

- W1 Incorrect answer, no work shown e.g. 5·17 **without** work
- W2 No work of merit

(c) (ii)

10 marks

Att 3

$\frac{3 \cdot 89 \times 7 \cdot 24 - \sqrt{8 \cdot 94}}{8 \cdot 52 - 3 \cdot 65}$	
$= (28 \cdot 1636 - 2 \cdot 989983278) \div 4 \cdot 87$	<b>4m</b>
$= 25 \cdot 17361672 \div 4 \cdot 87$	<b>7m</b>
$= 5 \cdot 16912$	<b>9m</b>
$= 5 \cdot 17$	<b>10m</b>

*Blunders (-3)*

- B1 Correct answer, no work shown ✍
- B2 Decimal error
- B3 Incorrect operation
- B4 Inversion
- B5 Mathematical error
- B6 Precedent error
- B7 Square root error
- B8 Sign error
- B9 Stops at  $25 \cdot 17361672 \div 4 \cdot 87$

*Slips (-1)*

- S1 Numerical errors to a max of -3
- S2 Stops at 5·169 or 5·1691 or 5·16912 or similar
- S3 Early rounding if it affects final answer, but note A2

*Misreadings (-1)*

- M1 Misreads a digit provided it doesn't oversimplify the question

*Attempts (3 marks)*

- A1 Some correct calculation
- A2 Rounds to whole numbers and continues
- A3 Any relevant step

*Worthless (0)*

- W1 Incorrect answer, no work shown
- W2 No work of merit

(c) (iii)

5 marks

Att 2

$\sqrt{5}(\sqrt{2} + \sqrt{5}) - \sqrt{8}(\sqrt{2} - \sqrt{5})$	
$= \sqrt{10} + \sqrt{25} - \sqrt{16} + \sqrt{40}$	2m
$= \sqrt{10} + 5 - 4 + 2\sqrt{10}$	2m
$= 1 + 3\sqrt{10}$	5m

*Blunders (-3)*

- B1 Correct answer, no work shown ✍
- B2 Distribution error
- B3 Sign error
- B4 Error in surds, once if consistent
- B5 Mathematical error
- B6 Fails to finish

*Slips (-1)*

- S1 Numerical error to a max of -3

*Misreadings (-1)*

- M1 Misreads a digit provided it doesn't oversimplify the question

*Attempts (2 marks)*

- A1 10·486832298, no surds used, with work shown
- A2 Any relevant attempt at handling surds
- A3  $\sqrt{\quad} =$  power of  $\frac{1}{2}$
- A4 Any relevant work

*Worthless (0)*

- W1 Incorrect answer no work shown
- W2  $\sqrt{5} = 2.236067977$  and/or  $\sqrt{8} = 2.828427125$  and/or  $\sqrt{2} = 1.414213562$  and stops
- W3 No work of merit

## QUESTION 2

<b>Part (a)</b>	<b>10 marks</b>	<b>Att 3</b>
<b>Part (b)</b>	<b>20 marks</b>	<b>Att (3,2,2)</b>
<b>Part (c)</b>	<b>20 marks</b>	<b>Att (3,2,2)</b>
<b>Part (a)</b>	<b>10 marks</b>	<b>Att 3</b>

Fuel consumption in a car is measured in litres per 100 km.  
 Alan's car travels 1250 km on a tank of 68 litres.  
 ✍ Calculate his car's fuel consumption in litres per 100 km.

<b>(a)</b>	<b>10 marks</b>	<b>Att 3</b>
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<b>I</b>		
1250 ÷ 100		<b>3m</b>
= 12.50		<b>4m</b>
68 litres for 12.50 '100km' →	$\frac{68}{12.50}$	<b>7m</b>
= 5.44 litres / 100km.		<b>10m</b>
<b>II</b>		
1250 km = 68 litres .....given in question		
1 km = 68 ÷ 1250		<b>3m</b>
= .0544		<b>4m</b>
100km = .0544 × 100		<b>7m</b>
= 5.44 litres / 100km		<b>10m</b>

\*  $\frac{68}{1250} \times 100$  and stops is worth 4m; .1838 with work is 7m; 18.38 with work is 3m

### Blunders (-3)

- B1 Correct answer, no work shown ✍
- B2 Decimal error
- B3 Incorrect multiplier (check method)
- B4 Incorrect division
- B5 Mathematical error
- B6 Inversion
- B7 Incorrect operation
- B8 Fails to complete last step

### Slips (-1)

- S1 Numerical errors to a max of -3

### Misreadings (-1)

- M1 Misreads a digit provided it doesn't oversimplify the question

### Attempts (3 marks)

- A1 Multiplies or divides by 100
- A2 1250 ÷ 68
- A3 Some knowledge of relationship between fuel consumption and distance indicated
- A4 Any relevant step

### Worthless (0)

- W1 Incorrect answer no work shown
- W2 No work of merit

Part (b)

20 (10,5,5) marks

Att (3,2,2)

$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$  is the universal set.

$P = \{3, 5, 6, 8, 10\}$ ,  $Q = \{2, 4, 6, 8, 10, 12\}$  and  $R = \{2, 5, 6, 7, 9, 12\}$  are three subsets of  $U$ .

(i) Represent the above information on a Venn diagram.

Hence list the elements of:

(ii)  $(P \cup Q \cup R)'$

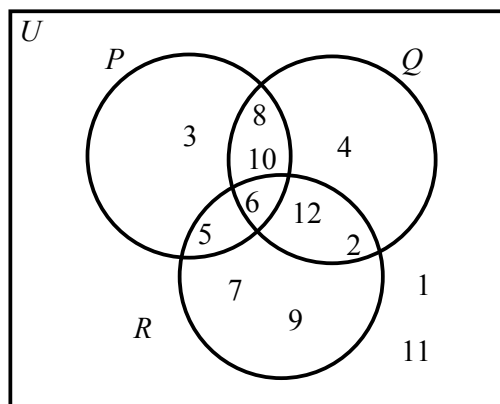
(iii)  $(P \cap Q) \setminus R$ .

(b) (i)

10 marks

Att 3

Venn diagram:



\* Ignore notation

\* Sets  $P$ ,  $Q$ ,  $R$  may be positioned differently from above

*Slips (-1)*

S1 Each incorrect or missing or misplaced element in Venn diagram each time but note A2

S2 Universal box not drawn on diagram

*Misreadings (-1)*

M1 Misreads a digit provided it doesn't oversimplify the question

*Attempts (3 marks)*

A1 Draws a Venn diagram with three intersecting circles and stops

A2 Any correct entry

A3 Universal box with two intersecting circles

A4 Three intersecting circles

A5 Universal box with a correct entry

*Worthless (0)*

W1 Rectangle only

W2 Circle with no correct entry

W3 Two intersecting circles with no correct entry

**(b) (ii)**

**5 marks**

**Att 2**

$(P \cup Q \cup R)'$	=	$\{1, 11\}$
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- \* Answer may be dependent on candidate's answer to **(b)(i)**
- \* Answer may be indicated on diagram
- \* Ignore notation

*Blunders (-3)*

B1 Elements of  $(P \cup Q \cup R)$  given as answer *i.e.*  $\{2, 3, 4, 5, 6, 7, 8, 9, 10, 12\}$

*Slips (-1)*

S1 Each incorrect or missing or misplaced element to a maximum of -3, must have at least one element correct; note B1

*Attempts (2 marks)*

- A1 One correct element
- A2 Any relevant step

*Worthless (0)*

- W1 Incorrect answer with no work of merit, note B1
- W2  $\{ \}$  but note \*1
- W3 Draws diagram again, with no further work of merit

**(b) (iii)**

**5 marks**

**Att 2**

$(P \cap Q) \setminus R$	=	$\{8, 10\}$
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- \* Answer may be dependent on candidate's answer to **(b)(i)**
- \* Answer may be indicated on diagram
- \* Ignore notation

*Slips (-1)*

S1 Each incorrect or missing or misplaced element to a maximum of -3, must have at least one element correct

*Attempts (2 marks)*

- A1 Shades/indicates the correct region on Venn diagram, but elements not clearly identified
- A2  $(R \cap Q) \setminus P = \{2, 12\}$  or candidate's equivalent or  $(P \cap R) \setminus Q = \{5\}$  or candidate's equivalent
- A3 Any relevant step

*Worthless (0)*

- W1  $\{ \}$  but note \*1
- W2 Draws diagram again, with no further work of merit
- W3 Incorrect answer with no work of merit, note A2

**Part (c)****20 (10,5,5) marks****Att (3,2,2)**

An electronics company imports tablet computers from China at a cost of 696 Yuan (元) per tablet.

(i) ✍ Find the cost of each tablet, in euro, if €1 = 8.7 元.

The company must also pay a shipping cost on each tablet imported.

By selling a tablet at €105.40, the company can make a profit of 24%.

(ii) ✍ Find the shipping cost per tablet.

The company imports 1000 tablets from China. It sells 600 of them at €105.40 each (i.e. at a profit of 24%) and the remainder at a profit of 15%.

(iii) ✍ Find the overall profit, in euro, made by the company.

**(c) (i)****10 marks****Att 3****I**

$$\begin{aligned} \text{Cost in euro} &= \frac{696}{8.7} && \mathbf{7m} \\ &= \text{€}80 && \mathbf{10m} \end{aligned}$$

**II**

€1 = 8.7 元 Given

$$\text{€}1 \div 8.7 = \text{元} \quad \mathbf{3m}$$

$$\cdot 114942528 \quad \mathbf{4m}$$

$$\cdot 114942528 \times 696 \quad \mathbf{7m}$$

$$= \text{€}80 \quad \mathbf{10m}$$

\* € symbol not necessary in answer

\* (€ 1 ÷ 8.7) × 696 is worth 4m

*Blunders (-3)*

B1 Correct answer, no work shown ✍

B2 Decimal error

B3 Incorrect numerator

B4 Incorrect denominator

B5 Mathematical error

B6 Inversion

B7 Fails to finish

*Slips (-1)*

S1 Numerical error to a max of -3

S2 Early rounding of decimal if it affects final answer

*Misreadings (-1)*

M1 Misreads a digit provided it doesn't oversimplify the question

*Attempts (3 marks)*

A1 Any relevant step

*Worthless (0)*

W1 Incorrect answer no work shown

W2 No work of merit



(c) (ii)

5 marks

Att 2

**I**

$$124\% = €105.40$$

$$1\% = 105.40 \div 124 = .85$$

$$100\% = .85 \times 100 = €85$$

$$€85 - €80 = €5 \text{ shipping cost}$$

**II**

$$(€80 + x)(1.24) = €105.40$$

$$99.20 + 1.24x = 105.40$$

$$1.24x = 105.40 - 99.20$$

$$1.24x = 6.20$$

$$x = 6.20 \div 1.24$$

$$x = €5$$

**III**

$$\text{Each tablet with profit} = €80 \times 124\% = €99.20$$

$$€105.40 - €99.20$$

$$\text{Shipping charge} = €6.20 \div 1.24 = €5$$

**IV**

$$\text{Profit} + 124\% \text{ shipping charge} = €105.40 - €80 = €25.40$$

$$\text{Profit} = €25.40 - 124\% \text{ shipping charge} \quad \text{Shipping charge} = x$$

$$\text{Profit} = €25.40 - 1.24x$$

$$\% \text{ Profit} = \frac{\text{Profit}}{\text{Cost Price}} \times 100$$

$$24 = \frac{25.40 - 1.24x}{80} \times 100$$

$$24 = (25.40 - 1.24x) \times \frac{100}{80}$$

$$24 \div \frac{100}{80} = 25.40 - 1.24\%$$

$$24 \times \frac{80}{100} = 25.40 - 1.24x$$

$$19.20 = 25.40 - 1.24x$$

$$19.20 - 25.40 = -1.24x$$

$$-6.20 = -1.24x$$

$$124\% \text{ shipping charge} = €6.20$$

$$\text{Shipping charge} = €6.20 \div 1.24$$

$$= €5$$

\* Accept candidate's figure from (c)(i)

*Blunders (-3)*

B1 Correct answer, no work shown ✍

B2 Decimal error

B3 Incorrect operation

B4 Distribution error

B5 Mathematical error

B6 Incorrect value for cost price based on previous figures

B7 Fails to finish

*Slips (-1)*

S1 Numerical errors to a max of -3

*Misreadings (-1)*

M1 Misreads a digit provided it doesn't oversimplify the question

*Attempts (2 marks)*

A1 Shows some knowledge of % profit

A2 Any relevant step

*Worthless (0)*

W1 Incorrect answer no work shown

W2 No work of merit

(c) (iii)

5 marks

Att 2

$600 \times 105.40$	=	63,240
$400 \times 85 \times 1.15$	=	<u>39,100</u>
Total	=	102,340
$85 \times 1000$	=	<u>85,000</u>
Overall profit	=	€17,340

\* Accept candidate's values from (c)(i) and (ii)

\* Candidates may use other variations in calculating the overall profit

*Blunders (-3)*

B1 Correct answer, no work shown ✍

B2 Decimal error

B3 Incorrect operation

B4 Mathematical error

B5 Fails to finish

*Slips (-1)*

S1 Numerical errors to a max of 3

*Misreadings (-1)*

M1 Misreads a digit provided it doesn't oversimplify the question

*Attempts (2 marks)*

A1 Multiplies by 1000

A2 Multiplies by 600

A3 Multiplies by 400

A4 Finds 15% or states  $15\% = \frac{15}{100}$

A5 Any relevant step

*Worthless (0)*

W1 Incorrect answer no work shown

W2 No work of merit

### QUESTION 3

<b>Part (a)</b>	<b>10 marks</b>	<b>Att 3</b>
<b>Part (b)</b>	<b>20 marks</b>	<b>Att (2,3,2)</b>
<b>Part (c)</b>	<b>20 marks</b>	<b>Att (2,2,2,2)</b>

**Part (a)** **10 marks** **Att 3**

Given that 1 billion is a thousand million, find the sum of €3.6 billion and €700 million.  
Give your answer in the form  $a \times 10^n$  where  $n \in \mathbb{N}$  and  $1 \leq a < 10$ .

**(a)** **10 marks** **Att 3**

<b>I</b>	$3.6 \times 10^9 + 700 \times 10^6 = 3.6 \times 10^9 + 0.7 \times 10^9$	<b>4m</b>	$= 4.3 \times 10^9$
<b>II</b>	$€3,600,000,000 + €700,000,000 = €4,300,000,000$	<b>4m</b>	$= 4.3 \times 10^9$
<b>III</b>	$€3.6 \text{ billion} + €0.7 \text{ billion} = €4.3 \text{ billion}$	<b>4m</b>	$= 4.3 \times 10^9$
<b>IV</b>	$3.6 \times 10^9 + 7 \times 10^8$ (or equivalent) $= 3,600,000,000 + 700,000,000$ $= 4,300,000,000$	<b>4m</b>	$= 4.3 \times 10^9$

*Blunders (-3)*

- B1 Correct answer, no work shown
- B2 Decimal error
- B3 Answer not given in correct form
- B4 Index error
- B5 Mathematical error

*Misreadings (-1)*

- M1 Misreads a digit provided it doesn't oversimplify the question

*Attempts (3 marks)*

- A1 Indicates some knowledge of indices e.g. gets  $10^9$
- A2 Converts either expression to a whole number and stops
- A3 Writes 0.7 billion and stops
- A4 Any relevant step

*Worthless (0)*

- W1 Incorrect answer, no work shown
- W2 Attempts to add and shows no knowledge of indices, but note A1
- W3 No work of merit

- (i) ✍ Simplify  $\frac{6x^2 - 17x + 12}{3x - 4}$ .
- (ii) ✍ Factorise  $4c^2 - 3d - 2cd + 6c$ .
- (iii) ✍ Express in its simplest form:  $\frac{5}{x-3} - \frac{3}{x-2}$ .

(b) (i)

5 marks

Att 2

**I**

$$\frac{(3x-4)(2x-3)}{3x-4} = 2x-3$$

**II**

$$3x-4 \overline{) \begin{array}{r} 6x^2 - 17x + 12 \\ \underline{6x^2 - 8x} \phantom{+ 12} \\ -9x + 12 \\ \underline{-9x + 12} \\ 0 \end{array}}$$

**III**

$$\begin{aligned} (6x^2 - 17x + 12) \div (3x - 4) \\ (6x^2 - 9x - 8x + 12) \div (3x - 4) \\ [3x(2x - 3) - 4(2x - 3)] \div (3x - 4) \\ [(3x - 4)(2x - 3)] \div (3x - 4) \\ = 2x - 3 \end{aligned}$$

**IV**

$$\begin{aligned} (6x^2 - 17x + 12) \div (3x - 4) \\ (6x^2 - 8x - 9x + 12) \div (3x - 4) \\ [2x(3x - 4) - 3(3x - 4)] \div (3x - 4) \\ [(2x - 3)(3x - 4)] \div (3x - 4) \\ = 2x - 3 \end{aligned}$$

\*  $(2x - 3)(3x + 4)$  and continues is one blunder - B4. It will also incur B6 or B7.  
All other attempts to factorise apply B2, B3 and/or B4.

*Blunders (-3)*

- B1 Correct answer, no work shown ✍
- B2 Incorrect factors of  $6x^2$  in method I
- B3 Incorrect factors of +12 in method I
- B4 Incorrect factors leading to an incorrect middle term in method I
- B5 Mathematical error
- B6 Incorrect cancellation
- B7 Fails to finish *i.e.* no cancellation in method I

*Slips (-1)*

S1 Numerical errors to a maximum of -3

*Attempts (2 marks)*

A1 Some effort at factorising

A2 Sets up division

A3 Multiplies instead of dividing, with at least one correct term

A4 Finds guide number (72) in methods **III** and **IV** and stops

A5 Quadratic with some correct substitution

A6 Sets up quadratic and identifies  $a$ ,  $b$  or  $c$

A7 Uses quadratic formula and stops at correct roots ( $x = \frac{3}{2}$  and  $x = \frac{4}{3}$ )

A8 Any relevant step

*Worthless (0)*

W1 Incorrect answer, no work shown

W2 ( ) ( )

W3 Work of no merit

**(b) (ii)**

**10 marks**

**Att 3**

<b>I</b>		<b>II</b>	
$4c^2 - 3d - 2cd + 6c$	Given	$4c^2 - 3d - 2cd + 6c$	Given
$4c^2 + 6c - 2cd - 3d$	<b>3m</b>	$4c^2 - 2cd + 6c - 3d$	<b>3m</b>
$2c(2c + 3) - d(2c + 3)$	<b>7m</b>	$2c(2c - d) + 3(2c - d)$	<b>7m</b>
$(2c + 3)(2c - d)$	<b>10m</b>	$(2c - d)(2c + 3)$	<b>10m</b>

\* Accept any of the following for full marks with work (with or without brackets):

$(2c - d)$  and  $(2c + 3)$  [the word 'and' is written down]

$(2c - d)$  or  $(2c + 3)$  [the word 'or' is written down]

$(2c - d)$ ,  $(2c + 3)$  [a comma is used]

*Blunders (-3)*

B1 Correct answer, no work shown ✗

B2 Error in factorising any pair of terms, apply once if consistent.

B3 Incorrect last step e.g.  $2cd(2c - 3)$

B4 Incorrect common factor and continues e.g.  $2c(2c - 3) + d(2c - 3)$

B5 Incorrect common factor and continues e.g.  $2c(2c + 3) - d(2c - 3)$ . B3 or B6 will also apply.

B6 Fails to finish, stops at  $2c(2c + 3) - d(2c + 3)$  or similar

*Slips (-1)*

S1  $(2c - d) + (2c + 3)$

S2  $(2c - d) - (2c + 3)$

*Attempts (3 marks)*

A1 Some effort at factorising e.g. groups or attempts to pair

*Worthless (0)*

W1 Incorrect answer, no work shown

W2 No work of merit

(b) (iii)

5 marks

Att 2

$$\begin{aligned} & \frac{5}{x-3} - \frac{3}{x-2} \\ = & \frac{5(x-2) - 3(x-3)}{(x-3)(x-2)} & \mathbf{2m} \\ = & \frac{5x - 10 - 3x + 9}{(x-3)(x-2)} & \mathbf{2m} \\ = & \frac{2x - 1}{(x-3)(x-2)} & \mathbf{5m} \end{aligned}$$

*Blunders (-3)*

- B1 Correct answer, no work shown ✍
- B2 Incorrect common denominator or mishandles common denominator
- B3 Mishandles numerator
- B4 Distribution error
- B5 Mathematical error
- B6 Fails to combine like terms in final answer
- B7 Reads as  $\frac{5}{x-3} + \frac{3}{x-2}$  and continues

*Slips (-1)*

- S1 Numerical slips to a max of -3

*Attempts (2 marks)*

- A1 Correct common denominator and stops
- A2 No denominator used
- A3 Any relevant step

*Worthless (0)*

- W1 Incorrect answer, no work shown
- W2  $\frac{5}{x-3} - \frac{3}{x-2} = \frac{2}{-1}$  or  $\frac{2}{2x-5}$  or  $\frac{2}{5}$  or  $\frac{2}{-5}$  etc
- W3 No work of merit

**Part (c)****20 (5,5,5,5) marks****Att (2,2,2,2)**

Roisín cycled from Wicklow to Bray, a distance of 30 km. She left Wicklow at 10:30 and arrived in Bray at 12:20, having stopped in Greystones for 20 minutes. Greystones is 22 km from Wicklow.

- (i) Roisín's average speed between Wicklow and Greystones was  $x$  km/h. Write an expression in  $x$  for the time taken for this part of her journey.
- (ii) Her average speed for the second part of her journey, between Greystones and Bray, was 6 km/h slower than her speed between Wicklow and Greystones. Write an expression in  $x$  for the time it took to complete the second part of her journey.
- (iii) Write an equation in  $x$  to represent the above information.
- (iv) ✍ Solve the equation to find Roisín's speed for each part of the journey.

**(c) (i)****5 marks****Att 2**

$$\text{Time (1)} = \frac{22}{x}$$

\* Accept correct answer for full marks. No work required, no ✍

*Blunders (-3)*

B1 Inversion  $\frac{x}{22}$

*Attempts (2 marks)*

A1 Writes 22 and/or  $x$

A2 Speed =  $\frac{\text{Distance}}{\text{Time}}$

*Worthless (0)*

W1 Incorrect answer, no work shown

**(c) (ii)****5 marks****Att 2**

$$\text{Time (2)} = \frac{8}{x-6}$$

\* Accept correct answer for full marks. No work required, no ✍

*Blunders (-3)*

B1 Inversion  $\frac{x-6}{8}$

B2 Uses  $x + 6$

B3 Incorrect operation

*Attempts (2 marks)*

A1 Any combination of two of the following  $x$ , 6, 8

A2 Speed =  $\frac{\text{Distance}}{\text{Time}}$

A3 30 – 22 or 8

A4 Any relevant step

*Worthless (0)*

W1 Incorrect answer, no work shown; note A3



**(c) (iii)**

**5 marks**

**Att 2**

12:20 – 20 minutes – 10:30 = 1 hour 30 minutes	<b>2m</b>
= 1.5 hours	<b>2m</b>
Total time $\frac{22}{x} + \frac{8}{x-6} = 1.5$	<b>5m</b>

- \* Accept candidates' expressions from **(c)(i)** and **(c)(ii)**
- \* Accept correct answer for full marks. No work required, no ✍
- \* If no work, or no work of merit, at parts (i) and/or (ii) but states above, award *Att 2* and *Att 2* from parts **(c)(i)**, and/or **(c)(ii)** here

*Blunders (-3)*

B1 Sign error in setting up equation *e.g.* has  $\frac{22}{x} - \frac{8}{x-6} = \frac{3}{2}$

B2 Expression not equal to 1.5 or  $\frac{3}{2}$ , but note S1

B3 Uses 1.3

*Slips (-1)*

S1  $\frac{22}{x} + \frac{8}{x-6} = 90$

*Attempts (2 marks)*

A1 Incorrect expression but uses data from **(c) (i)** and **(c) (ii)**

A2 Constructs an equation or expression using at least two of the following:  
 $\frac{3}{2}$ , answer **(c) (i)**, answer **(c) (ii)**

A3 Attempt to subtract times

A4 Any relevant step

*Worthless (0)*

W1 1.3 only

W2 No work of merit

(c) (iv)

5 marks

Att 2

<p><b>I</b></p> $\frac{22}{x} + \frac{8}{x-6} = 1.5$ $22x - 132 + 8x = 1 \cdot 5x^2 - 9x$ $1.5x^2 - 39x + 132 = 0$ $x^2 - 26x + 88 = 0$ $(x - 4)(x - 22) = 0$ $x = 4 \quad x = 22$	<p><b>II</b></p> $\frac{22}{x} + \frac{8}{x-6} = \frac{3}{2}$ $\frac{22(2)(x-6) + 8(2)(x) = 3x(x-6)}{x(x-6)(2)}$ $44x - 264 + 16x = 3x^2 - 18x$ $3x^2 - 78x + 264 = 0$ $x^2 - 26x + 88 = 0$ $(x - 4)(x - 22) = 0$ $x = 4 \quad x = 22$	<p><b>2m</b></p>
<p><math>x = 22 \rightarrow \text{speed (1)} = 22 \rightarrow \text{speed (2)} \quad x - 6 = 22 - 6 = 16</math> but <math>x = 4 \rightarrow \text{speed (1)} = 4 \rightarrow \text{speed (2)} \quad x - 6 = -2</math> not possible <b>5m</b></p>		

\* Accept candidate's equation from (c) (iii)

*Blunders (-3)*

- B1 Correct answer, no work shown ✍
- B2 Sign error
- B3 Distribution error
- B4 Transposition error
- B5 Mathematical error
- B6 Correct factors and stops
- B7 Incorrect factors
- B8 Errors using quadratic formula

*Slips (-1)*

- S1 Numerical errors to a max of -3
- S2 Does not (or cannot) conclude that speed of -2 is not possible
- S3 Doesn't find speeds between Greystones and Bray for second part of the journey

*Attempts (2 marks)*

- A1 Linear equation merits attempt at most
- A2 Any correct relevant step
- A3 Quadratic formula with some correct substitution
- A4 Attempt at factorising

*Worthless (0)*

- W1 Incorrect answer and no work shown
- W2 ( ) ( )
- W3 No work of merit

## QUESTION 4

<b>Part (a)</b>	<b>10 marks</b>	<b>Att 3</b>
<b>Part (b)</b>	<b>20 marks</b>	<b>Att (2,2,3)</b>
<b>Part (c)</b>	<b>20 marks</b>	<b>Att (3,2,2)</b>

**Part (a)** **10 marks** **Att 3**

Graph on the number line the solution set of

$$4 - x \geq 2x - 5, x \in \mathbb{N}.$$

**(a)** **10 marks** **Att 3**

**I**

$$4 - x \geq 2x - 5$$

$$4 + 5 \geq 2x + x$$

$$9 \geq 3x$$

$$9 \div 3 \geq x$$

$$3 \geq x$$

**7m**

**II**

$$4 - x \geq 2x - 5$$

$$-x - 2x \geq -5 - 4$$

$$-3x \geq -9$$

$$3x \leq 9$$

$$x \leq 9 \div 3$$

$$x \leq 3$$

**7m**

$$x \leq 3 \rightarrow \{1,2,3\}$$

**7m**



*Blunders (-3)*

- B1 Correct answer no work shown
- B2 Transposition error
- B3 Mishandles inequality
- B4  $x \in \mathbb{R}$  or  $x \in \mathbb{Z}$  indicated
- B5 Mathematical error
- B6 No number line drawn
- B7 Values outside of range graphed, note S2

*Slips (-1)*

- S1 Numerical errors to a maximum of -3
- S2 Includes 0

*Misreadings (-1)*

- M1 Excludes equals in inequality

Attempts (3 marks)

- A1 Tests any value in the inequality and stops
- A2 Draws a number-line
- A3 No inequality, solves equation to get  $x = 3$
- A4 Any relevant step

Worthless (0)

- W1 List given with no correct value
- W2 No work of merit

**Part (b)**

**20 (5,5,10) marks**

**Att (2,2,3)**

Electricity is charged to a consumer at a day rate and at a night rate.  
Day rate units are charged at 14 cent per unit  
and night rate units are charged at 7 cent per unit.  
A consumer uses a total of 1100 units for a billing period, at a cost of €129.50.

(i) By letting  $x$  equal the number of day rate units used and  $y$  equal the number of night rate units used, write two equations to represent the above information.

(ii) ✍ Solve these equations to find the number of each type of unit used.

**(b) (i)**

**10 (5,5)marks**

**Att 2,2**

$x + y = 1100$	<b>5m</b>
$0.14x + 0.07y = 129.50$ or $14x + 7y = 12950$	<b>5m</b>

- \* Two equations to mark in (b)(i)
- \* Each equation is marked separately
- \* Each equation is worth 5marks, attempt 2
- \* Equations sufficient, no ✍ in question

Blunders (-3)

- B1 Incorrect term
- B2 Decimal error

Attempts (2,2 marks)

- A1 Mentions  $x$  or  $y$  or  $14x$  or  $7y$  or  $\cdot 14x$  or  $\cdot 07y$
- A2 Effort at creating an equation equal to 1100 or 129.50 or 12950
- A3 Any relevant step

(b) (ii)

10 marks

Att 3

I

$$\begin{array}{r} x + y = 1100 \quad (-7) \\ 14x + 7y = 12950 \end{array}$$

$$\begin{array}{r} -7x - 7y = -7700 \\ 14x + 7y = 12950 \end{array}$$

$$7x = 5250$$

$$x = \frac{5250}{7}$$

$$x = 750$$

$$x + y = 1100$$

$$750 + y = 1100$$

$$y = 1100 - 750$$

$$y = 350$$

$$\begin{array}{r} x + y = 1100 \quad (-14) \\ 14x + 7y = 12950 \end{array}$$

$$\begin{array}{r} -14x - 14y = -15400 \\ 14x + 7y = 12950 \end{array}$$

$$-7y = -2450$$

$$y = \frac{-2450}{-7}$$

$$y = 350$$

$$x + y = 1100$$

$$x + 350 = 1100$$

$$x = 1100 - 350$$

$$x = 750$$

$$x = 750 \quad y = 350$$

$$\begin{array}{r} x + y = 1100 \quad (-1) \\ 14x + 7y = 12950 \quad (\div 7) \end{array}$$

$$-x - y = -1100$$

$$2x + y = 1850$$

$$x = 750$$

$$x + y = 1100$$

$$750 + y = 1100$$

$$y = 1100 - 750$$

$$y = 350$$

II

$$x + y = 1100$$

$$y = 1100 - x$$

$$14x + 7(1100 - x) = 12950$$

$$14x + 7700 - 7x = 12950$$

$$7x = 12950 - 7700$$

$$7x = 5250$$

$$x = 750$$

$$y = 350$$

$$x + y = 1100$$

$$x = 1100 - y$$

$$14(1100 - y) + 7y = 12950$$

$$15400 - 14y + 7y = 12950$$

$$-7y = 12950 - 15400$$

$$-7y = -2450$$

$$7y = 2450$$

$$y = 350$$

$$x = 750$$

- \*1 Accept candidate's answers from part (i) provided oversimplification does not occur
- \*2 Apply only one blunder deduction B1 or B2 to any errors in establishing the first equation
- \*3 Finding the second variable is subject to a maximum deduction of 3 marks
- \*4 Correct values of  $x$  and  $y$  without algebraic work, **both verified in both equations** merits full marks
- \*5 Correct values of  $x$  and  $y$  without algebraic **work not verified or not fully verified in both equations** merits attempt mark only
- \*6 Equations may also be solved by substituting  $x = \frac{12950 - 7y}{14}$  or  $y = \frac{12950 - 14x}{7}$

*Blunders (-3)*

B1 Error(s) in establishing the first equation in terms of  $x$  i.e. ( $7x = 5250$ ) through elimination by cancellation or elimination by substitution

B2 Error(s) in establishing the first equation in terms of  $y$  i.e. ( $7y = 2450$ ) through elimination by cancellation or elimination by substitution

B3 Distribution error

B4 Transposition error

B5 Mathematical error

B6 Fails to find second variable

*Slips (-1)*

S1 Numerical errors to a max of 3

*Misreadings (-1)*

M1 Misreads a digit provided it doesn't oversimplify, apply each time to a max of 3

*Attempts (3 marks)*

A1 Any correct manipulation of either equation and stops

*Worthless (0)*

W1 Incorrect answer, no work shown

W2 Trial and error, but note \*4 and \*5

W3 No work of merit

Part (c)

20 (10,5,5) marks

Att (3,2,2)

- (i) ✍ Solve the equation  $x^2 - 6x + 4 = 0$ ,  
giving your answer in the form of  $a \pm \sqrt{b}$ , where  $a, b \in \mathbb{N}$ .
- (ii) ✍ Hence, or otherwise, find two values for  $p$  for which  
 $(3 + p)^2 - 6(3 + p) + 4 = 0$ .
- (iii) ✍ Show that the sum of the two values of  $p$  is zero.

(c) (i)

10 marks

Att 3

**I**

$$x^2 - 6x + 4 = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$a = 1 \quad b = -6 \quad c = 4$$
$$\frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(4)}}{2(1)}$$
$$= \frac{6 \pm \sqrt{36 - 16}}{2}$$
$$= \frac{6 \pm \sqrt{20}}{2} = \frac{6 \pm 2\sqrt{5}}{2} = 3 \pm \sqrt{5}$$

**II**

$$x^2 - 6x + 4 = 0$$
$$x^2 - 6x + 9 - 5 = 0$$
$$(x - 3)^2 - (\sqrt{5})^2 = 0$$
$$(x - 3 - \sqrt{5})(x - 3 + \sqrt{5}) = 0$$
$$x - 3 - \sqrt{5} = 0 \quad \text{or} \quad x - 3 + \sqrt{5} = 0$$
$$x = 3 + \sqrt{5} \quad \text{or} \quad x = 3 - \sqrt{5}$$

- B1 Correct answer no work shown ✍
- B2 Error in quadratic formula once only
- B3 Error in substitution once only
- B4 Error when applying quadratic formula once only
- B5 Invalid cancelling or stops at  $\frac{6 \pm \sqrt{20}}{2}$  or similar
- B6 Error in completing the square in method **II**
- B7 Error in factors in method **II**
- B8 Error in establishing roots or incorrect format for roots in method **II**

Slips (-1)

- S1 Each numerical error to a max of -3

Attempts (3 marks)

- A1 Identifies  $a$ ,  $b$  or  $c$  correctly and stops
- A2 Some attempt at factorising e.g.  $(x \quad)(x \quad)$

Worthless (0)

- W1 Incorrect answer without work
- W2 No work of merit

(c) (ii)

5 marks

Att 2

**I**

$$(3 + p)^2 - 6(3 + p) + 4 = 0 \quad \text{Given}$$

$$\text{From c (i) } x = 3 + p \quad \text{2m}$$

$$\text{So } 3 + p = 3 \pm \sqrt{5}$$

$$p = \pm \sqrt{5} \quad \text{5m}$$

**II**

$$(3 + p)^2 - 6(3 + p) + 4 = 0 \quad \text{Given}$$

$$9 + 6p + p^2 - 18 - 6p + 4 = 0 \quad \text{2m}$$

$$p^2 - 5 = 0$$

$$(p - \sqrt{5})(p + \sqrt{5}) = 0$$

$$p = \sqrt{5} \quad \text{and} \quad p = -\sqrt{5} \quad \text{5m}$$

\*Accept candidate's answers from part (c)(i)

*Blunders (-3)*

- B1 Correct answer no work shown ✍
- B2 Sign error
- B3 Transposition error
- B4 Mathematical error
- B5 Distribution error
- B6 Finds one solution only

*Slips (-1)*

- S1 Numerical errors to a max of -3

*Attempts (2 marks)*

- A1 States  $x = 3 + p$  and stops
- A2 Some use of answer from part (i)
- A3 Some correct multiplication in **II**
- A4 Any relevant step

*Worthless (0)*

- W1 Incorrect answer without work
- W2 No work of merit



**(c) (iii)**

**5 marks**

**Att 2**

Sum of roots	=	$\sqrt{5} + (-\sqrt{5})$
=		0

- \* Accept candidate's answers from part **(c)(ii)** above
- \* If candidate's  $p_1 + p_2 \neq 0$  and candidate acknowledges this with work, award full marks

*Blunders (-3)*

- B1 States  $\sqrt{5} - \sqrt{5} \neq 0$
- B2 Incorrectly states that candidate's  $p_1 + p_2 = 0$
- B3 Decimal error
- B4 Fails to finish

*Slips (-1)*

- S1 Numerical errors to a max of -3

*Attempts (2 marks)*

- A1 Some use of candidate's answers from part **(ii)**
- A2 Some relevant step

*Worthless (0)*

- W1 0 only or = 0 only
- W2 No work of merit

## QUESTION 5

<b>Part (a)</b>	<b>10 marks</b>	<b>Att 3</b>
<b>Part (b)</b>	<b>10 marks</b>	<b>Att (2,2)</b>
<b>Part (c)</b>	<b>30 marks</b>	<b>Att (3,3,2,2)</b>
<b>Part (a)</b>	<b>10 marks</b>	<b>Att 3</b>

✎ Given that  $4d = \frac{2c}{3} + \frac{a}{5}$ , write  $a$  in terms of  $c$  and  $d$ .

**(a)** **10 marks** **Att 3**

**I**

$$4d = \frac{2c}{3} + \frac{a}{5} \quad \text{Given}$$

$$4d - \frac{2c}{3} = \frac{a}{5} \quad \text{7m}$$

$$5\left(4d - \frac{2c}{3}\right) = a \quad \text{or} \quad a = \frac{60d-10c}{3} \quad \text{or} \quad a = 20d - \frac{10c}{3} \quad \text{10m}$$

**II**

$$4d = \frac{2c}{3} + \frac{a}{5} \quad \text{Given}$$

$$LCM = 15 \quad \text{3m}$$

$$15(4d) = 15\left(\frac{2c}{3}\right) + 15\left(\frac{a}{5}\right) \quad \text{3m}$$

$$60d = 10c + 3a \quad \text{4m}$$

$$60d - 10c = 3a \quad \text{7m}$$

$$a = \frac{60d-10c}{3} \quad \text{or} \quad a = 20d - \frac{10c}{3} \quad \text{or} \quad a = 5\left(4d - \frac{2c}{3}\right) \quad \text{10m}$$

\* Other methods may be used

*Blunders (-3)*

- B1 Correct answer no work shown ✎
- B2 Mishandles numerator
- B3 Incorrect LCM in **II** (any multiple of 15 acceptable)
- B4 Transposition error
- B5 Mathematical error
- B6 Fails to finish

*Slips (-1)*

- S1 Numerical errors to a max of -3

*Misreadings (-1)*

- M1 Misreads a digit provided it doesn't oversimplify the question

*Attempts (3 marks)*

- A1 Correct LCM only
- A2 Any relevant step

*Worthless (0)*

- W1 Incorrect answer, no work shown
- W2 No work of merit

Part (b)

10 (5,5)marks

Att (2,2)

(i) ✎ Find the value of  $3x^2 - 5x + \frac{4}{x}$ , when  $x = \frac{2}{3}$ .

(ii) ✎ Solve the equation  $\frac{x-1}{3} - \frac{5x+2}{4} = 1$ .

(b) (i)

5 marks

Att 2

I

$$3\left(\frac{2}{3}\right)^2 - 5\left(\frac{2}{3}\right) + \frac{4}{\left(\frac{2}{3}\right)} = \quad \mathbf{2m}$$

$$\frac{4}{3} - \frac{10}{3} + \frac{12}{2}$$
$$\frac{-6}{3} + 6$$
$$-2 + 6$$
$$4$$

5m

OR

II

$$\frac{x(3x^2) - x(5x) + 4}{3x^3 - 5x^2 + 4}$$

$$3x^3 - 5x^2 + 4$$

$$3\left(\frac{2}{3}\right)^3 - 5\left(\frac{2}{3}\right)^2 + 4$$

2m

$$\frac{8}{9} - \frac{20}{9} + 4$$

$$\frac{-12}{9} + 4$$

$$\frac{-12}{9} + \frac{36}{9}$$

$$\frac{24}{9}$$

$$\frac{24}{9} \times \frac{3}{2}$$

$$= 4$$

5m

*Blunders (-3)*

- B1 Correct answer no work shown ✍
- B2 Incorrect substitution *e.g.*  $x = \cdot 6$ , gives 4.74, but see S2
- B3 Incorrect handling of fractions
- B4 Drops denominator or mishandles denominator
- B5 Mathematical error
- B6 Mishandles numerator
- B7 Distribution error
- B8 Fails to finish

*Slips (-1)*

- S1 Numerical errors to a max of -3
- S2 If decimal is used and answer can be rounded to 4 correctly, but is not; otherwise B2

*Misreadings (-1)*

- M1 Misreads a digit provided it doesn't oversimplify the question

*Attempts (2 marks)*

- A1 Some correct substitution
- A2 No denominator used **II**
- A3 Any relevant step

*Worthless (0)*

- W1 Incorrect answer, no work shown
- W2 No work of merit

(b) (ii)

5 marks

Att 2

**I**

$$\frac{(x-1)}{3} - \frac{(5x+2)}{4} = 1$$

Given

$$4(x-1) - 3(5x+2) = 12 \quad (1)$$

2m

$$4x - 4 - 15x - 6 = 12$$

$$-11x - 10 = 12$$

$$-11x = 12 + 10$$

$$-11x = 22$$

$$x = -2$$

5m

**II**

$$\frac{4(x-1)}{12} - \frac{3(5x+2)}{12} = \frac{1}{1}$$

2m

$$\frac{4x - 4 - 15x - 6}{12} = 1$$

$$\frac{-11x - 10}{12} = 1$$

$$-11x - 10 = 12 \quad (1)$$

$$-11x = 12 + 10$$

$$-11x = 22$$

$$x = -2$$

5m

\* Other methods may be used

\*  $x = -2$  **verified** is worth 5 marks

*Blunders (-3)*

- B1 Correct answer no work shown ✗
- B2 Sign error
- B3 Incorrect denominator
- B4 Mishandles numerator
- B5 Mathematical error
- B6 Transposition errors
- B7 Drops denominator or mishandles denominator
- B8 Fails to finish

*Slips (-1)*

- S1 Numerical error to a max of 3

*Attempts (2 marks)*

- A1 Correct denominator only
- A2 No denominator, oversimplified
- A3 Some relevant step

*Worthless (0)*


- W1 Incorrect answer no work shown
- W2 No work of merit

Part (c)

30 (10,10,5,5) marks

Att (3,3,2,2)

Let  $f$  be the function  $f: x \rightarrow 10 - x - 2x^2$ .

- (i)  Draw the graph of  $f$  for  $-3 \leq x \leq 3$ ,  $x \in \mathbb{R}$ .
- (ii) Use your graph to estimate the maximum value of  $f(x)$ .
- (iii) Use your graph to estimate the values of  $x$  for which  $f(x) = 6$ .

(c) (i) Function  $f$

20(10,10) marks

Att (3,3)

I

$x$	-3	-2	-1	0	1	2	3
$f(x)$	-5	4	9	10	7	0	-11

II

$$f(x) = 10 - x - 2x^2$$

$$f(-3) = 10 - (-3) - 2(-3)^2 = 10 + 3 - 2(9) = 10 + 3 - 18 = -5 \rightarrow (-3, -5)$$

$$f(-2) = 10 - (-2) - 2(-2)^2 = 10 + 2 - 2(4) = 10 + 2 - 8 = 4 \rightarrow (-2, 4)$$

$$f(-1) = 10 - (-1) - 2(-1)^2 = 10 + 1 - 2(1) = 10 + 1 - 2 = 9 \rightarrow (-1, 9)$$

$$f(0) = 10 - (0) - 2(0)^2 = 10 + 0 - 2(0) = 10 + 0 - 0 = 10 \rightarrow (0, 10)$$

$$f(1) = 10 - (1) - 2(1)^2 = 10 - 1 - 2(1) = 10 - 1 - 2 = 7 \rightarrow (1, 7)$$

$$f(2) = 10 - (2) - 2(2)^2 = 10 - 2 - 2(4) = 10 - 2 - 8 = 0 \rightarrow (2, 0)$$

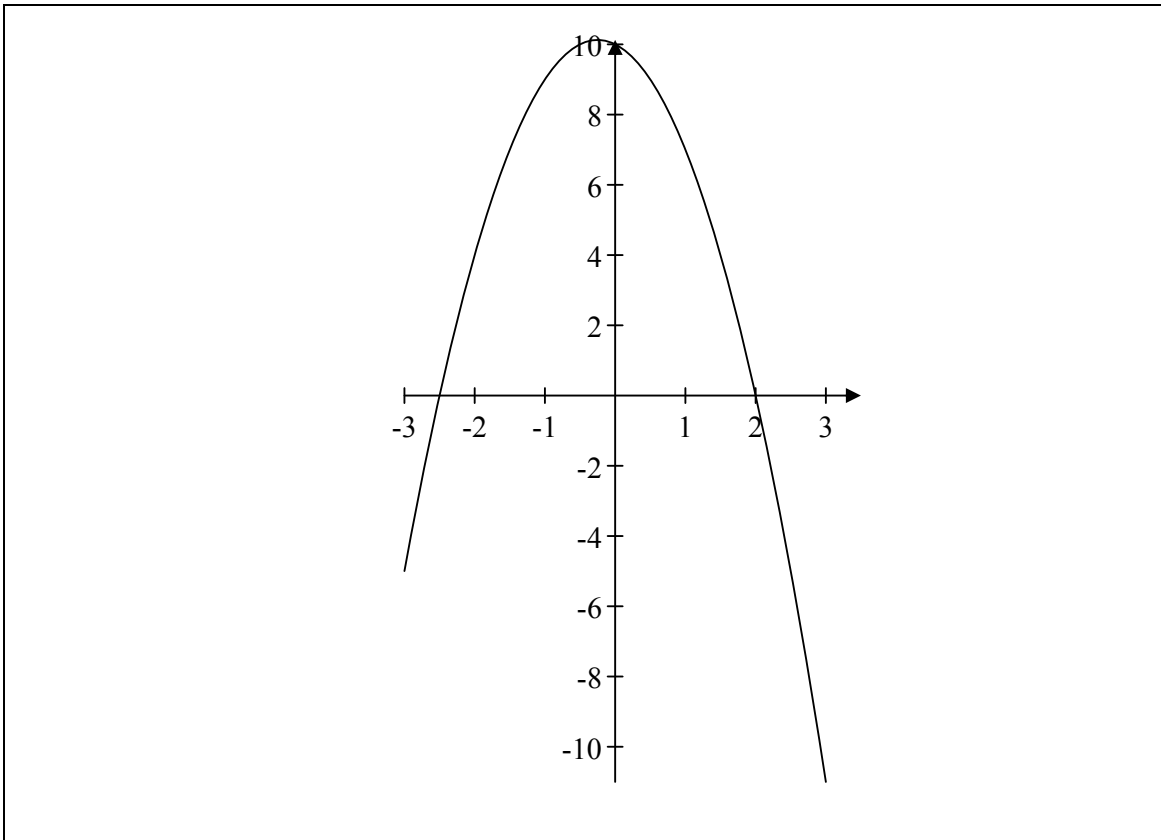
$$f(3) = 10 - (3) - 2(3)^2 = 10 - 3 - 2(9) = 10 - 3 - 18 = -11 \rightarrow (3, -11)$$

III

$x$	-3	-2	-1	0	1	2	3	
10	10	10	10	10	10	10	10	
$-x$	3	2	1	0	-1	-2	-3	
$-2x^2$	-18	-8	-2	0	-2	-8	-18	
$f(x)$	-5	4	9	10	7	0	-11	
Points	$(-3, -5)$	$(-2, 4)$	$(-1, 9)$	$(0, 10)$	$(1, 7)$	$(2, 0)$	$(3, -11)$	

- \* Table is worth 10 marks, graph is worth 10 marks
- \* Middle lines of table do not have to be shown
- \* Consistent error(s) in each row/column attract a maximum deduction of 3
- \* Points might not be listed, mark on position on graph
- \* Graph constitutes work in this question
- \* Candidates may choose not to use a table
- \* If graph is correct award 20 marks

Points	$(-3,-5)$	$(-2,4)$	$(-1,9)$	$(0,10)$	$(1,7)$	$(2,0)$	$(3,-11)$
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\* Accept candidate's values from table when plotting graph.

*Blunders (-3)*

- B1 Error in calculating  $-2x^2$ , once if consistent, note A2
- B2 Error in calculating  $-x$ , once if consistent
- B3 Adds in domain row when evaluating  $f(x)$
- B4 Each incorrect point without work
- B5 Each point plotted incorrectly, once if consistent
- B6 Each missing point
- B7 Axes reversed
- B8 Scale error, apply once
- B9 Points not joined to form curve or joined in incorrect order, apply once
- B10 Graph not extended to include full domain

*Slips (-1)*

- S1 Numerical errors to a max of -3

*Misreadings (-1)*

- M1 Incorrect digit provided it does not oversimplify the question

*Attempts (3,3 marks)*

- A1 Draws axes with some indication of scaling
- A2 Errors leading to a linear graph
- A3 Some correct substitution
- A4 Some relevant step

(c) (ii)

5 marks

Att 2

Max at  $f(x)$  or  $y = 10.1$

- \* Accept answer consistent with candidate's graph
- \* Tolerance =  $\pm 0.2$  of candidate's graph

*Blunders (-3)*

- B1 Maximum indicated on graph but no value stated
- B2 States or indicates  $x$  co-ordinate of maximum point

*Slips (-1)*

- S1 Writes maximum point instead of maximum value

*Attempts (2 marks)*

- A1 Some relevant work

(c) (iii)

5 marks

Att 2

$f(x) = 6 \rightarrow x = 1.2$  or  $-1.7$ .

- \* Accept answer consistent with candidate's graph
- \* Tolerance =  $\pm 0.2$  of candidate's graph

*Blunders (-3)*

- B1 One value only if two available, see \*1

*Misreadings (-1)*

- M1 Misreads a digit providing it does not oversimplify the question
- M2 Solves  $f(x) = -6$

*Attempts (2 marks)*

- A1 Some indication on graph at  $y = 6$
- A2 States  $y = 6$

*Worthless (0)*

- W1 Incorrect answer(s), no work shown
- W2 No work of merit



## QUESTION 6

<b>Part (a)</b>	<b>10 marks</b>	<b>Att 3</b>
<b>Part (b)</b>	<b>20 marks</b>	<b>Att (2,2,2,2)</b>
<b>Part (c)</b>	<b>20 marks</b>	<b>Att (3,2,2)</b>

**Part (a)** **10 marks** **Att 3**

Let  $g$  be the function  $g : x \rightarrow 2^{x-3}$ .

Find the value of  $g(3)$ .

**(a)** **10 marks** **Att 3**

<b>I</b>		
$g(3)$	$= 2^{3-3}$	<b>4m</b>
	$= 2^0$	<b>7m</b>
	$= 1$	<b>10m</b>
<b>II</b>		
$g : x \rightarrow 2^{x-3}$		
	$= \frac{2^x}{2^3}$	<b>3m</b>
	$= \frac{2^3}{2^3}$	<b>4m</b>
	$= \frac{8}{8}$	<b>7m</b>
	$= 1$	<b>10m</b>

*Blunders (-3)*

- B1 Correct answer no work shown
- B2 Mishandles  $2^0$
- B3 Mishandles indices, once if consistent
- B4  $2^{3-3} = 2^3 - 2^3$  and continues correctly
- B5 Fails to finish e.g. stops at  $2^0$

*Attempts (3 marks)*

- A1  $x = 3$  and stops
- A2 8

*Worthless (0)*

- W1 Incorrect answer no work shown, note A2
- W2  $2 \times 3 = 6$
- W3 No work of merit

**Part (b)**

**20 (5,5,5,5) marks**

**Att (2,2,2,2)**

Let  $f$  be the function  $f: x \rightarrow x^2 - 3x$ .

- (i) ✍ Express  $f(t)$  and  $f(2t+1)$  in terms of  $t$ .  
(ii) ✍ Hence, find the values of  $t$  for which  $f(t) = f(2t+1)$ .

**(b)(i)**

**10(5,5) marks**

**Att (2,2)**

$f: x \rightarrow x^2 - 3x$  Given

$$f(t) = t^2 - 3t \quad \mathbf{5m}$$

$$f(2t+1) = (2t+1)^2 - 3(2t+1) \quad \mathbf{5m}$$

- \* 2 parts to mark  $f(t)$  and  $f(2t+1)$  **5m** each
- \* Accept  $t^2 - 3t$  for 5 marks
- \* Accept  $(2t+1)^2 - 3(2t+1)$  for 5 marks
- \* Ignore notation if consistent

*Blunders (-3)*

- B1 Substitution error  
B2  $(2t+1)^2 + 3(2t+1)$

*Misreadings (-1)*

- M1 Misreads a digit providing it does not oversimplify the question

*Attempts (2,2 marks)*

- A1 Some correct substitution  
A2 States  $x = t$   
A3 States  $x = 2t + 1$   
A4 Any relevant step

*Worthless (0)*

- W1 No work of merit

(b) (ii)

(5,5) marks

Att (2,2)

$$t^2 - 3t = 4t^2 + 4t + 1 - 6t - 3 \quad \mathbf{2m}$$

$$t^2 - 3t = 4t^2 - 2t - 2$$

$$4t^2 - 2t - 2 - (t^2 - 3t) = 0$$

$$4t^2 - 2t - 2 - t^2 + 3t = 0$$

$$3t^2 + t - 2 = 0 \quad \mathbf{5m}$$

and

**I**

$$(t+1)(3t-2) = 0$$

$$\rightarrow t = -1, t = \frac{2}{3} \quad \mathbf{5m}$$

or

**II**

$$3t^2 + t - 2 = 0$$

$$3t^2 + 3t - 2t - 2 = 0$$

$$3t(t+1) - 2(t+1) = 0$$

$$(t+1)(3t-2) = 0$$

$$\rightarrow t = -1, t = \frac{2}{3} \quad \mathbf{5m}$$

or

**III**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad a = 3, b = 1, c = -2$$

$$x = \frac{-1 \pm \sqrt{1^2 - 4(3)(-2)}}{2(3)}$$

$$x = \frac{-1 \pm \sqrt{1^2 - (-24)}}{6}$$

$$x = \frac{-1 \pm \sqrt{25}}{6}$$

$$x = \frac{-1 \pm 5}{6}$$

$$x = \frac{-6}{6}$$

$$x = \frac{4}{6}$$

$$\rightarrow t = -1, t = \frac{2}{3} \quad \mathbf{5m}$$

\* Mark in 2 parts, **5m** for establishing an equation and **5m** for solving

\* Accept candidate's answers from (b)(i), but note A1

*Blunders (-3)*

- B1 Correct answer no work shown ✍
- B2 Sign error
- B3 Transposition error
- B4 Squaring error
- B5 Mathematical error
- B6 Substitution error
- B7 Error in quadratic formula
- B8 Distribution error
- B9  $(2t^2 + 1) + 3(2t + 1)$  and continues correctly, if not already penalised in **(b)(i)**
- B10 Error in solving
- B11 Finds one solution only
- B12 Fails to finish

*Slips (-1)*

- S1 Numerical error to a max of -3

*Misreadings (-1)*

- M1 Misreads a digit providing it does not oversimplify the question

*Attempts (2,2 marks)*

- A1 Linear equation merits *Att 2*, *Att 2* at most
- A2 Attempt to form equation
- A3 Attempt to solve
- A4 Some use of answer(s) from **(b)(i)**
- A5 Any relevant step

*Worthless (0)*

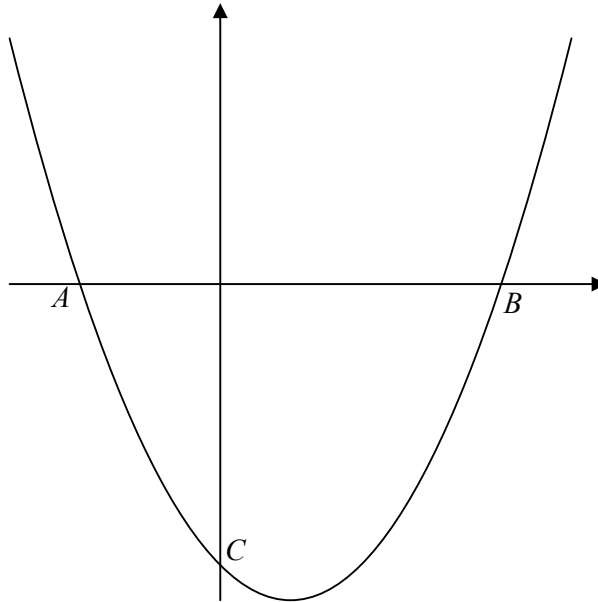
- W1 Incorrect answer(s), no work shown
- W2  $t = 2t + 1$  and stops
- W3 No work of merit

Part (c)

20 (10,5,5) marks

Att (3,2,2)

The diagram below shows part of the graphs of the function  $f : x \rightarrow x^2 - 2x - 8, x \in \mathbb{R}$ .



(i) The graph intersects the  $x$  axis at  $A$  and  $B$  and the  $y$  axis at  $C$ .

 Find the co-ordinates of  $A$ ,  $B$  and  $C$ .

(ii) Hence, write down the range of values of  $x$  for which  $x^2 - 2x - 8 \leq 0$ .

(c) (i)

15(10,5) marks

Att (3,2)

Intersects  $x$  axis:

<b>I</b>		
$x^2 - 2x - 8 = 0$		<b>3m</b>
$(x - 4)(x + 2) = 0$		<b>7m</b>
$x = 4, x = -2$		<b>9m</b>

→  $A(-2, 0) \quad B(4, 0) \quad \mathbf{10m}$

<b>II</b>		
$x^2 - 2x - 8 = 0$		<b>3m</b>
$x^2 - 4x + 2x - 8 = 0$		<b>3m</b>
$x(x - 4) + 2(x - 4) = 0$		<b>4m</b>
$(x - 4)(x + 2) = 0$		<b>7m</b>
$x = 4, x = -2$		<b>9m</b>

→  $A(-2, 0) \quad B(4, 0) \quad \mathbf{10m}$

<b>III</b>		
$x^2 - 2x - 8 = 0$		<b>3m</b>
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$		
$x = \frac{+2 \pm \sqrt{4 - 4(1)(-8)}}{2}$		<b>3m</b>
$x = \frac{+2 \pm \sqrt{4 + 32}}{2}$		<b>4m</b>
$x = \frac{+2 \pm 6}{2}$		<b>7m</b>
$x = 4, x = -2$		<b>9m</b>
$\rightarrow A(-2, 0) \quad B(4, 0)$		<b>10m</b>
<b><u>Intersects y axis:</u></b>		
$f(0) = (0)^2 - 2(0) - 8$		<b>2m</b>
$f(0) = -8$		<b>4m</b>
$\rightarrow C(0, -8)$		<b>5m</b>

- \* Mark in two separate parts. **10m** for finding where graph intersects x axis, **5m** for where graph intersects y axis
- \* Correct answer fully verified is full marks

*Blunders (-3)*

- B1 Correct answer no work shown ✗
- B2 Sign error
- B3 Incorrect operation
- B4 Incorrect factors
- B5 Mathematical error
- B6 Error in quadratic
- B7 Error in substitution
- B8 Transposition error
- B9 Square root error
- B10 Finds one solution only ( i.e.  $x = 4$  **or**  $x = -2$  )

*Slips (-1)*

- S1 Does not write co-ordinate
- S2 Does not label A or B, or labels them incorrectly. Apply once.

*Misreadings (-1)*

- M1 Misreads a digit providing it does not oversimplify the question

*Attempts (3,2 marks)*

- A1 Effort to solve equation ( $= 0$ ) or any indication of  $y = 0$
- A2 Effort to substitute ( $x = 0$ )
- A3 Quadratic with some correct substitution
- A4  $x = 4$  and/or  $x = -2$  only with no work
- A5 Some relevant step

*Worthless (0)*

- W1 Incorrect answer(s), no work shown
- W2 No work of merit

**(c) (ii)**

**5 marks**

**Att 2**

Range	$-2 \leq x \leq 4$
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- \* Accept candidate's  $x$  values from **(c)(i)**
- \* Accept "from  $-2$  to  $4$  inclusive" or similar for full marks

*Blunders (-3)*

- B1 Reversed inequalities
- B2  $2 \leq x \leq 4$ , minus sign omitted

*Slips (-1)*

- S1 Does not include equals in inequalities

*Attempts (2 marks)*

- A1 Some identification on graph
- A2  $f(x) \leq 0$
- A3 Some relevant step

*Worthless (0)*

- W1 Incorrect answer(s) with no work of merit
- W2 No work of merit



**Coimisiún na Scrúduithe Stáit**  
*State Examinations Commission*

# **JUNIOR CERTIFICATE EXAMINATION**

**2012**

**MARKING SCHEME**

**MATHEMATICS  
HIGHER LEVEL  
PAPER 2**



**MARKING SCHEME**  
**JUNIOR CERTIFICATE EXAMINATION 2012**  
**MATHEMATICS - HIGHER LEVEL - PAPER 2**

GENERAL GUIDELINES FOR EXAMINERS

1. Penalties of three types are applied to candidates' work as follows:
  - Blunders - mathematical errors/omissions (-3)
  - Slips- numerical errors (-1)
  - Misreadings (provided task is not oversimplified) (-1).

Frequently occurring errors to which these penalties must be applied are listed in the scheme. They are labelled: B1, B2, B3,..., S1, S2,..., M1, M2,...etc. These lists are not exhaustive.

2. When awarding attempt marks, e.g. Att(3), note that
  - any *correct, relevant* step in a part of a question merits at least the attempt mark for that part
  - if deductions result in a mark which is lower than the attempt mark, then the attempt mark must be awarded
  - a mark between zero and the attempt mark is never awarded.
3. Worthless work is awarded zero marks. Some examples of such work are listed in the scheme and they are labelled as W1, W2,...etc.
4. The phrase "hit or miss" means that partial marks are not awarded – the candidate receives all of the relevant marks or none.
5. The phrase "and stops" means that no more work is shown by the candidate.
6. Special notes relating to the marking of a particular part of a question are indicated by an asterisk. These notes immediately follow the box containing the relevant solution.
7. The sample solutions for each question are not intended to be exhaustive lists – there may be other correct solutions.
8. Unless otherwise indicated in the scheme, accept the best of two or more attempts – even when attempts have been cancelled.
9. The *same* error in the *same* section of a question is penalised *once* only.
10. Particular cases, verifications and answers derived from diagrams (unless requested) qualify for attempt marks at most.
11. A serious blunder, omission or misreading results in the attempt mark at most.
12. Do not penalise the use of a comma for a decimal point, e.g. €5.50 may be written as €5,50.

### BONUS MARKS FOR ANSWERING THROUGH IRISH

Bonus marks are applied separately to each paper as follows:

If the mark achieved is 225 or less, the bonus is 5% of the mark obtained, rounded **down**.  
(e.g. 198 marks  $\times$  5% = 9.9  $\Rightarrow$  bonus = 9 marks.)

If the mark awarded is above 225, the following table applies:

Bunmharc (Marks obtained)	Marc Bónais (Bonus Mark)	Bunmharc (Marks obtained)	Marc Bónais (Bonus Mark)
226	11	261 – 266	5
227 – 233	10	267 – 273	4
234 – 240	9	274 – 280	3
241 – 246	8	281 – 286	2
247 – 253	7	287 – 293	1
254 – 260	6	294 – 300	0

## QUESTION 1

<b>Part (a)</b>	<b>10 marks</b>	<b>Att 3</b>
<b>Part (b)</b>	20 (10,10) marks	Att (3,3)
<b>Part (c)</b>	20 (5,10,5) marks	Att (2,3,2)

<b>Part (a)</b>	<b>10 marks</b>	<b>Att 3</b>
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Find the perimeter of the shape shown in the diagram.

<b>(a)</b>	<b>10 marks</b>	<b>Att 3</b>
------------	-----------------	--------------

9 found Step 1

$$x^2 = 12^2 + 9^2$$

$$= 144 + 81$$

$$= 225$$

$$\Rightarrow x = 15 \quad \text{Step 2}$$

Perimeter = 21 + 12 + 12 + 15 = 60cm Step 3

*Blunders (-3)*

- B1 Correct answer without work shown ()
- B2 Error in Pythagoras' Theorem
- B3 Error in squaring
- B4 Each side omitted, having found x

*Slips (-1)*

- S1 Arithmetic slips to a maximum of (-3)

*Attempts (3 marks)*

- A1 Correct formula
- A2 Two or more correct sides added
- A3 Effort at Pythagoras' Theorem

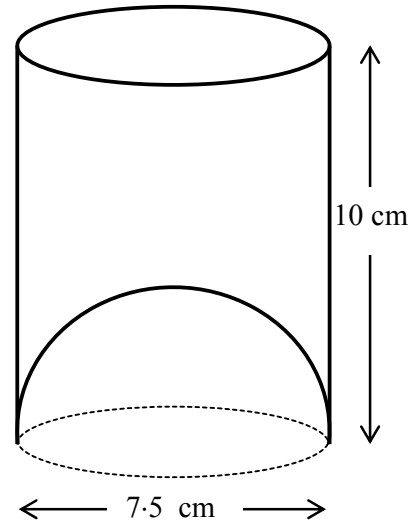
*Worthless (0)*

- W1 Area formula

**Part (b)****20 (10,10) marks****Att (3,3)**

A drinking glass is in the shape of a cylinder of diameter 7.5 cm and of height 10 cm. It has a hemispherical base as shown in the diagram.

- (i) ✍ Calculate the curved surface area of the cylindrical part of the glass, correct to two decimal places.
- (ii) ✍ Calculate the total surface area of the glass correct to two decimal places.

**(b) (i)****10 marks****Att 3**

$$\begin{aligned} \text{Curved surface area} &= 2\pi r h = 2 \times \pi \times 3.75 \times 10 \\ &= 75\pi = 235.619 \\ &= 235.62 \text{ cm}^2 \end{aligned}$$

\* Value of  $\pi$  used, other than the value of  $\pi$  from the calculator, giving an answer in the range 235.50 - 235.71 incurs -1

*Blunders (-3)*

- B1 Correct answer without work shown (✍)
- B2 Incorrect relevant formula
- B3 Incorrect substitution into correct formula
- B4 Incorrect  $r$
- B5 Incorrect  $h$
- B6 Answer in terms of  $\pi$
- B7 Value of  $\pi$  which affects the accuracy of the answer, other than \* above

*Slips (-1)*

- S1 Arithmetic slips to a maximum of (-3)
- S2 Answer not rounded or incorrectly rounded

*Misreadings (-1)*

- M1 Curved surface area of hemisphere

*Attempts (3 marks)*

- A1  $r = 3.75$
- A2 Indication that radius length is half of diameter length
- A3 Correct formula with some correct substitution

*Worthless (0)*

- W1 Volume formula for a cylinder

(b) (ii)

10 marks

Att 3

$$\begin{aligned}\text{Surface area of hemisphere} &= 2\pi r^2 = 2 \times \pi \times 3.75^2 \\ &= 28.125\pi \\ &= 88.357 \text{ or } 88.36 \text{ cm}^2\end{aligned}$$

$$\begin{aligned}\text{Total surface area} &= 75\pi + 28.125\pi \text{ or } 235.62 + 88.36 \\ &= 103.125\pi \quad \text{or} \quad 323.98 \\ &= 323.9767 \quad \text{or} \quad 323.98 \\ &= 323.98 \text{ cm}^2\end{aligned}$$

- \* Accept candidate's answer from (b) (i)
- \* Value of  $\pi$  used, other than the value of  $\pi$  from the calculator, giving an answer in the range 323.81 - 324.11 incurs -1, if not applied in (b) (i)

*Blunders (-3)*

- B1 Correct answer without work shown (✍)
- B2 Incorrect relevant formula
- B3 Incorrect substitution into correct formula
- B4 Incorrect  $r$  if different from (b) (i)
- B5 Answer in terms of  $\pi$ , if not penalised in (b) (i)
- B6 Value of  $\pi$  which affects the accuracy of the answer, if different from (b) (i)
- B7 Error in squaring

*Slips (-1)*

- S1 Arithmetic slips to a maximum of (-3)
- S2 Answer not rounded or incorrectly rounded
- S3 Failure to add areas
- S4 Areas subtracted

*Attempts (3 marks)*

- A1  $r = 3.75$
- A2 Indication that radius length is half of diameter length
- A3 Correct formula

*Worthless (0)*

- W1 Volume formula for a hemisphere

**Part (c)****20 (5,10,5) marks****Att (2,3,2)**

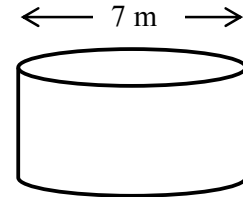
A large building has a flat roof of length 50 m and of width 40 m.  
On average there are 5 mm of rainfall on the roof in a week.



- (i) ✍ Calculate the average volume of rain that will fall on the roof in a week. Give your answer in  $\text{m}^3$ .

The rain is harvested in a cylindrical tank of diameter 7 m.

- (ii) ✍ Calculate the average rise in the level of the water in the tank in a week.  
Give your answer in metres correct to two decimal places.



The tank is emptied when the water reaches a height of 3.38 m.

- (iii) ✍ How many times a year, on average, will the tank be emptied?

**(c) (i)****5 marks****Att 2**

$$\begin{aligned} \text{Volume of rain} &= l \times b \times h \\ &= 50 \times 40 \times 0.005 \text{ m}^3 \\ &= 10 \text{ m}^3 \end{aligned}$$

*Blunders (-3)*

- B1 Correct answer without work shown (✍)  
B2 Incorrect substitution into correct formula  
B3 Answer not in  $\text{m}^3$   
B4  $5 \text{ mm} \neq 0.005 \text{ m}$

*Slips (-1)*

- S1 Arithmetic slips to a maximum of (-3)

*Attempts (2 marks)*

- A1 Correct formula  
A2 Indication of unit conversion  
A3  $50 \times 40$

(c) (ii)

10 marks

Att 3

$$\begin{aligned}\pi r^2 h &= 10 \\ \Rightarrow \pi \times (3.5)^2 \times h &= 10 \\ \Rightarrow 12.25\pi h &= 10 \\ \Rightarrow h &= \frac{10}{12.25\pi} \text{ or } \frac{10}{38.4845} \\ &= 0.2598 \\ &= 0.26 \text{ m}^2\end{aligned}$$

\* Accept candidate's answer from (c) (i)

*Blunders (-3)*

- B1 Correct answer without work shown (✓)
- B2 Incorrect relevant formula
- B3 Incorrect substitution into correct formula
- B4 Answer in terms of  $\pi$
- B5 Incorrect  $r$
- B6 Value of  $\pi$  which affects the accuracy of the answer
- B7 Error in squaring

*Slips (-1)*

- S1 Arithmetic slips to a maximum of (-3)
- S2 Answer not rounded or incorrectly rounded

*Attempts (3 marks)*

- A1 Correct formula with some correct substitution
- A2  $r = 3.5$
- A3 Answer from (c) (i) mentioned

*Worthless (0)*

- W1 Surface area formula for a cylinder

(c) (iii)

5 marks

Att 2

$$\text{Number of weeks} = \frac{3 \cdot 38}{0 \cdot 26}$$

$$= 13$$

$$\text{Number of times} = \frac{52}{13}$$
$$= 4$$

OR

$$0 \cdot 26 \times 52 = 13 \cdot 52$$

$$\text{Number of times} = \frac{13 \cdot 52}{3 \cdot 38}$$

$$= 4$$

OR

$$\text{Tank is emptied when Volume} = \pi r^2 h = \pi \times 3 \cdot 5^2 \times 3 \cdot 38$$
$$= 41 \cdot 405 \pi \text{ or } 130 \cdot 0776$$

$$\text{Number of weeks} = \frac{41 \cdot 405 \pi}{10} \text{ or } \frac{130 \cdot 0776}{10}$$

$$= 13$$

$$\text{Number of times} = \frac{52}{13}$$
$$= 4$$

\* Accept candidate's answers from (c) (i) and (ii)

*Blunders (-3)*

- B1 Correct answer without work shown (✍)
- B2 Number of times per year not calculated
- B3 Incorrect  $r$ , if not penalised in (c) (ii)
- B4 Incorrect  $h$
- B5 Value of  $\pi$  which affects the accuracy of the answer, if not penalised in (c) (ii)

*Slips (-1)*

- S1 Arithmetic slips to a maximum of (-3)

*Attempts (2 marks)*

- A1 Correct formula with some correct substitution
- A2  $r = 3 \cdot 5$
- A3 Multiplication or division with  $3 \cdot 38$
- A4 Some use of answer in (c) (ii)



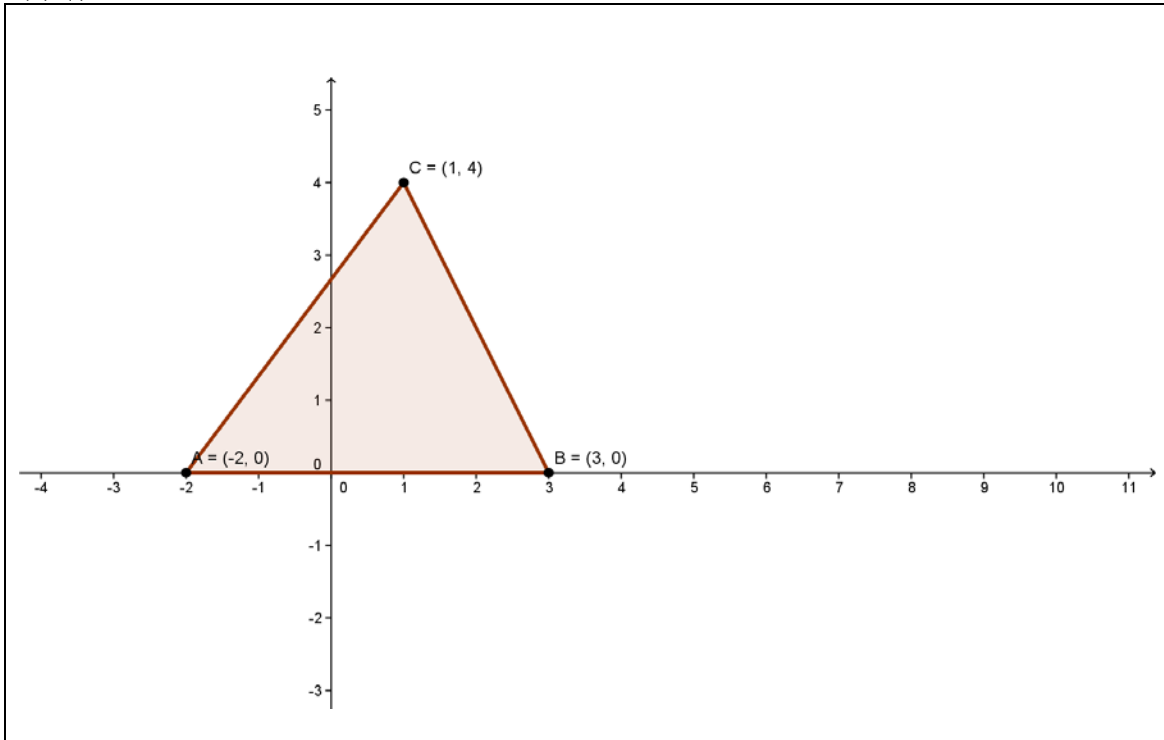
## QUESTION 2

Part (a)	25 marks	Att (5,3)
Part (b)	15 marks	Att (3,2)
Part (c)	10 marks	Att (2,2)

Part (a) 25 (15,10) marks (5,3)

- (i) ✍ Using graph paper, draw the triangle with vertices  $A(-2, 0)$ ,  $B(3, 0)$  and  $C(1, 4)$ .
- (ii) ✍ Calculate the area of the triangle  $ABC$ .

(a) (i) 15 marks Att 5



*Blunders (-3)*

- B1 Scale not uniform
- B2 Triangle not drawn
- B3 Each incorrect point
- B4 Both  $x$  and  $y$  coordinates switched

*Slips (-1)*

- S1 Not drawn on graph paper

*Attempts (5 marks)*

- A1 Scaled axes drawn
- A2 One point plotted

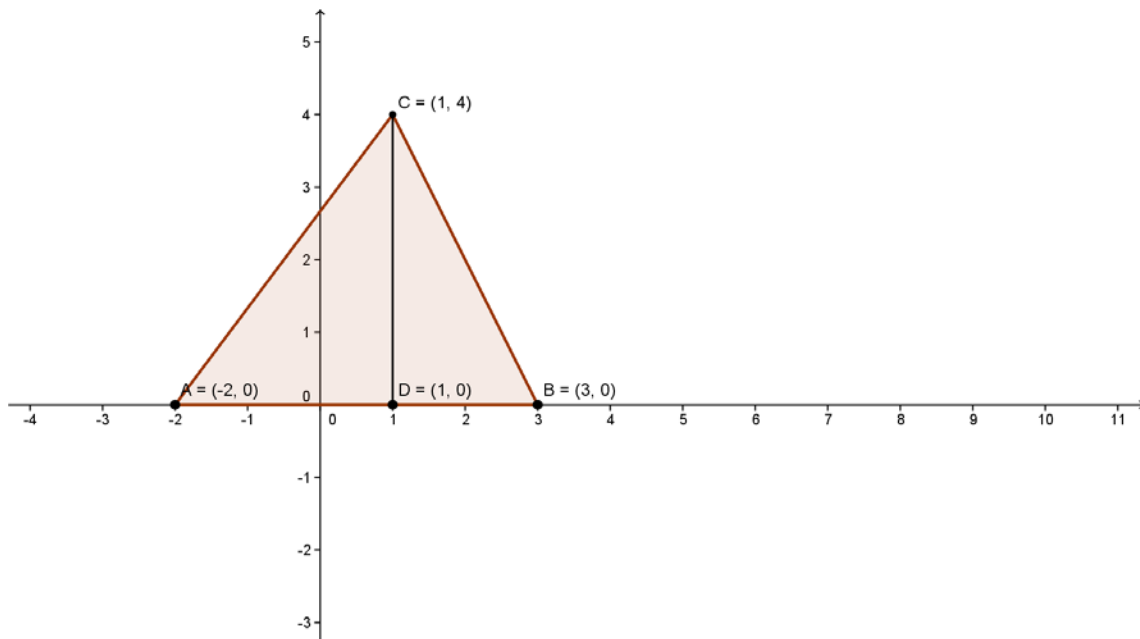
*Worthless (0)*

- W1 Unscaled axes drawn

(a) (ii)

10 marks

Att 3



$$\begin{aligned}\text{Area } \triangle ABC &= \frac{1}{2}(\text{base})(\text{perpendicular height}) \\ &= \frac{1}{2}|AB||CD| \\ &= \frac{1}{2}(5)(4) \\ &= 10 \text{ units}^2\end{aligned}$$

**OR**

$$|AC| = \sqrt{(1+2)^2 + (4-0)^2} = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$$

$$|CD| = 4$$

$$\sin \angle CAB = \frac{|CD|}{|AC|} = \frac{4}{5}$$

$$\begin{aligned} \text{Area } \Delta ABC &= \frac{1}{2} |AC| \cdot |AB| \cdot \sin \angle CAB \\ &= \frac{1}{2} (5)(5) \left( \frac{4}{5} \right) \\ &= 10 \text{ units}^2 \end{aligned}$$

**OR**

$$(-2,0) \rightarrow (0,0)$$

$$(1,4) \rightarrow (3,4) = (x_1, y_1)$$

$$(3,0) \rightarrow (5,0) = (x_2, y_2)$$

$$\begin{aligned} \text{Area } \Delta ABC &= \frac{1}{2} |x_1 y_2 - x_2 y_1| \\ &= \frac{1}{2} |(3)(0) - (5)(4)| \\ &= \frac{1}{2} |-20| = 10 \text{ units}^2 \end{aligned}$$

**OR**

$$|BC| = \sqrt{(3-1)^2 + (0-4)^2} = \sqrt{(2)^2 + (-4)^2} = \sqrt{4+16} = \sqrt{20}$$

The sides are 5, 5 and  $\sqrt{20}$  in length

$$a = 5, b = 5, c = \sqrt{20}$$

$$s = \frac{5+5+\sqrt{20}}{2} = 7.236$$

$$\begin{aligned} \text{Area } \Delta ABC &= \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{7.236(2.236)(2.236)(2.764)} \\ &= \sqrt{99.995} \\ &= 10 \text{ units}^2 \end{aligned}$$

\* Accept values consistent with candidate's graph

*Blunders (-3)*

- B1 Correct answer without work shown (✍)
- B2 Incorrect relevant formula
- B3 Incorrect base
- B4 Incorrect height
- B5 Triangle taken to be right angled at the point (1,4)
- B6 Incorrect sin ratio
- B7 Error in squaring
- B8 Error in translation

*Slips (-1)*

- S1 Arithmetic slips to a maximum of (-3)

*Attempts (3 marks)*

- A1  $\frac{1}{2}|x_1y_2 - x_2y_1|$  with some substitution
- A2 Correct base or height

*Worthless (0)*

- W1 Non - area formula with or without substitution

**Part (b)**

**15 (10,5) marks**

**Att (3,2)**

$l$  is the line  $2x - 11y = -16$  and  $k$  is the line  $x + 2y = -8$ .

(i) ✍ Find  $P$ , the point of intersection of  $l$  and  $k$ .

$Q(3, 2)$  is on the line  $l$  and  $R(2, -5)$  is on the line  $k$ .

(ii) ✍ Prove that the triangle  $PQR$  is isosceles.

**(b) (i)**

**10 marks**

**Att 3**

**OR**

$$l: 2x - 11y = -16$$

$$k: x + 2y = -8$$

$$l: 2x - 11y = -16$$

$$-2k: -2x - 4y = 16$$

$$\Rightarrow -15y = 0$$

$$\Rightarrow y = 0$$

$$\Rightarrow x = -8$$

$$k: x + 2y = -8$$

$$\Rightarrow x = -8 - 2y$$

$$l: 2(-8 - 2y) - 11y = -16$$

$$\Rightarrow -16 - 15y = -16$$

$$\Rightarrow -15y = 0$$

$$\Rightarrow y = 0$$

$$\Rightarrow x = -8$$

$P$  is the point  $(-8, 0)$ .

\* Accept  $(-8, 0) \in l$  and  $(-8, 0) \in k$  shown in each case

\* Accept  $x = -8$  and  $y = 0$  for full marks

*Blunders (-3)*

B1 Correct answer without work shown (✍)

B2 Transposition error

B3 Second value not found

*Slips (-1)*

S1 Arithmetic slips to a maximum of (-3)

*Misreadings (-1)*

M1 One value found and incorrectly substituted

*Attempts (3 marks)*

A1 Graphical solution correct

*Worthless (0)*

W1 Graphical solution incorrect

(b) (ii)

5 marks

Att 2

$$\begin{aligned} |PQ| &= \sqrt{(-8-3)^2 + (0-2)^2} \\ &= \sqrt{121+4} \\ &= \sqrt{125} \end{aligned}$$

$$\begin{aligned} |QR| &= \sqrt{(2-3)^2 + (-5-2)^2} \\ &= \sqrt{1+49} \\ &= \sqrt{50} \end{aligned}$$

$$\begin{aligned} |PR| &= \sqrt{(-8-2)^2 + (0+5)^2} \\ &= \sqrt{100+25} \\ &= \sqrt{125} \end{aligned}$$

$|PQ| = |PR|$   
Therefore the triangle  $PQR$  is isosceles

\* Accept candidate's answer from (b) (i)

*Blunders (-3)*

- B1 Correct lengths without work shown (✍)
- B2 Incorrect relevant formula
- B3 Both  $x$  and  $y$  switched in substitution
- B4 Error in squaring
- B5 Error in signs

*Slips (-1)*

- S1 Arithmetic slips to a maximum of (-3)
- S2 One incorrect substitution for  $x$  or  $y$
- S3 No conclusion or incorrect conclusion

*Misreadings (-1)*

- M1  $x$  and  $y$  switched from (b) (i)

*Attempts (2 marks)*

- A1 Correct formula with some correct substitution
- A2 Attempt at  $|PQ|$  or  $|PR|$  or  $|QR|$
- A3 Indication of some knowledge of an isosceles triangle
- A4 Attempt at difference of  $x$  values and/or difference of  $y$  values
- A5 Triangle correctly plotted

*Worthless (0)*

- W1 Incorrect formula with or without substitution

**Part (c)****10 (5,5) marks****Att (2,2)**

$S$  is the point  $(-4, -2)$  and  $T$  is the point  $(2, 6)$ .

- (i) ✍ Find the equation of the perpendicular bisector of  $[ST]$ .
- (ii) ✍ Verify that  $(-5, 5)$  is a point on the perpendicular bisector.
- (iii) ✍ Find the coordinates of the image of  $(-5, 5)$  under the axial symmetry in  $ST$ .

**(c) (i)****5 marks****Att 2**

$$\begin{aligned} \text{Midpoint } [ST] &= \left( \frac{-4+2}{2}, \frac{-2+6}{2} \right) \\ &= (-1, 2) \end{aligned}$$

$$\text{Slope } ST = \frac{6 - (-2)}{2 - (-4)} = \frac{8}{6} \text{ or } \frac{4}{3} \quad \text{Step 1}$$

$$\text{Slope of the perpendicular bisector} = -\frac{6}{8} \text{ or } -\frac{3}{4} \quad \text{Step 2}$$

$$\text{Equation of perpendicular bisector: } y - 2 = -\frac{6}{8}(x + 1) \text{ or } y - 2 = -\frac{3}{4}(x + 1) \quad \text{Step 3}$$

$$\begin{aligned} 8y - 16 &= -6x - 6 & \text{or} & & 4y - 8 &= -3x - 3 \\ 6x + 8y - 10 &= 0 & & & 3x + 4y - 5 &= 0 \end{aligned}$$

*Blunders (-3)*

- B1 Correct answer without work shown (✍)
- B2 Incorrect relevant formula
- B3 Both  $x$  and  $y$  switched in substitution
- B4 Midpoint not found or found graphically
- B5 Incorrect slope for perpendicular bisector

*Slips (-1)*

- S1 Arithmetic slips to a maximum of (-3)
- S2 One incorrect substitution for  $x$  or  $y$

*Attempts (2 marks)*

- A1 Correct formula with some correct substitution
- A2 Attempt at difference of  $x$  values and/or difference of  $y$  values
- A3 Indication that the product of the slopes of perpendicular lines is  $-1$
- A4 Midpoint or slope found
- A5 Graphical solution for slope correct
- A6 Indication of some knowledge of perpendicular bisector

*Worthless (0)*

- W1 Incorrect formula with or without substitution

**(c) (ii) & (iii)**

**5 marks**

**Att 2**

$\begin{aligned}(-5, 5) : y - 2 &= -\frac{3}{4}(x + 1) \\ 5 - 2 &= -\frac{3}{4}(-5 + 1) \\ 3 &= -\frac{3}{4}(-4) \\ 3 &= 3\end{aligned}$	<b>OR</b> 	$\begin{aligned}(-5, 5) : 3x + 4y - 5 &= 0 \\ 3(-5) + 4(5) - 5 &= 0 \\ -15 + 20 - 5 &= 0 \\ 0 &= 0\end{aligned}$
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- \* Accept candidates answer from (c) (i)
- \* Errors in simplifying equation in (c) (i) to a maximum of (-3)

*Blunders (-3)*

- B1 Transposition error
- B2 Both  $x$  and  $y$  switched in substitution

*Slips (-1)*

- S1 Arithmetic slips to a maximum of (-3)
- S2 Incorrect conclusion

*Attempts (2 marks)*

- A1 Graphical solution correct

*Worthless (0)*

- W1 Graphical solution incorrect

**(c) (iii)**

<p><math>(-5, 5)</math> is on the perpendicular bisector <math>\therefore</math> The image of <math>(-5, 5)</math> under axial symmetry in <math>ST</math> is the same as central symmetry in <math>(-1, 2)</math> <math>(-5, 5) \rightarrow (-1, 2) \rightarrow (3, -1)</math>  Answer <math>(3, -1)</math></p>
--

*Blunders (-3)*

- B1 Correct answer without work shown ( $\not\approx$ )
- B2 Wrong translation
- B3 One coordinate of image point only

*Slips (-1)*

- S1 Arithmetic slips to a maximum of (-3)

*Attempts (2 marks)*

- A1 Some correct work with a translation

*Worthless (0)*

- W1 The two given points plotted



### QUESTION 3

<b>Part (a)</b>	<b>20 marks</b>	<b>Att (5,2)</b>
<b>Part (b)</b>	<b>20 marks</b>	<b>Att (5,2)</b>
<b>Part (c)</b>	<b>10 marks</b>	<b>Att (2,2)</b>
<b>Part (a)</b>	<b>20 (15,5) marks</b>	<b>Att (5,2)</b>

In the diagram  $[MN]$  is parallel to  $[PQ]$ .  
 $|\angle POQ| = 43^\circ$  and  $|\angle OQP| = 70^\circ$ .  
 Find

(i) ✍ the value of  $x$

(ii) the value of  $y$ .

<b>(a) (i)</b>	<b>15 marks</b>	<b>Att 5</b>
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$$\begin{aligned}
 x &= 180 - (70 + 43) \\
 &= 180 - 113 \\
 &= 67
 \end{aligned}$$

\* Accept work on diagram

*Blunders (-3)*

B1 Correct answer without work shown (✍)

B2 Sum of angles in a triangle  $\neq 180^\circ$

*Slips (-1)*

S1 Arithmetic slips to a maximum of (-3)

*Attempts (5 marks)*

A1 Indication that the sum of the angles in a triangle =  $180^\circ$

A2 Indication that a straight angle =  $180^\circ$

A3 Indication of 113

*Worthless (0)*

W1 Diagram from examination paper either partially or fully drawn

<b>(a) (ii)</b>	<b>5 marks</b>	<b>Att 2</b>
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$$y = 67$$

\* Accept candidate's answer from (a) (i)

\* Accept work on diagram

*Blunders (-3)*

B1 Sum of angles in a triangle  $\neq 180^\circ$

B2 Sum of angles in a quadrilateral  $\neq 360^\circ$

*Attempts (2 marks)*

A1 Mention of corresponding angles

A2 Indication of 110 and/or 113

A3 Indication that the sum of the angles in a triangle =  $180^\circ$

A4 Indication that the sum of the angles in a quadrilateral =  $360^\circ$

*Worthless (0)*

W1 Diagram from examination paper either partially or fully drawn

W2  $y = 70$  or  $43$

**Part (b)**

**20 (15,5) marks**

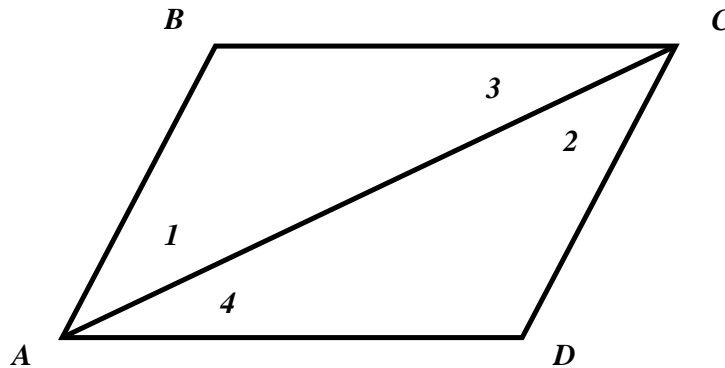
**Att (5,2)**

- (i) Prove that opposite sides and opposite angles of a parallelogram are respectively equal in measure.
- (ii) Show how to divide a line segment into three equal parts.  
All construction lines must be clearly shown.

**(b) (i)**

**15 marks**

**Att 5**



Given: Parallelogram  $ABCD$

To Prove:  $|AB| = |DC|$  and  $|BC| = |AD|$   
 $|\angle ABC| = |\angle ADC|$  and  $(|\angle BAD| = |\angle BCD|)$  Step 1

Construction: Join  $A$  to  $C$  Step 2  
 Label angles 1, 2, 3 and 4

Proof:  $|\angle 1| = |\angle 2|$ ..... alternate angles  
 $|\angle 3| = |\angle 4|$ ..... alternate angles  
 $|AC| = |AC|$ ..... common side Step 3  
 $\therefore \triangle ABC \equiv \triangle ADC$ ..... A.S.A. Step 4  
 $\Rightarrow |AB| = |DC|$  and  $|BC| = |AD|$ ..(corresponding sides)  
 and  $|\angle ABC| = |\angle ADC|$ .....(corresponding angles)

Also,  $|\angle 1| + |\angle 4| = |\angle 2| + |\angle 3|$   
 $\Rightarrow |\angle BAD| = |\angle BCD|$  Step 5

- \* Some steps may be indicated on candidate's diagram
- \* Must have one reason in Step 3 and A.S.A. in Step 4
- \* Accept other valid proofs

*Blunders (-3)*

- B1 Each step incorrect or omitted
- B2 Each step incomplete

*Attempts (5 marks)*

- A1 Parallelogram drawn with diagonal indicated
- A2 Parallelogram drawn with sides or angles to be proven equal indicated

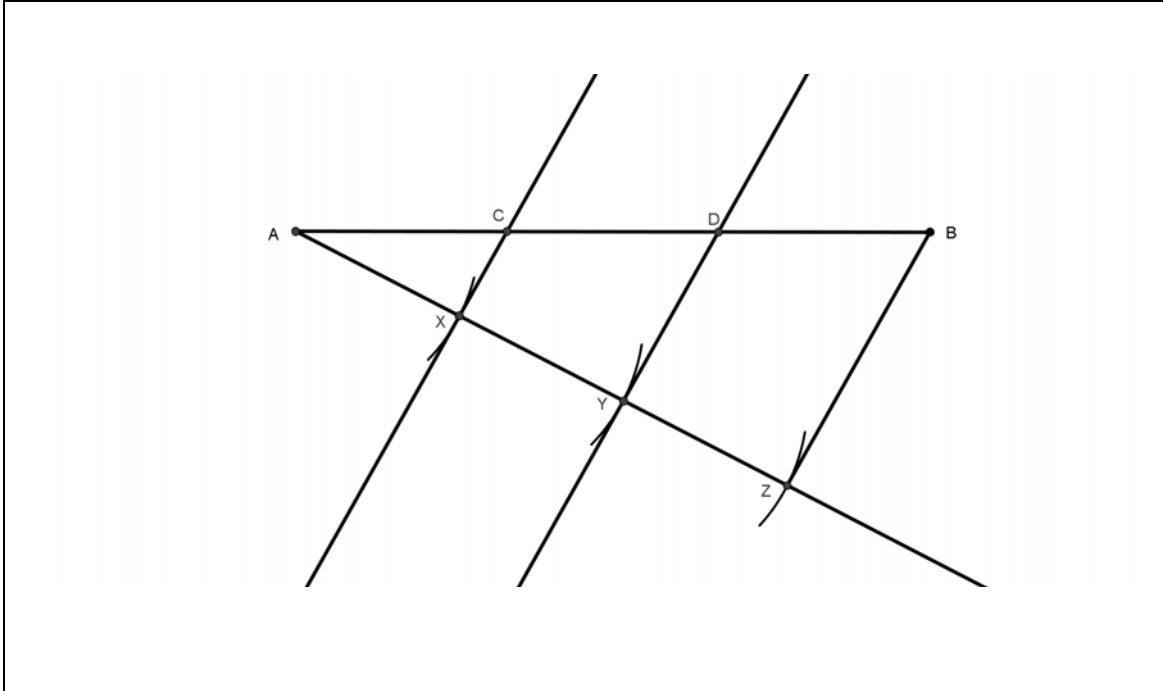
*Worthless (0)*

- W1 Wrong Theorem
- W2 Parallelogram drawn only
- W3 No diagram

(b) (ii)

5 marks

Att 2



\* Allow a tolerance of  $\pm 2$  mm in constructions

*Blunders (-3)*

B1 Third arc not joined to B

B2 Each parallel line not shown in construction

*Attempts (2 marks)*

A1 Line divided into three equal parts with no construction lines shown

*Worthless (0)*



W1 Line drawn

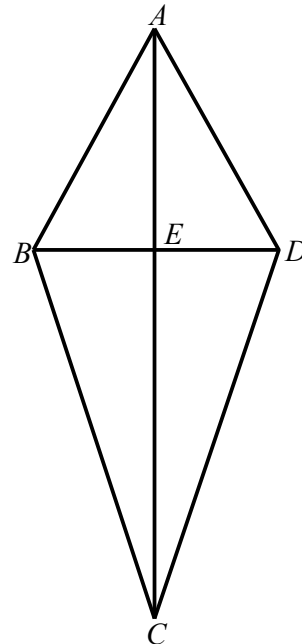
Part (c)

10 (5,5) marks

Att (2,2)

In the diagram  $|AB| = |AD|$  and  $|BC| = |DC|$ .  
 $AC$  intersects  $BD$  at  $E$ .

- (i)  Prove  $|\angle BAC| = |\angle DAC|$ .
- (ii)  Prove  $E$  is the midpoint of  $[BD]$ .



(c) (i)

5 marks

Att 2

Consider triangles  $ABC$  and  $ADC$

$ AB  =  AD $ .....	(given)	
$ BC  =  DC $ .....	(given)	Step 1
$ AC  =  AC $ .....	(common side)	Step 2
$\therefore \triangle ABC \equiv \triangle ADC$ .....	S.S.S.	Step 3
$\therefore  \angle BAC  =  \angle DAC $ .....	(corresponding angles)	

OR

$|\angle ABE| = |\angle ADE|$  ..... ( $|AB| = |AD|$ )  
 $|\angle CBE| = |\angle CDE|$  ..... ( $|BC| = |DC|$ )  
 $\Rightarrow |\angle ABE| + |\angle CBE| = |\angle ADE| + |\angle CDE|$   
 $\therefore |\angle ABC| = |\angle ADC|$

Consider triangles  $ABC$  and  $ADC$

$ \angle ABC  =  \angle ADC $		Step 1
$ AB  =  AD $ .....	(given)	
$ BC  =  DC $ .....	(given)	Step 2
$\therefore \triangle ABC \equiv \triangle ADC$ .....	S.A.S.	Step 3
$\therefore  \angle BAC  =  \angle DAC $ .....	(corresponding angles)	

- \* Some steps may be indicated on candidate's diagram
- \* Must have S.S.S. in Step 3 if Method 1 used
- \* Must have S.A.S. in Step 3 if Method 2 used

Blunders (-3)

- B1 Each step incorrect or omitted
- B2 Each step incomplete

Attempts (2 marks)

- A1 Both triangles indicated or redrawn separately
- A2 Indication of some knowledge of congruent triangles
- A3 Indication of some knowledge of an isosceles triangle
- A4 Equal sides or equal angles indicated on diagram

Worthless (0)

- W1 Diagram from examination paper either partially or fully drawn

(c) (ii)

5 marks

Att 2

Consider triangles  $ABE$  and  $ADE$

$ AB  =  AD $ .....(given)	
$ \angle BAE  =  \angle DAE $ .....(proven)	Step 1
$ AE  =  AE $ .....(common side)	
$\therefore \triangle ABE \equiv \triangle ADE$ .....S.A.S.	Step 2
$\therefore  BE  =  DE $ .....(corresponding sides)	Step 3
$\therefore E$ is the midpoint of $[BD]$	

**OR**

$\triangle ABC \equiv \triangle ADC \Rightarrow |\angle ACB| = |\angle ACD|$

Consider triangles  $BCE$  and  $CDE$

$ BC  =  DC $ .....(given)	
$ \angle BCE  =  \angle DCE $ .... (proven)	Step 1
$ CE  =  CE $ .....(common side)	
$\therefore \triangle BCE \equiv \triangle DCE$ .....S.A.S.	Step 2
$\therefore  BE  =  DE $ .....(corresponding sides)	Step 3
$\therefore E$ is the midpoint of $[BD]$	

- \* Some steps may be indicated on candidate's diagram
- \* Must have S.A.S. in Step 2

Blunders (-3)

- B1 Each step incorrect or omitted
- B2 Each step incomplete

Attempts (2 marks)

- A1 Both triangles indicated or redrawn separately
- A2 Indication of some knowledge of congruent triangles
- A3 Indication of some knowledge of an isosceles triangle
- A4 Equal sides or angles indicated on diagram

Worthless (0)

- W1 Diagram from examination paper either partially or fully drawn

## QUESTION 4

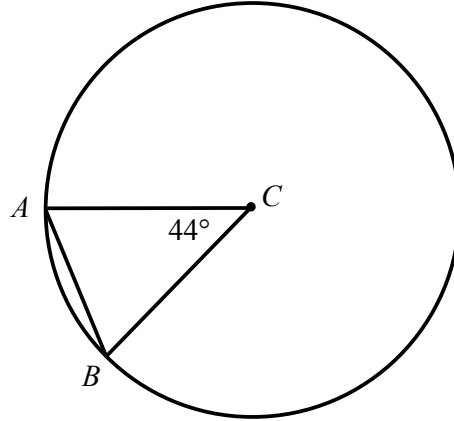
<b>Part (a)</b>	<b>25 marks</b>	<b>Att 8</b>
<b>Part (b)</b>	<b>15 marks</b>	<b>Att 5</b>
<b>Part (c)</b>	<b>10 marks</b>	<b>Att (2,2)</b>

**Part (a)** **25 marks** **Att 8**

$A$  and  $B$  are points on a circle  
with centre  $C$ .

$$|\angle BCA| = 44^\circ.$$

$\sphericalangle$  Find  $|\angle BAC|$ .



**(a)** **25 marks** **Att 8**

$$\begin{aligned} |\angle BAC| &= \frac{1}{2}(180 - 44)^\circ \\ &= \frac{1}{2}(136)^\circ \\ &= 68^\circ \end{aligned}$$

\* Some steps may be indicated on candidate's diagram

*Blunders (-3)*

- B1 Correct answer without work shown ( $\sphericalangle$ )
- B2 Sum of the angles in a triangle  $\neq 180^\circ$

*Attempts (8 marks)*

- A1 Indication that the sum of the angles in a triangle =  $180^\circ$
- A2 Indication of 136
- A3 Mention of an isosceles triangle


*Worthless (0)*

- W1 Diagram from examination paper either partially or fully drawn
- W2  $|\angle BAC| = 90^\circ$

Part (b)

15 marks

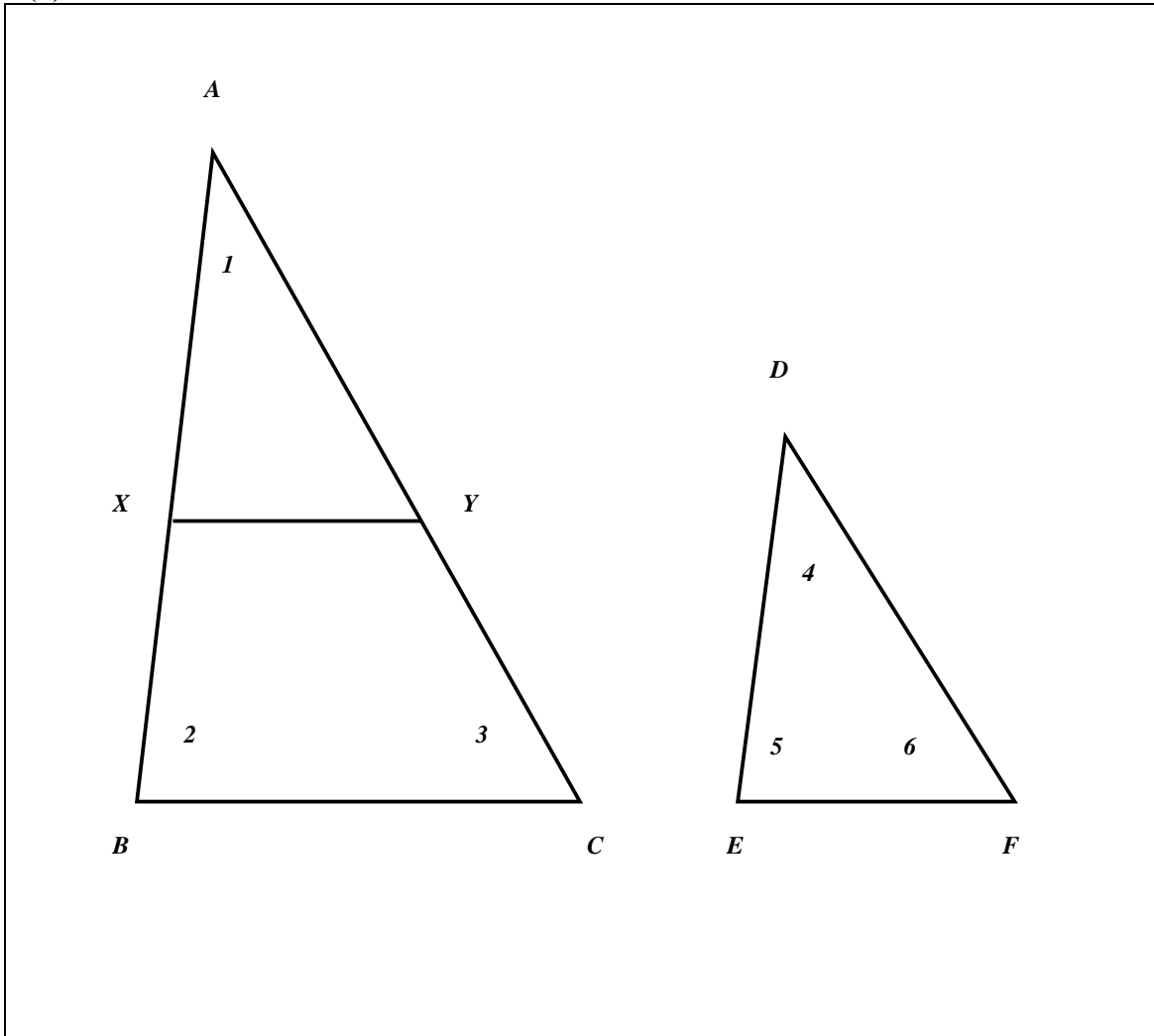
Att 5

 Prove that if two triangles are equiangular, the lengths of corresponding sides are in proportion.

(b)

15 marks

Att 5



**Given:** Two triangles  $ABC$  and  $DEF$  in which  $|\angle 1| = |\angle 4|$ ,  $|\angle 2| = |\angle 5|$  and  $|\angle 3| = |\angle 6|$

**To Prove:**  $\frac{|AB|}{|DE|} = \left(\frac{|BC|}{|EF|}\right) = \frac{|AC|}{|DF|}$  Step 1

**Construction:** Mark the point  $X$  on  $[AB]$  such that  $|AX| = |DE|$   
 Mark the point  $Y$  on  $[AC]$  such that  $|AY| = |DF|$   
 Join  $X$  to  $Y$  Step 2

**Proof:** Consider triangles  $AXY$  and  $DEF$   
 $|AX| = |DE|$  and  $|AY| = |DF|$ .....(construction)  
 $|\angle 1| = |\angle 4|$ .....(given)  
 $\therefore$  The triangles  $AXY$  and  $DEF$  are congruent.....S.A.S  
 $\therefore |\angle AXY| = |\angle DEF| = |\angle 5|$   
 ( But  $|\angle 2| = |\angle 5|$ .....given)  
 $\therefore |\angle AXY| = |\angle 2| = |\angle ABC|$   
 $\therefore XY \parallel BC$ .....(corresponding angles) Step 3  
 $\therefore \frac{|AB|}{|AX|} = \frac{|AC|}{|AY|}$  Step 4  
 $\therefore \frac{|AB|}{|DE|} = \frac{|AC|}{|DF|}$ .....( $|AX| = |DE|$  and  $|AY| = |DF|$ ) Step 5

Similarly, it can be proven that  $\frac{|AB|}{|DE|} = \frac{|BC|}{|EF|}$

$$\therefore \frac{|AB|}{|DE|} = \frac{|BC|}{|EF|} = \frac{|AC|}{|DF|}$$

- \* Some steps may be indicated on candidate's diagram
- \* Must have S.A.S in step 3
- \* Accept other valid proofs

*Blunders (-3)*

- B1 Each step incorrect or omitted
- B2 Each step incomplete

*Attempts (5 marks)*

- A1 Two separate diagrams drawn with equal angles indicated
- A2 The second diagram of the proof drawn

*Worthless (0)*

- W1 Wrong Theorem
- W2 Two triangles drawn
- W3 No diagram

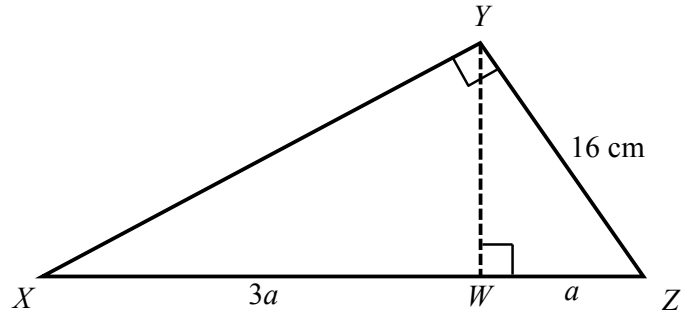


**Part (c)**

**10 (5,5) marks**

**Att (2,2)**

$XYZ$  is a right angled triangle with  $|\angle XYZ| = 90^\circ$ .  
 $W$  is a point on  $[XZ]$ , such that  $YW$  is perpendicular to  $XZ$ .



- (i) ✍ Prove  $\triangle XYZ$  and  $\triangle WYZ$  are equiangular.
- (ii) ✍ Given that  $|WZ| = a$  cm,  $|XW| = 3a$  cm and  $|YZ| = 16$  cm, find  $a$ .

**(c) (i)**

**5 marks**

**Att 2**

Consider $\triangle XYZ$ and $\triangle WYZ$	
$ \angle XYZ  =  \angle YWZ  \dots\dots\dots(90^\circ)$	Step 1
$ \angle XZY  =  \angle WZY  \dots\dots\dots(\text{common angle})$	Step 2
$ \angle YXZ  =  \angle WYZ  \dots\dots\dots(\text{third pair of angles are equal})$	
$\therefore \triangle XYZ$ and $\triangle WYZ$ are equiangular	Step 3

\* Some steps may be indicated on candidate's diagram

*Blunders (-3)*

- B1 Each step incorrect or omitted
- B2 Each step incomplete

*Attempts (2 marks)*

- A1 Both triangles indicated or redrawn separately

*Worthless (0)*

- W1 Diagram from examination paper either partially or fully drawn

(c) (ii)

5 marks

Att 2

$\triangle XYZ$  and  $\triangle WYZ$  are equiangular  
 $\therefore$  Corresponding sides are in proportion

$$\Rightarrow \frac{|XZ|}{|YZ|} = \frac{|YZ|}{|WZ|}$$

$$\Rightarrow \frac{4a}{16} = \frac{16}{a}$$

$$\Rightarrow 4a^2 = 256$$

$$\Rightarrow a^2 = 64$$

$$\Rightarrow a = 8$$

*Blunders (-3)*

- B1 Correct answer without work shown (✍)
- B2 Incorrect ratio
- B3 Error in cross multiplication
- B4 Error in square root

*Slips (-1)*

- S1 Arithmetic slips to a maximum of (-3)

*Attempts (2 marks)*

- A1 One correct relevant ratio
- A2 Indication of  $4a$

*Worthless (0)*

- W1 Diagram from examination paper either partially or fully drawn

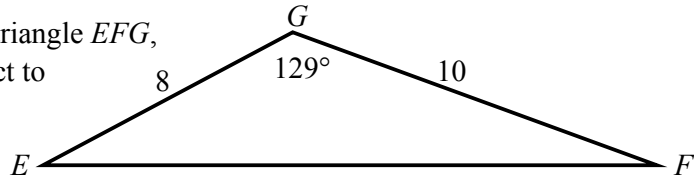
## QUESTION 5

<b>Part (a)</b>	<b>10 marks</b>	<b>Att 3</b>
<b>Part (b)</b>	<b>20 marks</b>	<b>Att (3,3)</b>
<b>Part (c)</b>	<b>20 marks</b>	<b>Att (3,3)</b>

**Part (a)** **10 marks** **Att 3**

In the diagram  $|\angle EGF| = 129^\circ$ ,  
 $|EG| = 8$  and  $|FG| = 10$ .

- $\pencil$  Calculate the area of the triangle  $EFG$ ,  
giving your answer correct to  
one decimal place.



**(a)** **10 marks** **Att 3**

$$\begin{aligned}\text{Area triangle } EFG &= \frac{1}{2}(8)(10)\sin 129^\circ = 40 \sin 129^\circ \\ &= 31.0858 \\ &= 31.1 \text{ units}^2\end{aligned}$$

*Blunders (-3)*

- B1 Correct answer without work shown ( $\pencil$ )
- B2 Incorrect relevant formula
- B3 Early rounding which affects the accuracy of the answer
- B4 Calculator in incorrect mode

*Slips (-1)*

- S1 Arithmetic slips to a maximum of (-3)
- S2 Answer not rounded or incorrectly rounded

*Attempts (3 marks)*

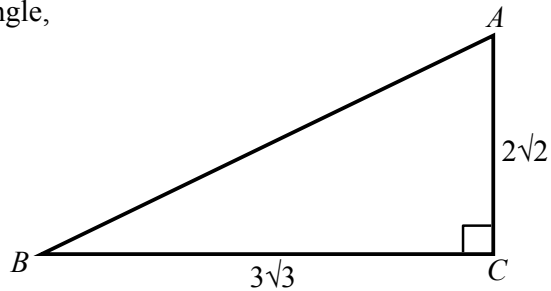
- A1 Correct formula with some correct substitution

*Worthless (0)*

- W1 Diagram from examination paper either partially or fully drawn
- W2 Triangle treated as right-angled
- W3 Incorrect formula with or without substitution

**Part (b)****20 (10,10) marks****Att (3,3)**

In the diagram  $ABC$  is a right angled triangle,  
with  $AC$  perpendicular to  $BC$ .  
 $|AC| = 2\sqrt{2}$  and  $|BC| = 3\sqrt{3}$ .



Calculate

- (i) ✎  $|AB|$ , leaving your answer in surd form  
(ii) ✎  $|\angle ABC|$ , correct to the nearest degree.

**(b) (i)****10 marks****Att 3**

$$\begin{aligned} |AB|^2 &= (3\sqrt{3})^2 + (2\sqrt{2})^2 \\ &= 27 + 8 \\ &= 35 \\ |AB| &= \sqrt{35} \end{aligned}$$

*Blunders (-3)*

- B1 Correct answer without work shown (✎)  
B2 Error in Pythagoras' Theorem  
B3 Error in squaring  
B4 Error in square root

*Slips (-1)*

- S1 Arithmetic slips to a maximum of (-3)  
S2 Answer not in surd form

*Attempts (3 marks)*

- A1 Effort at Pythagoras' Theorem

*Worthless (0)*

- W1 Diagram from examination paper either partially or fully drawn

(b) (ii)

10 marks

Att 3

$$\tan \angle ABC = \frac{2\sqrt{2}}{3\sqrt{3}} \text{ or } 0.54433$$

OR

$$\cos \angle ABC = \frac{3\sqrt{3}}{\sqrt{35}} \text{ or } 0.8783$$

OR  $\sin \angle ABC = \frac{2\sqrt{2}}{\sqrt{35}} \text{ or } 0.4781$

OR

$$\frac{\sin \angle ABC}{2\sqrt{2}} = \frac{\sin 90^\circ}{\sqrt{35}}$$

or

$$\sin \angle ABC = \frac{2\sqrt{2}}{\sqrt{35}} \text{ or } 0.4781$$

$$|\angle ABC| = 28.56^\circ = 29^\circ$$

\* Accept candidate's answer from (b) (i)

*Blunders (-3)*

- B1 Correct answer without work shown (✍)
- B2 Incorrect ratio for sin/cos/tan function
- B3 Incorrect ratio for Sine Rule
- B4 Calculator in incorrect mode
- B5 Early rounding which affects the accuracy of the answer

*Slips (-1)*

- S1 Arithmetic slips to a maximum of (-3)
- S2 Answer not rounded or incorrectly rounded

*Attempts (3 marks)*

- A1  $\tan A = \frac{\text{opposite}}{\text{adjacent}}$  or  $\cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$  or  $\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$
- A2 Sine rule with some correct substitution

*Worthless (0)*

- W1 Diagram from examination paper either partially or fully drawn

Part (c)

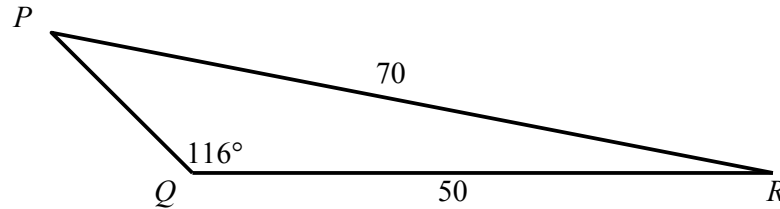
20 (10,10) marks

Att (3,3)

In the triangle  $PQR$ ,  $|PR| = 70$ ,  $|QR| = 50$  and  $|\angle PQR| = 116^\circ$ .

(i) ✎ Find  $|\angle QPR|$ , giving your answer correct to the nearest degree.

(ii) ✎ Find  $|PQ|$ , giving your answer correct to the nearest whole number.



(c) (i)

10 marks

Att 3

$$\frac{\sin \angle QPR}{50} = \frac{\sin 116^\circ}{70}$$

Step 1

$$\sin \angle QPR = \frac{50 \sin 116^\circ}{70} \text{ or } 0.641996$$

Step 2

$$|\angle QPR| = \sin^{-1} \left( \frac{50 \sin 116^\circ}{70} \right) \text{ or } \sin^{-1}(0.641996)$$

$$= 39.94^\circ$$

$$= 40^\circ$$

Step 3

*Blunders (-3)*

- B1 Correct answer without work shown (✎)
- B2 Incorrect ratio for Sine Rule
- B3 Calculator in incorrect mode
- B4 Error in transposition
- B5 Early rounding which affects the accuracy of the answer
- B6 Each step incorrect
- B7 Each step incomplete

*Slips (-1)*

- S1 Arithmetic slips to a maximum of (-3)
- S2 Answer not rounded or incorrectly rounded

*Attempts (3 marks)*

- A1 Sine Rule with some correct substitution

*Worthless (0)*

- W1 Diagram from examination paper either partially or fully drawn
- W2 Triangle treated as right-angled

(c) (ii)

10 marks

Att 3

$ \angle QRP  = 180^\circ - (40^\circ + 116^\circ) = 180^\circ - 156^\circ$ $= 24^\circ$		
$\frac{ PQ }{\sin 24^\circ} = \frac{70}{\sin 116^\circ}$	<b>OR</b> 	$\frac{ PQ }{\sin 24^\circ} = \frac{50}{\sin 40^\circ}$ Step 1
$ PQ  = \frac{70 \sin 24^\circ}{\sin 116^\circ}$		$ PQ  = \frac{50 \sin 24^\circ}{\sin 40^\circ}$ Step 2
$ PQ  = 31.6775$		$ PQ  = 31.6385$
$ PQ  = 32$		Step 3

\* Accept candidate's answer from (c) (i)

*Blunders (-3)*

- B1 Correct answer without work shown (✍)
- B2 Incorrect ratio for Sine Rule
- B3 Calculator in incorrect mode
- B4 Error in transposition
- B5 Early rounding which affects the accuracy of the answer
- B6 Each step incorrect
- B7 Each step incomplete

*Slips (-1)*

- S1 Arithmetic slips to a maximum of (-3)
- S2 Answer not rounded or incorrectly rounded

*Attempts (3 marks)*

- A1 Sine Rule with some correct substitution
- A2 Indication that the sum of the angles in a triangle =  $180^\circ$

*Worthless (0)*

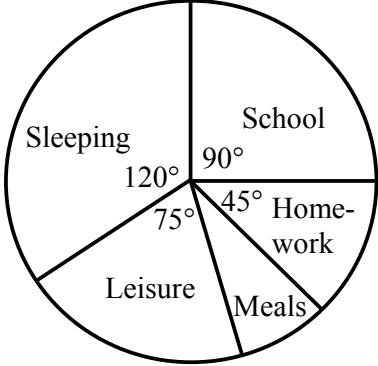
- W1 Diagram from examination paper either partially or fully drawn
- W2 Triangle treated as right-angled


## QUESTION 6

<b>Part (a)</b>	<b>10 marks</b>	<b>Att 3</b>
<b>Part (b)</b>	<b>20 marks</b>	<b>Att (3,3)</b>
<b>Part (c)</b>	<b>20 marks</b>	<b>Att (3,2,2)</b>

**Part (a)** **10 marks** **Att 3**

The pie chart shows how Mary spends her time over a typical 24 hour period.



 Copy and complete the following table in your answer book.

**(a)** **10 marks** **Att 3**

<p>Sleeping: <math>\frac{120}{360} \times 24 = 8</math></p> <p>School: <math>\frac{90}{360} \times 24 = 6</math></p> <p>Homework: <math>\frac{45}{360} \times 24 = 3</math></p> <p>Leisure: <math>\frac{75}{360} \times 24 = 5</math></p> <p>Meals:</p> <p style="margin-left: 20px;">Angle = <math>360^\circ - (120 + 90 + 45 + 75)^\circ</math></p> <p style="margin-left: 40px;"><math>= 360^\circ - 330^\circ</math></p> <p style="margin-left: 40px;"><math>= 30^\circ</math></p> <p style="margin-left: 20px;"><math>\frac{30}{360} \times 24 = 2</math></p>	<b>OR</b>	<p>Sleeping: <math>\frac{120}{360} \times 24 = 8</math></p> <p>School: <math>\frac{90}{360} \times 24 = 6</math></p> <p>Homework: <math>\frac{45}{360} \times 24 = 3</math></p> <p>Leisure: <math>\frac{75}{360} \times 24 = 5</math></p> <p>Meals: <math>24 - (8 + 6 + 3 + 5)</math></p> <p style="margin-left: 40px;"><math>= 24 - 22</math></p> <p style="margin-left: 40px;"><math>= 2</math></p>
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	Sleeping	School	Homework	Meals	Leisure
No. of hours	<b>8</b>	<b>6</b>	<b>3</b>	<b>2</b>	<b>5</b>



*Blunders (-3)*

- B1 Correct answer without work shown (✍)
- B2 Sum of angles  $\neq 360^\circ$
- B3 Incorrect fraction
- B4 Each entry omitted

*Slips (-1)*

- S1 Arithmetic slips to a maximum of (-3)

*Misreadings (-1)*

- M1 No table

*Attempts (3 marks)*

- A1 Indication of  $360^\circ$
- A2 Indication of  $330^\circ$  or  $30^\circ$

*Worthless (0)*

- W1 Diagram from examination paper either partially or fully drawn

**Part (b)****20 (10,10) marks****Att (3,3)**

The table below shows the results of a survey of the amount of money (in euro) that 150 people spent in a supermarket.

Amount (€)	0 – 10	10 – 15	15 – 20	20 – 30	30 – 50
No. of people	15	30	50	45	10

[Note: 10 – 15 means 10 or more but less than 15, etc.]

- (i) ✍ Taking mid-interval values, calculate the mean amount of money spent in the supermarket.
- (ii) ✍ Calculate the maximum percentage of the people who could have spent between €5 less than the mean and €5 more than the mean.

**(b) (i)****10 marks****Att 3**

The mid-interval values are: 5, 12.5, 17.5, 25, 40

$$\begin{aligned} \text{Mean} &= \frac{(15 \times 5) + (30 \times 12.5) + (50 \times 17.5) + (45 \times 25) + (10 \times 40)}{150} \\ &= \frac{75 + 375 + 875 + 1125 + 400}{150} \\ &= \frac{2850}{150} \\ &= 19 \end{aligned}$$

The mean amount spent was €19

*Blunders (-3)*

- B1 Correct answer without work shown (✍)
- B2 Consistent incorrect mid-interval values
- B3 Division by 5
- B4 Division by sum of mid-interval values
- B5 Mid-interval values added to frequencies instead of multiplied

*Slips (-1)*

- S1 Arithmetic slips to a maximum of (-3)

*Attempts (3 marks)*

- A1 One correct multiplication in numerator
- A2 Indication of division by 150
- A3 One correct mid-interval value
- A4 Sum of mid-interval values divided by 150 or 5

*Worthless (0)*

- W1 Sum of frequencies divided by 5

**(b) (ii)**

**10 marks**

**Att 3**

$\text{€5 less than mean: } \text{€19} - \text{€5} = \text{€14}$	
$\text{€5 more than mean: } \text{€19} + \text{€5} = \text{€24}$	Step 1
$\text{Maximum number} = 30 + 50 + 45 = 125$	Step 2
$\text{Maximum percentage} = \frac{125}{150} \times 100\%$	
$= \frac{250}{3}\% \text{ or } 83\frac{1}{3}\% \text{ or } 83.3\% \text{ or } 83\%$	Step 3

\* Accept candidate's answer from (b) (i)

*Blunders (-3)*

- B1 Correct answer without work shown ( $\not\approx$ )
- B2 Each step incorrect
- B3 Each step incomplete
- B4 Omission of a number
- B5 Extra number

*Slips (-1)*

- S1 Arithmetic slips to a maximum of (-3)

*Attempts (3 marks)*

- A1 Indication of 14 or 24 or equivalent
- A2 Use of 100

**Part (c)****20 (10,5,5) marks****Att (3,2,2)**

A speed camera, situated in a 50 km/h speed limit zone, recorded the speed of the cars, in km/h, passing it over a one hour period. The following are the results:

36 72 43 62 56 57 65 50 47 56 62 59 46 43  
25 54 47 51 56 52 48 53 49 39 57 76 37 49

(i) Copy and complete the cumulative frequency table in your answer book.

Speed	< 30	< 40	< 50	< 60	< 70	< 80
No. of cars		<b>4</b>		<b>23</b>		<b>28</b>

- (ii) ✍ Use your cumulative frequency table to construct the ogive.
- (iii) ✍ Use your ogive to estimate the number of cars with speeds between 45 and 55 km/h.
- (iv) ✍ What is the difference between your estimate and the actual number of cars with speeds between 45 and 55 km/h?

**(c) (i)****10 marks****Att 3**

Speed	< 30	< 40	< 50	< 60	< 70	< 80
No. of cars	1	<b>4</b>	12	<b>23</b>	26	<b>28</b>

*Blunders (-3)*

B1 Omission of a value

*Slips (-1)*

S1 Arithmetic slips to a maximum of (-3)

*Attempts (3 marks)*

A1 Any one value filled into table

A2 Indication of counting values

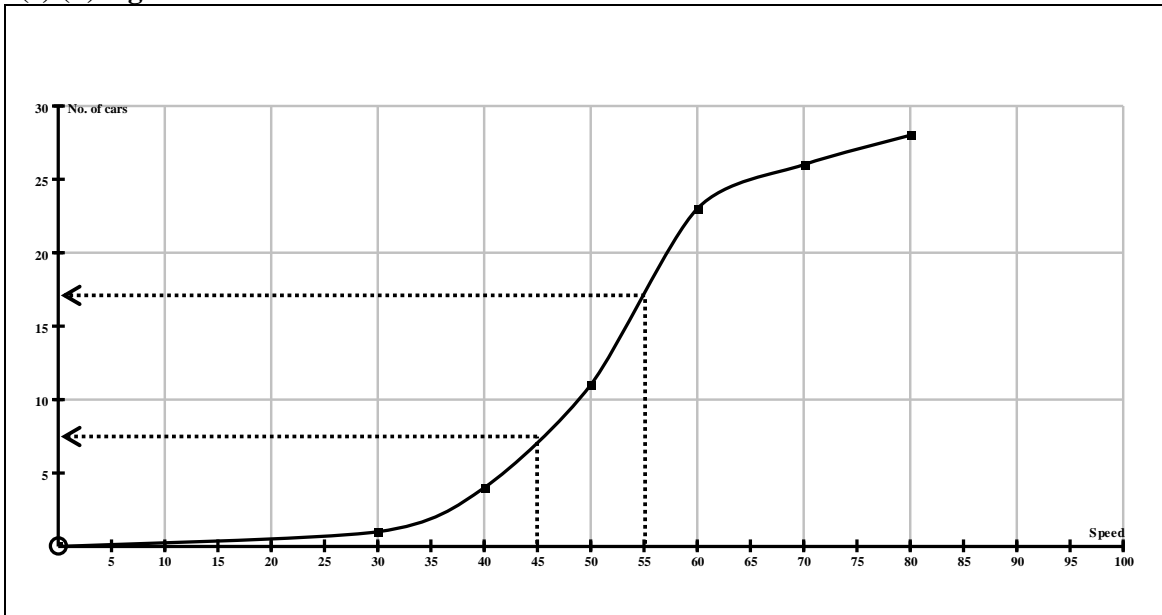
*Worthless (0)*

W1 Table or list copied from examination paper

(c) (ii) Ogive

5 marks

Att 2



\* Accept candidate's perpendicular axes

\* Accept candidate's cumulative frequency table from (c) (i)

*Blunders (-3)*

B1 Scale not uniform on y-axis or above 30 on x-axis

B2 Points plotted but not joined

B3 Consistent error in plotting points

*Slips (-1)*

S1 Each point incorrectly plotted

S2 Each point omitted

S3 Points joined with straight lines

S4 Graph not drawn from origin

*Attempts (2 marks)*

A1 Scaled axes drawn

*Worthless (0)*

W1 Unscaled axes drawn

**(c) (iii) & (iv)**

**5 marks**

**Att 2**

Estimate of number of cars, with speeds between 45 and 55 from ogive:

$$17 - 8 = 9$$

- \* Accept answer consistent with candidate's work in (c) (ii)
- \* Accept a tolerance of  $\pm 2$

*Blunders (-3)*

- B1 Correct answer without work shown (~~✗~~)
- B2 Line drawn from incorrect starting point on correct axis
- B3 Lines not drawn

*Slips (-1)*

- S1 Arithmetic slips to a maximum of (-3)
- S2 Incorrect reading from graph with work shown
- S3 Difference not found

*Attempts (2 marks)*

- A1 Vertical line from 45 and/or 55 drawn

**(c) (iv)**

Actual number of cars with speeds between 45 and 55 is 11

The difference is  $11 - 9 = 2$

- \* Accept candidate's answer from (c) (iii)