Coimisiún na Scrúduithe Stáit
State Examinations Commission

Scéim Mharcála *Scrúduithe Ardteistiméireachta, 2004*

Matamaitic *Gnáthleibhéal*

Marking Scheme *Leaving Certificate Examination, 2004*

Mathematics *Ordinary Level*

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MARKING SCHEME

LEAVING CERTIFICATE EXAMINATION 2004

MATHEMATICS

ORDINARY LEVEL

PAPER 1

GENERAL GUIDELINES FOR EXAMINERS - PAPER 1

1. Penalties of three types are applied to candidates' work as follows:
   - Blunders - mathematical errors/omissions (-3)
   - Slips - numerical errors (-1)
   - Misreadings (provided task is not oversimplified) (-1).

   Frequently occurring errors to which these penalties must be applied are listed in the scheme. They are labelled as B1, B2, B3,......, S1, S2, S3,...., M1, M2, etc. Note that these lists are not exhaustive.

2. When awarding attempt marks, e.g. Att(3), note that
   - any correct relevant step in a part of a question merits *at least* the attempt mark for that part
   - if deductions result in a mark which is lower than the attempt mark, then the attempt mark must be awarded
   - a mark between zero and the attempt mark is never awarded.

3. Worthless work is awarded zero marks. Some examples of such work are listed in the scheme and they are labelled as W1, W2,....etc.

4. The phrase “hit or miss” means that partial marks are not awarded – the candidate receives all of the relevant marks or none.

5. The *same* error in the *same* section of a question is penalised *once* only.

6. Special notes relating to the marking of a particular part of a question are indicated by an asterisk. These notes immediately follow the box containing the relevant solution.

7. Particular cases, verifications and answers derived from diagrams (unless requested) qualify for attempt marks at most.

8. A serious blunder, omission or misreading merits the ATTEMPT mark at most.

9. The phrase “and stops” means that no more work is shown by the candidate.

10. Accept the best of two or more attempts – even when attempts have been cancelled.

11. Allow comma for decimal point, e.g. €5.50 may be written as €5,50.

QUESTION 1

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 marks**	**Att 7**
Part (c)	**20 marks**	**Att 7**

Part (a) **10 marks** **Att 3**

There are 240 eggs in a box.
2.5% of the eggs are broken.
Find the number of eggs that are broken.

(a) **10 marks** **Att 3**

2.5% of 240	240 / 100	2.5% = 1/40	100 – 2.5 = 97.5 %	100 – 2.5 = 97.5 %	..3m
= 0.025 x 240	= 2.4	240 / 40	2.4 x 97.5 = 234	0.975 x 240 = 234	..7m
= 6	2.4 x 2.5 = 6	= 6	240 – 234 = 6	240 – 234 = 6	..10m

* Correct answer without work: 10 marks. Incorrect answer without work: no marks.
* 240 / 2.5 = 96: Apply B3+B4, i.e. 4m. Or, 240 x 100 = 24 000 is B3 + B4, i.e.4m.
* Allow candidates to use a comma instead of a decimal point, e.g. 2,5 for 2.5.

Blunders (-3)
B1 Decimal error, e.g. 2.5 x 240, or 0.25 x 240, or 0.0025 x 240, or gets 25%.
B2 Gets 1% = 2.4 and stops, or gets 97.5% = 234 and stops.
B3 Multiplies by 100, or omits the 100, in method I or II..
B4 Divides by 2.5, or omits 2.5 altogether, in method I or II.
B5 $\frac{240 \times 100}{2.5} = 9600$, i.e. inverted fraction.

Slips (-1)
S1 Numerical error.

Attempts (3 marks)
A1 Mentions 100 or 1/40 or 97.5.
A2 240 divided correctly by a number other than 40, 100 or 2.5.

Worthless (0)
W1 2.5% = 1/2.5 and stops, or 2.5 % = 1/25 and stops.
W2 2.5 / 240 = 0.0104 and stops.

Part (b) **20 (5, 10, 5) marks** **Att (2, 3, 2)**

The standard rate of income tax is 20% and the higher rate is 42%. Orla has a gross income of €58 000 for the year and a standard-rate cut-off point of €35 000. **(i)** Calculate the amount of tax due at the standard rate. **(ii)** Calculate the total amount of gross tax due. **(iii)** Orla has tax credits of €3 400 for the year. After tax is paid, what is Orla's income for the year?

(b)(i) **5 marks** **Att 2**

35 000 x 0.2 = 7 000	35 000 x 20/ 100 = 7 000	1/5 of 35000 = 7 000	1% of 35000 = 350 … 2m 20% = 7 000 …5m

* Correct answer without work: 5 marks. Incorrect answer without work: no marks.
* Marks are non-transferable between parts of (b), i.e. no retrospective marking allowed.
* Premature use of 3400 in (b)(i) and/or (b)(ii) -- apply one blunder in (b)(i) or (b)(ii).
* If candidates do (b) all in one, they can get the marks for the individual parts.

Blunders (-3)
B1 Finds 42% of 35 000 (= 14 700).
B2 Finds 20% of €58 000 or €23 000. These only.
B3 Multiplies by 100.
B4 Decimal error.
B5 Percentage-fraction error, e.g. 20% = 1/20th used.

Attempts (2 marks)
A1 20% = 20/100 or 1/5 or 0.2, and stops.
A2 Finds a percentage, other than 42% or 20%, of €35 000 correctly, e.g. gets 50%, or 10%. of €35 000 and stops. Could be used.
A3 *Uses* 35 000 in some way, e.g. 20% of 35 000 written, or 35 000 x 80 written, and stops.

Slips (-1)
S1 Numerical error, e.g. 35 000 x 0.2 = 7 500.

Worthless (0)
W1 20% = 1/20 and stops, or 20% = 100/20 and stops.
W2 Finds an irrelevant percentage of a sum of money other than €58 000, €35 000 or €23 000.

(b)(ii) **10 marks** **Att 3**

58 000 – 35 000 = 23 000 … 3m
23 000 × 0.42 = 9660 …7m
Total: 7000 + 9660 = 16 660 ..10m

* Correct answer without work: 10 marks. Incorrect answer without work: no marks.
* Accept without penalty the use of candidate's answer from (b)(i.)
* 3 steps: deduct 35000, get 42% of remainder, add 7000. B(-3) for each step missing, e.g. gets 42% of 58000 (= 24360) or of 35 000 (= 14 700), and stops => B1+B3=> 4m .
20% of 58 000 (= 11 600) and stops => B1 + B2 + B3 => att 3m.

Blunders (-3)
B1 Gets 42% of 35 000 or 58 000 and adds on 7 000, i.e. 35 000 not deducted from 58 000.
B2 Incorrect tax rate used, e.g. 20%. of 23 000 (= 4 600)
B3 Does not add on 7 000, or answer from (b)(i).
B4 Multiplies by 100.
B5 Decimal error.
B6 Percentage-fraction error, e.g. 42% = 1/42 used.

Attempt (3 marks)
A1 58 000 – 35 000 = 23 000 and stops.
A2 Mentions 42 / 100 or 0.42, and stops.
A3 Adds 7000, or the answer (b)(i), to a relevant sum of money.

Worthless (0)
W1 42% = 1/42 and stops.

(b)(iii) **5 marks** **Att 2**

16 660 − 3400 = 13 260	58 000 − 16 660 = 41 340 … 2m
58 000 − 13 260 = 44 740	41 340 + 3 400 = 44 740 …5m

Blunders (-3)
B1 Subtracts 3400 from incorrect amount, e.g. 58 000 – 3 400 = 54 600, and continues.
B2 Adds 3 400 to tax bill, or ignores 3 400 in this part.
B3 Finds 13 260 and stops, or finds 41 340 and stops.

Slips (-1)
S1 Misreads own work e.g. 16660 becomes 16600

Attempts (2 marks)
A1 Adds or subtracts 3 400 to/from a relevant sum of money.

Part (c)	**20 (5, 5, 10) marks**	**Att (2, 2, 3)**

A faulty petrol pump actually delivers 1.02 litres of petrol for every 1 litre that the pump registers. During one day the pump registers 2650 litres.
(i) What was the actual volume of petrol delivered?
(ii) Customers paid 85 cent for every litre of petrol registered.
Find the total amount paid for the petrol.
(iii) If the pump had registered the correct volume delivered, how much more would have been paid?

(c)(i)	**5 marks**	**Att 2**

Delivered: 2650 ×1.02 = 2703	2650 x .02 = 53 … 2m 2650 + 53 = 2703 …5m

* Correct answer without work: 5 marks. Incorrect answer without work: no marks.
* No retrospective marking or transfer of marks between parts of (c).

Blunders (-3)
B1 Divides by 1.02, e.g. 2650 / 1.02 = 2598.
B2 Decimal error, e.g. 2650 x 1.2 = 3180. (Caution: In (c)(iii), 3180 x 0.85 = 2703).
B3 2650 x 0.98 (= 2597), or 2650 / 0.98 (= 2704).
B4 Does not add on 53 (in method II).

(c)(ii)	**5 marks**	**Att 2**

Amount paid 2650 × 0.85 = €2252.50	Amount paid 2650 × 85 c …2m = 225250 c …5m

* Correct answer without work: 5 marks. Incorrect answer without work: no marks.
* If 2650 x 85 = 225250, assume answer is in cent: award 5m.
* Apply benefit of doubt re units (€ or c) but if units specified, mark accordingly.

Blunders (-3)
B1 Decimal error (e.g. €225250)
B2 Works on 2703 litres, e.g. 2703 x 0.85 (= 2297.55), or candidate's answer (c)(i) x 0.85.
B3 Incorrect operation used, e.g. divides instead of multiplies. Each time.

(c)(iii)	**10 marks**	**Att 3**

2703 × €0.85 = €2297.55 €2297.55 − €2252.50 = €45.05	2703 – 2650 = 53 litres 53 × €0.85 = €45.05	€0.85 x 0.02 …3m = €0.017 …7m 2650 x €0.017 = 45.05 ..10m
IV: 2650 x 1.02 x €0.85 …3m = 2297.55 …7m €2297.55 − €2252.50 = €45.05 ..10m		

* Accept use of candidate's answers from (c)(i) and (c)(ii).
* No penalty for incorrect order of subtraction of candidate's correct price difference is found, or for presenting a negative answer or discarding the minus sign.
* 45.05 or 4505 (these only), without work: 10 marks. (i.e. we assume units correct). Others without work: no marks.

Blunders (-3)
B1 Decimal error.
B2 Leaves answer at €2297.55. Unfinished.
B3 Incorrect operation used, e.g. adds instead of subtracts, or divides instead of multiplies. Each time

Attempts (3 marks)
A1 Step 1 of any of method above, and stops, e.g. gets 53 litres.

A2 Makes some use of 2703 or answer (c)(i), and stops.

QUESTION 2

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 marks**	**Att 6**
Part (c)	**20 marks**	**Att 7**

Part (a) **10 marks** **Att 3**

Find the value of $3(2p - q)$ when $p = -4$ and $q = 5$.

(a) **10 marks** **Att 3**

$3(2p - q) = 3.[2(-4) - 5]$	$3(2p - q) = 6p - 3q$ …3m
$= 3.[-13]$	$= 6(-4) - 3(5)$ …7m
$= -39$	$= -39$ ..10m

* Correct answer without work: 10 marks. Incorrect answer without work: 0 marks.

Blunders (-3)

B1 Substitution error, once for p and once for q; but if p and q swapped, then once overall.
B2 Bracket error, e.g. $3[-8 - 5] = -24 - 5$, or $3(2p - q) = 6p - q$. Apply once.
B3 Sign error, e.g. $-8 - 5 = +13$. Apply each time.
B4 $p = 4$ substituted (makes question easier).
B5 Adds or subtracts instead of multiplying, e.g. $6(-4) - 5 = 6 - 4 - 5$.

Misreadings (-1)

M1 $q = -5$ substituted, e.g. $3[2(-4) - (-5)]$ or $3[2(-4) + 5]$.

Attempts (3 marks)

A1 Some correct substitution, e.g. for p or q or both, and stops.
A2 Some correct multiplication of brackets, or some correct calculation.

Worthless (0)

W1 Solves an equation, unless marks gained for substitution.

Part (b) **20 (10, 10) marks** **Att (3, 3)**

(i)	Solve $2x^2 - 7x + 3 = 0$
(ii)	Show that $x - 2$ is a factor of $x^3 - 3x^2 - x + 6$.

(b)(i) **10 marks** **Att 3**

$(x - 3)(2x - 1) = 0$ …7m $x = 3,\ x = ½$ ..10m	$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ ...3m $x = \dfrac{-(-7) \pm \sqrt{7^2 - 4(2)(3)}}{2(2)}$ ..7m $= \dfrac{7 \pm \sqrt{49 - 24}}{4} = \dfrac{7 \pm \sqrt{25}}{4} = \dfrac{12}{4}$ or $\dfrac{2}{4} = 3$ or $\dfrac{1}{2}$ ..10m

* Both answers correct by Remainder Theorem or a graph:10 marks. See B4 and A4.

* Allow without penalty the implied use of " = 0".

* One answer correct (e.g. x = 3) without work: att 3m; but if verified, give 7m. (See B4).

* (x – 3)(2x – 1) and stops, 4m (i.e. Note 2 + B2) ; but m(x +3)(2x – 1) and stops, att 3 (i.e Note 2 + B1 + B2). But $2x^2 - 7x = -3$ and stops is worthless (insufficient for att.).

Blunders (-3)

B1 Incorrect factors. Apply once.

B2 Incorrect or no roots from factors. Apply once.

B3 Quadratic formula error (in formula, substitution or simplification). Each time to a maximum of 2 blunders.

B4 One answer correct by trial and error, or from a graph, e.g. finds f(3) = 0.

Attempts (3 marks)

A1 Correct quadratic formula and stops.

A2 Incorrect quadratic formula, with a max of one error, and with some correct substitution, and stops.

A3 Tries to find factors, e.g. (x …?)(2x …?), or $2x^2 - 6x - x + 3$.

A4 Effort to use Remainder Theorem, e.g. finds f(1) and stops.

Worthless (0)

W1 Solves linear equation.

(b)(ii) **10 marks** **Att 3**

I: x = 2 or f(2) …3m $= (2)^3 - 3(2)^2 - 2 + 6$ …7m $= 8 - 12 - 2 + 6$ . ..10m	II: $x - 2 \,/\, x^3 - 3x^2 - x + 6$ with quotient $x^2 - x - 3$ …set up 3m $x^3 - 2x^2$ …7m $-x^2 - x$ $-x^2 + 2x$ $-3x + 6$ $-3x + 6$ ..10m
III: 2 | 1 − 3 −1 6 …3m 2 −2 −6 …7m add −1 −3 0 ..10m	IV: −2 | 1 − 3 −1 6 …3m −2 2 6 …7m subtract −1 −3 0 ..10m

Blunders (–3)

B1 Error in algebraic division, to a max. of 2 blunders. e.g. error in mult., division, addition, subtraction (e.g. signs), cancellations etc. Caution: Note the 2nd step in II.

B2 Mathematical error in indices or brackets, e.g. $(2)^3 = 6$, or $3(2)^2 = 36$. Once per line if consistent.

B3 f(− 2) found, or divides by x + 2. Caution: Note the 2nd step in method II.

B4 Missing or incomplete step.
[Step could assume previous step, e.g. In method I, step 2 assumes Step 1 is fine].

Slips (–1)

S1 Arithmetical slips in method I (but not sign errors).

Attempts (3 marks)

A1 x = 2 and stops, or f(2) and stops, or sets up division and stops. (i.e. First line).

A2 Effort to use Remainder Theorem, e.g. substitutes 1 and stops.

A3 Differentiates the cubic, with any term correct. (Newton-Raphson relevant).

Worthless (0)

W1 Substitutes (x – 2), i.e. $f(x-2) = (x-2)^3 - 3(x-2)^2 - (x-2) + 6$ and continues.

Part (c) **20 (5, 5, 10) marks** **Att (- , - , 3)**

(i) Evaluate $8^{\frac{1}{3}}$.

(ii) Express $4^{\frac{1}{4}}$ in the form 2^k, $k \in Q$.

(iii) Solve for x the equation

$$\left(8^{\frac{1}{3}}\right)\left(4^{\frac{1}{4}}\right) = 2^{5-x}.$$

(c)(i) **5 marks** **Hit/Miss**

$8^{1/3} = (2^3)^{1/3}$ $= 2$	$8^{1/3} = \sqrt[3]{8}$ $= 2$ ...5m Hit / Miss	By calculator $8^{1/3} = 2$ ...5m Hit/Miss

* Correct answer: 5 marks. Incorrect answer: no marks.

(c)(ii) **5 marks** **Hit / Miss**

$4^{\frac{1}{4}} = \left(2^2\right)^{\frac{1}{4}}$ $= 2^{\frac{1}{2}}.$	$4^{1/4} = \sqrt[4]{4}$ or $\sqrt[4]{2^2}$ $= 2^{2/4}$	$4^{1/4} = \sqrt{\sqrt{4}} = \sqrt{2}$ $= 2^{1/2}$ ...5m Hit/Miss

* Correct answer: 5 marks. Incorrect answer: no marks.

* Allow $2^{0.5}$ full 5 marks.

(c)(iii) **10 marks** **Att 3**

$$2.2^{\frac{1}{2}} = 2^{5-x} \quad \text{...3m}$$
$$2^{1.5} = 2^{5-x}$$
$$1.5 = 5 - x$$
$$x = 3.5 \quad \text{..10m}$$

* Accept, without penalty, use of candidate's answers from (c)(i) and (ii) in this part.

* $2(1.414) = 2^{5-x}$ may be correctly solved using logs: 10m.

* N.B. Candidates can only get 0m, 3m or 10 m (fully correct) for this question.

Attempts (3 marks)

A1 Any correct substitution (i.e. the correct or candidate's values from previous work) and stops.

A2 Any correct and relevant use of indices, e.g. $32 = 2^5$.

A3 $2(1.414) = 2^{5-x}$ and stops, or continues with $2.818 = 2 - 5x$ and solves ($x = 2.172$).

QUESTION 3

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 marks**	**Att 7**
Part (c)	**20 marks**	**Att 6**

Part (a) **10 marks** **Att 3**

Solve for x

$$2x = 3(5 - x).$$

(a) **10 marks** **Att 3**

$2x = 15 - 3x$ ...3m $2x + 3x = 15$ $5x = 15$ ...7m $x = 3$ or $15/5$ ..10m	$\frac{2x}{3} = 5 - x$ ...3m $\frac{2x}{3} + x = 5$ $\frac{2x + 3x}{3} = 5$ $5x = 15$ ...7m $x = 3$ or $15/5$ ..10m	III: $2(3) = 3(5 - 3)$ ... 7m $6 = 6$ or $x = 3$ or 'true' ..10m

* Correct answer without work: 10 marks.
* Incorrect answer by trial and error: Att 3, e.g. see A3 below.
* Allow answer 15/5 for 10 marks even if subsequently given as 1/3. But see B4.

Blunders (-3)

B1 Error multiplying out brackets, e.g. $2x = 15 - x$, or $8 - x$, and continues. Once. See A1.
B2 Transposition error, e.g. $2x - 3x = 15$, or $2x - 3 = 5 - x$, or $2x - 3(x + 5) = 0$, or $5x = 15 \Rightarrow x = 5$.
B3 Incorrect evaluation of $(2x/3) + x$ in method II, e.g. $(2x/3) + x = 3x/3$.
B4 Finishes incorrectly, or fails to finish last step, e.g. $5x = 15 \Rightarrow x = 5/15$ or $x = 10$.

Misreadings (-1)

M1 Misreading if it doesn't oversimplify the problem, e.g. $2x = 3(5 + x)$.

Attempts (3 marks)

A1 One step or part of step correct, e.g. $2x = 15 - x$ and stops.
A2 Any correct transposition and stops; but see W2.
A3 Effort at trial and error, e.g. substitutes 1 for x.

Worthless (0)

W1 Incorrect answer without work.
W2 $5 - x = 0 \Rightarrow x = 5$, and/or $2x = 0 \Rightarrow x = 0$.

Part (b) **20 marks** **Att 7**

Solve for x and y

$$x + y = 1$$
$$x^2 + y^2 = 13.$$

(b) **20 marks** **Att 7**

$y = 1 - x$	… 7m	$x = 1 - y$	… 7m
$x^2 + (1 - x)^2 = 13$	…8m	$(1 - y)^2 + y^2 = 13$	…8m
$x^2 + 1 - 2x + x^2 = 13$		$1 - 2y + y^2 + y^2 = 13$	
$2x^2 - 2x - 12 = 0$	…11m	$2y^2 - 2y - 12 = 0$	…11m
$x^2 - x - 6 = 0$	…11m	$y^2 - y - 6 = 0$	…11m
$(x - 3)(x + 2) = 0$	…14m	$(y - 3)(y + 2) = 0$	…14m
$x = 3$ or $x = -2$	…17m	$y = 3$ or $y = -2$	…17m
$y = -2$ or $y = 3.$	…20m	$x = -2$ or $x = 3.$	…20m
$(3, -2), (-2, 3)$		$(3, -2), (-2, 3)$	

* Sets of coordinates found by graphical method, or trial and error, or without work:
Two correct sets: if verified in both equations, 20 marks; if not verified, Att 7.
One correct set: if verified in both equations, Att 7; if not verified, no marks.
Both sets incorrect: no marks, whether tried to verify of not.

* Candidate finds first variables, substitutes into 2nd degree equation, finds correct and incorrect values and presents them all as solutions: no penalty (ignore excess answers). However, if only the incorrect ones are offered as solutions, apply B(-3).

* No additional marks from the point where the equation becomes linear.

* Allow $y = x - 1$ or $x = y - 1$ without penalty if $x^2 + (x - 1)^2 = 13$ or $(y - 1)^2 + y^2 = 13$ [=8m]. But, if $y = x - 1$ or $x = y - 1$ and stops: 0m. For $(1 - y)^2 + (1 - x)^2 = 13$, allow att 7m.

Blunders (-3)

B1 Squaring error, e.g. $(1 - x)^2 = 1 + x^2$ or $1 - x^2$. Apply once. See Note 3.
B2 Algebraic error when totting/simplifying, etc. Example: $x^2 - 2x - 12 = 0$.
Candidate may solve the quadratic generated without further penalty ($x = 4.6, -2.6$).
B3 Quadratic formula error (in formula, substitution or simplification). Each time to a maximum of 2 blunders (equivalent to two steps).
B4 Incorrect factors. Apply once.
B5 Incorrect root(s) from candidate's factor(s). Apply once.
B6 One value for x when two available, or one value for y when two available.
B7 Fails to find values of second variable. (B6 and B7 could both apply).
B8 Finds x but substitutes back into y (or vice versa)
B9 Transposition error, e.g. signs. Each time.

Attempts (7 marks)

A1 Correct quadratic formula and stops.
A2 An effort to find the *second* variable, having found with work of no value (not invented) the first variable.

Worthless (0)

W1 Incorrect values without work.
W2 Invented values substituted, and continues, e.g. $x + y = 1 \Rightarrow x = 0, y = 1$ or some such.
However may have $y = 1 - x$ or $x = 1 - y$ (i.e. Att 7) before inventing value(s).
W3 $x + y = 1 \Rightarrow x^2 + y^2 = 1$; or $x^2 + y^2 = 13 \Rightarrow x + y = \sqrt{13}$. Even if continued. See A2.

Part (c)	**20 (10, 10) marks**	**Att (3, 3)**

p is a positive number and f is the function $f(x) = (2x + p)(x - p)$, $x \in \mathbf{R}$.
(i) Given that $f(2) = 0$, find the value of p.
(ii) Hence, find the range of values of x for which $f(x) < 0$.

(c)(i)	**10 marks**	**Att 3**

I:	II:
$f(x) = 2x^2 - 2px + px - p^2$ …3m $= 2x^2 - px - p^2$ …4m $f(2) = 2(2)^2 - p.2 - p^2 = 0$ $= 8 - 2p - p^2 = 0$ $p^2 + 2p - 8 = 0$ $(p + 4).(p - 2) = 0$ …7m $p = 2$ ..10m	$f(2) = [2(2) + p].(2 - p)$ …3m $(4 + p).(2 - p)$ …4m $(4 + p).(2 - p) = 0$ …7m $p = 2$ ..10m

III:	
$(2x + p)(x - p) = 0$	…3m
$x = p$	…7m
$f(x) = f(2) \Rightarrow 2 = p$ *OR* $\Rightarrow x = 2 = p$.	..10m

* The " = 0" may be implied by subsequent work.
* No retrospective marking or transfer of marks between parts of (c).
* Work must be shown in (c)(i). See W1.
* If the p equation becomes linear, no additional marks from that point on.

Blunders (-3)
B1 Substitution error, e.g. 2 for p and then calculates x = 2 (and x = –1).
B2 Solves f(2) = 2 correctly for p (= 1.6).
B3 Transposition error, e.g. $8 - 2p - p^2 = 0 \Rightarrow p^2 - 2p + 8 = 0$. Once.
B4 Incorrect factors. Apply once.
B5 Incorrect roots from factors. Apply once.
B6 Index error, e.g. $2(2)^2 = 4^2 = 16$.
B7 Multiplication error. See Note 2 in 3(c)(ii).
B8 f(-2) evaluated instead of f(2).
B9 Quadratic formula error (in formula, substitution or simplification). Each time to a maximum of 2 blunders (equivalent to two steps).

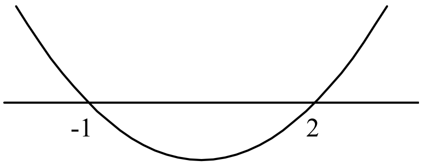
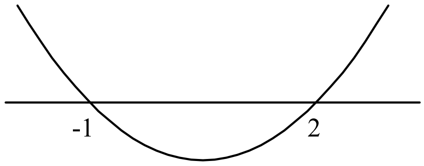
Misreadings (-1)
M1 p = 2, p = – 4. (Doesn't discard negative value).

Attempts (3 marks)
A1 Correct quadratic formula and stops.
A2 2 substituted for x, or partially substituted; or f(k) = 0 where $k \neq 2$ (i.e. trial and error).
A3 Effort to multiply out (2x + p)(x – p) with at least one term correct (including its sign).

Worthless (0)
W1 Answer without work, even if correct.
W2 2 substituted for p and stops. See B1 above.

(c)(ii) **10 marks** **Att 3**

<table>
<tr>
<td>I: $(2x + 2).(x - 2) < 0$ …3m

$f(x) = 0$ at $x = -1$ or $x = 2$

$f(x) < 0$ for $-1 < x < 2$ ..10m</td>
<td>II: $(2x + 2).(x - 2) = 0$ …3m
$2x^2 - 2x - 4 = 0$
$x^2 - x - 2 = 0$
$(x + 1)(x - 2) = 0$
$x = -1, \ x = 2$

$f(x) < 0$ for $-1 < x < 2$ ..10m</td>
</tr>
<tr>
<td colspan="2">III: $(2x + 2)(x - 2) < 0$ …3m
=> <u>*either*</u> $2x + 2 < 0$ and $x - 2 > 0$ <u>*or*</u> $2x + 2 > 0$ and $x - 2 < 0$
=> $x < -1$ and $x > 2$ ¦ $x > -1$ and $x < 2$ ..10m
impossible ¦ i.e. $-1 < x < 2$</td>
</tr>
</table>

* N.B. Candidates can only get 0m, 3m or 10m, unless M1 applies.
* Allow substitution of p from (c)(i), without penalty, even if p was incorrect.
* If f(x) was incorrectly multiplied out in (c)(i) and not re-done here, no penalty provided it was quadratic.
* Ignore inclusion of equals signs.
* Accept correct answer in non-technical language, e.g. "from –1 to 2".
* Allow candidate to indicate the answer on the graph or on a number line.
* Incorrect answer without work: no marks. Correct answer without work: att 3m.
* $(2x + p)(x - p) < 0$ and stops, is insufficient for attempt marks: 0m.
  $(2x + 2)(x + 4) < 0$: att 3m, even if finished.

Misreadings (-1)

M1 Misreading if not oversimplifying, e.g. solves $f(x) > 0$ or solves $f(x) < 0$ for $p = -4$.

Attempts (3 marks)

A1 Correct quadratic formula and stops.

A3 Effort to multiply out $(2x + p)(x - p)$ with at least one term correct (including its sign).

QUESTION 4

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 marks**	**Att 7**
Part (c)	**20 marks**	**Att 6**

Part (a) **10 marks** **Att 3**

Given that $i^2 = -1$, simplify

$$4(2-i) + i(3+5i)$$

and write your answer in the form $x + yi$, where $x, y \in \mathbf{R}$.

(a) **10 marks** **Att 3**

I:		II:		
$8 - 4i + 3i + 5i^2$	…7m	$4(2-i) = 8 - 4i$	…3m	Interchangeable…one right, 3m
$8 - 4i + 3i - 5$	…7m	$i.(3+5i) = 3i + 5i^2$	…7m	both right, 7m
$3 - i$	..10m	$= 3i - 5$	…7m	
		adding => $3 - i$	..10m	

* Correct answer without work: att 3. Incorrect answer without work: 0m.
* i = –1 or i = 1 used from the start: oversimplified, Att 3m is the maximum possible.
* If 3 – i = 0, ignore the "= 0".

Blunders (-3)

B1 Bracket error, e.g. 4(2 – i) = 8 – i, or 4(2 – i) = 6 – 4i. Apply once if consistent.
B2 $i^2 \neq -1$, or mishandles $5i^2$, e.g. $5i^2 = 5(-1) = -4$.
B3 $-4i + 3i = -12i^2$, or similar.
B4 Adds real and imaginary parts, e.g. $3i + 5i^2 = 8i$.
B5 Sign error when totting, e.g. – 4i + 3i = – 7i or i.
B6 Equates real and imaginary parts during process, e.g. answer 3 = i.
No penalty if rectified later to 3 – i.

Slips (-1)

S1 Numerical slip when adding real to real, or imaginary to imaginary. Each time.

Attempts (3 marks)

A1 Removes one bracket correctly, or partially correctly, and stops,
e.g. 4(2 – i) = 8 – 4i and stops, or = 8 – i and stops, or = 8 – 2i and stops.

Part (b) **20 (5, 5, 10) marks** **Att (2 ,2, 3)**

(i) Let $w = 1 - 2i$.
Plot w and $\overline{w}$ on an Argand diagram, where $\overline{w}$ is the complex conjugate of w.

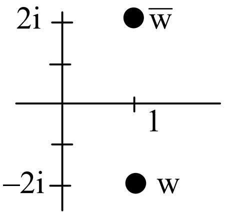
(ii) Solve $z^2 - 10z + 26 = 0$.
Write your answers in the form $a + bi$, where $a, b \in \mathbf{R}$.

(b)(i) **10 (5, 5) marks** **Att (2, 2)**

Plot w … 5m (Att 2m) (See A1)

$\overline{w} = 1 + 2i$ …2m

Plot $\overline{w}$ …5m (Att 2m) (See A2)



* If the axes are reversed they must be identified, or B1 applies.
* Unlabelled axes: assume horizontal axis is real, e.g. point (– 2, 1) plotted on unlabelled axes is B1.
* Points (– 2, 1) and (2, 1) plotted on unlabelled axes: award 2m for w and, 5 marks for $\overline{w}$, i.e. penalise reversal of axes once in (b)(i).
* Attempt marks *for drawing axes* can be awarded once only, for w or $\overline{w}$. See A1, A2.
* If 5m are gained for one part, candidate cannot get A1 for drawing axes in other part.
* One unnamed point plotted: if at (1, – 2) or (1,2), allow 5 marks; if not, assume it is w.
* If two unnamed dots in correct position on graph, apply benefit of doubt: award 5+5m.

Blunders (-3)
B1 Incorrect plotting of w, e.g. plots 1–2i at– i, or plots 1 – 2i as line from (1, 0) to (0, –2).
B2 Incorrect calculation of $\overline{w}$, e.g. $\overline{w} = -1 - 2i$, or $\overline{w} = -1 + 2i$.
B3 Incorrect plotting of candidate's $\overline{w}$.

Misreadings (-1)
M1 Takes w = 1 – 3i. Penalise once if w = 1 – 3i and $\overline{w}$ =1 + 3i correctly plotted. (4+5m)
M2 w and $\overline{w}$ correctly plotted but labels swapped. Apply misreading once. (4+5m).

Attempts (2 marks)
A1 A correct set of scaled axes (ticks sufficient). Apply once
A2 $\overline{w}$ incorrectly calculated, but result plotted correctly.

(b)(ii) **10 marks** **Att 3**

$$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ ...3m

$$= \frac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(26)}}{2(1)} \quad \textbf{or} \quad \frac{\mathbf{10} \pm \sqrt{\mathbf{100} - 104}}{2} \quad \textbf{or} \quad \frac{\mathbf{10} \pm \sqrt{-4}}{2}$$ ...7m

$$= \frac{10 \pm 2i}{2}$$ ...still 7m

$$= 5 \pm i$$ ...10m

Blunders (-3)
B1 Quadratic formula error (in formula, substitution or simplification). Each time to a maximum of two blunders.
B2 Sign error, e.g. $\sqrt{100 - 104} = \sqrt{4}$, and continues to answers of 4 and 6. (i.e. 7marks).
B3 $\sqrt{-4}$ = 2, giving answers of 4 and 6. (Award 7 marks). Relates to $i^2 \neq -1$.

Attempts (3 marks)
A1 Substitutes "correctly" into incorrect but relevant formula (with at most one error) and stops
A2 Effort at factorising, e.g. (z….?)(z….?), or z(z – 10) + 26.
A3 Effort at trial and error, e.g. z = 1 tried or z = 1 – 2i tried.

Part (c) **20 (10, 10) marks** **Att (3, 3)**

Let $z_1 = 5 + 12i$ and $z_2 = 2 - 3i$.

(i) Find the value of the real number k such that $|z_1| = k|z_2|$.

(ii) p and q are real numbers such that

$$\frac{z_1}{z_2} = p(q + i).$$

Find the value of p and the value of q.

(c)(i) **10 marks** **Att 3**

$\lvert z_1\rvert = \lvert 5+12i\rvert$ ...3m $= \sqrt{5^2+12^2}$ or $\sqrt{169}$ or 13 ...4m $\lvert z_2\rvert = \lvert 2-3i\rvert$ $= \sqrt{2^2+(-3)^2}$ or $\sqrt{13}$ ...7m $k = \frac{\sqrt{169}}{\sqrt{13}}$ or $\frac{13}{\sqrt{13}}$ or $\sqrt{13}$ or $\frac{13}{3.6}$ or 3.6 ..10m	II: $k = \frac{\lvert z_1\rvert}{\lvert z_2\rvert}$ ...3m $= \frac{\sqrt{5^2+12^2}}{\sqrt{2^2+(-3)^2}}$ …4m …7m $= \frac{13}{\sqrt{13}}$ ..10m
III: $\lvert z_1\rvert^2 = k^2.\lvert z_2\rvert^2$ ...3m $5^2+12^2 = k^2.[2^2+(-3)^2]$ ...7m $169 = k^2.[13]$ $k^2 = 13 \Rightarrow k = \sqrt{13}$ ..10m	IV: $z_1.\bar{z}_1 = k^2(z_2.\bar{z}_2)$ ...3m $25-60i+60i-144i^2 = k^2(4+6i-6i-9i^2)$ ...7m $169 = k^2(13)$ $k^2 = 13 \Rightarrow k = \sqrt{13}$ ..10m

* In methods I and II, finding $|z_1|$ and $|z_2|$ are interchangeable. First one correct, 4 marks; second one correct, 7 marks.

* Allow, without penalty, $\pm\sqrt{13}$ or equivalent answers.

* $z_2 = 2 + 3i$: treat as M(–1) each time (i.e. in both parts).

Blunders (-3)

B1 Incorrect modulus formula, i.e. error in $|z| = \sqrt{a^2+b^2}$ or in $|z|^2 = z.\bar{z}$.

B2 Incorrect substitution in correct formula, e.g. $|z_1| = \sqrt{5^2+144i^2} = \sqrt{25-144} = \sqrt{-119}$.

B3 Square root error, e.g. $\sqrt{25+144} = 5+12$. Apply each time.

B4 Adds real and imaginary parts.

B5 Not squaring k if using method III or IV, e.g. 169 = k(13). Apply once.

Attempts (3 marks)

A1 Substitutes correctly for z_1 <u>or</u> z_2 <u>into</u> $|z_1| = k.|z_2|$, and stops.

A2 $\sqrt{a^2+b^2}$ and stops, or Coordinate Geometry distance formula correct and stops.

A3 $\sqrt{a^2-b^2}$ *with* some correct substitution, or $a^2 + b^2$ *with* some correct substitution or distance formula with *one* error and *with* some correct substitution, and stops. These only.

A4 $|z|^2 = z.\bar{z}$ (or equivalent), or states $\bar{z}_1 = 5-12i$ and stops, or $\bar{z}_2 = 2+3i$ and stops.

A5 No modulus found, att 3 is the maximum mark – unless method III or IV is used.

Worthless (0)

W1 $\sqrt{a^2-b^2}$ *without* substitution, or $a^2 + b^2$ *without* substitution.

W2 Other incorrect formula with/without substitution.

(c)(ii) **10 marks** **Att 3**

$$\frac{z_1}{z_2} = \frac{5+12i}{2-3i} = \frac{5+12i}{2-3i} \times \frac{2+3i}{2+3i} \quad \text{... 3m}$$

$$= \frac{10+15i+24i+36i^2}{4+9} \quad \text{...4m}$$

$$= \frac{-26+39i}{13} \text{ or } -2+3i \quad \text{...still 4m}$$

$pq + pi = -2 + 3i \Rightarrow p = 3$ ...7m

$3q = -2 \Rightarrow q = -2/3$ ..10m

* Correct values for p and q without work, but both fully tested: 10 marks.
If both not fully tested: no marks.

* May multiply across by z_2 (Att 3) and correctly form the resulting simultaneous equations [= 4m], solve correctly for one variable [=7m], and finish correctly [=10m]: mark on slip and blunder. See A4.

Blunders (–3)

B1 Incorrect conjugate, e.g. multiplies by (2 – 3i) / (2 + 3i) to get (46 + 9i) / 13.
B2 $i^2 \neq -1$. Apply once in (c)(ii).
B3 Each omitted or incorrect term when multiplying; max of 2 (1 on num., 1 on denom.).
B4 Incorrect adding of terms, e.g. not real to real, or imaginary to imaginary.
B5 Denominator not real after multiplication, or forgets to multiply denom. by conjugate.
B6 Multiplies out numerators and denominators and stops. Implies B10 also.
B7 Inverts the fraction (e.g. when top and bottom done separately), e.g. 13/(– 26 + 39i).
B8 Error multiplying out brackets, e.g. p(q + i) = q + pi => q = – 2 and p = 3.
B9 (– 26 + 39i) / 13 = –2 + 39i, or –26 + 3i.
B10 Doesn't (or cannot) solve simultaneous equations (for p and q) or error solving them. e.g. from –2 + 3i concludes that p = –2 and q = 3. (i.e. loses final 3 marks).

Slips (–1)

S1 Numerical slip when adding real to real, or imaginary to imaginary.
S2 p = 3 and q = (–2/3)*i*, retains the *i* in answer.

Misreadings (–1)

M1 z_1 = 5 – 12i used, or similar misreading.

Attempts (3 marks)

A1 Substitutes for z_1, z_2, or z_1/z_2, and stops, e.g. (5 + 12i)/ (2 – 3i) and stops.
A2 Correct conjugate of denominator, and stops.
A3 Any correct and relevant multiplication.
A4 Multiplies across by z_2 and stops. See Note 2 above.

QUESTION 5

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 marks**	**Att 7**
Part (c)	**20 marks**	**Att 7**

Part (a) **10 marks** **Att 3**

> The first term of an arithmetic sequence is 40 and the common difference is –5.
> Write down the first five terms of the sequence.

(a) **10 marks** **Att 3**

a = 40, d = – 5 … 3m Terms: 40, 35, 30, 25, 20 ..10m	$Tn = a + (n-1)d$ … 3m $T_1 = 40$ $T_2 = 40 + 1(-5) = 35$ $T_3 = 40 + 2(-5) = 30$ $T_4 = 40 + 3(-5) = 25$ $T_5 = 40 + 4(-5) = 20$ => Terms: 40, 35, 30, 25, 20 ..10m

* Correct terms without work: 10 marks.
* No penalty for notation errors , e.g. r written instead of d, but otherwise correct.

Blunders (-3)

B1 Adds instead of subtracts, i.e. 40, 45, 50, 55, 60, with or without work.
B2 Incorrect *a* or *d* used, unless rectified later. Or swaps *a* and *d* (one blunder).
B3 Incorrect *Tn* of AP formula, namely:
$Tn = a + nd$, $Tn = a + (n+1)d$, $Tn = a - (n-1)d$, $Tn = a - (n+1)d$. Others worthless.
B4 Each incorrect or missing term without work, but note that the first term might be implied e.g. $40 - 5 = 35 = T_2$.

Slips (-1)

S1 Arithmetic error.

Misreadings (–1)

M1 Misreads as a different five (consecutive) terms, e.g. from T_2 to T_6, or from T_6 to T_{10}.

Attempts (3 marks)

A1 $a = 40$ or $T_1 = 40$, and/or $d = -5$ and stops.
A2 Correct *Tn* of AP and stops.
A3 Correct *Sn* of AP with some correct substitution, and stops. See W3.
A4 $T_2 = 35$ and stops.

Worthless (0)

W1 All terms incorrect
W2 Any use of GP or ratio, unless mentions $a = 40$ and/or $d = -5$.
W3 *Sn* formula of AP (even if correct) unless some correct substitution redeems it.

Part (b) **20 (10, 5, 5) marks** **Att (3, 2, 2)**

The *n*th term of an arithmetic series is given by

$$T_n = 1 + 5n.$$

(i) The first term is *a* and the common difference is *d*. Find the value of *a* and the value of *d*.

(ii) Find the value of *n* for which $T_n = 156$.

(iii) Find S_{12}, the sum of the first 12 terms.

(b)(i) **10 marks** **Att 3**

a or $T_1 = 1 + 5 = 6$	…3m	} Interchangeable
$T_2 = 1 + 2(5) = 11$	…7m	
$\therefore d = 11 - 6 = 5$	..10m	

* Finding T_1 and T_2 are interchangeable: One term correct, 3m; both terms correct, 7m.

* Correct answers without work: full marks. d = 5 without work merits 7 marks.

* May use $d = T_n - T_{n-1} = 1+5n - \{1 + 5(n-1)\} = 5$. Mark on slip/blunder

Blunders (-3)

B1 $d = 6 - 11 = -5$, but treat d = 6 – 11 = 5 as an error rectified and don't penalise.

B2 $1 + 5n = 6n$.

Slips (-1)

S1 Arithmetic error calculating T_1 or T_2..

Attempts (3 marks)

A1 $d = T_2 - T_1$ and stops, or $T_2 = a + d$ and stops.

A2 $Tn = a + (n - 1)d$ and stops.

A3 Some substitution into $Tn = 1 + 5n$ and stops.

Worthless (0)

W1 Formula for *Sn* of an AP or GP, and stops.

(b)(ii) **5 marks** **Att 2**

1 + 5n = 156	a + (n – 1) d = 156 6 + (n – 1)5 = 156 1 + 5n = 156	156 – 6 = 150	…2m	6, 11, 16, 21, 26, 31, 36, 41, …., 156 Lists all the terms and counts them to get 31 …5m
5n = 155 n = 31	5n = 155 n = 31	÷ 5 = 30 + 1 = 31	..5m	

* Correct answer without work: 5 marks.

* Accept candidate's *a* and *d* from (b)(i) above

Blunders (-3)

B1 Incorrect substitution of *a* or *d* or *n* into correct AP formula.

B2 Incorrect *Tn* of AP formula, namely:
$Tn = a + nd$, $Tn = a + (n+1)d$, $Tn = a - (n - 1)d$, $Tn = a - (n + 1)d$. Others worthless.

B3 Each blunder in solving equations, e.g. transposition, or $1 + 5n = 156 \Rightarrow 6n = 156$.

B4 Incorrect procedure (e.g. fails to add 1), if method III used (rule of thumb method).

B5 Incorrect term or incorrect total when counted, if method IV used.

B6 $Tn \neq 156$ and *continues*.

Attempts (2 marks)
A1 Correct *a* or *d* values stated or substituted.
A2 $Tn = a + (n - 1)d$ and stops.
A3 Finds T_{156}.

Worthless (0)
W1 Any use of GP or ratio, provided no mention of candidate's values of *a* and/or *d*.
W2 Incorrect answer without work.

(b)(iii) **5 marks** **Att 2**

I: $S_n = \frac{n}{2}\{2a + (n-1)d\}$ ...2m $S_{12} = \frac{12}{2}\{2(6) + 11(5)\}$ …still 2m $= 402$ … 5m	II: $Sn = \frac{n}{2}\{a + T_n\}$ ...2m $S_{12} = (12/2)\{6 + T_{12}\}$ ...still2m $T_{12} = a + 11d = 6 + 11(5) = 61$.: $S_{12} = 6\{6 + 61\} = 402$ ...5m
III: $S_{12} = 6 + 11 + 16 + 21 + 26 + 31 + 36 + 41 + 46 + 51 + 56 + 61$ …2m $= 402$ …5m	

* Candidate may use values of *a* and *d* found in (b)(i). Correct answer without work: 5m.

Blunders (–3)
B1 Incorrect relevant *Sn* of AP formula *used*. Incorrect relevant = one error only. See W2
B2 Incorrect substit. of *a, n* or *d* into correct AP formula, or *a* and *d* swapped. Apply once.
B3 Bracket error in simplifying.
B4 Each error or omission in list (excluding knock-on errors).

Slips (-1)
S1 Arithmetic error, e.g. totting in method III.

Attempts (2 marks)
A1 Correct *a*, and/or *d* values stated or substituted; or correct *n* value substituted.
A2 Correct *Sn* formula of AP and stops, including $Sn = (n/2)\{ a + L\}$ where *L* is the last term (T_{12} in this case).
A3 $T_2 = \underline{11}$ and stops, or $T_{12} = 61$ and stops.
A4 $S_{12} = 6 + \underline{11} + ….$ and stops.

Worthless (0)
W1 Incorrect answer without work.
W2 Incorrect *Sn* formula for AP with one or more errors, and stops.
W3 Formula for GP and stops, but if *a* = 6 and/or '*r*' = 5 mentioned, then award att 2m.

Part (c)	**20 (10, 5, 5) marks**	**Att (–, 2, 2)**

> The first term of a geometric series is 1 and the common ratio is −4.
> **(i)** Write down the first three terms of the series.
> **(ii)** Find S_6, the sum of the first 6 terms.
> **(iii)** Show that $16S_4 - 3 = S_6$, where S_4 is the sum of the first 4 terms.

(c)(i)	**10 marks**	**Hit / Miss**

$T_n = ar^{n-1} \Rightarrow T_1 = a = 1$ …3m Hit/Miss	Terms are: 1 …3m Hit/Miss
$T_2 = ar = 1(-4) = -4$ + 4m Hit/Miss = 7m	-4 + 4m Hit/Miss = 7m
$T_3 = ar^2 = 1(-4)^2 = 16$ + 3m Hit/Miss = 10m	16 + 3m Hit/Miss = 10m

* e.g. : 1, 4, 16 merits 3m + 0m + 3m = 6 marks. 1 – 4 – 16 merits 3m + 4m + 0m = 7 marks.

(c)(ii)	**5 marks**	**Att 2**

$S_6 = \frac{1-(-4)^6}{1+4}$ $= \frac{1-4096}{5}$	$S_6 = \frac{(-4)^6 - 1}{-4-1}$ $= \frac{4096-1}{-5}$	$S_6 = 1 - 4 + 16 - 64 + 256 - 1024$ …2m
$= -819$	$= -819$	$= -819$ …5m

(c)(iii)	**5 marks**	**Att 2**

$S_4 = \frac{1-(-4)^4}{1+4} = \frac{1-256}{5} = -51$ …2m	$1 - 4 + 16 - 64 = -51$ …2m
$16.S_4 = -816$ ...still 2m	16.S4 = -816 …still 2m
$-816 - 3 = -819$ ...5m	$-816 - 3 = -819$ …5m

* The same marking scheme applies, *separately*, to (c)(ii) and (c)(iii);
i.e. the same error made in both parts should be penalised in both parts.

* No penalty if $r = 4$ is used in the *numerator* in method I or II of (ii) or method I of (iii).

Blunders (–3)

B1 $r = 4$ used in denominator. See note 2 above.

B2 Incorrect substitution into correct *Sn* of GP formula, including *a* and *r* swapped.

B3 Each incorrect or omitted term using the list method.

B4 Error in indices, e.g. $1 - (-4)^6 = 5^6$ in (ii), or $1 - (-4)^4 = 5^4$ in (iii).

B5 Error in signs.

Attempts (2 marks)

A1 $a = 1$ and/or $r = -4$ or candidate's value of *r,* or correct value of *n* substituted.

A2 Correct *Sn* formula of GP and stops.

A3 $S_6 = T_1 + T_2 + T_3 + \ldots + T_6$ and stops, or $S_6 = a + ar + ar^2 + \ldots + ar^5$ and stops.

A4 $Sn = \frac{a(r^n + 1)}{r+1}$ or $Sn = \frac{a(1-r)^n}{1-r}$; or $Sn = \frac{a(r-1)^n}{r-1}$. These only and with some substitution.

A5 Correct *Tn* of GP and stops.

Worthless (0)

W1 Incorrect answer without work.

W2 Any use of AP or difference, but if $a = 1$ or $r = -4$ is stated/used, apply A1 (2 marks).

QUESTION 6

Part (a)	**10 marks**	**Att 3**
Part (b)	**40 marks**	**Att 11**

Part (a) **10 marks** **Att 3**

> Let $g(x) = 1 - kx$.
> Given that $g(-3) = 13$, find the value of k.

(a) **10 marks** **Att 3**

g(–3) = 1 – k(–3) *or* 1 + 3k …3m 13 = 1 – k(–3) *or* 1 + 3k …7m 3k = 12 k = 4 or 12/3 ..10m	k = 0 => 1 – 0(–3) = 1 k = 1 => 1 – 1(–3) = 4 k = 2 => 1 – 2(–3) = 7 k = 3=> 1 – 3(–3) = 10 k = 4 => 1 – 4(–3) = 13 ..10m

* 3 steps in method I: substitution, equating 13, solving.
* Correct answer without work: 10 marks.
* May err and rectify: cancel penalty, e.g. g(13) = 1 + 3k => 13 = 1 + 3k, etc.
* – 3 = 1 – k(13) => 13k = 4 => k = 4/13 : apply B4 *once* (i.e. swaps the –3 and 13).
* – 3 = 1 – kx => kx = 4 and stops is B4 + B3 + B5 => att 3m.
* 13 = 1 – kx => k = –12/x, and stops, is B3 + B5.

Blunders (-3)
B1 Sign error, each time. Note: Finds – k = – 4 correctly, and stops: apply B1.
B2 Transposition error, e.g. 3k = 12 => k = 3/12.
B3 Each omitted substitution
B4 Each incorrect substitution, apart from M1 below, and note 4 (swapping substitutions).
B5 Missing or incomplete step, e.g. 3k = 12 and stops.

Misreadings (-1)
M1 Substitutes – 3 for k in 1 – kx, i.e. 13 = 1 – (-3)x => x = 4

Attempts (3 marks)
A1 g(–3) = 1 – k(–3) and stops.
A2 Any value substituted for k and stops, i.e. effort at trial and error,
e.g. 1 – 1x or 1 – 1(– 3).
A3 One correct substitution for x or g(x), and stops.
A4 $g(x) = 1 + 3k \Rightarrow k = \dfrac{g(x) - 1}{3}$ and stops.

Part (b) **40 (5, 5; 10, 5, 5, 5, 5) marks** **Att (2, 2; 3, 2, 2, - , -)**

Let $f(x) = x^3 - 3x^2 + 1, \quad x \in \mathbf{R}$.

(i) Find $f(-1)$ and $f(3)$.

(ii) Find $f'(x)$, the derivative of $f(x)$.

(iii) Find the co -ordinates of the local maximum point and of the local minimum point of the curve $y = f(x)$.

(iv) Draw the graph of the function f in the domain $-1 \le x \le 3$.

Use your graph to:

(v) estimate the range of values of x for which $f(x) < 0$ and $x > 0$

(vi) estimate the range of values of x for which $f'(x) < 0$.

(b)(i) **10 (5, 5) marks** **Att (2, 2)**

$$f(-1) = (-1)^3 - 3(-1)^2 + 1 = -1 - 3 + 1 \quad \text{...2m}$$
$$= -3 \quad \text{...5m}$$

$$f(3) = (3)^3 - 3(3)^2 + 1 = 27 - 27 + 1 \quad \text{...2m}$$
$$= 1 \quad \text{...5m}$$

- * Correct answer without work: 5m. Incorrect answer without work: 0m.
- * Misreading f(x), if not oversimplified (see M1), is penalised once in all of part (b).
- * Marking scheme below applies separately to each part of (b)(i).
- * Takes $f(x) = x^2 - 3x^2 + 1$ in each part of (b): oversimplified in each part (att 2 max.) apart from (b)(ii) where it is a misreading and 9 marks are possible.

Blunders (–3)

B1 Sign error.

B2 Index error, e.g. $(-1)^3 = -3$, or $(3)^3 = 9$.

Misreadings (–1)

M1 Misread as $f(x) = x^3 - 3x^2 - 1$, or $f(x) = x^3 + 3x^2 - 1$, etc.

Attempts (2 marks)

A1 Some correct substitution.

A2 Incorrect substitution, e.g. f(1) substituted or calculated.

A3 Differentiates before substituting. Each time. (Oversimplified).

Worthless (0)

W1 No substitution attempted, e.g. writes "f(-1)" and stops.

(b)(ii) **10 marks** **Att 3**

$$f'(x) = 3x^2 - 6x + 0 \quad \textit{or} \quad f'(x) = 3x^2 - 6x$$

- * dy/dx = 0 or $f'(x) = 0$ merits 4 marks. (2 terms incorrect, 1 correct)

Blunders (-3)

B1 Differentiation error, once per term. 3 terms to check.

Attempt (3 marks)

A1 Mentions dy/dx.

A2 Effort at first principles, e.g. $y + \Delta y$, or $x + \Delta x$, or $x + h$, or $x - h$, etc. mentioned.

Worthless (0)

W1 No term differentiated correctly, but see A1.

(b)(iii) **5 marks** **Att 2**

$3x^2 - 6x = 0$	… 2m
$3x.(x - 2) = 0$	
$x = 0, \quad x = 2$	…still 2m
$\Rightarrow y = 1, \quad y = -3$	…5m

- * No need to distinguish between max and min points – not asked in question.
- * Ignore errors in candidate's work relating to 2nd derivative.
- * If points not calculated, but read from a graph or by trial and error: 5m for (0,1) *and* (2, –3) isolated; 2m for a (0,1) *or* (2, –3) isolated; 0m for neither of these two points isolated.
- * In line 1, an implied use of '= 0' is acceptable when factors found etc.
- * If (b)(ii) is not attempted and $f'(x)$ is found in (b)(iii) then mark (b)(iii) out of 10m (for derivative) + 5m (for max and min).
- * $d^2y/dx^2 = 6x - 6$, or a correct $f''(x)$ statement, merits att. 2m.

Blunders (-3)

B1 $f'(x)$ not equated to zero (but see Note 4 above), or $f'(0)$ found.
B2 Incorrect factors.
B3 Incorrect roots from factors.
B4 Quadratic formula error (in formula, substitution or simplification). Max of 2 blunders.
B5 Calculates only one turning point and stops, e.g. x = 2, y = – 3 and stops
B6 Error(s) in calculating y (apart from a totting error which is a slip; see S1).
B7 y coordinates calculated incorrectly or not found, e.g. x = 0, x = 2 becomes (0, 2). Or, calculates x and substitutes the x into $f'(x)$ to get y.

Slips (-1)

S1 Numerical, e.g. totting y values.

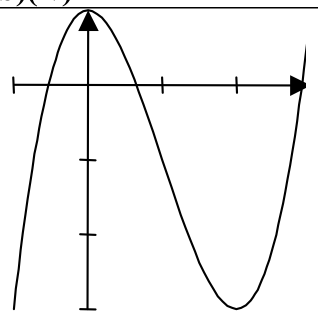
Attempts (2 marks)

A1 Candidate's $f'(x) = 0$, or states "$f'(x) = 0$" or "$dy/dx = 0$".
A2 f(0) = 1 or f(2) = –3 and stops.
A3 Quadratic formula correct and stops.

Worthless (0)

W1 Effort to solve f(x) = 0.

(b)(iv)	**5 marks**	**Att 2**



For reference:

x	–1	0	2	3
x^3	–1	0	8	27
$-3x^2$	–3	0	–12	– 27
1	1	1	1	1
y	–3	1	–3	1

* Candidate transfers information correctly from (b)(i) and (b)(iii) to a graph: award 5m. Reasonable degree of accuracy only.
* 4 points needed , i.e. candidate's answers (b)(i) and (b)(iii), or these points from a table.
* Accept candidate's answers from (i) and (iii) unless graph oversimplified (e.g. linear).

Blunders (-3)
B1 Error transferring information from (b)(i) or (b)(iii), or error from/in table.
B2 Serious inaccuracy of scale(s). Apply once.

Slips (–1)
S1 Four points plotted but not joined. Apply once. Or joins points with straight lines.

Attempts (2 marks)
A1 Scaled axes drawn and stops.
A2 Effort to find point on graph, e.g. f(1) or other points calculated.
A3 Graphs $f'(x)$. A serious misreading (oversimplified to a quadratic graph).

(b)(v)	**5 marks**	**Hit/Miss**

$0.7 < x < 2.9$

* Correct answer according to candidate's cubic graph: 5 marks Hit/Miss. [No graph of f => no marks in (v)].
* Ignore inclusion of equals signs.
* Mark according to candidate' graph.
* Allow accuracy of ± 0.2
* Accept correct answer in non-technical language, e.g. from 0.7 to 2.9.
* Allow candidate to indicate answers on the graph or on a number line.

(b)(vi)	**5 marks**	**Hit/Miss**

$0 < x < 2$

* Correct answer according to candidate's cubic graph: 5 marks Hit/Miss. [No graph of f => no marks in (vi)].
* Ignore inclusion of equals signs.
* Mark according to candidate's graph.

QUESTION 7

Part (a)	**10 marks**	**Att 4**
Part (b)	**20 marks**	**Att 6**
Part (c)	**20 marks**	**Att 8**

Part (a) **10 (5, 5) marks** **Att (2, 2)**

Differentiate with respect to x:

(i) $2x^5$

(ii) $4(3-x^2)$.

(a)(i) **5 marks** **Att 2**

$10x^4$

(a)(ii) **5 marks** **Att 2**

$4(-2x)$ or $-8x$

* Accept, for full marks, $2(5x^4)$ or $5(2x^4)$in (i), and $4(-2x) + (3-x^2)(0)$ in (ii).
* Correct answer without work or notation: full marks.
* If done from first principles, ignore errors in procedure – just mark the answer.
* In (i), for coefficient correct only, or power correct only: allow 2m. (Only exception).

Blunders (–3) ...Applying to each part of (a).

B1 Differentiation error. Once per term.

B2 dy/dx = $(0).(5x^4)$ in (i), or dy/dx = $(0).(0-2x)$ in (ii).

B3 Each error in u.v formula.

Attempts (2 marks for each part)

A1 In (a)(ii), any correct multiplication prior to differentiation, e.g. $12 - x^2 = 0 - 2x$

A2 Any term (given *or* candidate's) differentiated correctly -- including the 4, i.e. d(4)/dx = 0, or if $-2x$ appears to be the derivative of $-x^2$ in candidate's work.

A3 Unsuccessful effort at first principles, e.g. $y + \Delta y$ on L.H.S., or x replaced by $x + \Delta x$ on R.H.S., 'limit' mentioned, $\Delta x \rightarrow 0$, $f(x+h)$, etc.

A4 Writes down the notation *dy/dx* or $f'(x)$ and stops.

Worthless (0)

W1 No term differentiated correctly, but check attempts A1 to A4 first, and note 4 for (i).

Part (b) **20 (10, 10) marks** **Att (3, 3)**

(i) Differentiate $(x^2-4)(x^2+3x)$ with respect to x.

(ii) Given that $y=(x^2-2x-3)^3$, show that $\dfrac{dy}{dx}=0$ when $x = 1$.

(b)(i) **10 marks** **Att 3**

	$y = x^4 + 3x^3 - 4x^2 - 12x$ …3m
$(x^2-4)(2x+3) + (x^2+3x)(2x)$ ..10m	$dy/dx = 4x^3 + 9x^2 - 8x - 12$ ..10m

* In method I, no penalty for omission of brackets as long as multiplication is implied.
* If u/v used, apply B2 twice (central sign, division by v^2). There may be other errors.
* dy/dx = (2x)(2x+3) merits 3 marks, i.e. 10 – B1 – B1 – B2.

Blunders (–3)

B1 Differentiation error. Once per term. (Two terms to check in I, four terms in II).
B2 Error in u.v formula, e.g. central sign.
B3 Does u.(du/dx) + v.(dv/dx), i.e. dy/dx = $(x^2 – 4)(2x) + (x^2 + 3x)(2x + 3)$. Apply once.
B4 In II, each omitted or incorrect term in the expansion (line 1) to a max of 2 blunders.

Attempts (3 marks)

A1 Any correct derivative, e.g. an implied "0".
A2 $u = x^2 – 4$ or $v = x^2 + 3x$, or vice versa, and stops.
A3 One or more terms multiplied correctly in method II.

Worthless (0)

W1 u.v or u/v rule written down (from Tables) and stops.

(b)(ii) **10 marks** **Att 3**

I: dy/dx = $3(x^2 – 2x– 3)^2.(2x – 2)$ …7m x = 1 => $3(x^2 – 2x– 3)^2.(2 – 2)$ …9m = 0 ..10m	II: $u = x^2 – 2x– 3$ and $y = u^3$ du/dx = 2x – 2 and dy/du = $3u^2$ dy/dx = (dy/du).(du/dx) = $3u^2.(2x – 2)$ = $3(x^2 – 2x– 3)^2.(2x – 2)$ …7m x = 1 => $3(x^2 – 2x– 3)^2.(2 – 2)$ …9m = 0 ..10m..
III: y = $x^6 – 6x^5 + 3x^4 + 28x^3 – 9x^2 – 54x – 27$ …3m dy/dx = $6x^5 – 30x^4 + 12x^3 + 84x^2 – 18x– 54$ …7m x = 1 => 6 – 30 + 12 + 84 – 18 – 54 …9m = 0 ..10m	

* Treat $3(x^2 – 2x – 3)^2$ and (2x – 2) as two separate terms. See B1 and B2 below.
* No penalty for omission of brackets, as long as multiplication is implied.
* Ans $3(x^2 –2x– 3)^2 = 48$: B2 => <u>7m</u>. Ans $3(x^2 – 2x– 3)^2$: B2 +B4 => <u>4m</u>
  Ans $3(2x – 2)^2 = 0$: B1 + B2 => <u>4m</u>. Ans $3(2x – 2)^2$: B1+B2+B4 => <u>3m</u>
  Ans 2x – 2 = 0: oversimplified => <u>3m</u>. Ans 2x – 2: oversimplified => <u>3m.</u>
* If candidate tries to multiply out first, mark using slips and blunders.
* In method I or II, needn't substitute into quadratic, if multiplied by (0) = answer 0.
* Substitutes derivative directly, e.g. $3[1^2 – 2(1) – 3]^2.(2 – 2)$ =>9m. If it = 0, then 10m.

Blunders (–3)

B1 Differentiation error(s) in $3(x^2 – 2x– 3)^2$ part of derivative. Apply once.
B2 Differentiation error(s) in (2x – 2) part of derivative. Or, not multiplied by (2x – 2) or candidate's equivalent, but *implied multiplication* is acceptable. Apply B2 once.
e.g. $3(x^2 – 2x–3)^2\ 2x – 2$ is an implied multiplication.
B3 In method II, each error applying the chain rule.
B4 Doesn't evaluate derivative at x = 1, or puts dy/dx = 1, i.e. loses last three marks.

Attempts (3 marks)

A1 Any correct relevant derivative, e.g. the "0" in the second term.
A2 $u = x^2 – 2x– 3$ and stops.
A3 Some correct element of *chain rule*, e.g. coefficient 3, or power 2.
A4 $y = 3x^2 – 6x – 9$ => dy/dx = 6x – 6. Oversimplified.
A5 dy/dx = $(2x – 2)^3$.
A6 At least one term correctly multiplied out in method III.

Worthless (0)

W1 Substitutes x = 1 into f(x), and stops.

Part (c) **20 (5, 5, 5, 5) marks** **Att (2, 2, 2, 2)**

A jet is moving along an airport runway. At the instant it passes a marker it begins to accelerate for take-off. From the time the jet passes the marker, its distance from the marker is given by

$$s = 2t^2 + 3t$$

where s is in metres and t is in seconds.

(i) Find the speed of the jet at the instant it passes the marker ($t = 0$).

(ii) The jet has to reach a speed of 83 metres per second to take off. After how many seconds will the jet reach this speed?

(iii) How far is the jet from the marker at that time?

(iv) Find the acceleration of the jet.

(c)(i) **5 marks** **Att 2**

$\frac{ds}{dt} = 4t + 3 \text{ ms}^{-1}$ …2m

t = 0 => speed = 4(0) + 3 …4m

= 3 …5m

* Correct answer without work: 5 marks. (i.e. we assume differentiation was done).
* Marks are non-transferable between parts of (c), i.e. no retrospective marking allowed.
* No differentiation or reference to differentiation: award 0 marks.
* No penalty for incorrect notation.
* If the parts of (c) are unlabelled, and the context doesn't identify which part is which, assume the questions were answered in sequence from (c)(i) to (c)(iv).

Blunders (-3)

B1 Differentiation error, once per term. (Two terms to check).
B2 Incorrect value for t substituted into speed (ds/dt) equation.
B3 Speed = $d^2s/dt^2 = 4$. If rectified (in this part), withdraw penalty.
B4 4t + 3 = 0 and stops or continues.

Slips (–1)

S1 4(0) + 3 = 4 + 3 = 7, or 4(0) + 3 = 7.

Attempts (2 marks)

A1 One term correctly differentiated and stops.
A2 ds/dt or $f'(x)$ mentioned, or differentiation implied.

Worthless (0)

W1 Incorrect answer without work.
W2 States speed = d^2s/dt^2 and stops.
W3 t = 0 substituted into the s equation.
W4 Effort to use Speed = Distance $\div$ Time.

(c)(ii) **5 marks** **Att 2**

$4t + 3 = 83$ …2m $t = 20$ …5m

* Correct answer without work: 5 marks. Incorrect answer without work: no marks.
* If derivative not used, found or mentioned: no marks.
* Accept candidate's ds/dt from (c)(i) provided it was a derivative.
* No retrospective award of marks from (c)(ii) to (c)(i).

Blunders (-3)
B1 ds/dt $\neq$ 83 solved, or t = 83 substituted into 4t + 3 (getting 335 seconds).
B2 Transposition error.

Attempts (2 marks)
A1 Mentions ds/dt in this part, or ds/dt found again.
A2 ds/dt = 83 and stops, or candidate's ds/dt from (c)(i) = 83 and stops.
A3 Using 4t + 3 or candidate's derivative, an effort to tabulate, graph or trial and error.

Worthless (0)
W1 Solves $2t^2 + 3t = 83$, or t = 83 substituted into $2t^2 + 3t$.

(c)(iii) **5 marks** **Att 2**

$s = 2t^2 + 3t = 2(20)^2 + 3(20)$ … 2m $= 800 + 60 = 860$ …5m

* Accept candidate's value of *t* from (c)(ii).
* If distance formula is not substituted in this part, award no marks unless A1 applies.

Blunders (-3)
B1 Incorrect *t* substituted into distance formula, i.e. $t \neq$ ans (c)(ii) substituted.
B2 Mathematical errors, e.g. $2(20)^2 = 1600$.

Slips (-1)
S1 Numerical slips

Attempts (2 marks)
A1 $2t^2 + 3t =$ *some number*, e.g. $2t^2 + 3t = 20$, i.e. for using the distance formula.

Worthless (0)
W1 First derivative used.

(c)(iv) **5 marks** **Att 2**

I: d^2s/dt^2 …2m $= 4$ …5m	II: $a = (v - u)/t$ correctly substituted for speed, e.g. $(83 - 3)/20$ …2m $= 4$ …5m

* Correct answer without work: 5 marks. Incorrect answer without work: no marks.
* $v = u + f.t$: no marks (Formula in Tables). But see A3 below.
* In method I, 2nd derivative not mentioned or used: no marks, unless A2 below applies, i.e. uses distance formula => 0 marks, or uses speed formula *again* => 0 marks.

Blunders (-3)
B1 Differentiation error. Once per term. Two terms to check.

Attempts (2 marks)
A1 Mentions d^2s/dt^2 or dv/dt or similar, i.e. 2nd derivative.
A2 Finds ds/dt *for the first time* and stops. Even partially correct, e.g. ds/dt = 4t and stops.
A3 $v = u + a.t$ and stops, or $a = (v - u)/t$ and stops. (i.e. *a* substituted for *f* in formula).

Worthless (0 marks)
W1 2nd derivative not found or mentioned, unless A2 applies or Method II is used.

QUESTION 8

Part (a)	**10 marks**	**Att 4**
Part (b)	**20 marks**	**Att 7**
Part (c)	**20 marks**	**Att 7**

Part (a) **10 (5, 5) marks** **Att (2, 2)**

Let $g(x) = 3x - 7$.
(i) Find $g(7)$.
(ii) Find the value of k for which $g(7) = k[g(0)]$.

(a)(i) **5 marks** **Att 2**

$g(7) = 3(7) - 7$ … 2m
$= 21 - 7$ …still 2m
$= 14$ … 5m

(a)(ii) **5 marks** **Att 2**

$g(0) = 3(0) - 7 = -7$ …2m
$14 = k(-7)$ …still 2m
$k = -2$ …5m

* Correct answers without work: full marks.
* No penalty for omission of brackets, provided the blunders below are avoided.
* No penalty for notation, e.g. in (i), g(x) = 3(7) – 7 = 14 merits full 5 marks.
* In (ii), g(0) = 3(0) – 7 = 3 – 7 = – 4 is S1, but g(0) = 3 – 7 = 4 is S1 + B4 => att 2m.
* $k = \frac{14}{-7}$ and stops: still 2 marks.
* g(0) = –7 and then 14 = k(7): apply B(–3).

Blunders (-3)
B1 In (i), solves 3x – 7 = 7 => x = 14/3.
B2 In (ii), solves 3x – 7 = 0 => x = 7/3.
B3 Incorrect substitution for x. Apply once in (i), once in (ii).
B4 Sign error.
B5 Serious error in calculation, e.g. 3(7) – 7 = 0, i.e. cancels the sevens.

Slips (-1)
S1 3(0) – 7 = 3 – 7 = – 4

Attempts (2 marks)
A1 $g(7)$ = 3x – 7 and stops, or g(x) = 3(7) – 7 and stops, (i.e. some substitution).
A2 3(7) and stops.

Worthless (0)
W1 Incorrect answer without work, e.g. 21 without work.

Part (b) **20 marks** **Att 7**

Differentiate $x^2 + 3x$ with respect to x from first principles.

(b) **20 marks** **Att 7**

$f(x+h) = (x+h)^2+3(x+h)$	...7m...	$y + \Delta y = (x + \Delta x)^2 + 3(x+\Delta x)$
$f(x+h) = x^2 +2hx + h^2 + 3x + 3h$	@ 11m	$y + \Delta y = x^2 + 2x\Delta x + (\Delta x)^2 + 3x + 3\Delta x$
$f(x+h)-f(x) = x^2 +2hx + h^2 + 3x + 3h - x^2-3x$		$y = x^2 + 3x$
$f(x+h)-f(x) = 2hx + h^2 + 3h$	@ 14m	$\Delta y = 2x\Delta x + (\Delta x)^2 + 3\Delta x$
$\frac{f(x+h)-f(x)}{h} = 2x + h + 3$	@ 17m	$\frac{\Delta y}{\Delta x} = 2x + \Delta x + 3$
$\lim_{h \to 0} \frac{f(x+h)-f(x)}{h} = 2x + 3$	@ 20m	$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = 2x + 3$

* Overlook $\Delta x = 0$ or $h = 0$ in limit; and use of dy/dx instead of lim $\Delta y/\Delta x$.
* If first mention of LHS is in last line, then B4 and B5 apply; i.e. 14m for RHS correct.
* Perfectly correct RHS but <u>no</u> LHS => B4 + B5 + B6 apply.
* After substit. and further work, B(-3) for each major step omitted; see steps @ above.
* f(x – h) used: no penalty.

Blunders (-3)

B1 Error multiplying out $(x + \Delta x)^2$ or $(x + h)^2$. Apply a maximum of two blunders.
B2 Error multiplying out , e.g. $3(x + \Delta x) = 3x + \Delta x$, or omits the multiplication by 3.
B3 $(\Delta x)^2 = \Delta^2.x^2$ (if it affects the solution); or $2x\Delta x = 2\Delta x^2$, <u>but</u> allow Δx^2 for $(\Delta x)^2$.
B4 Omits $y + \Delta y$, or Δy, or $f(x + h)$, or $f(x + h) - f(x)$ on LHS. Apply once.
B5 Omits $\Delta y/\Delta x$ or $\{ f(x + h) - f(x)\}/h$ on L.H.S.
B6 Omits limiting idea (word "lim" unnec.) or has other than $\Delta x \to 0$ or $h \to 0$ on L.H.S., i.e. should have "lim", or "$\Delta x \to 0$" (allow "$\Delta x = 0$" instead), or "dy/dx".
B7 Evaluates limit where Δx or h will not divide, e.g. no Δx on R.H.S. at that stage.
B8 Limit error, e.g. $\Delta y/\Delta x = 2x + \Delta x + 3$ but $\lim \Delta y/\Delta x \neq 2x + 3$.
B9 Differentiates from first principles x^2 or $x^2 + 3$.

Slips (-1)

S1 Correct term such as $2x\Delta x$ subsequently "becomes" $2\Delta x$. (Misreads own work.)

Attempts (7 marks)

A1 $y+\Delta y$ or $f(x+h)$ on LHS; or $x+\Delta x$ or $x+h$ or $x – h$ substituted somewhere on RHS.
A2 Linear function differentiated from first principles: even if correct, award Att 7.

Worthless (0)

W1 Answer $2x + 3$ without work (i.e. not from first principles).

Part (c) **20 (10, 5, 5) marks** **Att (3, 2, 2)**

Let $f(x) = \dfrac{1}{x+3}$, $x \in \mathbf{R}$, $x \neq -3$.

(i) Find $f'(x)$, the derivative of $f(x)$.

(ii) There are two points on the curve $y = f(x)$ at which the slope of the tangent is -1. Find the co-ordinates of these two points.

(iii) Show that no tangent to the curve $y = f(x)$ has a slope of 1.

(c)(i) **10 marks** **Att 3**

$f'(x) = \dfrac{(x+3).0 - (1)(1)}{(x+3)^2}$ ..10m	$f(x) = (x+3)^{-1}$ …3m $f'(x) = -1.(x+3)^{-2}$ ..10m

* No penalty for omission of brackets as long as multiplication is implied.

* 1/(x + 3) is clearly identified by candidate as u/v, but uv formula is used: apply B2 +B3. Other penalties may/may not arise.

* Candidates need not simplify the answer. Don't penalise, in this part, an error doing so. Penalise, if necessary, in (c)(ii), i.e. if incorrect (simplified) expression is used in (c)(ii) or (c)(iii).

Blunders (-3)

B1 Differentiation error, once per term. Method I has two terms to check, II has one term.
B2 Central sign error in u/v formula.
B3 No (or incorrect) division by v^2, whether v^2 mentioned in formula or not.
B4 Vice versa switching of derivatives, e.g. does $[v.dv/dx) - u.(du/dx)]/v^2$. Apply once.
B5 $f(x) \neq (x+3)^{-1}$ if using method II. But if f(x) taken as $(x+3)^1$. See A3 and W2.

Attempts (3 marks)

A1 Any correct derivative and stops, e.g. $f'(x) = 0/1$, or 0.
A2 $f(x) = (x+3)^{-1}$ and stops.
A3 f(x) = x + 3 and differentiates this correctly. (Oversimplified).
A4 Mentions dy/dx.
A5 u = 1 and/or v = x + 3, and stops.

Worthless (0)

W1 No differentiation done in method I, or step 1 missing (and not subsumed) in method II. But See A4.
W2 f(x) = x + 3 and stops.

(c)(ii) **5 marks** **Att 2**

$-1.(x+3)^{-2} = -1$ or $\dfrac{-1}{(x+3)^2} = -1$ …2m

$(x+3)^2 = 1$

	$x^2 + 6x + 9 = 1$	$x^2 + 6x + 9 = 1$
$x + 3 = \pm 1$	$x^2 + 6x + 8 = 0$	$x^2 + 6x + 8 = 0$
		$x = \dfrac{-6 \pm \sqrt{6^2 - 4(1)(8)}}{2}$
$x = \pm 1 - 3$	$(x+2)(x+4) = 0$	
$x = -2, x = -4$	$x = -2, \ x = -4$	$x = -2, \ x = -4$
$y = 1, \ y = -1$	$y = 1, \ y = -1$	$y = 1, \ y = -1$ …5m
or $(-2, 1), (-4, -1)$	or $(-2, 1), (-4, -1)$	or $(-2, 1), (-4, -1)$

* Allow candidates to use $f'(x)$ from previous part; however, if this oversimplifies the question then the max mark attainable is Att 2. See Note 3 in (c)(i).

* Use of $m = (y_2 - y_1) / (x_2 - x_1)$ is worthless in (c)(ii) and/or (c)(iii).

Blunders (–3)

B1 Index or inversion error.

B2 Only one root extracted, e.g. $(x+3)^2 = 1 \Rightarrow x + 3 = 1 \Rightarrow x = -2$.

B3 Transposition or sign error.

B4 Incorrect factors. Apply once

B5 Incorrect roots from factors. Apply once.

B6 y value(s) not found

B7 Quadratic formula error (in formula, substitution or simplification).

Attempts (2 marks)

A1 Candidate's answer to (c)(i) equated to –1, and stops.

A2 Tidies up the $f'(x)$ obtained in (c)(i), or re-writes $f'(x)$ or answer (c)(i) in this part.

A3 dy/dx or $f'(x)$ mentioned in this part, and stops.

A4 Quadratic formula correct, and stops.

Worthless (0)

W1 $1/(x+3) = -1$ and stops, or continues (i.e. derivative not used).

(c)(iii) **5 marks** **Att 2**

$\dfrac{-1}{(x+3)^2} = 1$ *or* $-1 = (x+3)^2$ ...2m impossible because …. (Accept any valid reason, …5m e.g. $(x+3)^2$ can't be negative).	$(x+3)^2 = -1$ …2m $x^2 + 6x + 9 = -1$ $x^2 + 6x + 10 = 0$ $b^2 - 4ac < 0$ or quadratic no real roots …5m => no tangent with slope 1.

* $(x+3)^2$ cant be negative, or can't be –1, or must be positive: award 5m

* Allow candidate's $f'(x)$ from (c)(i), but if the question is oversimplified: att 2m max.

Blunders (-3)

B1 Cross–multiplication error, e.g. $(x+3)^2 = 1$.

Attempts (2 marks)

A1 Candidate's answer to (c)(i) equated to 1, and stops.

A2 Effort to sketch *f(x)*.

A3 dy/dx or $f'(x)$ mentioned in this part, and stops.

A4 Quadratic formula correct, and stops.

MARKING SCHEME

LEAVING CERTIFICATE EXAMINATION 2004

MATHEMATICS

ORDINARY LEVEL

PAPER 2

GENERAL GUIDELINES FOR EXAMINERS - PAPER 2

1. Penalties of three types are applied to candidates' work as follows:
   - Blunders - mathematical errors/omissions (-3)
   - Slips - numerical errors (-1)
   - Misreadings (provided task is not oversimplified) (-1).

   Frequently occurring errors to which these penalties must be applied are listed in the scheme. They are labelled as B1, B2, B3,......, S1, S2, S3,...., M1, M2, etc. Note that these lists are not exhaustive.

2. When awarding attempt marks, e.g. Att(3), note that
   - any correct relevant step in a part of a question merits *at least* the attempt mark for that part
   - if deductions result in a mark which is lower than the attempt mark, then the attempt mark must be awarded
   - a mark between zero and the attempt mark is never awarded.

3. Worthless work is awarded zero marks. Some examples of such work are listed in the scheme and they are labelled as W1, W2,....etc.

4. The phrase "hit or miss" means that partial marks are not awarded – the candidate receives all of the relevant marks or none.

5. The *same* error in the *same* section of a question is penalised *once* only.

6. Special notes relating to the marking of a particular part of a question are indicated by an asterisk. These notes immediately follow the box containing the relevant solution.

7. Particular cases, verifications and answers derived from diagrams (unless requested) qualify for attempt marks at most.

8. A serious blunder, omission or misreading merits the ATTEMPT mark at most.

9. The phrase "and stops" means that no more work is shown by the candidate.

10. Accept the best of two or more attempts – even when attempts have been cancelled.

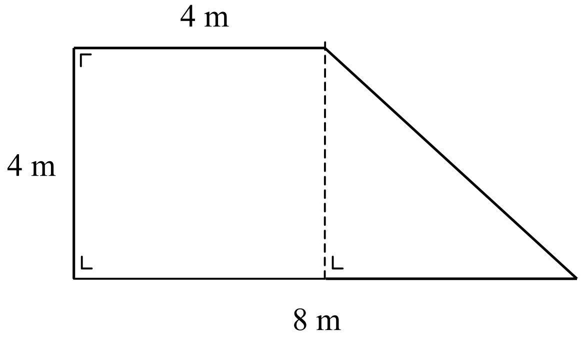
11. Allow comma for decimal point, e.g. €5.50 may be written as €5,50.

QUESTION 1

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 marks**	**Att 7**
Part (c)	**20 marks**	**Att 7**

Part (a) **10 marks** **Att 3**

Calculate the area of the figure in the diagram.



(a) **10 marks** **Att 3**

I	II	III
Area of square = 4 × 4	Rectangle = 8 × 4	Trapezium = ½ h[side one + side two] 3 m
Area of triangle = ½ × 4 × 4	Triangle = ½ × 4 × 4	= ½ × 4 × (4 + 8) 7 m
Area of figure = 16 + 8	Figure = 32 – 8	= 2 × 12 9 m
= 24 m^2	= 24 m^2	= 24 m^2 10 m

* Accept correct answer without work.
* Not more than 3 marks may be deducted for errors in calculations.

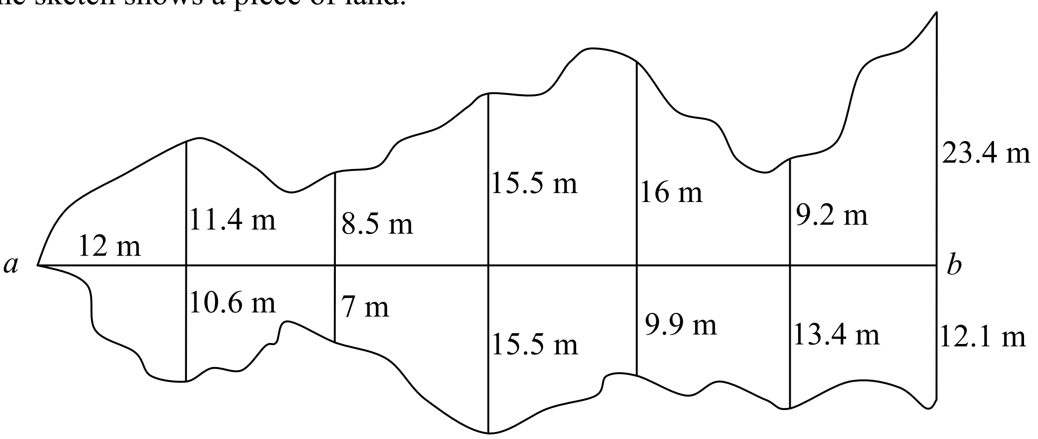
Blunders (-3)
B1 Incorrect relevant formula which does not simplify the task, e.g. omits ½ for triangle or trapezium.
B2 Each arbitrary dimension or each different blunder in dimension, subject to attempt mark.
B3 Mathematical error, e.g. adds instead of multiplies or vice versa or mishandles the ½.
B4 No calculations, i.e. Area of figure = 4×4 + 0·5×4×4 and stops merits 7 marks.
Case 1: Area square = 4 × 4 =16 and stops merits 4 marks.
Case 2: Area of triangle = ½ × 4 × 4 = 8 and stops merits 4 marks.
Case 3: 4 × 4 × 8 = 128 or ½ × 4 × 4 × 8 = 64 merits 4 marks.

Slips (-1)
S1 Each numerical slip to a maximum of 3.
S2 Leaves answer as 16 + 8.
S3 16 and 8 given with work shown.
Misreadings (-1)
M1 Each obvious misreading of a dimension to a maximum of 3.
Attempts (3 marks)
A1 Some relevant work, e.g. 4 × 4 or 8 – 4 [= base of triangle] or 8 × 4 or something relevant added to the diagram and stops.
A2 Correct relevant formula not transcribed from the tables.
A3 Some correct substitution into reasonable formula.
A4 Perimeter, e.g. 4 + 4 + 8 or 4 + 4 + 4 + 4.
A5 Some application of Pythagoras' theorem to find slant height.

Worthless (0)
W1 Incorrect answer without work, subject to identified attempts.

Part (b) **20 (15, 5)marks** **Att (5, 2)**

The sketch shows a piece of land.



At equal intervals of 12 m along $[ab]$, perpendicular measurements are made to the boundary as shown on the sketch.

Use Simpson's Rule to estimate the area of the piece of land.

* Note: If 22 is taken as F, this is B2(-3) and consequent errors in TOFE is B3(-3).

(b) Use of formula **15 marks** **Att 5**
Calculations **5 marks** **Att 2**

I Area = $^{h}/_{3}$ {F + L + 2(odds) + 4(evens)}……………………….5 marks
Top = $^{12}/_{3}${0 + 23·4 + 2(8·5 + 16) + 4(11·4 + 15·5 + 9·2)}
or Bottom = $^{12}/_{3}${0 + 12·1 +2(7 + 9·9) + 4(10·6 + 15·5 + 13·4)}…… 12 m
or Both
$^{12}/_{3}${0+35·5+2(8·5+7+16+9·9+ 4(11·4+10·6+15·5+15·5+9·2+13·4)}..15 m

Top = 4{23·4 + 2(24·5) + 4(36·1)}	Together
+Bottom 4{12·1 + 2(16·9) +4(39·5)}	= 4{35·5 +2(41·4) +4(75·6)}
= 4{216·8} + 4{203·9}	= 4{420·7}
= 867·2 + 815·6 = 1682·8 m^2	= 1682·8 m^2

II $^{h}/_{3}${F + L + TOFE}

F/L	O	E
0	8·5	11·4
23·4	16	15·5
		9·2
0	7	10·6
12·1	9·9	15·5
		13·4
$^{12}/_{3}${	× 2	× 4}

* Candidate must not lose more than 5 marks for calculations.
* Allow $^{h}/_{3}$ = {F + L + TOFE} and penalise later, if used.

Blunders (-3)
B1 Incorrect $^{h}/_{3}$ (once).
B2 Incorrect F and / or L or extra term with F and / or L (once).
B3 Incorrect TOFE (once).
B4 E **or** O omitted (once).
B5 Mathematical blunder, e.g. distribution error (once).
B6 Finds area of top **or** bottom only.
B7 Misplaced decimal point.

Slips (-1)
S1 Each numerical slip to a maximum of 3.

Attempts (5 marks for substituting into formula, 2 marks for calculations)

A1	Some relevant step, e.g. identifies F and / or L or odds or evens and stops.	(5 m)
A2	Statement of Simpson's Rule not transcribed from tables.	(5 m)
A3	E **and** O omitted (candidate may be awarded attempt at most).	(Max. 5 m and/or 2 m)
A4	Completes all rectangles but no calculations.	(5 m)
A5	Completes all rectangles and adds areas.	(5 m + 2 m)
A6	Correct answer without work.	(5 m + 2 m)
A7	Some correct calculation only.	(2 m)

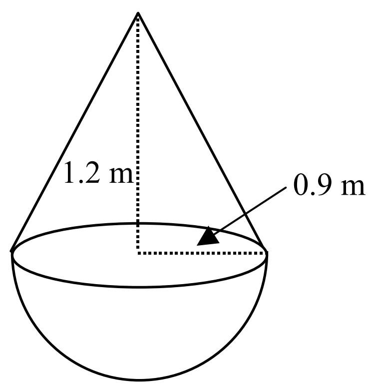
Worthless (0)

W1 Incorrect answer without work.

W2 Formula transcribed from tables and stops.

Part (c) **20 (5, 5, 10) marks** **Att (2, 2, 3)**

A buoy at sea is in the shape of a hemisphere with a cone on top, as in the diagram.
The radius of the base of the cone is 0·9m and its vertical height is 1·2 m.



(i) Find the vertical height of the buoy.

(ii) Find the volume of the buoy, in terms of π.

(iii) When the buoy floats, 0·8 m of its height is above water. Find, in terms of π, the volume of that part of the buoy that is above the water.

(c)(i) **5 marks** **Att 2**

Vertical height of buoy = 1·2 + 0·9 = 2·1 m

* Accept correct answer without work.

Blunders (-3)

B1 Misplaced decimal point, subject to S1.

B2 1·2 + x, for x ≠ 0·9.

B3 1·2 × 0·9 = 1·08 or similar.

Slips (-1)

S1 Each numerical slip to a maximum of 3.

S2 1·2 + 0·9 and stops.

Attempts (2 marks)

A1 Applies Pythagoras' theorem to calculate the slant height = 1·5 m.

A2 Some relevant step.

(c)(ii) **5 marks** **Att 2**

Volume of cone = $\frac{1}{3}\pi(0\cdot9)^2(1\cdot2)$
Volume of Hemisphere = $\frac{1}{2}\times\frac{4}{3}\pi(0\cdot9)^3$
Volume of buoy = $\frac{1}{3}\pi(0\cdot9)^2(1\cdot2)+\frac{2}{3}\pi(0\cdot9)^3$
$= 0\cdot324\pi + 0\cdot486\pi$
$= 0\cdot81\pi \text{ m}^3$

* Candidate may not lose more than 3 marks for calculations.
* Allow candidate to use values from part (i).
* Accept volume of sphere read as $\frac{4}{8}\pi r^3$

Blunders (-3)
B1 Incorrect formula for volume of cone (once for cone), e.g. $\frac{1}{3}\pi rh$, $\frac{1}{3}r^2h$, πr^2h, πrl,
B2 Incorrect formula for volume of hemisphere (once for hemisphere), e.g. omits the ½.
B3 Incorrect substitution.
B4 Mathematical blunder, e.g. $(0\cdot9)^2 = 1\cdot8$.
B5 Misplaced decimal point, e.g. $(0\cdot9)^2 = 8\cdot1$ or candidate drops decimal points without identifying new units.
B6 Obvious value of π outside tolerance, subject to full marks for volume = 0·81π.
B7 Incorrectly drops π, e.g. $\text{vol} = 0.81\text{m}^3$ or inserts an additional π, e.g. $\text{vol} = 2.5\pi\,\text{m}^3$, subject to marks already secured.

Slips (-1)
S1 Each numerical slip to a maximum of 3.
S2 Early or incorrect rounding off that affects the answer. (once only)
S3 Inserts a value of π from 3.1 to 3.2 inclusive.
S4 Volume of cone = 0·324π and volume of hemisphere = 0·486π , with work.

Attempts (2 marks)
A1 Some relevant step, e.g. ½ of sphere or indicates addition of volumes.
A2 Correct relevant formula not transcribed from tables, e.g. volume of hemisphere = $\frac{2}{3}\pi r^3$.
A3 1·2 m = 120 cm or similar.
A4 A correct substitution, if a formula is written.
A5 Correct answer without work.

(c)(iii) **10 marks** **Att 3**

$\frac{0\cdot8}{1\cdot2}=\frac{r}{0\cdot9}$	3m
$\Rightarrow r=\frac{0\cdot8\times0\cdot9}{1\cdot2}=0\cdot6$	4m
$\frac{1}{3}\times\pi\times0\cdot6^2\times0\cdot8$	7m
$=0\cdot096\pi \text{ m}^3$	10m

* Accept candidate's values from previous parts, if used.
* $3\cdot1 \le \pi \le 3\cdot2$ (Volume = 0·2976, Volume = 0·3015…, Volume = 0·3072…) ➔ S3.

- * Note: any arbitrary *r* without work, including 0·9, ➔ attempt mark.
- * Note: Vol of cone $\times \frac{8}{27} = 0{\cdot}096\pi \text{ m}^3 \Rightarrow$ full marks.

Blunders (-3)

B1 Blunder in finding radius of cone, with work shown.
B2 Incorrect relevant formula.
B3 Incorrect and inconsistent substitution into correct formula.
B4 Mathematical blunder.
B5 Takes height of buoy above water as 0·8 × 1·2 [h = 0·96, r = 0·72, vol of cone = 0·5211…].
Note: S3 also applies if value of π is used.

Slips (-1)

S1 Each numerical slip to a maximum of 3.
S2 Early or incorrect rounding off that affects the answer.
S3 Inserts a value for π where $3{\cdot}1 \le \pi \le 3{\cdot}2$

Misreadings (-1)

M1 Takes height of buoy above water as 0·8 × 2·1 [h = 1·68] and continues.

Attempts (3 marks)

A1 Some relevant step, e.g. some correct substitution.
A2 Correctly fills in formula and stops.
A3 Correct answer without work.

QUESTION 2

Part (a)	10 marks	Att 3
Part (b)	40 (10,10,10,5,5) marks	Att (3,3,3,2,2)

Part (a) **10 marks** **Att 3**

$p(5, -8)$ and $q(11, 10)$ are two points.
Find the co-ordinates of the midpoint of $[pq]$.

(a) **10 marks** **Att 3**

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{5+11}{2}, \frac{-8+10}{2}\right) = (8,1)$$

3 m 7 m 10 m

* Accept $(^{16}/_2, ^{2}/_2)$ or (8, 1) without work.
* If a formula for midpoint is not written, any sign or substitution error is at least a blunder.

Blunder (-3)

B1 Incorrectly treats couples as (x_1, x_2) and (y_1, y_2) or switches x and y (once).
B2 Incorrect relevant formula [<u>two</u> or more signs incorrect], e.g.

$$\left(\frac{x_1-x_2}{2}, \frac{y_1-y_2}{2}\right) \text{or} \left(\frac{y_1+y_2}{2}, \frac{x_1+x_2}{2}\right) \text{or} \left(\frac{x_1+y_1}{2}, \frac{x_2+y_2}{2}\right).$$

B3 Two or more signs incorrect in substitution, if formula is written.
B4 Mathematical error, e.g. -8 + 10 = -2 or a blunder in use of fractions, e.g. $^{(-8+10)}/_2 = -4 + 10$.
B5 Last step omitted.

Slips (-1)

S1 Each numerical slip to a maximum of 3, e.g. 5 + 11 = 17.
S2 <u>One</u> incorrect sign in formula or substitution, if formula is written.
S3 <u>One</u> incorrect substitution, if formula is written.

Attempts (3 marks)

A1 Some relevant step, e.g. (5, -8) with x_1 or y_1 identified.
A2 Plots (5, -8) and / or (11, 10) correctly.
A3 Correct relevant formula and stops.
A4 Diagram with correct midpoint indicated, but co-ordinates not written.

Worthless (0)

W1 Irrelevant formula, even if completed, e.g. distance or $\sqrt{x_2y_1 - x_1y_2}$ or similar, subject to A1.

Part (b) **40 (10, 10, 10, 5, 5)marks** **Att (3, 3, 3, 2, 2)**

$a(-1, -2)$, $b(3, 1)$, $c(0, 4)$ are three points.

(i) Find the length of $[ab]$.

(ii) Calculate the area of the triangle *abc.*

(iii) The line L is parallel to ab and passes through the point c. Find the equation of L.

(iv) Show that the point $d(-4, 1)$ is on L.

(v) Investigate whether *abcd* is a parallelogram.

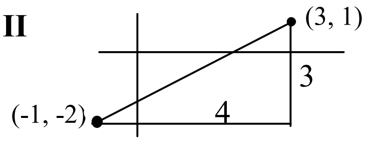
(b)(i) **10 marks** **Att 3**

I

$|ab| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(3-(-1))^2 + (1-(-2))^2} = \sqrt{4^2 + 3^2} = \sqrt{16+9} = \sqrt{25}$ or 5

3 m 7 m 10 m

II



Distance $= \sqrt{4^2 + 3^2} = \sqrt{25}$ *or* 5

* Correct substitution into a correct formula and fails to finish, merits 7 marks.

* 2nd step presupposes 1st step.

* If a formula for distance is **<u>not written</u>**, any sign or substitution error is, at least a blunder, e.g.

Distance $= \sqrt{(3+1)^2 - (1+2)^2} = \sqrt{7}$ ……………..one blunder

Distance $= \sqrt{(3-1)^2 + (1-2)^2} = \sqrt{5}$ ………… …...one blunder

Distance $= \sqrt{(3+1)^2 + (1-2)^2} = \sqrt{17}$ ………...……one blunder

Blunders (-3)

B1 Incorrectly treats couples as (x_1, x_2) *and* (y_1, y_2) or switches x and y (once).

B2 Incorrect relevant formula [<u>two</u> or more signs incorrect], e.g.

$\sqrt{(x_2 - x_1)^2 - (y_2 - y_1)^2}$ *or* $\sqrt{(x_2 + x_1)^2 + (y_2 + y_1)^2}$ *or* $\sqrt{(x_2 + x_1)^2 - (y_2 + y_1)^2}$

B3 Two or more incorrect substitutions, if formula is written.

B4 Mathematical error, e.g. $4^2 = 8$.

B5 Square root omitted, e.g. distance = 25.

B6 Last step omitted.

Slips (-1)

S1 Each numerical slip to a maximum of 3.

S2 <u>One</u> incorrect sign in $(x_2 - x_1)$ or $(y_2 - y_1)$ part of formula.

S3 <u>One</u> incorrect substitution, if correct formula is written.

S4 Obvious misreading of co-ordinate, or finds $|ac|$ or $|bc|$

Attempts (3 marks)

A1 Some relevant step, e.g. (3, 1) with x_1 or y_1 identified.

A2 Plots (3, 1) and / or (-1, -2) reasonably well..

A3 Correct relevant formula and stops.

A4 Formula with $(x_2 - x_1)$ or $(y_2 - y_1)$ and some correct substitution.

A5 Oversimplifies, e.g. $\sqrt{(x_2 - x_1) + (y_2 - y_1)}$ with some correct substitution, even if completed.

A6 States Pythagoras' Theorem or $\sqrt{a^2 + b^2}$.

A7 $\sqrt{25}$ or 5 without work.

A8 Uses translation, e.g. (4, 3) and stops.

Worthless (0)

W1 Irrelevant formula and stops.

(b)(ii) **10 marks** **Att 3**

I $(-1, -2) \to (-1, -6)$ Area = ½| $x_1y_2 - x_2y_1$ | $(3, 1) \to (3, -3)$ = ½|(−1× −3) − (3 ×−6)| $(0, 4) \to (0, 0)$ = ½| 3 + 18| = ½|21| or 10·5	**II** Area = ½| $x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)$| 3m = ½| 1(1 - 4) + 3(4 + 2) + 0(-2 - 1)| 7m = ½|-1(-3) + 3(6) +0(-3)| = ½|21| or 10·5 10m

III $\frac{1}{2}\begin{vmatrix} -1 & -2 \\ 3 & 1 \\ 0 & 4 \\ -1 & -2 \end{vmatrix}$ 3 m

Area = ½| (-1)(1) + (3)(4) + (0)(-2) − (3)(-2) − (0)(1) − (-1)(4)| 7 m

= ½| -1 +12 +0 +6 +0 +4|

= ½|21| or 10·5 10 m

* Area = ½| -21| or −10·5 incurs no penalty.

Blunders (-3)

B1 Incorrect relevant formula and continues, e.g. ½|$x_1y_2 + x_2y_1$| or ½|$x_1y_2 \times x_2y_1$| or omits ½.

B2 Two or more incorrect substitutions, if formula is written.

B3 Mathematical error, e.g. (-1)(-3) = -3.

B4 No necessary translation, i.e. calculates area of Δaob or Δaoc or Δ boc, or blunder in translation

B5 Area = ½ |ab|.|bc| and continues.

B6 Last step omitted.

Slips (-1)

S1 Each numerical slip to a maximum of 3.

S2 <u>One</u> incorrect sign in formula for method II.

S3 <u>One</u> incorrect substitution, if correct formula is written or in method III..

S4 Error in one ordinate having used correct translation in method I.

Attempts (3 marks)

A1 Some relevant step, e.g. (3, 1) with x_1 or y_1 identified, in this part.

A2 Plots (-1, -2) and / or (3, 1) and / or (0, 4) reasonably well for this part.

A3 Correct relevant formula and stops.

A4 Correct answer without work.

A5 ½|$x_1y_2 \times x_2y_1$| or similar with some correct substitution and stops.

Worthless (0)

W1 ½ on its own.

(b)(iii) **10 marks** **Att 3**

I Slope *ab* $= \dfrac{y_2 - y_1}{x_2 - x_1}$ $= \dfrac{1+2}{3+1} = \dfrac{3}{4}$ Equation of *L*: $y - 4 = \dfrac{3}{4}(x - 0)$ $\Rightarrow 4y - 16 = 3x \Rightarrow 3x - 4y + 16 = 0$	**II** Slope *ab* $= \dfrac{y_2 - y_1}{x_2 - x_1}$ $= \dfrac{1+2}{3+1} = \dfrac{3}{4}$ or 3x - 4y = c $3(0) - 4(4) = c \Rightarrow c = -16$	**III** Slope *ab* $= \dfrac{y_2 - y_1}{x_2 - x_1}$ 3m $= \dfrac{1+2}{3+1} = \dfrac{3}{4}$ or y = ¾ x + c 7m $4 = ¾ (0) + c \Rightarrow c = 4$ 10m or y = ¾ x + 4

* Errors in simplifying equation of *L* to be penalised in later part, if used.

* Answers without work

$y - 4 = ¾(x - 0)$ or any correct variation……………………….award full marks.

$y - 4 = -^4/_3(x - 0)$ or equivalent..……………………………….award 7 marks.

$y - 4 = m(x - 0)$ or equivalent, m not relevant…………………award 4 marks.

$y - 1 = m(x - 3)$, line with neither slope nor point correct……..award 3 marks.

Blunders (-3)

B1 Incorrect relevant formula for slope and continues, e.g.

$$\frac{x_2 - x_1}{y_2 - y_1} \text{ or } \frac{y_2 + y_1}{x_2 + x_1} \text{ or } \frac{y_2 - y_1}{x_1 - x_2}.$$

B2 Incorrect relevant formula for line and continues, e.g. $x - x_1 = m(y - y_1)$ or $y + y_1 = m(x + x_1)$.

B3 Two or more incorrect substitutions, if formula is written.

B4 Switches *x* and *y*, e.g. $y - 0 = ¾(x - 4)$.

B5 Uses a point, other than *a* or *b,* which is not on *L*.

B6 *L* not parallel to *ab* with work.

B7 Last step omitted, e.g. $y = ¾ x + c$ and stops.

Slips (-1)

S1 Each numerical slip to a maximum of 3.

S2 <u>One</u> incorrect sign in formula.

S3 <u>One</u> incorrect substitution, if correct formula is written.

Misreadings (-1)

M1 Uses point *a* or *b* instead of *c*.

Attempts (3 marks)

A1 Some relevant step, e.g. indicates that slope of L is equal to slope of *ab*.

A2 Draws a line through *c* parallel to *ab* and stops.

A3 Correct relevant formula and stops.

A4 Formula with $x_2 - x_1$ and / or $y_2 - y_1$ and some correct substitution.

A5 $m = \tan\theta$ or $\tan = \dfrac{vertical}{horizontal}$ and stops.

(b)(iv) **5 marks** **Att 2**

I	II	III
$d(-4, 1)$ $L: 3x - 4y + 16 = 0$ 2m $\Rightarrow 3(-4) - 4(1) + 16 = 0$ $\Rightarrow -12 - 4 + 16 = 0$ 5m $\Rightarrow d \in L$	$y - 1 = \frac{3}{4}(x + 4)$ $3x - 4y + 16 = 0$ $\Rightarrow d \in L$	$3(-4) - 4(1) = c$ $c = -16$ or $3x - 4y = -16$ $\Rightarrow d \in L$
IV Slope of $dc = \frac{4-1}{0--4} = \frac{3}{4}$ 2m, slope $ab = \frac{3}{4}$, $\Rightarrow$ slopes equal, 5m $\Rightarrow d \in L$		

* Accept candidate's line L from part (iii), subject to slips and blunders with correct conclusion.

* Correct step at simplifying the equation of L in b(iii) gets at least att. 2 here.

Blunders (-3)

B1 Mixes up x and y entries.

B2 Mathematical error, e.g. $(3)(-4) = \pm 7$.

B3 Transposing error in method II, e.g. $3x + 4y + 16 = 0$.

B4 Incorrect relevant formula, e.g. $x - x_1 = m(y - y_1)$ or $y + y_1 = m(x + x_1)$.

Slips (-1)

S1 Each numerical slip to a maximum of 3.

S2 <u>One</u> incorrect sign in line formula, e.g. $y + y_1 = m(x - x_1)$.

S3 <u>One</u> incorrect substitution, if correct line formula is written.

S4 No conclusion or incorrect conclusion in case where candidates $d \notin L$.

Attempts (2 marks)

A1 Some relevant step, e.g. some effort at substitution.

A2 Point (-4, 1) plotted reasonably well.

A3 Correct relevant formula, e.g. for slope or line.

A4 Writes, "if a point is on a line, it must satisfy its equation", or similar.

(b)(v) **5 marks** **Att 2**

I	II	III
$ab \parallel cd$ from above Slope of $ad := \frac{1+2}{-4+1} = \frac{3}{-3} = -1$ Slope of $bc = \frac{4-1}{0-3} = \frac{3}{-3} = -1$ Hence, $ad \parallel bc$. Hence *abcd* a parallelogram	$\|ab\| = 5$ and $ab \parallel cd$ from above $\|cd\| = \sqrt{(-4-0)^2 + (1-4)^2} = \sqrt{25}$ or 5 $\Rightarrow \|ab\| = \|cd\|$ Hence *abcd* a parallelogram	$\vec{bc} = (0\vec{i} + 4\vec{j}) - (3\vec{i} + \vec{j})$ $\vec{bc} = -3\vec{i} + 3\vec{j}$ $\vec{ad} = -3\vec{i} + 3\vec{j}$ $\Rightarrow$ *abcd* a parallelogram

IV	V
$ab \parallel cd$ from above Area of $\Delta abc = 10.5$ from above Area of $\Delta acd = 10.5$ Area of Δabc = Area of Δacd Hence *abcd* a parallelogram	Midpoint of $[ac]$ is $(-\frac{1}{2}, 1)$ Midpoint of $[bd]$ is $(-\frac{1}{2}, 1)$ Hence *abcd* a parallelogram

* Accept correct answers (values) and conclusions without work.

* Note: Omission of first line from either method I, II or IV is a blunder.

Blunders (-3)

B1 Incorrect relevant formula.

B2 $\overrightarrow{bc}$ written as $\vec{b} - \vec{c}$ or $\vec{b} + \vec{c}$ in III or uses wrong direction if using translations.

Slips (-1)

S1 Incorrect or no conclusion.

S2 Each numerical slip to a maximum of 3.

Attempts (2 marks)

A1 Some correct step, e.g. b $\rightarrow$ a $\Rightarrow$ -4, -3.

A2 Draws a reasonable diagram for this part.

A3 Correct relevant formula and stops.

A4 "Yes, it is a parallelogram", without work.

Worthless (0)

W1 "No" without work.

QUESTION 3

Part (a)	**10 marks**	**Att (2, 2)**
Part (b)	**20 marks**	**Att (3, 3)**
Part (c)	**20 marks**	**Att (3, 3)**

Part (a)	**10 (5, 5) marks**	**Att (2, 2)**

The circle *C* has equation $x^2 + y^2 = 36$.

(i) Write down the radius of *C*.

(ii) The radius of another circle is twice the radius of *C*.
The centre of this circle is (0, 0). Write down its equation.

(a)(i)	**5 marks**	**Att 2**

$$r^2 = 36 \Rightarrow r = 6$$

* Accept $r = 6$ without work.

Blunders (– 3)

B1 $r^2 = 36$ and stops.
B2 $r^2 = 36$ ➔ $r = 18$.
B3 Uses √ incorrectly.
B4 Incorrect relevant formula and continues, e.g. $x^2 + y^2 = r \Rightarrow r = 36$.

Slips (-1)

S1 Draws a circle with centre (0, 0) and radius obviously = 6 but does not write r = 6.
S2 $x^2 + y^2 = 6^2$ without $x^2 + y^2 = r^2$.

Attempts (2 marks)

A1 Some relevant step e.g. mentions (0,0) / draws graph with centre at (0,0).
A2 Correct relevant formula and stops $x^2 + y^2 = r^2$.
A3 Gets a point that is on the circle e.g. (6,0) etc.
A4 Writes down the formula for distance $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.
A5 Any mention of $x_1 = 0$ or $y_1 = 0$.
A6 $r = 36$ with or without work.

(a)(ii)	**5 marks**	**Att 2**

$2r = 12$, Equation $x^2 + y^2 = 12^2$ *or* 144

* Accept candidate's equation consistent with part (i) above without work
* Accept $x^2 + y^2 = r^2$, $r^2 = 12^2$ *or* 144.

Blunders (– 3)

B1 $r^2 = 2r$, i.e. 24 not 144.
B2 Incorrect relevant formula for circle $x^2 - y^2 = r^2$ with correct substitution.
B3 Leaves out squares, i.e. $x + y = 144$.
B4 Just doubles answer from part (i), i.e. $x^2 + y^2 = 12$

Attempts (2 marks)

A1 Correct relevant formula $x^2 + y^2 = r^2$.
A2 Writes down 144 and stops.

A3 Writes down distance formula and stops.
A4 Some relevant step.
A5 $x^2 + y^2 = 72$.

Worthless (0)
W1 Linear equation for circle, subject to attempt.

Part (b) **20 (10, 10) marks** **Att(3, 3)**

A circle has equation $x^2 + y^2 = 13$.
The points $a(2, -3)$, $b(-2, 3)$ and $c(3, 2)$ are on the circle.

(i) Verify that $[ab]$ is a diameter of the circle.

(ii) Verify that $\angle acb$ is a right angle.

(b)(i) **10 marks** **Att 3**

I $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$|ab| = \sqrt{(-2-2)^2 + (3+3)^2} = \sqrt{16+36} = \sqrt{52} = 2\sqrt{13}$

Radius of circle $r = \sqrt{13}$

$|ab| = 2r$
(➔ $[ab]$ is a diameter).

II

Midpoint of $[ab] = \left(\frac{x_1 + x_2}{2}, \frac{y \ + y_2}{2}\right)$

$= \left(\frac{2-2}{2}, \frac{-3+3}{2}\right) = (0, 0)$

Centre of circle is (0,0)

(➔ $[ab]$ is a diameter.)

III

Centre of circle is (0,0)
Image of $a(2, -3)$ under S_o
is $b(-2, 3)$
(➔ $[ab]$ is a diameter.)

IV

$y - y_1 = m(x - x_1)$	3m
Equation ab ➔ $y - -3 = \frac{-3}{2}(x - 2)$	4m
Substitute (0, 0) ➔ $0 - -3 = -\frac{3}{2}(0 - 2)$	7m
➔ $3 = 3$	10m

(➔ $[ab]$is a diameter.)

* Award 0 marks for this part (i) if candidate omits it and does not address *diameter* in part (ii). See B6 of this part (i).

Blunders (- 3)
B1 Incorrectly treats couples as (x_1, x_2) and (y_1, y_2).
B2 Incorrect relevant formula e.g. $\sqrt{(x_2 - x_1)^2 - (y_2 - y_1)^2}$ etc.
B3 Two or more signs incorrect in substitution, if formula is written.

B4 Mathematical error, e.g. $(-3)^2 = 6$.
B5 Failure to state last line.
B6 No calculation in this part but correct conclusion from calculations in part (ii).
B7 Incorrect radius, if it is not a slip.
B8 Uses square root incorrectly.
B9 Error in translation, Method III.

Slips (– 1)
S1 One incorrect sign in $(x_2 - x_1)$ or $(y_2 - y_1)$ part of formula.
S2 One incorrect substitution, if formula is written.
S3 Error in one ordinate, having used correct translation in method III

Attempts (3 marks)
A1 Writes down correct relevant formula and stops.
A2 Plots *a* or *b* , correctly.
A3 States (0,0) or $r = \sqrt{13}$.
A4 Substitutes any one (or more) of the three points into the given equation of the circle.
A5 States Theorem of Pythagoras and stops.
A6 Some relevant step, e.g. some statement indicating that the diameter is twice the radius or the centre of the circle is the midpoint of the diameter.

(b)(ii) **10 marks** **Att 3**

I	II
3m 4m	$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ 3m
Slope $ac = \frac{y_2-y_1}{x_2-x_1} = \frac{2+3}{3-2} = \frac{5}{1} = 5$,	$\|ab\|^2 = (-2-2)^2 + (3+3)^2 = 52$ 4m
Slope $cb = \frac{3-2}{-2-3} = \frac{1}{-5}$. 7m	$\|ac\|^2 = (3-2)^2 + (2+3)^2 = 26$
	$\|cb\|^2 = (-2-3)^2 + (3-2)^2 = 26$ 7m
Hence $ac \perp cb$;	$\Rightarrow \|ab\|^2 = \|ac\|^2 + \|cb\|^2$ 10m
hence $\|\angle acb\|$ is a right angle	$\Rightarrow \|\angle acb\| = 90^o$

* Award 0 marks for this part (ii) if candidate omits it and does not address $|\angle acb| = 90^o$ in part (i). See B6 of this part (ii).

Blunders (- 3)
B1 Incorrectly treats couples as (x_1, x_2) and (y_1, y_2).
B2 Incorrect relevant formula e.g. $\sqrt{(x_2-x_1)^2-(y_2-y_1)^2}$ or $\frac{y_2+y_1}{x_2+x_1}$ or $\frac{y_2-y_1}{x_1-x_2}$ or $\frac{x_2-x_1}{y_2-y_1}$ etc.
B3 Two or more signs incorrect in substitution if formula written.
B4 Mathematical error $-(-3) = -3$ or similar.
B5 Failure to state last line.
B6 No calculation in this part but correct conclusion from calculations in part (i).

Slips (-1)
S1 One incorrect sign in $(y_2 - y_1)$ part of formula.
S2 One incorrect substitution if formula written.
S3 Each numerical slip to a maximum of three.

Attempts (3 marks)
A1 Correct relevant step, e.g. formula and stops.

A2 Any formula with $(x_2 - x_1)$ and / or $(y_2 - y_1)$ and some correct substitution.
A3 States Theorem of Pythagoras and stops.
A4 Plots c correctly.
A5 States $m_1 \times m_2 = -1$ and stops.
A6 Correct step, e.g. (3, 2) with x_1 and y_1 clearly identified.

Worthless (0)
W1 Irrelevant formula and no substitution.

Part (c) **20 (10, 10) marks** **Att (3, 3)**

K is a circle with centre (−2, 1). It passes through the point (−3, 4).

(i) Find the equation of K.

(ii) The point $(t, 2t)$ is on the circle K.
Find the two possible values of t.

(c)(i) **10 marks** **Att 3**

I	II	III
$(x-h)^2 + (y-k)^2 = r^2$ …… 3m	$(x-h)^2 + (y-k)^2 = r^2$	(-3, 4) is an element of $x^2 + y^2 + 2gx + 2fy + c = 0$, 3m
4m 7m $r = \sqrt{(-3+2)^2 + (4-1)^2} = \sqrt{1+9} = \sqrt{10}$ $(x+2)^2 + (y-1)^2 = 10$ 10m	$(x+2)^2 + (y-1)^2 = r^2$ 7m $\Rightarrow (-3+2)^2 + (4-1)^2 = r^2$ $\Rightarrow r^2 = 10$ 10m	$g = 2, f = -1$ 7m $9 + 16 - 12 - 8 + c = 0$ $x^2 + y^2 + 4x - 2y - 5 = 0$, 10m

* $(-2-h)^2 + (1-k)^2 = 10$ ➔ 7 marks.
* Note : In method I, line 2 without line 1 ➔ 3 marks only.

Blunders (-3)
B1 Incorrect relevant formula, e.g. $(x - h)^2 - (y - k)^2 = r^2$, i.e. incorrect central sign, etc.
B2 Incorrectly treats couples as (x_1, x_2) and (y_1, y_2).
B3 Two or more signs incorrect in substitution if formula written.
B4 Mathematical error, e.g. $(3)^2 = 6$ or $-(-2) = -2$.
B5 Uses or omits√ incorrectly.
B6 Incorrect centre or radius, subject to M1.
B7 Blunder in expanding $(x + 2)^2$ or $(y - 1)^2$ if used, subject to marks already secured.

Slips (-1)
S1 One incorrect sign or substitution in $(x - h)$ or $(y - k)$ part.
S2 Numerical slips to a maximum of three.

Misreadings (-1)
M1 Uses (-3, 4) as centre and (-2, 1) as point on the circumference.

Attempts (3 marks)
A1 Correct relevant formula and stops.
A2 Any formula with $(x_1 - x_2)$ or $(y_1 - y_2)$ and some correct substitution.
A3 Diagram plots (-2, 1) and (-3, 4) correctly and / or draws a circle.

A4 States the formula $(x - h)^2 + (y - k)^2 = r^2$ or $x^2 + y^2 + 2gx + 2fy + c = 0$.
A5 Linear equation merits attempt mark at most.

(c)(ii) **10 marks** **Att 3**

I

From $(x+2)^2 + (y-1)^2 = 10$,

$(t+2)^2 + (2t-1)^2 = 10 \quad \Rightarrow t^2 + 4t + 4 + 4t^2 - 4t + 1 = 10 \quad \Rightarrow 5t^2 = 5 \quad \Rightarrow t = \pm 1$

3m 7m 10m

II

From $x^2 + y^2 + 4x - 2y - 5 = 0$,

$t^2 + 4t^2 + 4t - 4t - 5 = 0 \quad \Rightarrow 5t^2 = 5 \quad \Rightarrow t = \pm 1$

* Accept candidates answer for part (i), if not linear.

Blunders (-3)

B1 Blunder in expanding $(t + 2)^2$ or $(2t - 1)^2$. (once only)
B2 Transposing error.
B3 Mathematical error.
B4 Incorrect relevant formula e.g. $\sqrt{(x_2 - x_1)^2 - (y_2 - y_1)^2}$ etc.
B5 Two or more signs incorrect in substitution, if formula written.
B6 Error in factorising or in application of quadratic formula.
B7 Gets only one solution for t.
B8 Confuses x and y co-ordinates, i.e. $x = 2t$ and $y = t$.

Slips (-1)

S1 Each numerical slip to a maximum of three.

Attempts (3 marks)

A1 Some relevant step, e.g. $x = t$ or $y = 2t$ or $r = \sqrt{10}$, in this part.
A2 Substitutes t in for x or $2t$ in for y in equation of circle.
A3 Any formula with $(x_2 - x_1)$ or $(y_2 - y_1)$ and some correct substitution.
A4 One or both correct values for t, with or without verification is an attempt.
A5 Linear equation merits at most the attempt.

QUESTION 4

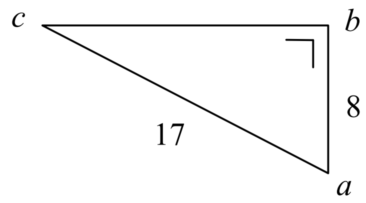
Part (a)	**10 marks**	**Att 3**
Part (b)	**20 marks**	**Att 7**
Part (c)	**20 marks**	**Att 7**

Part (a) **10 marks** **Att 3**

In the triangle *abc*,

$|ab| = 8, \quad |ac| = 17 \quad \text{and} \quad |\angle abc| = 90°.$

Find $|bc|$.



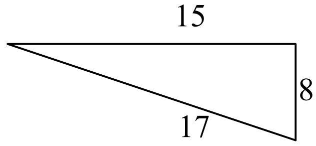
(a) **10 marks** **Att 3**

$|bc|^2 + 8^2 = 17^2 \quad \Rightarrow |bc|^2 + 64 = 289 \quad \Rightarrow |bc|^2 = 289 - 64 = 225 \quad \Rightarrow |bc| = \sqrt{225} \;\; or \;\; 15$

3m 4m 7m 10m

* Accept correct trigonometrical method.

* Accept Pythagorean triple 8, 15, 17 explicitly written or



Blunders (-3)

B1 Blunder in Pythagoras' Theorem.

B2 Blunder in indices.

B3 Transposition error.

Attempts (3 marks)

A1 Statement or use of any relevant theorem.

A2 Scaled diagram giving |*bc*| = 15.

A3 15 without work.

A4 Some relevant step.

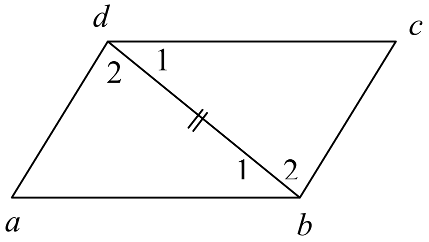
Worthless (0)

W1 Example 17 – 8.

Part (b) **20 marks** **Att 7**

Prove that the opposite sides of a parallelogram have equal lengths.

(b) **20 marks** **Att 7**



abcd is a parallelogram

To Prove: $|ab| = |dc|$ and $|ad| = |bc|$

Join *db* Step 1, 7m

Proof: Consider triangles *abd* and *cdb*

$|\angle abd| = |\angle cdb|$ … alternate Step 2, 10m

$|\angle bda| = |\angle dbc|$ …alternate Step 3, 13m

$|bd|$ common Step 4, 16m

Hence the triangles are congruent Step 5, 19m

Hence, $|ab| = |cd|$ and $|ad| = |cb|$, corresponding sides . 20m

* Diagram with angles clearly marked and side indicated as in solution ➔ 16 marks.

* Proof, without diagram ➔ Att 7, if proof can be reconciled with a diagram.

Blunders (-3)

B1 Each step omitted or incorrect.

B2 Steps in an illogical order, once only, but steps 2 and 3 and 4 may be interchanged.

Note: in cases where steps are missing from the text, B2 may or may not apply.

Attempts (7 marks)

A1 Any relevant step stated or indicated.

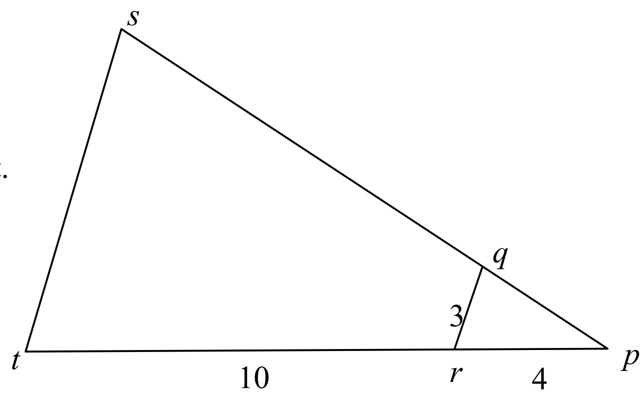
Worthless (0)

W1 Incorrect, irrelevant theorem, subject to the attempt mark.

Part (c) **20 (5, 10, 5) marks** **Att (2, 3, 2)**

The triangle *pst* is the image of the triangle *pqr* under an enlargement with centre *p*.

$|pr| = 4$, $|rt| = 10$ and $|qr| = 3$.



(i) Find the scale factor of the enlargement.

(ii) Find $|st|$.

(iii) The area of the triangle *pqr* is 5 square units.
Find the area of the quadrilateral *qstr*.

(c)(i) **5 marks** **Att 2**

$$|pt| = k\,|pr| \qquad \Rightarrow k = \frac{|pt|}{|pr|} \qquad = \frac{14}{4} = 3\cdot 5$$

* Accept correct answer without work.

Blunders (-3)

B1 Blunder in finding scale factor, e.g. writes scale factor is 2.5 or increases by 250%.

B2 Incorrect ratio in finding scale factor or writes $\frac{4}{14}$.

B3 Incorrect centre of enlargement.

Attempts (2 marks)

A1 Attempt at ratio, e.g. $|pt|:|pr|$ or $\Delta pts:\Delta prq$ and stops.

A2 Some relevant step, e.g. 14 and stops.

(c)(ii) **10 marks** **Att 3**

I

$$|st| \quad = 3\cdot 5|qr| \quad = \quad 3\cdot 5(3) \quad = 10\cdot 5$$

3m 7m 10m

II

$$\frac{|st|}{|qr|} = \frac{|pt|}{|pr|} \qquad \Rightarrow \frac{|st|}{3} = \frac{14}{4} \qquad \Rightarrow |st| = \frac{42}{4} \quad \text{or } 10\cdot 5$$

3m 7m 10m

* Accept candidate's scale factor, k, from (i).

* Accept correct answer without work.

Blunders (-3)

B1 Incorrect and inconsistent k.

B2 Mathematical blunder, e.g. in isolating $|st|$.

B3 Misplaced decimal point.

Attempts (3 marks)

A1 Some relevant step.

(c)(iii) **5 marks** **Att 2**

Area triangle $pst \quad = \quad 3\cdot 5^2 \times 5 \quad = \quad 61\cdot 25$

Area quadrilateral $= \quad 61\cdot 25 \ - \ 5 \quad = \quad 56\cdot 25$

* Accept candidate's scale factor from previous part.

Blunders (-3)

B1 $5 * (3\cdot 5)^2$, where * is not multiply.

B2 $5 \times 3\cdot 5 = 17\cdot 5$.

B3 Failure to get k^2 from k.
B4 Incorrect and inconsistent scale factor.

Slips (– 1)
S1 Finds area of Δpst correctly and stops.

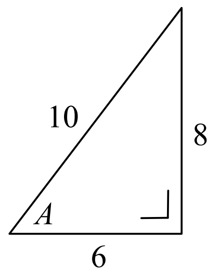
Attempts (2 marks)
A1 Area of triangle formula from tables with some substitution.
A2 Some relevant step, e.g. (3·5) squared.
A3 Relevant formula not transcribed from tables, e.g. area triangle = ½ base × h.
A4 Consistent answer without work.

QUESTION 5

Part (a)	**10 marks**	**Att (2, 2)**
Part (b)	**20 marks**	**Att (2,3,2)**
Part (c)	**20 marks**	**Att (3,3)**

Part (a) **10 (5, 5) marks** **Att (2,2)**

The lengths of the sides of a right-angled triangle are shown in the diagram and A is the angle indicated.



(i) Write down the value of cos A.

(ii) Hence, find the angle A, correct to the nearest degree.

(a)(i) **5 marks** **Att 2**

$$\cos A = \frac{6}{10} \text{ or equivalent}$$

* Accept correct answer without work.
* Accept use of 3, 4, 5, as length of sides.

Blunders (-3)

B1 Incorrect Ratio, e.g. $\cos A = \frac{8}{10}$

Misreadings (-1)

M1 Sin A = $\frac{8}{10}$ or Tan A = $\frac{8}{6}$ and stops.

Attempts (2 Marks)

A1 Any trigonometric function defined correctly, e.g. Tan A = $\frac{opposite}{adjacent}$.

A2 Identifies any correct side as opposite, adjacent, or hypotenuse.

A3 Any correct work with Pythagoras.

A4 Some relevant step.

Worthless (0)

W1 SOHCAHTOA stated.

(a)(ii) **5 marks** **Att 2**

$$A = 53°$$

* Accept an answer consistent with candidate's answer from part (i).
* Accept use of 3, 4, 5, as length of sides.
* Accept use of Sine or Cosine Rule.
* Accept correct answer without work.

Blunders (-3)
B1 Uses calculator in Rad / Grad Mode. (0·927) / (59·033).
B2 Error in use of inverse function, e.g. Cos 0·6 = 0·999° ≈ 1°.

Slips (-1)
S1 Failure to round off or rounds off too early, if it affects the answer.
S2 Each numerical slip to a maximum of 3.

Attempts (2 Marks)
A1 A correct step.

Worthless (0 Marks)
W1 Incorrect answer without work

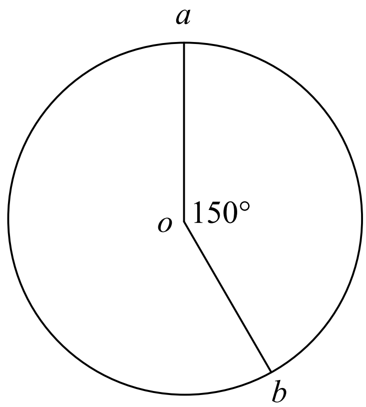
Part (b) **20 (5, 10, 5) marks** **Att (2, 3, 2)**

A circle has centre *o* and radius 4 cm.
a and *b* are two points on the circle and
$|\angle aob| = 150°$.

(i) Find the area of the circle, correct to the nearest cm^2.

(ii) Find the area of the sector *aob*, correct to the nearest cm^2.

(iii) Find the length of the shorter arc *ab*, correct to the nearest cm.



(b)(i) **5 marks** **Att 2**

$$\pi r^2 = 3\cdot 14 \times 4^2 = 50\cdot 24 = 50 \text{ cm}^2$$

* Accept 3·1 ≤ π ≤ 3·2
* No more than 3 marks may be lost for the calculations.
* Areas of both sectors added in part (ii) merits 5 Marks in part (i).
* Accept correct answer without work.

Blunders (-3)
B1 Treats $r^2 = 2r$.
B2 Blunder in applying formula.
B3 Leaves answer in terms of π.
B4 Incorrect relevant formula, e.g. $\frac{1}{2}|ab|$ Sin$C$.

Slips (-1)
S1 Failure to round off or rounds off too early, if it affects the answer, e.g. $49.6 \le \text{area} \le 51.2$, without work.
S2 Each numerical slip to a maximum of 3.

Misreadings (-1)
M1 $r \neq 4$

Attempts (2 Marks)
A1 Some correct substitution into a reasonable formula.
A2 Some relevant step, e.g. diagram with $r = 4$, in this part.

(b)(ii) **10 marks** **Att 3**

3m 7m 9m 10m **I** $\frac{150}{360}\pi r^2 = \frac{150}{360}(50 \cdot 24) = 20 \cdot 93 = 21\ \text{cm}^2$	**II** Area = $\frac{1}{2} r^2\theta = \frac{1}{2} \times 4^2 \times {}^{150\pi}/_{180} = 20{\cdot}93 = 21$
III $\frac{210}{360}\pi r^2 = \frac{210}{360}(50 \cdot 24) = 29 \cdot 3 \approx 29\ \text{cm}^2$	**IV** Area = $\frac{1}{2} r^2\theta = \frac{1}{2} \times 4^2 \times {}^{210\pi}/_{180} = 29{\cdot}3 = 29$

* Accept an answer consistent with candidate's answer from part (i).
* Accept either sector.
* Both sector areas calculated in part (ii) but not added merits 2 marks in part (i).

Blunders (-3)
B1 Incorrect fraction of circle.
B2 Incorrect substitution into formula.
B3 Fails to convert θ to radians correctly.
B4 Incorrect relevant formula.
B5 Degree measure of circle ≠ 360°.
B6 Mathematical error.

Slips (-1)
S1 Failure to round off or rounds off too early, if it affects the answer.
S2 Each numerical slip to a maximum of 3.

Misreadings (-1)
M1 $r \neq 4$

Attempts (2 Marks)
A1 Identifies 210° as the reflexive angle.
A2 $\frac{5}{12}$ or $\frac{7}{12}$ stated and stops.
A3 Some relevant step, e.g. 360° – 150° and stops.
A4 Correct answer without work.

(b)(iii) **5 marks** **Att 2**

I $\frac{150}{360}2\pi r = \frac{150}{360} \times 2 \times 3 \cdot 14 \times 4 = 10 \cdot 46 \approx 10$ cm	**II** Arc = $r\theta = 4 \times {}^{150\pi}/_{180} = 10{\cdot}46 \approx 10$ cm

* Note: Incorrect answer without work e.g. 600 is worthless.

Blunders (-3)
B1 Incorrect fraction of circumference.
B2 Incorrect substitution into formula.
B3 Fails to convert θ to Radians.

B4 Incorrect relevant formula, with some substitution.
B5 Degree measure of circle $\neq 360°$.

Slips (-1)
S1 Failure to round off or rounds off too early, if it affects the answer.
S2 Each numerical slip to a maximum of 3.

Misreadings (-1)
M1 $r \neq 4$
M2 Works with major arc. (14·66 → 15cm).

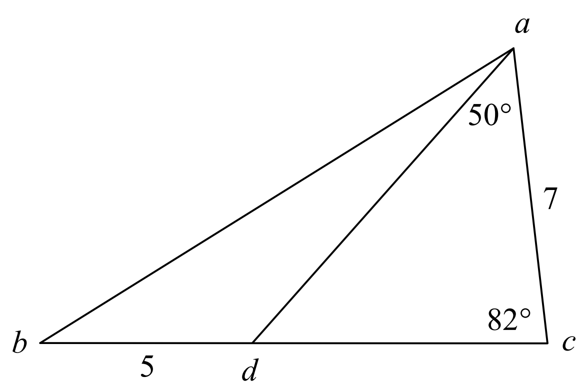
Attempts (2 Marks)
A1 $\frac{5}{12}$ or $\frac{7}{12}$ stated and stops.
A2 Some relevant step, e.g. diagram with $r = 4$, in this part.
A3 Correct answer without work.

Part (c) **20 (10, 10) marks** **Att (3, 3)**

In the triangle abc, d is a point on $[bc]$.
$|bd| = 5$ cm, $|ac| = 7$ cm,
$|\angle dca| = 82°$ and $|\angle cad| = 50°$.

(i) Find $|dc|$, correct to the nearest cm.

(ii) Find $|ab|$, correct to the nearest cm.



(c)(i) **10 marks** **Att 3**

$|\angle adc| = 180 - (50 + 82) = 48°$ 3m

$$\frac{|dc|}{\sin 50} = \frac{7}{\sin 48} \quad (7\text{m}) \quad \Rightarrow \quad |dc| = \frac{7\sin 50}{\sin 48} = \frac{7(0\cdot7660)}{0\cdot7431} = 7\cdot21 \ (9\text{m}) = 7 \text{ cm} \ (10\text{m})$$

* 7 marks for correct formula with correct substitution.
* Calculation errors incur at most -3 marks.

Blunders (-3)
B1 Error in Sine Rule (once).
B2 Incorrect substitution and continues.
B3 Incorrect function read, e.g. reads cosine instead of sine and continues.
B4 Uses radian (or gradient) mode incorrectly.
B5 Mathematical Error, e.g. Error in cross multiplying.
B6 Sum of angles in a triangle $\neq 180°$.
Slips (-1)

S1 Failure to round off or rounds off too early, if it affects the answer.
S2 Each numerical slip to a maximum of -3.
Misreading (-1)
M1 Finds $|ad|$.(9·327→ 9).

Attempts (3 Marks)
A1 Identifies $|\angle adc|$ = 48° and stops.
A2 Incorrect relevant formula, e.g. area of triangle with some correct substitution.
A3 Some relevant step, e.g. external angle = sum of opposite internal angles.
A4 Correct answer without work.

(c)(ii) **10 marks** **Att 3**

$$|ab|^2 = 12^2 + 7^2 - 2(12)(7)\cos 82 \quad ...7m$$
$$|ab|^2 = 144 + 49 - 168(0\cdot1392) = 193 - 23\cdot3856 = 169\cdot6144$$
$$|ab| = 13\cdot02 \quad ...9m$$
$$|ab| = 13 \quad ...10m$$

* 7 marks for correct formula with correct substitution.
* Calculation errors incur at most -3 marks.
* Accept an answer consistent with candidate's answer from part (i).

Blunders (-3)
B1 Error in Cosine Formula (once).
B2 Incorrect substitution & continues.
B3 Incorrect function read, e.g. reads sine instead of cosine and continues.
B4 Uses radian (or gradient) mode incorrectly. (5·78 → 6, 12·08 → 12)
B5 Mathematical error, e.g. error in transposing.
B6 Sum of angles in triangle ≠ 180°.

Slips (-1)
S1 Failure to round off or rounds off too early, if it affects the answer.
S2 Each numerical slip to a maximum of -3.
Misreading (-1)
M1 Finds $|ad|$.(9·18 → 9). Distinct from possible work in (c) part (i).

Attempts (3 Marks)
A1 $|bc|$ = 12 and stops.
A2 Identifies $|\angle adb|$ = 132° and stops.
A3 Correct formula with some but not all substitution.
A4 Incorrect relevant formula, e.g. area of triangle with some substitution.
A5 Some relevant step, e.g. sum of angles in triangle = 180°.
A6 Correct answer without work.
A7 Incorrectly avoids use of cosine rule ➔ attempt 3 at most

Worthless (0)
W1 Identifies $|\angle abd|$ = 32° or $|\angle bad|$ = 16° without work. (Measurement from diagram)

QUESTION 6

Part (a)	**10 marks**	**Att (2,2)**
Part (b)	**20 marks**	**Att(2,2,2,2)**
Part (c)	**20 marks**	**Att (2,3,2)**

Part (a) **10 (5, 5) marks** **Att (2, 2)**

The letters of the word CUSTOMER are arranged at random.

(i) How many different arrangements are possible?
(ii) How many of these arrangements begin with the letter C?

(a)(i) **5 marks** **Att 2**

8! or 40320 or 8.7.6.5.4.3.2.1 or ${}^{8}P_{8}$

* Accept correct answer without work.
* Multiplication must be clearly indicated, i.e. 8,7,6,5,4,3,2,1 or | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 | and stops merits Att 2 marks.
* If parts of (a) are not identified, and if it is not obvious which part is being attempted treat each part in order.
* Penalise repeated errors in each section, but allow the candidate to use result from one section in a later one.
* Note: Any relevant integer from the solutions written down or used is A3.(i.e.1,2,3,4,5,6,7 or 8)
  e.g. 3 × 6 gets Att 2.
  18 without work gets zero.
  64 without work gets zero.

Blunders (-3)

B1 Each 'box' omitted, (but allow omission of 1 e.g. 8.7.6.5.4.3.2 is full marks for (a)(i)).
B2 Each incorrect 'box'.
B3 Addition instead of multiplication.

Slips (-1)
S1 Numerical errors to a maximum of 3.

Attempts (2 marks)
A1 One correct step (e.g. partial list) and stops.
A2 At least one correct permutation listed, e.g. C,U,S,T,O,M,E,R (not CUSTOMER).
A3 Any relevant <u>integer</u> from the solutions written down and stops.
A4 Writes any permutation or factorial or combination symbol and stops.
A5 One or more boxes drawn.

(a)(ii) **5 marks** **Att 2**

$7!$ *or* 5040 *or* $7.6.5.4.3.2.1$ *or* ${}^{7}P_{7}$ *or* $\frac{1}{8} \times \text{ans(i)}$

* Accept correct or consistent answer without work.

* Multiplication must be clearly indicated, i.e. 7,6,5,4,3,2,1 or | 7 | 6 | 5 | 4 | 3 | 2 | 1 | and stops merits Att 2 marks.

* If parts of (a) are not identified, and if it is not obvious which part is being attempted treat each part in order.

* Penalise repeated errors in each section, but allow the candidate to use result from one section in a later one.

* Note: Relevant integers for A3 are 1,2,3,4,5,6,7,8.

Blunders(-3)

B1 Each 'box' omitted, (but allow omission of 1 e.g. 7.6.5.4.3.2 is full marks for (a)(ii)).

B2 Each incorrect 'box'.

B3 Addition instead of multiplication.

Slips (-1)

S1 Numerical errors to a maximum of 3.

Attempts (2 marks)

A1 One correct step (e.g. partial list) and stops.

A2 At least one correct permutation listed, e.g. C,U,S,T,O,M,E,R (not CUSTOMER), for this part.

A3 Any relevant <u>integer</u> from the solutions written down and stops.

A4 Writes any permutation or factorial or combination symbol and stops.

A5 One or more boxes drawn.

A6 Fraction $\frac{1}{8}$ in part (ii).

Part (b) **20 (5, 5, 5, 5) marks** **Att (2, 2, 2, 2)**

A committee of 3 people is selected from a group of 15 doctors and 12 dentists.

In how many different ways can the 3 people be selected

(i) if there are no restrictions

(ii) if the selection must contain exactly 2 doctors

(iii) if the selection must contain at least 1 doctor and at least 1 dentist

(iv) if the selection must contain one specific doctor and one specific dentist?

(b)(i) **5 marks** **Att 2**

$\binom{27}{3}$ *or* $\binom{27}{24}$ *or* ${}^{27}C_{3}$ *or* ${}^{27}C_{24}$ *or* 2925

or $\frac{27!}{24!.3!}$ *or* $\frac{27.26.25}{3.2.1}$ *or* $\frac{{}^{27}P_{3}}{3!}$ *or* $\frac{17550}{6}$

* Note: In (b)(i) relevant integers for A2 are 1,2,3,6,24,25,26,27,17550.

(b)(ii) **5 marks** **Att 2**

$$\binom{15}{2}\times\binom{12}{1} \quad or \quad 105\times 12 \quad or \quad 1260$$

* Note: In (b)(ii) relevant integers for A2 are 1,2,12,15,105.

(b)(iii) **5 marks** **Att 2**

$$\binom{15}{1}\times\binom{12}{2}+\binom{15}{2}\times\binom{12}{1} \quad or \quad 990+1260 \quad or \quad 2250 \quad or \quad ans(b)(ii)+\binom{15}{1}\times\binom{12}{2}$$

$$or \quad ans(b)(i)-\binom{15}{3}-\binom{12}{3}$$

* Note: In (b)(iii) relevant integers for A2 are 1,2,3,12,15,990,1260.

(b)(iv) **5 marks** **Att 2**

$$1\times 1\times\binom{25}{1} \quad or \quad \binom{25}{1} \quad or \quad {}^{25}C_1 \quad or \quad \binom{25}{24} \quad or \quad {}^{25}C_{24} \quad or \quad 25 \quad or \quad \left[\binom{14}{1}+\binom{11}{1}\right]$$

* Note: In (b)(iv) relevant integers for A2 are 1,11,14,24.
* Accept correct answer without work.
* No penalty for $\left(\frac{27}{3}\right)$, but $\frac{27}{3}$ is 2 Blunders. $\therefore$ Award *Att 2*; apply in each section.
* If parts of (b) are not identified, and if it is not obvious which part is being attempted treat each part in order.
* Penalise repeated errors in each section, but allow the candidate to use result from one section in a later one, where applicable, e.g. in (b) (iii).

Blunders(-3)
B1 ${}^{27}P_3$ or 27.26.25 or 17550 and stops.
B2 'Top' section incorrect, e.g. $\binom{15}{3}$ or $\binom{12}{3}$.
B3 'Bottom' section incorrect, e.g. $\binom{27}{2}$.
B4 Inverted, e.g. $\binom{3}{27}$ in (b) (i).
B5 Addition instead of multiplication or vice-versa.

Slips(-1)
S1 Numerical errors to a maximum of 3.

Attempts(2 marks)
A1 One relevant step e.g. 27! in (b)(i).
A2 Any relevant integer from the possible solutions, for each separate part.
A3 Any use of !, P, C, $\binom{}{}$.
A4 A statement such as 1 doctor and 2 dentists + 2 doctors and 1 dentist.
A5 A partial list.

Part (c) **20 (5, 10, 5) marks** **Att (2, 3, 2)**

Four cards, numbered 2, 3, 4, 5 respectively, are shuffled and then placed in a row with the numbers visible.

Find the probability that

(i) the numbers shown are in the order: 5, 4, 3, 2

(ii) the first and second numbers are both even

(iii) the sum of the two middle numbers is 7.

(c)(i) **5 marks** **Att 2**

$\frac{1}{4!}$ *or* $\frac{1}{24}$ *or* $1-\frac{23}{24}$ *or* $0\cdot 041\dot{6}$ *or* $4\cdot 1\dot{6}\%$ *or* 1 *in* 24 *or* $1:24$ *or* $\frac{1}{4}\times\frac{1}{3}\times\frac{1}{2}\times\frac{1}{1}$

or

Sample space (S)

2,3,4,5	3,2,4,5	4,2,3,5	5,2,3,4
2,3,5,4	3,2,5,4	4,2,5,3	5,2,4,3
2,4,3,5	3,4,2,5	4,5,2,3	5,3,2,4
2,4,5,3	3,4,5,2	4,5,3,2	5,3,4,2
2,5,4,3	3,5,2,4	4,3,2,5	5,4,2,3
2,5,3,4	3,5,4,2	4,3,5,2	**5,4,3,2**

$p=\frac{\#E}{\#S}=\frac{1}{24}$

* Note: In (c)(i) relevant integers for A2 are 1,2,3,4,23,24.
* Accept correct answer without work.
* Once correct ratio appears, ignore subsequent work.
* Accept $\left(\frac{1}{24}\right)$, but $\begin{pmatrix}1\\24\end{pmatrix}$ is B1+B2, i.e. award attempt 2.
* Accept decimal answers correct to 2 decimal places.
* If sections of (c) are not identified, treat each section in order, if it is not obvious which part is being attempted.
* Penalise repeated errors in each section, but allow the candidate to use result from one section in a later one, where applicable.
* Note: Sample space may be drawn up only once and appropriate event for each part highlighted clearly merits **at least** 2+7+2 marks i.e. 11 marks for part (c).

Blunders (-3)

B1 $\begin{pmatrix}24\\1\end{pmatrix}$ *or* ${}^{24}C_1$ and stops.
B2 4! or 4x3x2x1 and stops.
B3 Inverted fractions: $\frac{24}{1}$ *or* $\frac{24}{23}$ *or* $24:1$.
B4 Incorrect #E, e.g. #E = 4.
B5 Incorrect #S. e.g. #S = 120.
B6 Draws up the sample space and highlights correct event, but does not write down the probability.

B4 + B5; e.g. $\frac{4}{120}$ *or* $\frac{1}{30}$, i.e. award attempt 2

B2 + B5: $2\times3\times4\times5$ and stops. i.e. award attempt 2

Slips(-1)
S1 Numerical errors to a maximum of 3.
S2 Each omission from Sample space to a maximum of 3.

Attempts(2 marks)
A1 Any relevant step.
A2 Any relevant integer from possible solutions.
A3 Any arbitrary $\frac{a}{b}$ or $a{:}b$ and stops, where $0 \le \frac{a}{b} \le 1$
A4 Any definition of probability, e.g. $\frac{\#E}{\#S}$ and stops.
A5 Any use of !, P,C, $\binom{}{}$

(c)(ii) **10 marks** **Att 3**

$\frac{4}{4!}$ *or* $\frac{4}{24}$ *or* $\frac{1}{6}$ *or* $1-\frac{20}{24}$ *or* $0\cdot1\dot{6}$ *or* $1\dot{6}\cdot6\%$ *or* 1 *in* 6 *or* $1:6$ *or* $4\times ans(i)$

or

Sample space (S)

2,3,4,5	3,2,4,5	**4,2,3,5**	5,2,3,4
2,3,5,4	3,2,5,4	**4,2,5,3**	5,2,4,3
2,4,3,5	3,4,2,5	4,5,2,3	5,3,2,4
2,4,5,3	3,4,5,2	4,5,3,2	5,3,4,2
2,5,4,3	3,5,2,4	4,3,2,5	5,4,2,3
2,5,3,4	3,5,4,2	4,3,5,2	5,4,3,2

$p = \frac{\#E}{\#S} = \frac{4}{24}$ *or* $\frac{1}{6}$

or $\frac{2}{4}\times\frac{1}{3}$

* Note: In (c)(ii) relevant integers for A2 are 1,2,3,4,6,20,24.
* Note: There are three elements to the calculations:
 (i) Identifying the total number of outcomes.
 (ii) Identifying the number of outcomes of interest.
 (iii) Forming the fraction.
Any one step missing or incorrect is 7 marks.
Any two steps missing, provided (i) and (ii) are not arbitrary is 4 marks
If candidate relies on an incorrect Sample Space from part (i), then it must be clearly identified as such. e.g. candidate writes “ number of outcomes = 120” or similar.
i.e. Mark as follows:
 10 marks for a correct answer.
 10 marks for $4\times$ans(i).
 7 marks for replacement case, but otherwise fully correct i.e. $\frac{2\times2}{4\times4}$ (shown)
 7 marks for fully correct Sample Space and fully correct outcomes identified and stops.
* Note: $\frac{2}{4}$ is an example of A3 i.e. 3 marks.
* Accept correct or consistent answer without work.
* Once correct ratio appears, ignore subsequent work.

* Accept $\left(\frac{4}{24}\right)$, but $\binom{4}{24}$ is 2 Blunders i.e. award 4 marks.
* Accept decimal answers correct to 2 decimal places.
* Accept candidate's #S from part (i).

Blunders (-3)

B1 Inverted fractions: $\frac{24}{4}$ *or* $\frac{24}{20}$ *or* $24:4$.
B2 Incorrect #E, e.g. #E = 8.
B3 Incorrect #S [if different to answer (i)].
B4 Draws up the sample space and highlights correct event, but does not write down the probability
B2 + B3; e.g. $\frac{8}{120}$ *or* $\frac{1}{15}$, i.e. award 4 marks but if #S from part (i) used, apply only B2.

Slips(-1)

S1 Numerical errors to a maximum of 3.

Attempts(3 marks)

A1 Any relevant step.
A2 Any relevant integer from possible solutions.
A3 Any arbitrary $\frac{a}{b}$ or *a*:*b* and stops, where $0 \le \frac{a}{b} \le 1$, written for this part.
A4 Any definition of probability for this part, e.g. $\frac{\#E}{\#S}$ and stops.
A5 Any use of !, P,C, $\binom{}{}$ for this part.

(c)(iii) **5 marks** **Att 2**

$\frac{8}{4!}$ *or* $\frac{8}{24}$ *or* $\frac{1}{3}$ *or* $1-\frac{16}{24}$ *or* $0\cdot\dot{3}$ *or* $33\cdot\dot{3}\%$ *or* 1 *in* 3 *or* $1:3$ *or* $8\times ans(i)$ *or* $2\times ans(ii)$

or

Sample space (S)

2,3,4,5	3,2,4,5	4,2,3,5	5,2,3,4
2,3,5,4	**3,2,5,4**	**4,2,5,3**	5,2,4,3
2,4,3,5	3,4,2,5	**4,5,2,3**	5,3,2,4
2,4,5,3	3,4,5,2	4,5,3,2	**5,3,4,2**
2,5,4,3	**3,5,2,4**	4,3,2,5	5,4,2,3
2,5,3,4	3,5,4,2	4,3,5,2	**5,4,3,2**

$p=\frac{\#E}{\#S}=\frac{8}{24}$ *or* $\frac{1}{3}$

* Note: In (c)(iii) relevant integers for A2 are 1,2,3,4,8,16,24.
* Accept correct or consistent answer without work.
* Once correct ratio appears, ignore subsequent work.
* Accept $\left(\frac{8}{24}\right)$, but $\binom{8}{24}$ is 2 Blunders i.e. award Att 2.
* Accept decimal answers correct to 2 decimal places.
* Accept candidate's #S from part (i) or part (ii).

Blunders (-3)

B1 $\binom{24}{8}$ *or* ${}^{24}C_8$ and stops.

B2 4! or 4x3x2x1 and stops, written for this section.

B3 Inverted fractions: $\frac{24}{8}$ *or* $\frac{24}{16}$ *or* $24:8$.

B4 Incorrect #E, e.g. #E = 16.

B5 Incorrect #S [if different to answer (i) or answer (ii)].

B6 Draws up the sample space and highlights correct event, but does not write down the probability.

B4 + B5; e.g. $\frac{16}{120}$ *or* $\frac{2}{15}$, but if #S from part (i) or part (ii) used apply only B4, i.e. award att 2.

B2 + B5: $2\times3\times4\times5$ and stops, i.e. award attempt 2.

Slips(-1)

S1 Numerical errors to a maximum of 3.

S2 Each omission from Sample space to a maximum 3, if #S different from part (i).

Attempts(2 marks)

A1 Any relevant step.

A2 Any relevant integer from possible solutions.

A3 Any arbitrary $\frac{a}{b}$ or *a*:*b* and stops, where $0 \le \frac{a}{b} \le 1$, written for this part.

A4 Any definition of probability for this part, e.g. $\frac{\#E}{\#S}$ and stops.

A5 Any use of !, P,C, $\binom{}{}$ for this part.

QUESTION 7

Part (a)	10 marks	Att 3
Part (b)	40 marks	Att (2,5,3,2,2)

Part (a) **10 marks** **Att 3**

The mean of the set of numbers {1, 3, 7, 9} is 5.
Find the standard deviation, correct to one decimal place.

(a) **10 marks** **Att 3**

Deviations $5-1, \quad 5-3, \quad 5-7, \quad 5-9,$

$$\sigma = \sqrt{\frac{16+4+4+16}{4}} = \sqrt{\frac{40}{4}} = \sqrt{10} = 3.16 = 3.2$$

or

X	$d = \lvert x-\bar{x}\rvert$	d^2
1	4	16
3	2	4
7	2	4
9	4	16
		40

$$\sigma = \sqrt{\frac{\Sigma d^2}{n}} = \sqrt{\frac{40}{4}} = \sqrt{10} = 3\cdot 2$$

* Candidates may assign a frequency of k (likely 1) to **each** x-value and use $\sqrt{\frac{\Sigma fd^2}{\Sigma f}}$. This is acceptable.

* Accept correct answer without work. (may have used a calculator)

Blunders(-3)

B1 No d column $\Rightarrow \sqrt{\frac{12}{4}}$ or $\sqrt{3} = 1.7$

B2 No $\sqrt{\ }$ used $\Rightarrow \frac{40}{4} = 10$ [Note: Mean deviation: $\frac{12}{4} \Rightarrow$ apply B1 and B2.]

B3 Denominator $\neq 4$ e.g. $\sqrt{40} = 6\cdot 3$.

B4 Mathematical error e.g. $4^2 = 8$ or $(-4)^2 = -16$ (Apply once only).

B5 Inconsistent k values for frequencies, if used, i.e. k values not same for each X value.

Slips(-1)

S1 Rounding error or fails to round.

Misreadings(-1)

M1 Any obvious misreading which does not oversimplify or change the task.

M2 Uses a mean other than 5.

Attempts(3 marks)

A1 Correct relevant formula and stops. e.g. $\sqrt{\frac{\Sigma fd^2}{\Sigma f}}$ *or* $\sqrt{\frac{\Sigma d^2}{n}}$ *or* $\frac{\Sigma x}{n}$ *or* $\frac{\Sigma fx}{\Sigma f}$

A2 Any relevant step.

Part (b) **40 (5, 15, 10, 5, 5)marks** **Att (2, 5, 3, 2, 2)**

The following table shows the time in minutes spent by customers in a cafeteria.

Time in minutes	0 – 10	10 – 20	20 – 40	40 – 70
Number of customers	80	100	160	60

[Note that 10 − 20 means at least 10 but less than 20 minutes etc.]

(i) Find the total number of customers.

(ii) Draw a histogram to represent the data.

(iii) By taking the data at the mid-interval values, calculate the mean number of minutes per customer.

(iv) What is the greatest number of customers who could have spent more than 30 minutes in the cafeteria?

(v) What is the least number of customers who could have spent more than 30 minutes in the cafeteria?

(b)(i) **5 marks** **Att 2**

Total number of customers = 80 + 100 + 160 + 60 = 400

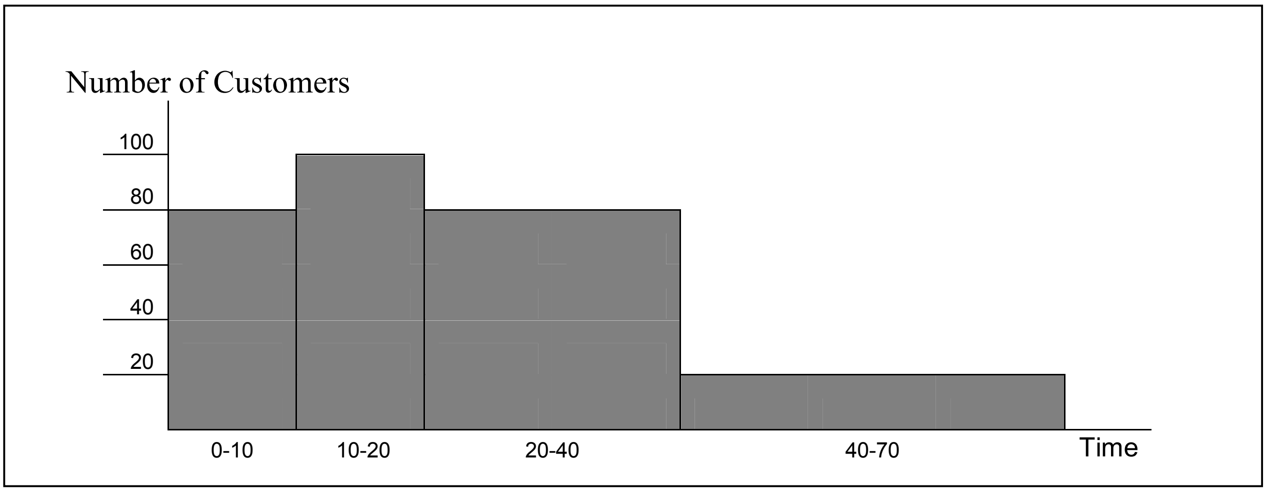
* Accept correct answer without work.

Blunders(-3)
B1 Multiplies instead of adds.

Slips(-1)
S1 Numerical errors to a maximum of 3.
S2 Each value omitted in the sum, but work must be shown
S3 Leaves as 80 + 100 + 160 + 60, i.e. does not add the values.

Attempts(2 marks)
A1 Any one of the values written down.

(b)(ii) **15 marks** **Att 5**



* Accept areas of rectangles proportional to frequencies, but note B2.
* Note: each rectangle may be blundered once only.

Blunders(-3)
B1 Scale not indicated or incorrect frequency scale.
B2 Each incorrect rectangle or rectangle omitted, subject to B1.
B3 Puts spaces between rectangles.

Attempts (5 marks)
A1 Draws axes and stops, even without labels or scales.
A2 Treats 0-80, 80-100, etc as intervals and 10,20,etc as frequencies.
A3 Any relevant step, e.g. draws a frequency polygon, or cumulative frequency curve.

(b)(iii) **10 marks** **Att 3**

x	f	fx
5	80	400
15	100	1500
30	160	4800
55	60	3300
	400	10000

$$\bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{10000}{400} \quad or \quad 25$$

or

Mid-interval values: 5, 15, 30, 55

$$\text{Mean} = \frac{5\times 80 + 15\times 100 + 30\times 160 + 55\times 60}{400} = \frac{400+1500+4800+3300}{400} = \frac{10000}{400} = 25$$

* Accept correct answer without work. i.e. uses calculator.
* All answers (except those in B1) must be consistent with **written** mid-interval and frequency values, otherwise incorrect answer without work merits zero.
* Leaves answer as $\frac{10000}{400}$ is acceptable for full marks.

Blunders (-3)
B1 Mid-interval values not used. [Taking the lower endpoint of each interval gives $\bar{x} = 16\cdot 5$: the upper one gives $33\cdot 5$.]
B2 Multiplies instead of adds in denominator, but do not penalise again if this blunder was in (b)(i). [$\frac{10000}{76800000}$ *or* $0\cdot 00013$]
B3 Gets $\Sigma(\text{f} + \text{x})$ in numerator.[$\frac{505}{400}$ *or* $1\cdot 2625$]
B4 Uses 4 as denominator. [$\frac{10000}{4}$ *or* 2500]
B5 Inverts, i.e. $\frac{400}{10000}$ *or* $0\cdot 04$
B6 No frequencies, i.e. $\frac{5+15+30+55}{400} = \frac{105}{400}$ *or* $0\cdot 2625$

B7 Omits a class, if not already penalised.

B2 + B3 $\Rightarrow \dfrac{505}{76800000}$ *or* $0 \cdot 0000066$

B4 + B6 $\Rightarrow \dfrac{5+15+30+55}{4} = \dfrac{105}{4}$ *or* $26 \cdot 25$

B1 + B6 $\Rightarrow$ e.g. $\dfrac{0+10+20+40}{400} = \dfrac{70}{400}$ or $0 \cdot 175$ or similar.

Slips(-1)

S1 Each numerical error to a maximum of 3.

S2 Each incorrect mid-interval value to a maximum of 3.

Attempts(3 marks)

A1 Mean = $\dfrac{\Sigma fx}{\Sigma f}$ *or* $\dfrac{\Sigma x}{n}$ and stops.

A2 One or more correct mid-interval value and stops.

A3 A correct relevant multiplication and stops.

A4 $\Sigma f = 400$ stated in this part and stops.

A5 Some relevant step, e.g. finds the median or modal class, or draws a cumulative frequency curve.

(b)(iv) **5 marks** **Att 2**

Greatest number of customers = 160 + 60 = 220

* Accept correct answer without work.

* Note: Award 2 marks for 160, 60, 340 with, or without work.
Award 2 marks for 100 with work, i.e. 160 – 60.

Slips (-1)

S1 Answer left as 160 + 60, i.e. no addition.

Attempts (2 marks)

A1 Some effort to read answer from ogive.

A2 " 60 + x", x < 160 (with work shown).

(b)(v) **5 marks** **Att 2**

Least number of customers = 60

* Accept correct answer without work.

* Note: Award 2 marks for 220, 340.
Award 2 m for 100 with work, i.e. 160 – 60, written here, if not already awarded in b (iv), or " 60 + x", x < 160(with work shown), if not already awarded in b (iv).

Attempts (2 marks)

A1 Some effort to read answer from ogive.

QUESTION 8

Part (a)	**10 marks**	**Att(2,2)**
Part (b)	**20 marks**	**Att 7**
Part (c)	**20 marks**	**Att (2,3,2)**

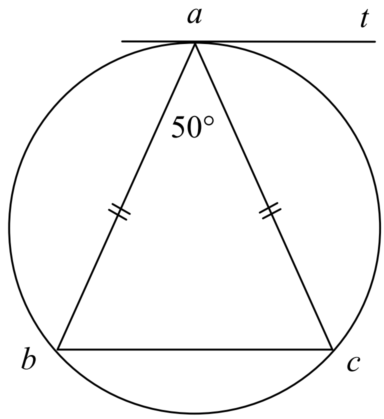
Part (a) **10 (5,5) marks** **Att (2,2)**

The points a, b and c lie on a circle.
ta is a tangent to the circle.

$|ab| = |ac|$ and $|\angle cab| = 50°$.

(i) Find $|\angle abc|$.

(ii) Find $|\angle tac|$.

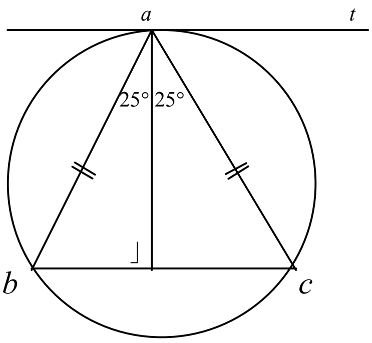


(a)(i) **5 marks** **Att 2**

$$|\angle abc| + |\angle bca| = 180° - 50° = 130°$$
$$2|\angle abc| = 130° \Rightarrow |\angle abc| = 65°$$

or

$|\angle abc| + 90° + 25° = 180°$
$|\angle abc| + 115° = 180°$
$|\angle abc| = 65°$



* Accept answers to (a) (i) and (a) (ii) clearly indicated on a diagram.
* Accept correct answer without work.

Blunders (-3)
B1 Angle sum of triangle ≠ 180°.
B2 Incorrect base angles of isosceles triangle, e.g. gives $|\angle abc| = 50°$ (or 80°) .
B3 Transposition error.
Slips (-1)
S1 Numerical errors to a maximum of 3.

Attempts (2 marks)
A1 Angle sum of triangle = 180° and stops.

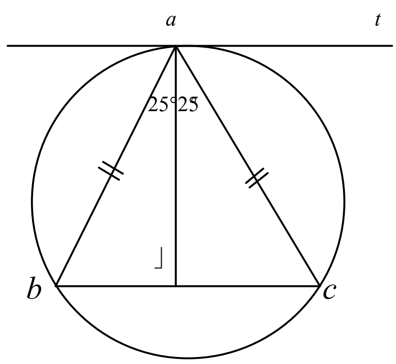
A2 States base angles of an isosceles triangle are equal or marks correct equal base angles on a diagram.
A3 Draws in mediator of [bc].

Worthless (0 marks)
W1 Diagram reproduced without modification.

(a)(ii) **5 marks** **Att 2**

$$|\angle tac| = |\angle abc| = 65°$$
or

$$|\angle tac| + 25° = 90°$$
$$|\angle tac| = 90° - 25°$$
$$|\angle tac| = 65°$$



* Accept correct answer without work.
* Accept candidate's answer for $|\angle abc|$ from (a) (i).
* Accept correct answer clearly indicated on a diagram.
* Candidate must explicitly give $|\angle tac|$ to gain full marks in (a) (ii). i.e. one answer of 65° is worth 5 marks only.
* Accept $|\angle tac| = |\angle abc|$.

Blunders (-3)
B1 Transposition error.
B2 Treats $\angle tac$ as exterior angle of triangle *abc* i.e. $|\angle tac| = 130°$.

Slips(-1)
S1 Numerical errors to a maximum of 3.

Attempts (2 marks)
A1 Any mention of angle in alternate segment.
A2 States *bc* is parallel to *at* and stops.

Worthless(0)
W1 Diagram reproduced without modification.

Part (b) **20 marks** **Att 7**

Prove that if [ab] and [cd] are chords of a circle and the lines *ab* and *cd* meet at the point *k*, which is outside the circle, then $|ak|.|kb| = |ck|.|kd|$.

(b) **20 marks** **Att 7**

The chords [*ab*] and [*cd*] intersect at *k*, outside the circle.

To prove: $|ak|.|kb| = |ck|.|kd|$

Join *a* to *d* and *c* to *b*…………….. ….step 1, 7m

Proof: Consider triangles *adk* and *cbk*.

$|\angle dka| = |\angle bkc|$ ....*step* 2, 10*m*

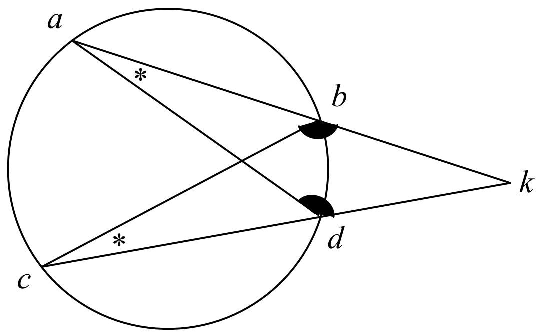
$|\angle kad| = |\angle kcb|$ ....*step* 3, 13*m*

Hence the triangles are similar

or $|\angle kda| = |\angle kbc|$………………. step 4, 16m

$\frac{|ak|}{|ck|} = \frac{|kd|}{|kb|}$ ……………….……….. step 5, 19m

Hence, $|\text{ak}|.|\text{kb}| = |\text{ck}|.|\text{kd}|$, 20m



* Accept steps stated or clearly indicated.

* Note: Proof without diagram, merits att 7, if proof can be reconciled with a diagram.

Blunders (-3)

B1 Incorrect step or part of a step. [Award 0 marks for a step which is omitted]

B2 Proves the internal case.

B3 Steps in incorrect order.

Attempts (7 marks)

A1 Outline diagram and stops (Circle with two intersecting chords).

A2 Attempt only if special "tangent case" is used for this part.

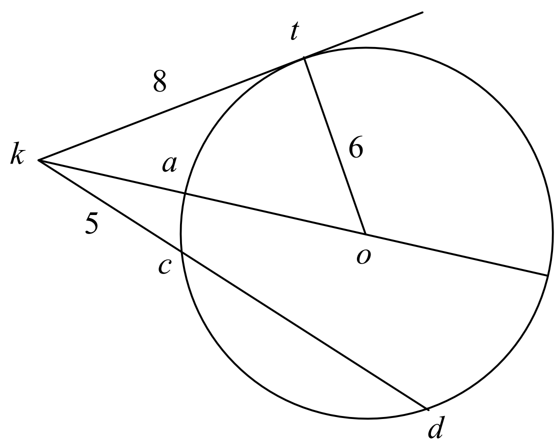
Part (c) **20 (5, 10, 5) marks** **Att (2, 3, 2)**

kt is a tangent to the circle, centre *o*.
$[ab]$ is a diameter and $[cd]$ is a chord of the circle.
ab and *cd* meet at the point *k*.
$|kt| = 8$, $|ot| = 6$
and $|kc| = 5$.

(i) Find $|ko|$.

(ii) Verify that $|kt|^2 = |ka|.|kb|$.

(iii) Find $|cd|$.



(c)(i) **5 marks** **Att 2**

I
$\|ko\|^2 = \|kt\|^2 + \|to\|^2 = 8^2 + 6^2 = 64 + 36 = 100 \Rightarrow \|ko\| = \sqrt{100}\ or\ 10$
II
Let $\|ak\| = x \Rightarrow x(x+12) = 64 \Rightarrow x^2 + 12x - 64 = 0 \Rightarrow (x+16)(x-4) = 0 \Rightarrow x = 4 \Rightarrow \|ko\| = 10$

* Accept correct answer without work.
* Accept use of Pythagorean triple 6,8,10.
* Accept correct trigonometric method.

Blunders (-3)
B1 Pythagoras' Theorem used incorrectly. [e.g. $|ko| = 8 + 6 = 14$]
B2 Mathematical blunder, e.g. $8^2 = 16$ or similar.
B3 Errors in solving quadratic in II.
B4 Transposition error.

Slips(-1)
S1 Numerical errors to a maximum of 3.
S2 Stops at $x = 4$.

Attempts (2 marks)
A1 Marks 90° in correct position on the diagram.
A2 Some relevant step, e.g. mentions Pythagoras or 8^2 or similar.

(c)(ii) **10 marks** **Att 3**

$\|kt\|^2 = 8^2 = 64$..............................3 *marks*
$\|ka\| = \|ko\| - \|ao\| = 10 - 6 = 4$.......6 *marks*
$\|kb\| = \|ka\| + \|ab\| = 4 + 12 = 16$.....7 *marks*
$\|ka\|.\|kb\| = 16 \times 4 = 64$..............10 *marks*
$(\Rightarrow \|kt\|^2 = \|ka\|.\|kb\|)$

* Accept candidate's answer for $|ko|$ from (c) (i).
* Steps one and two are interchangeable.
* Accept values for $|ka|$ and/or $|kb|$ written on a diagram.
* $|kt|^2 = 8^2 = 64.$ in part (c) (i) merits at least the attempt mark here and in (c) (iii) and vice-versa, if explicitly written for those parts.

Blunders (-3)
B1 Mathematical blunder, e.g. $8^2 = 16$ or similar.
B2 Proves the general case, $|kt|^2 = |ka|.|kb|$, in this part.

Slips(-1)
S1 Numerical errors to a maximum of 3.

(c)(iii) **5 marks** **Att 2**

I
$\|kt\|^2 = \|kc\|.\|kd\| \Rightarrow 64 = 5\|kd\| \Rightarrow \|kd\| = 12.8 \Rightarrow \|cd\| = 12.8 - 5 = 7.8$
II
$\|kc\|.\|kd\| = \|ka\|.\|kb\| = 64 \Rightarrow 5.\ \|kd\| = 64 \Rightarrow \|kd\| = 12 \cdot 8 \Rightarrow \|cd\| = 12.8 - 5 = 7.8$
III
let $\|cd\| = x \Rightarrow 5(x+5) = 64 \Rightarrow 5x + 25 = 64 \Rightarrow 5x = 39 \Rightarrow x = 7 \cdot 8$

* Accept candidate's value for $|kt|^2$ from (c) (ii).

Blunders(-3)

B1 Transposing error.

B2 Distributive error. e.g. $5(x+5) = 5x + 5$

B3 $|kt|^2 = |kc|.|cd|$ or similar

Slips(-1)

S1 Numerical errors to a maximum of 3.

S2 Calculates $|kd| = 12 \cdot 8$ and stops.

Attempts (2 marks)

A1 States $|kt|^2 = |kc|.|kd|$ or $|kc|.|kd| = |ka|.|kb|$ and stops.

A2 Correct answer without work.

QUESTION 9

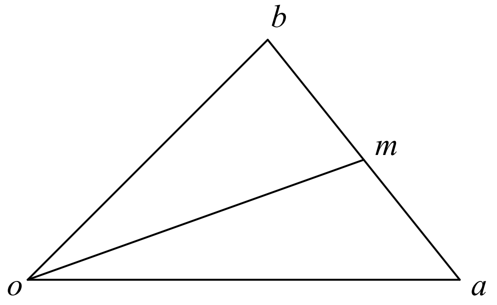
Part (a)	**10 marks**	**Att (2,2)**
Part (b)	**20 marks**	**Att (3,3)**
Part (c)	**20 marks**	**Att (3,3)**

Part (a) **10 (5, 5) marks** **Att (2, 2)**

oab is a triangle.
o is the origin and *m* is the midpoint of $[ab]$.

(i) Express $\overrightarrow{ba}$ in terms of $\vec{a}$ and $\vec{b}$.

(ii) Express $\vec{m}$ in terms of $\vec{a}$ and $\vec{b}$.



(a)(i) **5 marks** **Att 2**

I

$$\overrightarrow{ba} = \vec{a} - \vec{b}$$

II

$$\vec{b} + \overrightarrow{ba} = \vec{a} \;\Rightarrow\; \overrightarrow{ba} = \vec{a} - \vec{b}$$

III

$$\vec{a} + \overrightarrow{ab} = \vec{b} \;\Rightarrow\; \overrightarrow{ab} = \vec{b} - \vec{a} \;\Rightarrow \overrightarrow{ba} = \vec{a} - \vec{b}$$

* Accept correct answer with no work shown.
* Allow $\overrightarrow{oa}$ for $\vec{a}$ and $\overrightarrow{ob}$ for $\vec{b}$ in parts (i) and (ii).
* Accept letters without arrows.

Blunders (-3)

B1 Incorrect rule e.g $\overrightarrow{ba} = \vec{b} - \vec{a}$

B2 Error in using triangle law e.g. $\overrightarrow{ba} = \vec{b} + \vec{a}$

Attempts (2)

A1 Correct relevant step e.g. relevant arrow added to given diagram

A2 Correct relevant application of vectors, e.g. $\vec{bo} + \overrightarrow{oa}$.

Misreadings (-1)

M1 Any obvious misreading which does not oversimplify or change the task, e.g. $\overrightarrow{ab} = \vec{b} - \vec{a}$

Worthless (0)

W1 Diagram reproduced without modifications

(a)(ii)	**5 marks**	**Att 2**

I $\vec{m} = ½(\vec{a} + \vec{b})$
II $\vec{m} = \vec{b} + \overrightarrow{bm} = \vec{b} + ½\overrightarrow{ba} = \vec{b} + ½(\vec{a} - \vec{b})$ **or** $½(\vec{a} + \vec{b})$
III $\vec{m} = \vec{a} + \overrightarrow{am} = \vec{a} + ½\overrightarrow{ab} = \vec{a} + ½(\vec{b} - \vec{a})$ **or** $½(\vec{a} + \vec{b})$

* Accept correct answer with no work shown.

Blunders(-3)

B1 Incorrect rule e.g $\overrightarrow{ba} = \vec{b} - \vec{a}$

B2 Error in using triangle law e.g. $\overrightarrow{ba} = \vec{b} + \vec{a}$

Attempts (2)

A1 Correct relevant step e.g. relevant arrow added to given diagram

A2 Correct relevant application of vectors, e.g. $\vec{bo} + \overrightarrow{oa}$.

A3 $\vec{m} = \frac{1}{2}\overrightarrow{ab}$ or $½\vec{a} + \vec{b}$ or similar.

Misreadings (-1)

M1 Any obvious misread which does not oversimplify or change the task.

Worthless (0)

W1 Diagram reproduced without modifications

W2 Any combination of 3 letters, e.g. $\overrightarrow{mba}$

Part (b)	**20 (10, 10) marks**	**Att (3, 3)**

Let $\vec{p} = 5\vec{i} + 2\vec{j}$ and $\vec{q} = 3\vec{i} - 6\vec{j}$.

(i) Express $2\vec{p} - 3\vec{q}$ in terms of $\vec{i}$ and $\vec{j}$.

(ii) Find the scalars *k* and *t* such that
$k(5\vec{i} + 2\vec{j}) + t(3\vec{i} - 6\vec{j}) = 7\vec{i} - 26\vec{j}$.

(b)(i)	**10 marks**	**Att 3**

$$2\vec{p} - 3\vec{q} = 2(5\vec{i} + 2\vec{j}) - 3(3\vec{i} - 6\vec{j}) = 10\vec{i} + 4\vec{j} - 9\vec{i} + 18\vec{j} = \vec{i} + 22\vec{j}$$
3m 7m 10m

Blunders(-3)

B1 Mixes up $\vec{i}$'s and $\vec{j}$'s.

B2 Algebraic error, e.g. $\vec{i}^2 + 22\vec{j}^2$ or a sign error.

B3 Distributive law error, e.g. $-3(3\vec{i} - 6\vec{j}) = -9\vec{i} - 18\vec{j}$

B4 Stops at $10\vec{i} + 4\vec{j} - 9\vec{i} + 18\vec{j}$.

Slips (-1)

S1 Numerical errors to a maximum of 3

Misreading (-1)

M1 $2\vec{q} - 3\vec{p}$ and continues. (Answer: $-9\vec{i} - 18\vec{j}$)

Attempts (3)

A1 $\vec{i}$ or $22\vec{j}$ with work shown and stops.

A2 Either bracket multiplied out correctly and stops.

A3 Plots one or more relevant vectors

A4 Correct answer without work.

Worthless (0)

W1 Incorrect answer without work.

(b)(ii) **10 marks** **Att 3**

i components: $5k + 3t = 7$ j components: $2k - 6t = -26$ 3m

$10k + 6t = 14$

$\underline{2k - 6t = -26}$ 7m 10m

$12k = -12 \Rightarrow k = -1$ and $-5 + 3t = 7 \Rightarrow t = 4$

Blunders (-3)

B1 Mixes up $\vec{i}$'s and $\vec{j}$'s .

B2 Distributive error, e.g. $5k\vec{i} + 2\vec{j}$ or a sign error.

B3 Algebraic error, e.g. $10k + 6t = 7$

B4 Stops at $k = -1$ <u>or</u> $t = 4$

B5 Transpositin error e.g. $t = -4$

Slips (-1)

S1 Numerical errors to a maximum of 3

Attempts (3)

A1 $5k\vec{i} + 2k\vec{j} + 3t\vec{i} - 6t\vec{j} = 7\vec{i} - 26\vec{j}$ and stops.

A2 $5k + 3t = 7$ <u>and / or</u> $2k - 6t = -26$ and stops.

A3 Some relevant step.

Part (c) **20 (10, 10) marks** **Att (3, 3)**

Let $\vec{x} = 2\vec{i} + 3\vec{j}$ and $\vec{y} = 5\vec{i} + \vec{j}$.

(i) Show that $|\vec{x} - \vec{y}| < |\vec{x}| + |\vec{y}|$.

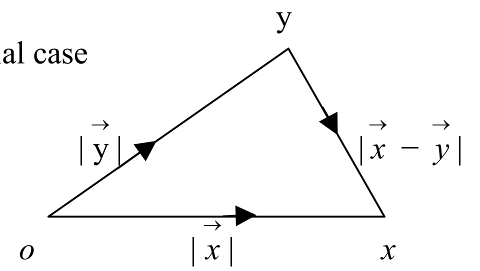
(ii) Write $\vec{x}^{\perp}$ in terms of $\vec{i}$ and $\vec{j}$.

Hence, calculate the dot product $\vec{y}.(\vec{x} + \vec{x}^{\perp})$.

(c)(i) **10 marks** **Att 3**

$\vec{x} - \vec{y} = 2\vec{i} + 3\vec{j} - (5\vec{i} + \vec{j})$ <u>or</u> $-3\vec{i} + 2\vec{j}$	3m
$\lvert\vec{x} - \vec{y}\rvert = \sqrt{9+4} = \sqrt{13}$	4m
$\lvert\vec{x}\rvert = \sqrt{4+9} = \sqrt{13}$	7m
$\lvert\vec{y}\rvert = \sqrt{25+1} = \sqrt{26}$	9m
$\sqrt{13} < \sqrt{13} + \sqrt{26}$	10m

Special case



➔ $|\vec{x} - \vec{y}| < |\vec{x}| + |\vec{y}|$

* Apply coordinate geometry principles if candidate uses length of line segment formula

* Note: $|\vec{x}|$ and $|\vec{y}|$ and stops ➔ 4 marks.

$|\vec{x} - \vec{y}|$ and one of $|\vec{x}|$ or $|\vec{y}|$ ➔ 7 marks.

Blunders (-3)

B1 Incorrect relevant formula e.g. $\sqrt{x^2 - y^2}$ or $\sqrt{(x_2 - x_1)^2 - (y_2 - y_1)^2}$

B2 Mathematical error, e.g $(-3)^2 = -9$

Slips (-1)

S1 Each numerical slip to a maximum of 3

Attempts (3)

A1 Correct formula and stops.

A2 $(-3)^2 = 9$ <u>or</u> $(2)^2 = 4$ and stops.

A3 Some relevant step, e.g. plots 1 or more relevant points

Misreadings (-1)

M1 Any obvious misread which does not oversimplify or change the task.

(c)(ii)	**10 marks**	**Att 3**

$$\vec{x}^{\perp} = -3\vec{i}+2\vec{j}$$

$$\vec{y}.(\vec{x}+\vec{x}^{\perp}) = (5\vec{i}+\vec{j}).(2\vec{i}+3\vec{j} -3\vec{i}+2\vec{j}) = (5\vec{i}+\vec{j}).(-\vec{i}+5\vec{j}) = -5+5 = 0$$

3m 4m 7m 9m 10m

Blunders (-3)

B1 Sign error e.g. $\vec{x}^{\perp} = 3\vec{i}+2\vec{j}$.

B2 Mixes up $\vec{i}$'s and $\vec{j}$'s .

B3 $\vec{i}^2 \neq 1$ and / or $\vec{j}^2 \neq 1$.

B4 $\vec{i}.\vec{j} \neq 0$.

Slips (-1)

S1 Each numerical slip to a maximum of 3.

Attempts (3 marks)

A1 Correct relevant formula and stops, e.g. $\vec{x}.\vec{y} = |\vec{x}||\vec{y}|\cos\theta$ and stops.

A2 $\vec{i}^2 = 1$ and / or $\vec{j}^2 = 1$ and / or $\vec{i}.\vec{j} = 0$ and stops.

A3 $|\vec{y}| = \sqrt{25+1} = \sqrt{26}$ or similar and stops.

A4 Any relevant step, e.g. $-2\vec{i}-3\vec{j}$ and stops.

QUESTION 10

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 marks**	**Att (3,2,2)**
Part (c)	**20 marks**	**Att (3,3)**

Part (a) **10 marks** **Att 3**

Expand $(1+x)^4$ fully.

(a) **10 marks** **Att 3**

$$(1+x)^4 = 1+\binom{4}{1}x+\binom{4}{2}x^2+\binom{4}{3}x^3+\binom{4}{4}x^4 \text{............}7\,marks$$

$$= 1+4x+6x^2+4x^3+x^4 \text{..}10\,marks$$

* Accept long multiplication or Pascal's triangle.

* Accept $(x+1)^4$ expanded correctly.

* Accept correct answer without work.

Note: 2 terms correct → 4 marks: 3 or 4 terms correct → 7 marks: 5 terms correct → 10 marks

Blunders (-3)

B1 Error in powers (once only).

B2 Error in working out binomial coefficients (apply once), subject to attempt.

B3 Puts powers of x as denominators e.g. $\binom{4}{2}\left(\frac{x}{2}\right)$ *or* $\frac{4}{2}x^2$ (apply once).

B4 Puts a + sign between coefficient and power of x e.g. $\binom{4}{2}+x^2$ (apply once).

B5 Does not work out binomial coefficients(once). i.e. first line in solution box.

Slips(-1)

S1 Numerical errors to a maximum of 3.

Misreadings(-1)

M1 $(1-x)^4$ *or* $(x-1)^4$ and continues correctly.

Attempts (3 marks)

A1 Any term written down correctly or part of Pascal's Triangle or coefficients only.

A2 Any step towards getting a binomial coefficient e.g. $\binom{4}{2}$.

A3 Any correct step towards long multiplication.

Part (b) **20 (10, 5, 5) marks** **Att (3, 2, 2)**

The fourth term of an arithmetic sequence is −4.
The seventh term of the sequence is −16.
(i) Find the common difference, d.
(ii) Find the first term, a.
(iii) Show that the difference between the fourth term and the twenty-ninth term is 100.

(b)(i) **10 marks** **Att 3**

I	II
$T_n = a+(n-1)d$ … 3 marks $a+3d = -4$ ….4 marks $a+6d = -16$ ….7 marks $3d = -12$ ….9 marks $\Rightarrow d = -4$ ….10 marks	$T_7 - T_4 = 3d$ …. 3 marks $-4 + 3d = -16$ $\Rightarrow$ $3d = -16+4 = -12$ $\Rightarrow d = -4$ 7 m 9 m 10 m

* Allow candidate to find value of *a* first.

Blunders (-3)
B1 Error in T_n formula(once only).
B2 Uses formula for S_n of an arithmetic series. (Gives $d = -\frac{6}{7}$)
B3 Transposition error.
B4 Fails to multiply / divide both sides of equation(s) correctly when eliminating variable.
B5 Treats as a geometric sequence and finishes correctly.(Gives $r = \sqrt[3]{4}$)
B6 Incorrect coefficient for *d* in II. e.g. $-4 + 4d = -16$ and continues.
B7 Each missing equation.
B8 Does not solve equations.

Slips(-1)
S1 Numerical errors to a maximum of 3.

Attempts (3 marks)
A1 Correct relevant formula (either arithmetic or geometric) and stops.
A2 Some relevant step.

Worthless (0 marks)
W1 Incorrect answer without work.

(b)(ii) **5 marks** **Att 2**

I	II	III
$a+3d = -4$ $a+3(-4) = -4$ $a = -4+12=8$	$a+6d = -16$ $a+6(-4) = -16$ $a = -16+24 = 8$	$T_4 = -4 \Rightarrow T_3 = 0$ $T_2 = 4 \Rightarrow T_1 = 8$

* Accept correct answer without work.
* Accept candidate’s answer from part (b) (i).

Blunders (-3)
B1 Incorrect value substituted for d.
B2 Transposition error.
B3 Uses Geometric sequence if not already penalised in part (b) (i) [Gives a = -1]

Slips(-1)
S1 Numerical errors to a maximum of 3.

Attempts (2 marks)
A1 Correct relevant formula (either arithmetic or geometric) written for this part and stops.
A2 Some relevant step.

Worthless (0 marks)
W1 Incorrect answer without work.

(b)(iii) **5 marks** **Att 2**

I	II
$T_4 = a + 3d = -4$	29 - 4 = 25
$T_{29} = a + 28d = 8 + 28(-4) = 8 - 112 = -104$	$25 \times 4 = 100$
$T_4 - T_{29} = -4 - (-104) = 100$	

* Accept candidate's answers from (b) (i) and (b) (ii).
* Accept -100 as answer.

Blunders (-3)
B1 Incorrect value substituted for d, if not already penalised.
B2 Transposition error.
B3 Uses Geometric sequence if not already penalised in part (b) (i) .

Slips(-1)
S1 Numerical errors to a maximum of 3.

Attempts (2 marks)
A1 Correct relevant formula (either arithmetic or geometric) written for this part and stops.
A2 Some relevant step.

Worthless (0 marks)
W1 Incorrect answer without work.

Part (c)	**20 (10, 10) marks**	**Att (3, 3)**

(i) The sum to infinity of a geometric series is 4.
The first term, a, is twice the common ratio, r.
Find r.

(ii) €500 is invested at 7.5% per annum compound interest.
Show that after 10 years the value of the investment is greater than €1000.

(c)(i)	**10 marks**	**Att 3**

$S_\infty = \dfrac{a}{1-r} = 4$	➔ $\dfrac{2r}{1-r} = 4$	➔ $2r = 4 - 4r$	➔ $6r = 4$	➔ $r = \dfrac{2}{3}$
3m	4m	7m	9m	10m

* Accept use of $\lim_{n\to\infty} S_n$ to get S_∞.

Blunders(-3)
B1 $a \neq 2r$ and continues.

B2 Incorrect relevant formula e.g. $\dfrac{a}{1+r}$. [Gives r = -2]

B4 Transposition error.

Slips(-1)
S1 Numerical errors to a maximum of 3.

Attempts (3 marks)

A1 Correct relevant formula written for this part and stops, e.g. $S_n = \dfrac{a(1-r^n)}{1-r}$.

A2 Some relevant step.
A3 States $|r| < 1$ and stops.

Worthless (0 marks)
W1 Incorrect answer without work.

(c)(ii)	**10 marks**	**Att 3**

I

$A = P\left(1+\dfrac{r}{100}\right)^n$	$\Rightarrow A = 500(1\cdot075)^{10}$	$= 500(2\cdot06)$	$=$ €$1030\cdot52$	(>€1000)
3m	7m	9m	10m	

II

P_1 = 500·00	P_2 = 537·50	P_3 = 577·81	P_4 = 621·15	P_5 = 667·74
I_1 = 37·50	I_2 = 40·31	I_3 = 43·34	I_4 = 46·59	I_5 = 50·08
P_2 = 537·50	P_3 = 577·81	P_4 = 621·15	P_5 = 667·74	P_6 = 717·82
P_6 = 717·82	P_7 = 771·66	P_8 = 829·53	P_9 = 891·74	P_{10} = 958·62
I_6 = 53·84	I_7 = 57·87	I_8 = 62·21	I_9 = 66·88	I_{10} = 71·90
P_7 = 771·66	P_8 = 829·53	P_9 = 891·74	P_{10} =958·62	Value = 1030·52

€$1030\cdot52$ (>€1000)

* Accept long method of working from year to year. i.e. method II.

Blunders(-3)
B1 Incorrect *r*.
B2 Decimal error.
B3 $€500(1\cdot075)^{10}$ and stops.
B4 Serious numerical error e.g. $(1\cdot075)^{10}$ = 10.75.
B5 Subtracts in long method.
B6 Sign error in formula.

Slips(-1)
S1 Numerical errors to a maximum of -3.
S2 Premature rounding that affects the final amount, to a maximum of -3
S3 Each year omitted in long method, subject to the attempt mark.

Misreadings(-1)
M1 Any obvious misreading which does not oversimplify or change the task.

Attempts (3 marks)
A1 Mention of $1\cdot075$ *or* $\frac{7\cdot5}{100}$.
A2 $7\cdot5\%$ *of* $€500 = €37\cdot50$ and stops or $\frac{\text{PTR}}{100}$ used.
A3 Correct Compound Interest formula or S_n for a geometric series.
A4 Correct answer without work.

Worthless(0 marks)
W1 $\frac{500}{7\cdot5} = 66\cdot67$

QUESTION 11

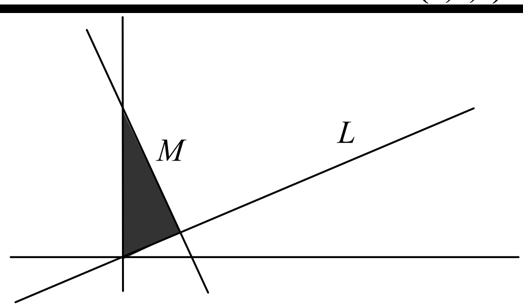
Part (a)	**15 marks**	**Att (2,2,2)**
Part (b)	**35 marks**	**Att (2,2,2,2,2,2,2)**

Part (a) **15(5, 5, 5) marks** **Att (2,2,2)**

The equation of the line L is $x - 2y = 0$.

The equation of the line M is $2x + y = 4$.

Write down the three inequalities that together define the shaded region in the diagram.



(a) **15 marks** **Att (2, 2, 2)**

L :	$x - 2y \leq 0$	5 marks , Att 2
M :	$2x + y \leq 4$	5 marks , Att 2
Y-axis :	$x \geq 0$	5 marks , Att 2

* Accept correct inequalities without work.
* Accept $x - 2y < 0$, $2x + y < 4$, $x > 0$.
* Note: The attempt A3 is an attempt at the inequality $x \geq 0$ only.
* It is possible to award 2 marks (only) for part (a).

Blunders (- 3)
B1 Incorrect inequality sign, each time.
B2 Mathematical error when testing a point, e.g. sign error.
B3 Incorrect or no conclusion, e.g. $2x + y = 4 \Rightarrow 2(0) + 0 = 4$ [Att 2]
[Note : $2x + y = 4 \Rightarrow 2(0) + 0 < 4$ merits 5 marks]

Slips (- 1)
S1 Numerical slips to a maximum of 3, e.g. $2 \times 0 = 2$.
Attempts (2 marks for each inequality)

A1 Finds, or plots, one or more points on given line(s) each line.
Note : (0,0) and (0,4) and (1.6, 0.8) , each merits 2 x Att 2 [points are on two lines].
A2 Substitutes any point and stops (each inequality).
A3 $y \leq 0$ or $y \geq 0$ and stops (without work).
A4 Some correct step in solving simultaneous equations (once).
A5 $L \leq 0$ merits Att 2 and $M \leq 4$ merits Att 2, each without work.
A6 Some relevant step, e.g. adds something relevant, such as half-plane arrow, to the diagram, each line.

Worthless (0 marks)
W1 $M \leq 0$, without work.
W2 Copies the given diagram and adds nothing to it.

Part (b) **35 (20, 10, 5) marks** **Att (2,2,2,2,2,2,2)**

A shop-owner displays videos and DVDs in his shop.
Each video requires 720 cm^3 of display space and each DVD requires 360 cm^3 of display space. The available display space cannot exceed 108 000 cm^3. The shop-owner buys each video for €6 and each DVD for €8. He does not wish to spend more than €1200.

(i) Taking x as the number of videos and y as the number of DVDs, write down two inequalities in x and y and illustrate these on graph paper.

During a DVD promotion the selling price of a video is €11 and of a DVD is €10.
Assuming that the shop-owner can sell all the videos and DVDs,

(ii) how many of each type should he display in order to maximise his income

(iii) how many of each type should he display in order to maximise his profit?

(b)(i) Inequalities **10 (5,5)marks** **Att (2,2)**

I

Space: $720x + 360y \le 108000 \Rightarrow 2x + y \le 300$

Cost: $6x + 8y \le 1200 \Rightarrow 3x + 4y \le 600$

Accept II

	Video	DVD	Maximum
Space Cost	$720x$ $6x$	$360y$ $8y$	108 000 1 200

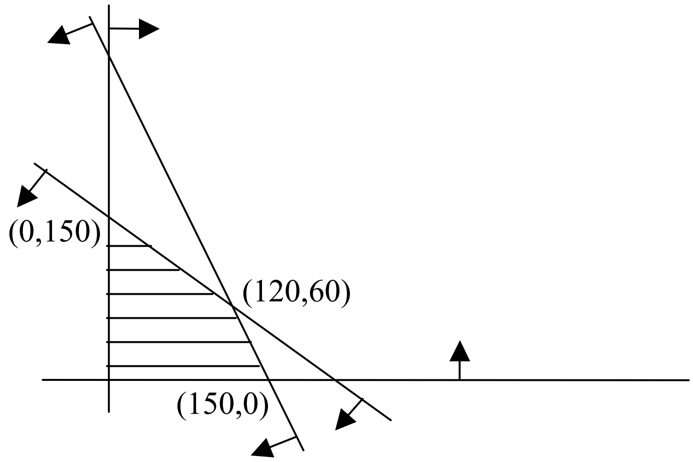
* Accept $2x + y \le 300$ and $3x + 4y \le 600$ or equivalent or different letters.
* Do not penalise here for incorrect or no inequality sign. Penalise in graph, if used.
* Case 720 360 108 000
6 8 1200 Award 10 marks. Penalise in graph, if linkup is incorrect.

Blunders (- 3)
B1 Mixes up x's and y's (once only, if consistent error).
B2 Confuses rows and columns in table, e.g. $720x + 6y \le 108\,000$ and/or $360x + 8y \le 1200$.
B3 Misplaced decimal point, e.g. $2x + y \le 30$.

Attempts (2 marks for each inequality)
A1 Incomplete relevant data in table and stops (each inequality).
A2 $720x$ and/or $360y$ and stops, ($1 \times$ att 2)
A3 $6x$ and/or $8y$ and stops, ($1 \times$ att 2)
A4 Some variable $\le$ 108 000, some variable $\le$ 1200, each time.
A5 Any other correct inequality, e.g. $x \ge 0$, $y \ge 0$ each time

(b)(i) Graph **10(5,5) marks** **Att (2,2)**

(b) (i)



* Each half-plane merits 5 marks, attempt 2 marks.
* Points or scales required.
* Half-planes required but no penalty for not indicating intersection if half-planes are indicated.
* If half-planes are indicated correctly, do not penalise for incorrect shading.
* Accept correct shading of intersection for half-planes, but candidates may shade out areas that are not required and leave intersection blank.
* Correct shading over-rules arrows.
* Two lines drawn **and no shading**, only one of the following applies :

Case 1: Two sets of arrows in expected direction ……………....10 marks
Case 2: Two sets of arrows in unexpected direction …………. ...10 marks
Case 3: One sets of arrows "correct" and the other "incorrect"……7 marks(5 + Att 2)
Case 4: One line with and the other without arrows………………7 marks (5 + Att 2)
Case 5: No arrows ………………………………………...4 marks (Att 2, Att 2)

Blunders (-3)
B1 No half-plane indicated (each time)
B2 Blunder in plotting a line or calculations (each line).
B3 Incorrect shading (once), e.g. one or both of the small triangles shaded.

Attempts (2 marks each half-plane)
A1 Some relevant work towards a point on a line, i.e. 2 m for each line attempted.
A2 Draws axes or axes and one line (1 × Att 2 m).
A3 Draws axes and two lines reasonably accurately (award Att 2 + Att 2).

(b) (ii) Intersection of lines **5 marks** **Att 2**

$2x + y = 300 \quad \Rightarrow \quad 8x + 4y = 1200$
$3x + 4y = 600 \qquad\qquad \underline{3x + 4y = 600}$
$5x = 600 \qquad x = 120, \quad y = 60$

* Accept candidate's own equations from previous parts.
* If y is calculated, accept consistent value for x without further work and vice versa.

Blunders (-3)
B1 Fails to multiply / divide both sides of equation(s) correctly when eliminating variable.
B2 Sign error.

B3 x or y value only.
B4 Transposing error.

Slips (-1)
S1 Numerical slips to a maximum of 3.

Attempts(2 marks)
A1 Any relevant step towards solving equations.
A2 Correct or consistent answer without work or from a graph.
[Should get same values from graph as if they had been found algebraically e.g. (121, 60) on its own gets zero.]

Worthless(0 marks)
W1 Incorrect answer without work and inconsistent with graph.

(b)(ii) Income **5 marks** **Att 2**

Step 1	Vertices	$11x + 10y$	Income
Step 2	(0,150)	0 + 1500	1500
Step 3	(120,60)	1320 + 600	1920
Step 4	(150,0)	1650 + 0	1650

Step 5 120 videos and 60 DVDs to maximise income

* Accept point of intersection from previous part.
* Information does not have to be in table form.
* Award 1 mark for each consistent step, subject to the attempt mark, using $11x + 10y$.
* Accept only vertices consistent with previously accepted work, not arbitrary ones.
  If (0,300) is tested and result is used to give max .income, apply -1. Otherwise ignore.
* Accept correct vertices or vertices from candidate's indicated area on non-simplified graph.
* Accept any correct multiple or fraction of 11x + 10y in this part of (b) (ii).
* If no marks have been awarded for intersection of lines and this point is written here award Att 2 for the previous part and also reward it here if the step is correct.
* Answer must be explicit, e.g. award 4 marks if step 3 is indicated but step 5 not written.
* Testing **only** (120,60) to get 1920 merits Att 2 for this part of (ii) even if the candidate writes 120 videos and 60 DVDs .
  No comparison means the attempt mark at most.

Slips (-1)
S1 Each arithmetic slip to a maximum of 3.
S2 Each step of the solution omitted, subject to the attempt mark [Step 1 may be implied].

Attempts (2 marks)
A1 Uses $6x + 8y$ as income expression
A2 Any relevant work involving x or y and / or 11, 10 or similar.
A3 Any attempt at substituting coordinates into some expression.
A4 States 120 videos and / or 60 DVDs with no other work.

(b) (iii) Profit **5 marks** **Att 2**

Step 1	Vertices	$5x + 2y$	Profit
Step 2	(0,150)	0 + 300	300
Step 3	(120,60)	600 + 120	720
Step 4	(150,0)	750 + 0	750

Step 5 150 videos and no DVDs to maximise his profit

or

1500-6(0)-8(150) = 300
1920-6(120)-8(60) = 720
1650-6(150)-8(0) = 750
⇒ 150 videos and no DVDs to maximise his profit

* Accept point of intersection from part (ii)
* Information does not have to be in table form.
* Award 1 mark for each consistent step, subject to the attempt mark, using $5x + 2y$.
* Accept only vertices consistent with previously accepted work, not arbitrary ones.
  If (200,0) is tested and result is used to give max. profit, apply -1. Otherwise ignore.
* Accept correct vertices or vertices from candidate's indicated area on non-simplified graph.
* Accept any correct multiple or fraction of $5x + 2y$ in this part of (b) (iii).
* If no marks have been awarded for intersection of lines, in part (ii), and this point is written here, award Att 2 for intersection part and also reward it here if the step is correct.
* Answer must be explicit, e.g. award 4 marks if step 3 is indicated but step 5 not written.
* Testig only (150,0) to get 750 merits Att 2 for this part, even if the candidate writes 150 videos and no DVDs .
* No comparison means the attempt mark at most.
* If $11x + 10y$ is used here as the profit expression, and not in part (ii),
  candidate can receive 5 marks for part (ii).
* Candidate may use answers from Income to calculate Profit and hence maximum profit. [2nd Method above]
* Case: uses $6x + 8y$ and does not subtract from income, merits attempt mark at most.

Slips (-1)
S1 Each arithmetic slip to a maximum of 3.
S2 Each step of the solution omitted, subject to the attempt mark
[Step 1 may be implied].

Attempts (2 marks)
A1 Uses $11x + 10y$ as profit expression, if not already penalised.
A2 Any relevant work involving x or y and / or 5, 2 or similar.
A3 Any attempt at substituting coordinates into some expression.
A4 States 150 videos and / or no DVDs without work.

BONUS MARKS FOR ANSWERING THROUGH IRISH

Bonus marks are applied separately to each paper as follows:

If the mark achieved is less than 226, the bonus is 5% of the mark obtained, rounding *down*.
(e.g. 198 marks × 5% = 9.9 ⇒ bonus = 9 marks.)

If the mark awarded is 226 or above, the following table applies:

Marks obtained	Bonus
226 – 231	11
232 – 238	10
239 – 245	9
246 – 251	8
252 – 258	7
259 – 265	6
266 – 271	5
272 – 278	4
279 – 285	3
286 – 291	2
292 – 298	1
299 – 300	0