



Coimisiún na Scrúduithe Stáit State Examinations Commission

JUNIOR CERTIFICATE EXAMINATION 2006 MATHEMATICS - HIGHER LEVEL - PAPER 1 MARKING SCHEME

GENERAL GUIDELINES FOR EXAMINERS

- Penalties of three types are applied to candidates' work as follows:
 - Blunders - mathematical errors/omissions (-3)
 - Slips- numerical errors (-1)
 - Misreadings (provided task is not oversimplified) (-1).Frequently occurring errors to which these penalties must be applied are listed in the scheme. They are labelled: B1, B2, B3,..., S1, S2,..., M1, M2,...etc. These lists are not exhaustive.
- When awarding attempt marks, e.g. Att(3), note that
 - any *correct, relevant* step in a part of a question merits at least the attempt mark for that part
 - if deductions result in a mark which is lower than the attempt mark, then the attempt mark must be awarded
 - a mark between zero and the attempt mark is never awarded.
- Worthless work is awarded zero marks. Some examples of such work are listed in the scheme and they are labelled as W1, W2,...etc.
- The phrase "hit or miss" means that partial marks are not awarded – the candidate receives all of the relevant marks or none.
- The phrase "and stops" means that no more work is shown by the candidate.
- Special notes relating to the marking of a particular part of a question are indicated by an asterisk. These notes immediately follow the box containing the relevant solution.
- The sample solutions for each question are not intended to be exhaustive lists – there may be other correct solutions.
- Unless otherwise indicated in the scheme, accept the best of two or more attempts – even when attempts have been cancelled.
- The *same* error in the *same* section of a question is penalised *once* only.
- Particular cases, verifications and answers derived from diagrams (unless requested) qualify for attempt marks at most.
- A serious blunder, omission or misreading results in the attempt mark at most.
- Do not penalise the use of a comma for a decimal point, e.g. €5.50 may be written as €5,50.

QUESTION 1

Part (a)	10 marks	Att 3
Part (b)	20 marks	Att 6
Part (c)	20 marks	Att 6

Part (a) **10 marks** **Att 3**

- (a) The price of a DVD increases from €12·50 to €13·75.
~~✍~~ Express this increase as a percentage of the original price.

(a) **10 marks** **Att 3**

I	II
Increase = €13·75 - €12·50 = €1·25 Percentage Increase = $\frac{1.25}{12.50} \times 100 = 10\%$	$\frac{13.75}{12.50} = 1.10 \Rightarrow 110\%$ Increase = 10%

Blunders (-3)

- B1 Correct answer but no work shown (~~✍~~)
- B2 Each error in decimal point
- B3 Stops at $\frac{1}{10}$ (Method I) or 1.10 (Method II)
- B4 Expresses as a percentage of €13·75. Note: With work stops at 0·909 incurs B3 also

Slips (-1)

- S1 Numerical errors to a max of 3

Attempts (3 marks)

- A1 Finds €1·25 and stops - with work

Worthless (0)

- W1 Incorrect answer and no work shown

Part (b)

20 (10,10) marks

Att 3,3

Part (b)(i)

10 marks

Att 3

- (b) (i)** The time taken by Jack to travel from Derry to Waterford, a distance of 378 km, is 6 hours.
His return journey from Waterford to Derry, by the same route, takes an extra 45 mins.
 By how many km/h is his average speed slower on the return journey?

<p>(i) Outward journey (Derry -Waterford) Speed = $\frac{378\text{km}}{6\text{h}} = 63\text{km/h}$</p>	<p>Inward journey (Waterford -Derry) Speed = $\frac{378\text{km}}{6.75\text{h}} = 56\text{km/h}$</p>
Difference 63 km/h – 56 km/h = 7 km/h	

* Candidates may offer other correct versions

Blunders (-3)

- B1 Correct answer but no work shown ()
- B2 Error in S/D/T formula - once only
- B3 Each different conversion error e.g. uses 6.45h for inward journey
- B4 Fails to calculate reduction in speed
- B5 Each error in decimal point

Slips (-1)

- S1 Numerical errors to a max of 3
- S2 Stops at 63 km/h-56 km/h

Misreadings (-1)

- M1 Correct answer but not in km/h e.g. 116.666 m/min or 1.9444 m/s

Attempts (3 marks)

- A1 Defines outward and/or inward speed only e.g. $\frac{378\text{km}}{6\text{h}}$ and/or $\frac{378\text{km}}{6.75\text{h}}$ and stops
- A2 Correct expression for S/D/T
- A3 Converts 45 mins to 0.75 h and stops
- A4 States 1 hour = 60 minutes
- A5 Defines and calculates one speed only

Worthless (0)

- W1 Incorrect answer and no work shown

Part (b) (ii)**10 marks****Att 3**

- (b) (ii)** Jill has a gross income of €50 000.
 Her total income tax payable amounts to €10 460.
 The standard rate cut off point is €32 000.
 The standard rate of tax is 20% and the higher rate is 42%.
 ✍ What are Jill's tax credits for the year?

Part b (ii)**10 marks****Att 3**

(ii)	€32000 @ 20% =	€6400
	€18000 @ 42% =	€7560
	Total Tax =	€ 13960
	Tax Paid =	€ 10460
	Tax Credits =	€3500

* Candidates may offer other correct versions

Blunders (-3)

- B1 Correct answer, but no work shown (✍)
- B2 Each error in decimal point.
- B3 Mathematical error, e.g. 20% or 42% taken as some incorrect fraction. Note: May incur 2 blunders.
- B4 20% of some incorrect figure or 42% of some incorrect figure. Note: May incur 2 blunders.
- B5 Error in calculating total tax
- B6 Incorrect or no tax credits calculated

Slips (-1)

- S1 Numerical errors to a max of 3
- S2 Stops at €13960 - €10460

Attempts (3 marks)

- A1 Finds 20% correctly of some figure and stops -with work shown
- A2 Writes $20\% = \frac{1}{5}$
- A3 Shows some knowledge of tax credits e.g. writes Tax Payable = Total tax - Tax credits

Worthless (0)

- W1 Incorrect answer and no work shown

(c)

10(5, 5) marks

Att (2,2)

(c) (i) ✍ By rounding to the nearest whole number, estimate the value of

$$\frac{\sqrt{42 \cdot 91 + 21 \cdot 3}}{17 \cdot 56 - 3 \cdot 7 \times 4 \cdot 2}$$

Then, evaluate $\frac{\sqrt{42 \cdot 91 + 21 \cdot 3}}{17 \cdot 56 - 3 \cdot 7 \times 4 \cdot 2}$, correct to two decimal places.

(c) (i) Estimate

5 marks

Att 2

(i) $\frac{\sqrt{43+21}}{18-16} = \frac{\sqrt{64}}{2} = \frac{8}{2} = 4.$

* Accept rounding down / up of final answer.

Blunders (-3)

B1 Correct answer, but no work shown (✍).

B2 Each error in decimal point.

B3 Each different error in precedent.

B4 Mathematical error eg $\sqrt{43+21}$ as $\sqrt{43} + \sqrt{21}$ or $\sqrt{\quad}$ taken as $\frac{1}{2}$ or invalid cancelling

Slips (-1)

S1 Numerical errors to max of 3

S2 Error in rounding to the nearest whole number, (e.g. 42.91 rounds to 42) each time to a max of 3

S3 With work stops at $\frac{8}{2}$

Attempts (2 marks)

A1 Some correct rounding to nearest whole number.

Worthless (0)

W1 Incorrect answer and no work shown

(c) (i) Evaluate

5 marks

Att 2

(i)	$\frac{\sqrt{64.21}}{17.56 - 15.54} = \frac{8.013}{2.02} = 3.966 = 3.97.$
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- * Do not penalise same error if already penalised in **Estimate** above.
- * Accept correct answer without work

Blunders (-3)

- B1 Each error in decimal point
- B2 Each different error in precedent.
- B3 Mathematical error

Slips (-1)

- S1 Numerical errors to max of 3
- S2 Fails to round off or incorrect rounding or, early rounding off, which affects answer
- S3 Answer givens as 3.96 or 3.966 with no work shown

Attempts (2 marks)

- A1 Some correct calculation done

Worthless (0)

- W1 Incorrect answer and no work shown but, note S3 above
- W2 3.9 or 4.0 without work

(c) (ii)

10 marks

Att 3

(ii)

A sum of money was invested at compound interest for two years.

The interest rate for each year was 5%.

After the two years the sum amounted to €5512.50.

 Calculate the original sum of money invested.

(c) (ii)

10 marks

Att 3

	I	
(ii)	€1 amounts to $(1.05)^2 =$	€1.1025
	Sum invested	$= \frac{€5512.5}{1.1025} = €5000$

II	
A year 2 = €5512.50	105% of P year 2
$P \text{ year } 2 = \frac{€5512.50}{1.05} = €5250$	
A year 1 = €5250	105% of P year 1
$P \text{ year } 1 = \frac{€5250}{1.05} = €5000$	

III	
P year 1 = 100%	
I year 1 = 5%	
P year 2 = 105%	
I year 2 = 5.25%	
Amount = 110.25%	
110.25% = €5512.5	
$100\% = \frac{€5512.5}{110.25} \times 100$	
= €5000	

* Candidates may offer other correct versions- e.g. may use Compound Interest formula
Blunders (-3)

B1 Correct answer, but no work shown (.

B2 Each error in decimal point

B3 Working backwards subtracts 5% for each year, once only, with work (Answer €4975.03)

B4 Stops at €5250 with work

B5 Mathematical error

Slips (-1)

S1 Numerical errors to max of 3

Attempts (3 marks)

A1 Ignores cumulating - subtracts 10% of €5512.50 with work shown

A2 States correct CI formula and stops -even though not required by syllabus

A3 Mentions 105%

Worthless (0)

W1 Incorrect answer and no work shown

QUESTION 2

Part (a)	10 marks	Att 3
Part (b)	20 marks	Att 6
Part (c)	20 marks	Att 6

Part (a) **10 marks** **Att 3**

- (a) Derek processed 390 passport applications during the month of July.
He processed 10% fewer applications during the month of August.
 How many applications did he process in August?

Part (a) **10 marks** **Att 3**

I		II
$390 \times 0.9 = 351$	or	$10\% = 39$ $390 - 39 = 351$

Blunders (-3)

- B1 Correct answer, but no work shown (.
- B2 Each error in decimal point
- B3 Expresses 10% or 90% as incorrect fraction
- B4 Fails to subtract 39 or adds 39 to 390

Slips (-1)

- S1 Numerical errors to a max of 3

Attempts (3 marks)

- A1 States $10\% = \frac{1}{10}$ or similar
- A2 Mentions 90% with work

Worthless (0)

- W1 Incorrect answer and no work shown

Part (b)

20 (10, 10) marks

Att 3, 3

(b) 70 teenagers responded to a survey about holiday destinations.
30 had travelled to France,
26 had travelled to Spain
and 28 had travelled to Italy.
12 had travelled to both France and Spain,
8 had travelled to both Spain and Italy,
while x had travelled to France and Italy only.
4 teenagers had travelled to all three countries.
Twice as many had never travelled to any of these destinations as had travelled to France and Italy only.

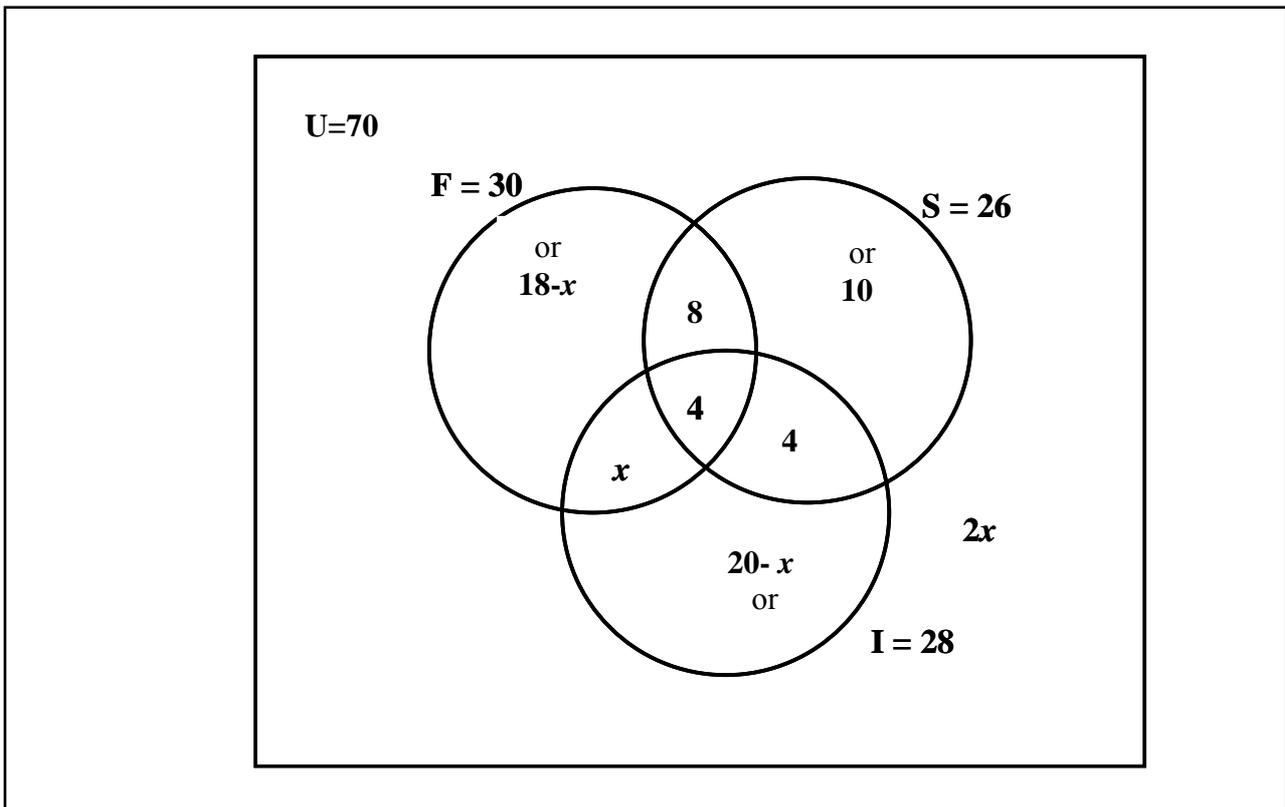


- (i) ✍ Represent the above information on a Venn diagram.
- (ii) ✍ Find the number of teenagers who had travelled to France only.

Part (b) (i)

10 marks

Att 3



* **Special Case Marking**

Blunders (-3)

B1 Each incorrectly placed or missing cardinal number from above

Attempts (3 marks)

A1 Any one correct item in above

A2 Draws Venn diagram of three intersecting circles and stops

(b) (ii)

10 marks

Att 3

(ii)

$$26 + (18 - x) + x + (20 - x) + 2x = 70 \text{ or equivalent}$$

$$64 + x = 70$$

$$x = 70 - 64$$

$$x = 6$$

Number who travelled to, France only $18 - 6 = 12$

* Accept candidates' answers from above even if generates fraction and/or negative answer, but see A1 below

Blunders (-3)

- B1 Correct answer, but no work shown (~~✗~~).
- B2 Each different error in forming equation
- B3 Each different error in balancing equation
- B4 Fails to find number who travelled to 'France only'
- B5 Mathematical error

Slips (-1)

- S1 Numerical errors to a max of 3

Attempts (3 marks)

- A1 Answer consistent with incorrect Venn diagram work at (i), but oversimplified

Worthless (0)

- W1 Incorrect answer and no work shown

Part (c)**20 (10,10) marks****Att (3,3)**

- (c) (i) A merchant buys tea for €3.29 per kg and then sells it at a profit of 60% of the cost price to a customer in England.
The exchange rate is £1 (sterling) = €1.46.
- ✍* Calculate the selling price of the tea in £ sterling, correct to two decimal places.
- (ii) The exchange rate changes to £1 (sterling) = €1.50. The selling price, in sterling, remains the same.
- ✍* Calculate the merchant's percentage profit in this case, correct to the nearest whole number.

(c) (i)**10 marks****Att 3**

(i)	I	II
	$€3.29 \times 1.6 = €5.264$	Works in £ sterling from start
	$\frac{€5.264}{1.46} = £3.605 = £3.61$	$\frac{€3.29}{1.46} = £2.2534$
	<i>or</i>	$£2.2534 \times 1.6 = £3.6054 = £3.61$
	$60\% = €1.974$	<i>or</i>
	$€3.29 + €1.974 = €5.264$	$60\% \text{ of } £2.2534 = £1.352$
	$\frac{€5.264}{1.46} = £3.605 \approx £3.61$	<i>Selling price</i> = $£1.352 + £2.2534$
		= $£3.6054 \approx £3.61$

* Candidates may offer other correct versions

Blunders (-3)

- B1 Correct answer, but no work shown (*✍*)
- B2 Each error in decimal point
- B3 Incorrect fraction for 60%
- B4 Error in exchange rate e.g. multiplies to get £ sterling
- B5 Mishandle profit e.g. subtracts or fails to add
- B6 Mixes or works with two currencies - each time.
- B7 Leaves answer as €5.26 or €5.264, fails to convert.

Slips (-1)

- S1 Numerical errors to a max of 3
- S2 Early rounding off that affects answer - each time. Note: needs 4 decimal points in intermediate stages
- S3 Fails to round off or rounds off incorrectly

Attempts (3 marks)

- A1 Finds 60% of €3.29 with work and stops
- A2 Write 60% as a fraction/ decimal and stops
- A3 Calculates the exchange rate for €1 and stops -with work

Worthless (0)

- W1 Incorrect answer and no work shown

(c) (ii)

10 marks

Att 3

	New rate	Sterling
New income		$\frac{€3 \cdot 29}{1 \cdot 5} = £2 \cdot 193$
$£3 \cdot 61 \times 1 \cdot 50 = €5 \cdot 415$		
Profit = $€5 \cdot 415 - €3 \cdot 29$		New Profit = $£3 \cdot 61 - £2 \cdot 193 = £1 \cdot 416$
% Profit = $\frac{2 \cdot 125}{3 \cdot 29} \times 100 = 64 \cdot 58\% \approx 65\%$		% Profit = $\frac{1 \cdot 416}{2 \cdot 193} \times 100 = 64 \cdot 56\% \approx 65\%$

- * Accept candidates' answer from part (i)
- * Candidates may offer other correct versions
- * Do not penalise the same error again in this section but see B6

Blunders (-3)

- B1 Correct answer, but no work shown (✗)
- B2 Each error in decimal point
- B3 Error in handling exchange rate
- B4 Calculates % of incorrect figure
- B5 Fails to calculate %, stops at $\frac{2 \cdot 125}{3 \cdot 29} \times 100$ or equivalent
- B6 Mixes or works with two currencies - each time.
- B7 Error in finding new profit

Slips (-1)

- S1 Numerical errors to a max of 3
- S2 Early rounding off that affects answer - each time
- S3 Fails to round off or rounds off incorrectly

Attempts (3 marks)

- A1 Correctly converts £3·61 or €3·29 using the new rate and stops - with work shown

Worthless (0)

- W1 Incorrect answer and no work shown

QUESTION 3

Part (a)	10 marks	Att (2,2)
Part (b)	20 marks	Att (3,3)
Part (c)	20 marks	Att (3,3)

Part (a) **5,5 marks** **Att 2,2**

(i)	Simplify $(64)^{\frac{3}{2}}$.
(ii)	Simplify $(64)^{\frac{2}{3}}$.

(a)(i) **5 marks** **Att 2**

<p style="text-align: center;">I</p> $(64)^{\frac{3}{2}} = \left(64^{\frac{1}{2}}\right)^3 = 8^3$ <p style="text-align: center;">(or) $= 2^9$ (or) $= 512$</p>	<p style="text-align: center;">II</p> $(64)^{\frac{3}{2}} = (64^3)^{\frac{1}{2}} = 262144^{\frac{1}{2}} = 512$	<p style="text-align: center;">III</p> <p style="text-align: center;">Calculator</p> $64^{1.5} = 512$
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- * Accept correct answer without work, no (✍)
- * Accept 8^3 , 2^9 or 512 for full marks

Blunders (-3)

- B1 Each index error
- B2 Finds 8 (I) or 262144 (II) only and stops

Attempts (2 marks)

- A1 Writes $\frac{3}{2} = 1.5$
- A2 Some correct work with indices

Worthless (0)

- W1 $\frac{64 \times 3}{2}$

(a)(ii)

5 marks

Att 2

I	II	III
$(64)^{\frac{2}{3}} = \left(64^{\frac{1}{3}}\right)^2 = 4^2$ (or) $= 2^4$ (or) $= 16$	$(64)^{\frac{2}{3}} = (64^2)^{\frac{1}{3}} = 4096^{\frac{1}{3}} = 16$	Calculator $64^{0.66} = 15.56$ accept

* Accept correct answer without work no (~~✓~~)

* Accept 4^2 , 2^4 and 16 for full marks.

Blunders (-3)

B1 Each index error

B2 Finds 4 (I) or 4096 (II) only and stops

Slip (-1)

S1 Uses 0.6 or 0.7 in calculator - gives 12.125 and 18.38 respectively

Attempts (2 marks)

A1 Writes $\frac{2}{3} = 0.6$ or 0.7

A2 Some correct work with indices

Worthless (0)

W1 $\frac{64 \times 2}{3}$

(b)

(10, 10) marks

Att(3,3)

(b) Let the cost of a meal for an adult be € x and the cost of a meal for a child be € y .

The cost of a meal for 3 adults and 2 children amounts to €125.

The cost of a meal for 2 adults and 3 children amounts to €115.

(i) Write down two equations in x and y to represent this information.

(ii)  Solve these equations to find the cost of an adult's meal and the cost of a child's meal.

(b) (i)

10 marks

Att 3

(i) $3x + 2y = 125$

$2x + 3y = 115$

* One correct equation merits 7 marks

Blunders (-3)

B1 One correct equation only and stops

B2 Incorrect or missing term, once per equation

B3 Mathematical error

Attempts (3 marks)

A1 Writes $3x$ and /or $2x$ only

A2 Writes $2y$ and/or $3y$ only

(b) (ii)

10 marks

Att 3

(ii)

$$3x + 2y = 125 \quad \mathbf{x -2}$$

$$-6x - 4y = -250$$

$$2x + 3y = 115 \quad \mathbf{x 3}$$

$$6x + 9y = 345$$

$$5y = 95$$

$$y = 19$$

$$3x + 2(19) = 125$$

$$3x = 125 - 38$$

$$3x = 87$$

$$x = 29$$

$$\text{Cost of adult's meal} = \text{€}29$$

$$\text{Cost of child's meal} = \text{€}19$$

* Accept candidates answers from (i) even if leads to negative values

* No standard work but verifies correct values in both equations accept for 10 marks

Blunders (-3)

B1 Correct answer, but no work shown (~~✗~~)

B2 Each different error in balancing equations e.g. fails to multiply across completely

B3 Mathematical error

B4 Calculates the value of either x or y correctly and stops

B5 Substitutes for incorrect variable at final step

Slips (-1)

S1 Numerical errors to a max of 3

S2 Finds $y = 19$ but substitutes some other value eg 1·9

Attempts (3 marks)

A1 Some correct relevant work

A2 $5x = 125$ $x = 25$ or similar, oversimplified

A3 Writes x in terms of y or vice versa and stops

Worthless (0)

W1 Incorrect answer and no work shown

Part (c)

20 (10,10) marks

Att (3,3)

(c) (i) ✍ Express in its simplest form:

$$\frac{1}{2x-3} - \frac{1}{2x+3}$$

(ii) ✍ Hence, or otherwise, solve the equation:

$$\frac{1}{2x-3} - \frac{1}{2x+3} = \frac{6}{7}, \quad x \in \mathbf{Z}.$$

Part (c)(i)

10 marks

Att 3

$$(i) \quad \frac{1}{(2x-3)} - \frac{1}{(2x+3)} = \frac{1(2x+3) - 1(2x-3)}{(2x-3)(2x+3)} = \frac{2x+3-2x+3}{(2x-3)(2x+3)} = \frac{6}{(2x-3)(2x+3)}$$

* Accept common denominator as $(2x-3)(2x+3)$ - penalise incorrect multiplication in part c(ii)

Blunders (-3)

- B1 Correct answer, but no work shown (✍)
- B2 Incorrect common denominator or mishandles common denominator
- B3 Each different error in distributive law
- B4 Mathematical /sign errors
- B5 Fails to add like terms in simplified numerator

Slips (-1)

- S1 Numerical slips to a max of 3

Attempts (3 marks)

- A1 Correct common denominator and stops
- A2 Leaves out denominator

Worthless (0)

- W1 Incorrect answer and no work shown
- W2 Subtracts to get $\frac{0}{-6x}$

$$\frac{6}{(2x-3)(2x+3)} = \frac{6}{7}$$

$$\frac{6}{4x^2-9} = \frac{6}{7}$$

$$24x^2 - 54 = 42$$

$$24x^2 = 96$$

$$x^2 = 4 \quad \text{or} \quad x^2 - 4 = 0 \quad \text{or} \quad \text{Formula}$$

$$x = \pm 2 \quad (x-2)(x+2) = 0$$

$$x = 2 \quad \text{or} \quad x = -2$$

* If part (i) mishandled but correct common denominator in (ii) award attempt (3marks)
for (i) - assuming 0 at part (i)

Blunders (-3)

- B1 Correct answer, but no work shown (~~✗~~)
- B2 Incorrect common denominator e.g. error in multiplying out
- B3 Each different error in balancing the equation
- B4 Mathematical /sign errors
- B5 Incorrect factors
- B6 Incorrect roots from factors
- B7 Stops at $x^2 = 4$ or equivalent. Max loss after this point is 3 marks

Slips (-1)

- S1 Numerical errors to a max of 3
- S2 Only finds one solution

Attempts (3 marks)

- A1 Multiplies out common denominator correctly and stops
- A2 Solves a linear equation
- A3 Carries down answer from (i)

Worthless (0)

- W1 Incorrect answer and no work shown

QUESTION 4

Part (a)	10 marks	Att 3
Part (b)	20 marks	Att 7
Part (c)	20 marks	Att 8

Part (a)	10 marks	Att 3
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 Express in its simplest form:

$$2x - [3 - (4 - 3x)] + 6.$$

Part (a)	10 marks	Att 3
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$$2x - [3 - (4 - 3x)] + 6 = 2x - [3 - 4 + 3x] + 6 = 2x - 3 + 4 - 3x + 6 = 7 - x$$

$$2x - [3 - (4 - 3x)] + 6 = 2x - 3 + (4 - 3x) + 6 = 2x - 3 + 4 - 3x + 6 = 7 - x$$

Blunders (-3)

- B1 Correct answer, but no work shown ()
- B2 Each different error in distributive law
- B3 Treats as an equation but award full marks if Left Hand Side is $7-x$
- B4 Order of operation error if affects answer
- B5 Mathematical error e.g. gives $2x-3+4-3x+6$ as $6x$, or similar, but if happens **after** $7-x$ no penalty

Slips (-1)

- S1 Numerical slips to a max of 3

Misreading (-1)

- M1 Copies incorrectly but does not oversimplify

Attempts (3 marks)

Note: There must be some work of merit to award attempt

- A1 Simplifies/removes one bracket and stops
- A2 Substitutes a number for x and calculates value of expression

Worthless (0)

- W1 Incorrect answer and no work shown

Part (b)

20 (10,5,5)marks

Att (3,2,2)

- (i) Solve $x^2 - 4x - 8 = 0$, giving your answer in the form $a \pm a\sqrt{b}$, where $a, b \in \mathbf{N}$.
- (ii) Factorise $9x^2 - 16y^2$.
- (iii) The length of one side of a rectangle is $x + 4$.
The area of the rectangle is $x^2 + 16x + 48$.
Find an expression in x for the length of the other side.

Part (b)(i)

10marks

Att 3

$$\begin{aligned}x^2 - 4x - 8 &= 0 \\a = 1 \quad b = -4 \quad c = -8 \\(i) \quad \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} &= \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \cdot 1 \cdot -8}}{2 \cdot 1} = \frac{4 \pm \sqrt{16 + 32}}{2} = \frac{4 \pm \sqrt{48}}{2} \\&= \frac{4 \pm 4\sqrt{3}}{2} = 2 \pm 2\sqrt{3}\end{aligned}$$

* Can be done by completing the square

Blunders (-3)

- B1 Correct answer, but no work shown (✗)
- B2 Each error in quadratic formula
- B3 Each different incorrect substitution but $b = 4$ and $c = 8 \rightarrow$ one blunder
- B4 Mathematical error in sign e.g. $(-4 \cdot 1 \cdot -8)$ as -32
- B5 Mathematical error in squaring $(-4)^2$ as -8 or similar
- B6 Ignores a minus in square root e.g. $\sqrt{-48}$ taken as $\sqrt{48}$
- B7 Error in handling surds or no work with surds
- B8 Invalid cancelling answer given as $2 \pm 4\sqrt{3}$ or similar
- B9 Stops at $\frac{4 \pm 4\sqrt{3}}{2}$ (not in required form) or correct decimal answer

Slips (-1)

- S1 Numerical errors to a max of 3

Attempts (3 marks)

- A1 Correct formula or incorrect relevant formula with some correct substitution and stops

Worthless (0)

- W1 Incorrect answer and no work shown
- W2 Tries to factorise

Part (b)(ii)**5 marks****Att 2**

(ii) $9x^2 - 16y^2 = (3x - 4y)(3x + 4y)$
--

* Accept correct answer and no work – no (✍).

Blunders (-3)

B1 Errors in sign. e.g. $(3x+4y)(3x+4y)$

B2 Stops at $3x(3x+4y) - 4y(3x+4y)$.

B3 Incorrect factorisation of one term, e.g. $(3x-16y)(3x+16y)$.

B4 Answer given as $(3x-4y) + (3x+4y)$, but $(3x-4y)$ and $(3x+4y)$ merits full marks

Attempts (2 marks)

A1 Any correct factors of $9x^2$ and / or $16y^2$.

A2 $(9x-16y)(9x+16y)$.

A3 Indicates some knowledge of the difference of two squares

Part (b)(iii)**5 marks****Att 2**

(iii)

$$x^2 + 16x + 48 = (x + 4)(x + 12)$$

$(x+12)$ other side

or

By formula, roots to find factors

or

$$x^2 + 16x + 48 \div (x + 4) = (x + 12)$$

Blunders (-3)

B1 Correct answer, but no work shown (✍)

B2 Incorrect other factor of $x^2 + 16x + 48$ with work shown

B3 Each different error in indices and signs when dividing.

B3 Each term omitted or incorrect when dividing

B4 Each error in multiplication/subtraction when dividing

B5 Stops at roots when using quadratic formula

Note: If uses formula for solving a quadratic equation, apply blunders (-3) as in part b(i)

Attempts (2 marks)

A1 States area = lb

A2 Some effort at factorising

A3 Indicates division

A4 Pick a specific value for x to find other side - not in terms of x

Worthless (0)

W1 Incorrect answer and no work shown

Part (c)**20 (5, 5, 5, 5) marks****Att (2,2,2,2)**

- (c) A farmer must feed bales of hay to his cattle for a total of 90 days.
 He feeds the cattle 540 bales of hay over a number of days.
 Their average consumption over this period is x bales per day.
- (i) Write an expression in x for the number of days taken to consume the 540 bales.
- If the average consumption is increased by 1 bale per day, then the cattle consume 300 bales in the remaining days.
- (ii) Write an expression in x for the number of days taken to consume the 300 bales.
- (iii) Using the above information, write an equation in x .
- (iv)  Solve this equation to find the value of x and the number of days taken to consume the first 540 bales.

Part (c)(i)**5 marks****Att 2**

(i) $\frac{540}{x}$

Blunders (-3)

B1 Number of days = $\frac{x}{540}$

Attempts (2 marks)

A1 Gives answer as $540 \cdot x$

A2 Any correct relevant step, e.g. trial and error, award once only in (i) and (ii) e.g. $\frac{540}{90} = 6$

Part (c)(ii)**5 marks****Att 2**

(ii) $\frac{300}{x+1}$ or $90 - \frac{540}{x}$

Blunders (-3)

B1 Inverts e.g. $\frac{x+1}{300}$ but do not penalised if already blundered in (i)

B2 Uses $x - 1$

B3 Sign error in $90 - \frac{540}{x}$

Attempts (2 marks)

A1 Gives answer as $300(x+1)$

Part (c)(iii)**5 marks****Att 2**

(iii)	$\frac{540}{x} + \frac{300}{x+1} = 90$	or equivalent
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* Accept candidate's answer from (i) and (ii) above

* If no work, or no work of merit, at parts (i) and/or (ii) but states above, award *Att 2* and *Att 2* from parts (i) and/or (ii) here*Blunders (-3)*

B1 Sign error in setting up equation has $\frac{540}{x} - \frac{300}{x+1} = 90$

B2 Expression not equal to 90

Slips (-1)

S1 Writes correct expression with required terms but no equal sign

Attempts (2 marks)

A1 Incorrect expression but uses data from (i) and (ii)

A2 $\frac{540}{x} + 90 - \frac{540}{x} = 90$

Part (c)(iv)**5 marks****Att 2**

(iv)

$$\frac{540}{x} + \frac{300}{x+1} = 90$$

$$540(x+1) + 300x = 90(x^2 + x)$$

$$540 + 540x + 300x = 90x^2 + 90x$$

$$90x^2 - 750x - 540 = 0$$

$$3x^2 - 25x - 18 = 0$$

$$(3x+2)(x-9) = 0$$

$$x = 9$$

$$\frac{540}{9} = 60$$

Number of days = 60

Blunders (-3)

B1 Correct answer, but no work shown (✗)

B2 Mathematical /sign error

B3 Each different error in distributive law

B4 Each different error in balancing equation

B5 Correct factors and stops

B6 Incorrect factors

B7 Errors using quadratic formula - see question 4(b) (i)

Slips (-1)

S1 Numerical errors to a max of 3

S2 Leaves answer as 9

Attempts (2 marks)

A1 No quadratic due to previous errors merits attempt at most

Worthless (0)

W1 Incorrect answer and no work shown

QUESTION 5

Part (a)	10 marks	Att 3
Part (b)	20 marks	Att 7
Part (c)	20 marks	Att 8

Part (a) **10 marks** **Att 3**

- (a) The temperature on Sunday is x° .
 The temperature rose by 3° each day for the next two days.
 The temperature then dropped by 4° each day for the next three days.
 ✍ Derive an expression in x for the temperature on the fifth day (i.e. Friday).



Part (a) **10 marks** **Att 3**

- | | | |
|--|---|---|
| (a) Sunday | = | x° |
| Monday | = | $x^\circ + 3^\circ$ |
| Tuesday | = | $x^\circ + 3^\circ + 3^\circ = x^\circ + 6^\circ$ |
| Wednesday | = | $x^\circ + 6^\circ - 4^\circ = x^\circ + 2^\circ$ |
| Thursday | = | $x^\circ + 2^\circ - 4^\circ = x^\circ - 2^\circ$ |
| Friday | = | $x^\circ - 2^\circ - 4^\circ = x^\circ - 6^\circ$ |
| or | | |
| $x^\circ + 2(3^\circ) - 3(4^\circ) = x^\circ + 6^\circ - 12^\circ = x^\circ - 6^\circ$ | | |

* Uses a specific value for x° on Sunday and gives the correct value for Friday, attempt (3) only, however if rewrites Friday value as a correct expression in x° accept for full mark

Blunders (-3)

- B1 Correct answer, but no work shown (✍)
- B2 Each day omitted or incorrect
- B3 Mathematical error
- B4 Fails to simplify

Slips (-1)

- S1 Numerical errors to max of 3

Attempts (3 marks)

- A1 Takes a particular value for x see note above *
- A2 Special cases $3+3-4-4-4 = -6$ or $3x, 6x, 2x, -2x, -6x$

Worthless (0)

- W1 Incorrect answer and no work shown

Part (b)

20 marks

Att 7

(b) Let f be the function $f: x \rightarrow 35x - 5x^2$.
 Draw the graph of f for $0 \leq x \leq 7$, $x \in \mathbf{R}$.

Part (b)

20 marks

Att 7

	x	0	1	2	3	4	5	6	7	
	$f(x)$	0	30	50	60	60	50	30	0	

$f(x) = 35x - 5x^2$									
$f(0) = 35(0) - 5(0)^2 = 0$					$f(1) = 35(1) - 5(1)^2 = 30$				
$f(2) = 35(2) - 5(2)^2 = 50$					$f(3) = 35(3) - 5(3)^2 = 60$				
$f(4) = 35(4) - 5(4)^2 = 60$					$f(5) = 35(5) - 5(5)^2 = 50$				
$f(6) = 35(6) - 5(6)^2 = 30$					$f(7) = 35(7) - 5(7)^2 = 0$				
x	0	1	2	3	4	5	6	7	
$35x$	0	35	70	105	140	175	210	245	
$-5x^2$	0	-5	-20	-45	-80	-125	-180	-245	
$f(x)$	0	30	50	60	60	50	30	0	

Values for quadratic graph

Blunders (-3)

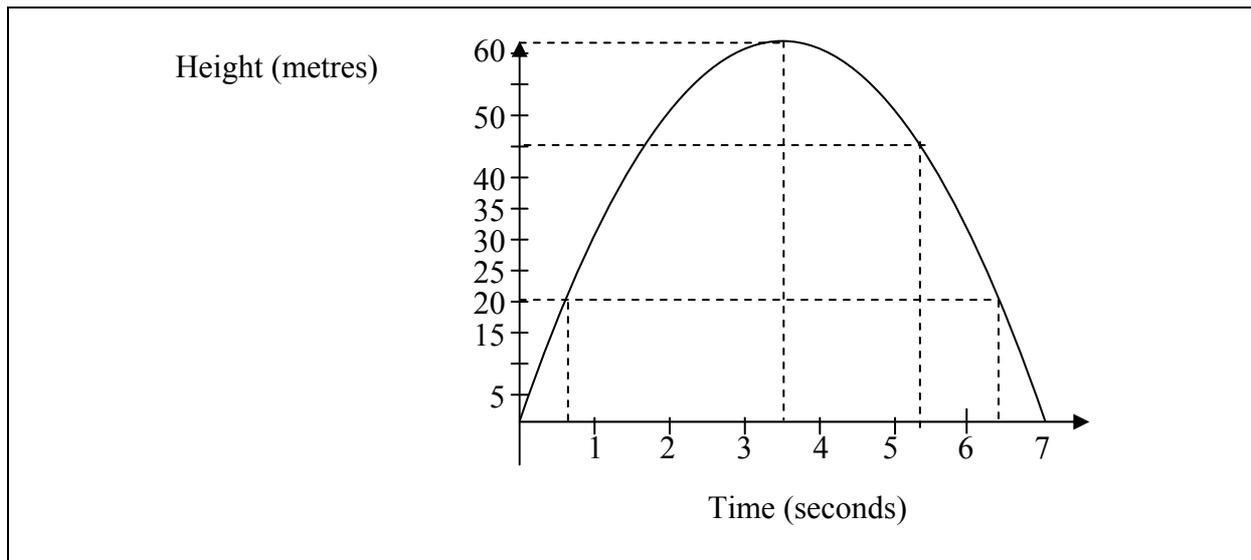
- B1 Each incorrect $f(x)$ without work.
- B2 x row added in, i.e. top row, or adds in extra row.
- B3 Treating the domain as $0 < x < 7$, can incur 2 Blunders if both omitted.
- B4 Each different blunder which yields an incorrect row (full or part),
eg. $5(-x)^2$ or $(-5x)^2$ for $-5x^2$.
- B5 Avoids square for some (not all) values. See Attempts below
- B6 Mathematical errors in tots, e.g. $105 - 45 = -60$, but apply once only.
- B7 Uses graph of $f: x \rightarrow 5x^2 - 35x$, avoids $-5x^2$ or graph of $7x - x^2$

Slips (-1)

- S1 Numerical errors to a max. of 3.

Attempts (7 marks)

- A1 Omits $-5x^2$ or does not treat as $5x^2$, linear expression
- A2 Correct or partly correct table / values but no graph drawn.



Plotting the quadratic graph

* Accept candidate's values from the table.

Blunders (-3)

- B1 Points not joined to form a reasonable graph or 'flat top'.
- B2 (x, y) plotted as (y, x) , but apply once only, or reverses axes.
- B3 Scale not reasonably uniform can apply to each axis
- B4 Each different blunder in plotting points from candidate's table / values.
- B5 Each point omitted
- B6 Points joined with straight lines.

Attempts (7 marks)

- A1 Scaled axes drawn.

Part (c)**20 (5,5,5,5) marks****Att (2,2,2,2)**

- (c) The formula for the height, y metres, of a ball above ground level, x seconds after it is fired vertically into the air, is given by:

$$y = 35x - 5x^2.$$

Use your graph from part (b) to estimate:

- (i) ✎ the maximum height reached by the ball
 (ii) ✎ the height of the ball after 5.5 seconds.

On two occasions the ball is 20 metres above the ground.

- (iii) ✎ Use your graph from part (b) to estimate the two times when this occurred.

* Note: If graph is linear award at most attempts marks (2,2,2,0) in (c)

Part (c)(i)**5 marks****Att 2**

- (i) Max at $x = 3.5$ s $f: 3.5 \rightarrow 61$ m

* Accept answer consistent with candidates curve tolerance ± 2 but must be greater than 60, see B4

Blunders (-3)

B1 No indication on graph

B2 Indication on graph but no value given or value outside tolerance

B3 $x(\text{time})$ value of maximum only

B4 Answer 60 if not already penalised for 'flat top' in part (b), graph

Slips (-1)

S1 Gives coordinates of maximum rather than maximum values

Attempts (2marks)

A1 Point indicated on graph only

Worthless (0)

W1 Answer inconsistent with candidate's graph

Part (c)(ii)**5 marks****Att 2**

- (ii) $x = 5.5$ s $f: 5.5 \rightarrow 41$ m

* Accept answer consistent with candidates curve tolerance ± 2

Blunders (-3)

B1 No indication on graph

B2 Indication on graph but no value given or value outside tolerance

Attempt (2)

A1 Finds from function only

Part (c)(iii)**10 (5, 5) marks****Att (2, 2)**

- (iii) $f(x) = 20$ m \rightarrow $x = 0.6$ s and $x = 6.4$ s

* Accept answer consistent with candidates curve tolerance ± 0.3

Blunders (-3)

B1 No indication on graph, each value

B2 Indication on graph but no value given or value outside tolerance, each time

Attempts (2 marks)

A1 Correctly solves $f(x) = 20$ by formula (Att 2 + Att 2)

QUESTION 6

Part (a)	10 marks	Att 3
Part (b)	20 marks	Att 7
Part (c)	20 marks	Att 8

Part (a) **10 marks** **Att 3**

(a) Given that:

$$v^2 = u^2 + 2as.$$

Write s in terms of v , u and a .

Part (a) **10 marks** **Att 3**

$$v^2 = u^2 + 2as$$

$$v^2 - u^2 = 2as$$

$$\frac{v^2 - u^2}{2a} = s$$

$$s = \frac{v^2 - u^2}{2a} \text{ or } s = \frac{u^2 - v^2}{-2a} \text{ or } s = \frac{v^2}{2a} - \frac{u^2}{2a} \text{ or equivalent}$$

Blunders (-3)

B1 Correct answer, but no work shown ().

B2 Each different transposition error

B3 Stops at $-s = \frac{u^2 - v^2}{2a}$

B4 Replaces u , v and/or a by a numerical value - each variable replaced, and then solves correctly

B5 Mathematical error

Slips (-1)

S1 Stops at $2s = \frac{v^2 - u^2}{a}$

Misreadings (-1)

M1 Solves correctly for a

Attempts (3 marks)

A1 Solves correctly for u or v

A2 Assigns numerical values to a , u and v to find s correctly

A3 Any correct transposition and stops

Part (b)

20 (10,5, 5) marks

Att (3,2,2)

(i) ✍ Factorise $2l - kl + km - 2m$.

(ii) Factorise $6x^2 - 19x + 10$.

(iii) Factorise $17x - 5x^2$.

Part (b)(i)

10 marks

Att 3

(i)

$$2l - kl + km - 2m = l(2-k) - m(2-k) = (l-m)(2-k)$$

$$2l - kl + km - 2m = 2l - 2m + km - kl = 2(l-m) - k(l-m) = (2-k)(l-m)$$

Blunders (-3)

B1 Correct answer, but no work shown (✍).

B2 Each different error in sign when factorising and/or regrouping

B3 Stops at $l(2-k) - m(2-k)$ or similar e.g. $l(2-k) - m(-k+2)$

B4 Answer given as $(l-m) + (2-k)$ but $(l-m)$ and $(2-k)$ merits full marks

B5 Error in factors $l(2-k) + m(k-2)$ given as $(2-k)(l+m)$ or similar

Attempts (3 marks)

A1 Some effort at regrouping

A2 Stops at $l(2-k) + m(k-2)$ or similar

Worthless (0)

W1 Incorrect answer and no work shown

Part (c)**20 (5, 5, 5, 5) marks****Att (2,2, 2, 2)****(c)** Let f be the function $f: x \rightarrow 1 - 3x$ and g be the function $g: x \rightarrow 1 - x^2$.

- (i)** ✍ Find $f(-2)$ and $g(5)$.
- (ii)** ✍ Express $f(x+1)$ in the form $ax + b$, a and $b \in \mathbf{Z}$.
- (iii)** ✍ Solve for $x: f(x+1) = f(-2) + g(5)$.

Part (c)(i)**(5,5) marks****Att 2,2****(i)** $f(-2) = 1 - 3(-2) = 7$ $g(5) = 1 - (5)^2 = 1 - 25 = -24$

* Slips and blunders apply to each calculation

Blunders (-3)

- B1 Correct answer without any work (✍) each part
- B2 Each different mathematical error e.g. 5^2 as 10
- B3 Sign error

Slips (-1)

- S1 Numerical errors to a max of 3, each part

Attempts (2 marks)

- A1 Stops at $f(-2) = 1 - 3(-2)$
- A2 Stops at $g(5) = 1 - 5^2$

Worthless (0)

- W1 Incorrect answer and no work shown - each part
- W2 Solves $f(x) = 2$ and/or $g(x) = 5$ - each part

Part (c)(ii)**5 marks****Att 2****(ii)** $f(x+1) = 1 - 3(x+1) = 1 - 3x - 3 = -3x - 2$ *Blunders (-3)*

- B1 Correct answer without any work (✍)
- B2 Distribution error
- B3 Sign error
- B4 Mathematical error

Slips (-1)

- S1 Numerical errors to a max of 3

Attempts (2marks)

- A1 Special case, lets $x =$ numerical value and finds $f(x+1)$ correctly
- A2 $f(x+1) = f(x) + 1 = 2 - 3x$ or $f(x+1) = f(x) + f(1)$ found correctly

Worthless (0)

- W1 Incorrect answer and no work shown
- W2 Solves $f(x) = x + 1$

Part (c)(iii)

5 marks

Att 2

(iii)	$-3x - 2 = 7 + (-24)$
	$-3x = 7 - 24 + 2$
	$-3x = -15$
	$x = 5$

* Accept candidates answers from parts (i) and (ii)

Blunders (-3)

B1 Correct answer without any work (✍)

B2 Each different error balancing equation

B3 Sign errors

Slips (-1)

S1 Numerical errors to max of 3

Attempts (2 marks)

A1 Any correct substitution in given equation

Worthless (0)

W1 Incorrect answer and no work shown