



**Coimisiún na Scrúduithe Stáit
State Examinations Commission**

LEAVING CERTIFICATE EXAMINATION, 2013

APPLIED MATHEMATICS – ORDINARY LEVEL

FRIDAY, 21 JUNE – MORNING, 9.30 to 12.00

Six questions to be answered. All questions carry equal marks.

A *Formulae and Tables* booklet may be obtained from the Superintendent.

Take the value of g to be 10 m s^{-2} .

\vec{i} and \vec{j} are unit perpendicular vectors in the horizontal and vertical directions, respectively, or eastwards and northwards, respectively, as appropriate to the question.

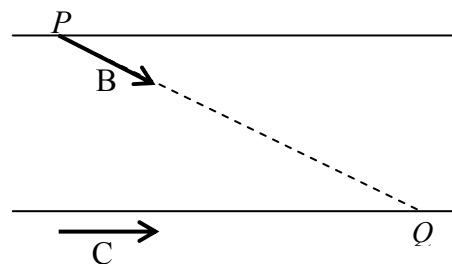
Marks may be lost if necessary work is not clearly shown.

1. The points P and Q lie on a straight level road.
 A car passes P with a speed of 28 ms^{-1} and decelerates uniformly for 6 seconds to a speed of 16 ms^{-1} .
 It then travels at a constant speed of 16 ms^{-1} for 8 seconds.
 The car now accelerates uniformly from 16 ms^{-1} to a speed of 24 ms^{-1} and then passes Q . The car travels 40 metres while accelerating.

- Find (i) the deceleration
 (ii) the acceleration
 (iii) $|PQ|$, the distance from P to Q
 (iv) the speed of the car 12 seconds before it passes Q
 (v) the average speed of the car between P and Q .

2. A river is 57 metres wide and has parallel banks.

Boat B departs from point P on its northern bank and lands at point Q on its southern bank.



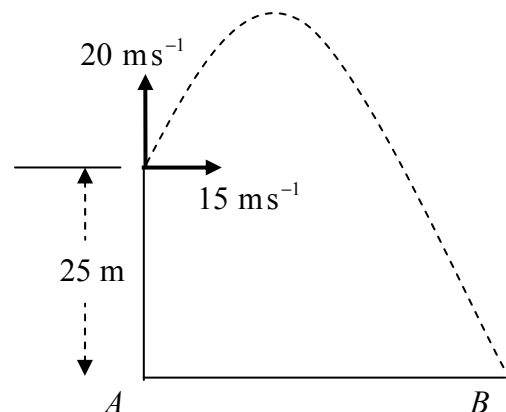
The actual velocity of the boat is $4\vec{i} - 3\vec{j} \text{ ms}^{-1}$.

Car C travels due east at a constant speed of 7 ms^{-1} along the southern bank of the river.

- Find (i) the velocity of C in terms of \vec{i} and \vec{j}
 (ii) the velocity of B relative to C in terms of \vec{i} and \vec{j}
 (iii) the magnitude and direction of the velocity of B relative to C
 (iv) the time it takes B to cross the river
 (v) $|PQ|$, the distance from P to Q .

3. A particle is projected from the top of a straight vertical cliff of height 25 m with velocity $15\vec{i} + 20\vec{j}$.
 It strikes the horizontal ground at B .

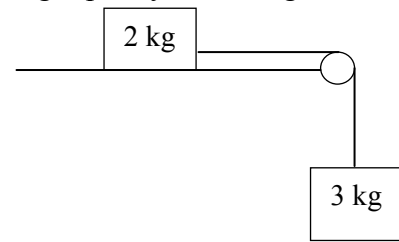
- Find (i) the time taken to reach the maximum height
 (ii) the maximum height above ground level
 (iii) the time of flight
 (iv) $|AB|$, the distance from A to B
 (v) the speed of the particle as it strikes the ground.



4. (a) A particle of mass 2 kg is connected to another particle of mass 3 kg by a taut light inelastic string which passes over a smooth light pulley at the edge of a rough horizontal table.

The coefficient of friction between the 2 kg mass and the table is $\frac{1}{2}$.

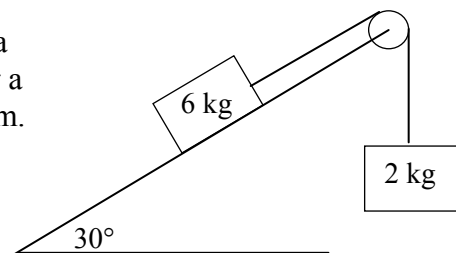
The system is released from rest.



- (i) Show on separate diagrams the forces acting on each particle.
 (ii) Find the common acceleration of the particles.
 (iii) Find the tension in the string.

- (b) Masses of 6 kg and 2 kg are connected by a taut light inelastic string which passes over a light smooth pulley as shown in the diagram.

The 6 kg mass lies on a smooth plane inclined at 30° to the horizontal. The 2 kg mass hangs vertically.



The system is released from rest.

- Find (i) the common acceleration of the particles
 (ii) the tension in the string.

5. A smooth sphere A, of mass 4 kg, collides directly with another smooth sphere B, of mass 1 kg, on a smooth horizontal table.

A and B are moving in opposite directions with speeds of 3 ms^{-1} and 2 ms^{-1} , respectively.



The coefficient of restitution for the collision is $\frac{1}{2}$.

- Find (i) the speed of A and the speed of B after the collision
 (ii) the loss in kinetic energy due to the collision
 (iii) the magnitude of the impulse imparted to B due to the collision.

6. (a) Particles of weight 5 N, 1 N, x N and 6 N are placed at the points $(2, q)$, $(-7, q)$, $(3, 3)$ and $(9, 1)$, respectively.

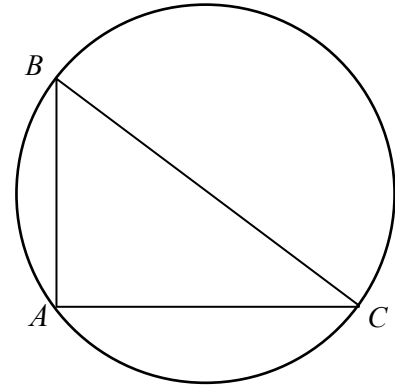
The co-ordinates of the centre of gravity of the system are $(4, 3)$.

- Find (i) the value of x
(ii) the value of q .

- (b) A circular lamina has the triangular portion with vertices A , B , and C removed.

A , B and C lie on the circumference of the circle and $[BC]$ is a diameter.

The co-ordinates of the vertices are $A(0, 0)$, $B(0, 18)$ and $C(24, 0)$.

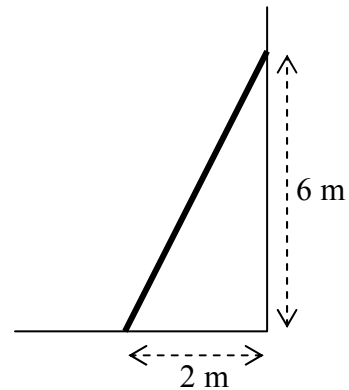


Find the co-ordinates of the centre of gravity of the remaining lamina.

7. (a) A uniform ladder, of weight 150 N, rests on rough horizontal ground and leans against a smooth vertical wall.

The foot of the ladder is 2 m from the wall and the top of the ladder is 6 m above the ground.

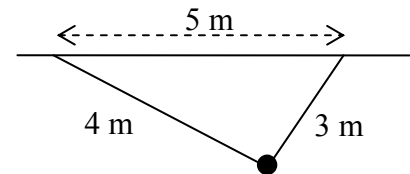
The ladder is in equilibrium and is on the point of slipping.



Find the coefficient of friction between the ladder and the ground.

- (b) Two light inextensible strings, of lengths 4 m and 3 m respectively, are tied to a particle weighing 45 N.

The other ends of the strings are tied to two points 5 m apart on a horizontal ceiling.



- (i) Show on a diagram the forces acting on the particle.
(ii) Write down the two equations that arise from resolving these forces horizontally and vertically.
(iii) Solve these equations to find the tension in each of the strings.

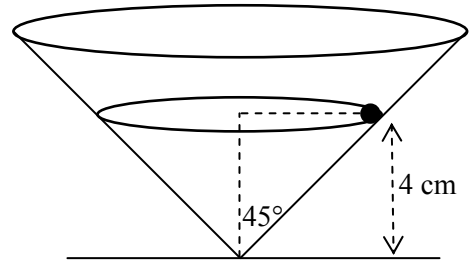
8. (a) A particle describes a horizontal circle of radius 3 m with uniform angular velocity ω radians per second. Its speed is 6 m s^{-1} .

Find (i) the acceleration of the particle
(ii) the time taken to complete one revolution.

- (b) A right circular hollow cone is fixed to a horizontal surface. Its semi-vertical angle is 45° and its axis is vertical.

A smooth particle of mass 2 kg describes a horizontal circle of radius r cm on the smooth inside surface of the cone.

The plane of the circular motion is 4 cm above the horizontal surface.



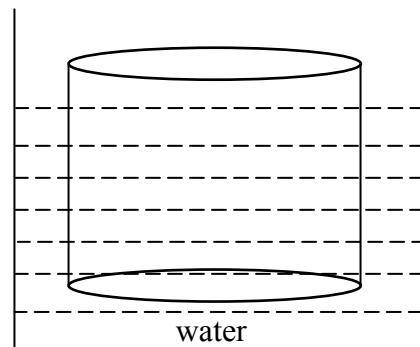
- (i) Find the value of r .
(ii) Show on a diagram all the forces acting on the particle.
(iii) Find the reaction force between the particle and the surface of the cone.
(iv) Calculate the speed of the particle.

9. (a) A right circular solid cylinder floats at rest in water with its axis vertical.

The radius of the cylinder is 5 cm and its height is 12 cm.

95% of the cylinder lies below the surface of the water.

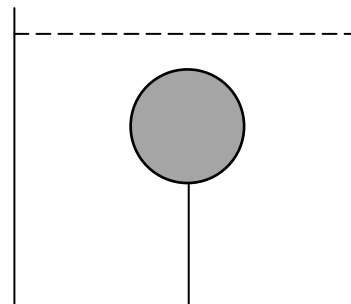
Find the weight of the cylinder.



- (b) A solid sphere has a radius of 2 cm.

The relative density of the sphere is 0.7 and it is completely immersed in a liquid of relative density 1.2.

The sphere is held at rest by a light inelastic vertical string which is attached to the base of the tank.



Find the tension in the string.

[Density of water = 1000 kg m^{-3}]

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