Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate 2013

Marking Scheme

Mathematics
(Project Maths – Phase 2)

Higher Level

Note to teachers and students on the use of published marking schemes

Marking schemes published by the State Examinations Commission are not intended to be standalone documents. They are an essential resource for examiners who receive training in the correct interpretation and application of the scheme. This training involves, among other things, marking samples of student work and discussing the marks awarded, so as to clarify the correct application of the scheme. The work of examiners is subsequently monitored by Advising Examiners to ensure consistent and accurate application of the marking scheme. This process is overseen by the Chief Examiner, usually assisted by a Chief Advising Examiner. The Chief Examiner is the final authority regarding whether or not the marking scheme has been correctly applied to any piece of candidate work.
Marking schemes are working documents. While a draft marking scheme is prepared in advance of the examination, the scheme is not finalised until examiners have applied it to candidates' work and the feedback from all examiners has been collated and considered in light of the full range of responses of candidates, the overall level of difficulty of the examination and the need to maintain consistency in standards from year to year. This published document contains the finalised scheme, as it was applied to all candidates' work.

In the case of marking schemes that include model solutions or answers, it should be noted that these are not intended to be exhaustive. Variations and alternatives may also be acceptable. Examiners must consider all answers on their merits, and will have consulted with their Advising Examiners when in doubt.

Future Marking Schemes

Assumptions about future marking schemes on the basis of past schemes should be avoided. While the underlying assessment principles remain the same, the details of the marking of a particular type of question may change in the context of the contribution of that question to the overall examination in a given year. The Chief Examiner in any given year has the responsibility to determine how best to ensure the fair and accurate assessment of candidates' work and to ensure consistency in the standard of the assessment from year to year. Accordingly, aspects of the structure, detail and application of the marking scheme for a particular examination are subject to change from one year to the next without notice.

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Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate Examination 2013

Mathematics
(Project Maths – Phase 2)

Paper 1

Higher Level

Friday 7 June   Afternoon 2:00 – 4:30

300 marks

Model Solutions – Paper 1

Note: The model solutions for each question are not intended to be exhaustive – there may be other correct solutions. Any examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his / her advising examiner.

Instructions

There are **three** sections in this examination paper:

Section A	Concepts and Skills	100 marks	4 questions
Section B	Contexts and Applications	100 marks	2 questions
Section C	Functions and Calculus (old syllabus)	100 marks	2 questions

Answer all eight questions.

Write your answers in the spaces provided in this booklet. You may lose marks if you do not do so.There is space for extra work at the back of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

Marks will be lost if all necessary work is not clearly shown.

Answers should include the appropriate units of measurement, where relevant.

Answers should be given in simplest form, where relevant.

Write the make and model of your calculator(s) here:

Section A	Concepts and Skills	100 marks

Answer **all four** questions from this section.

Question 1 **(25 marks)**

$z = \dfrac{4}{1+\sqrt{3}i}$ is a complex number, where $i^2 = -1$.

(a) Verify that z can be written as $1-\sqrt{3}i$.

$$z = \frac{4}{1+\sqrt{3}i} = \frac{4}{1+\sqrt{3}i} \times \frac{1-\sqrt{3}i}{1-\sqrt{3}i} = \frac{4-4\sqrt{3}i}{1+3} = 1-\sqrt{3}i$$

OR

If $z = \dfrac{4}{1+\sqrt{3}i} = 1-\sqrt{3}i$

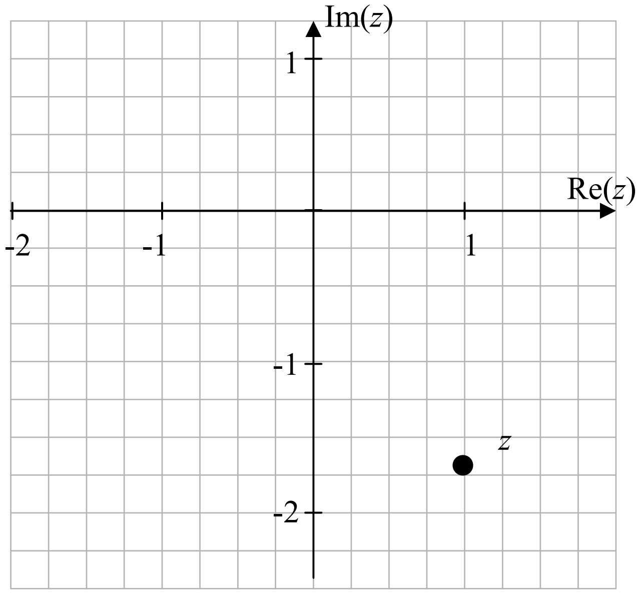
then $4 = (1+\sqrt{3}i)(1-\sqrt{3}i) = (1)^2 + (\sqrt{3})^2 = 4$
$\Rightarrow$ True

(b) Plot z on an Argand diagram and write z in polar form.

$$\tan\alpha = \frac{\sqrt{3}}{1} \Rightarrow \alpha = \frac{\pi}{3} \Rightarrow \theta = \frac{5\pi}{3}$$

$$r = |1-\sqrt{3}i| = \sqrt{1+3} = \sqrt{4} = 2$$

$$z = 2\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right)$$



(c) Use De Moivre's theorem to show that $z^{10} = -2^9\left(1-\sqrt{3}i\right)$.

$$z^{10} = \left[2\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right)\right]^{10}$$

$$= 2^{10}\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right)^{10} = 2^{10}\left(\cos\frac{50\pi}{3} + i\sin\frac{50\pi}{3}\right)$$

$$= 2^{10}\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right) = 2^{10}\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) = -2^9\left(1-\sqrt{3}i\right)$$

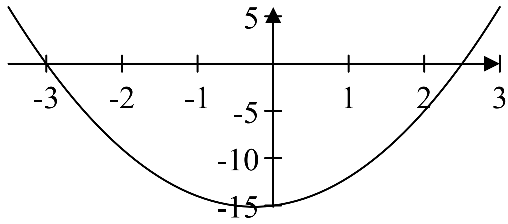
Question 2 **(25 marks)**

(a) Find the set of all real values of x for which $2x^2 + x - 15 \geq 0$.

$2x^2 + x - 15 = 0$

$\Rightarrow (2x-5)(x+3) = 0 \Rightarrow x = 2\tfrac{1}{2}$ or $x = -3$

$2x^2 + x - 15 \geq 0$ for $\{x \mid x \leq -3\} \cup \{x \mid x \geq 2\tfrac{1}{2}\}$



OR

$f(x) = 2x^2 + x - 15 = (2x-5)(x+3)$

$(2x-5)(x+3) = 0$

$\Rightarrow x = \dfrac{5}{2}$ or $x = -3$

(i): $x \geq -3$ and $x \geq \dfrac{5}{2} \Rightarrow x \geq \dfrac{5}{2}$

(ii): $x \leq -3$ and $x \leq \dfrac{5}{2} \Rightarrow x \leq -3$

Solution Set: $\{x \mid x \leq -3\} \cup \{x \mid x \geq \tfrac{5}{2}\}$

(b) Solve the simultaneous equations;

$$x + y + z = 16$$
$$\frac{5}{2}x + y + 10z = 40$$
$$2x + \frac{1}{2}y + 4z = 21.$$

$$\begin{array}{r} x + y + z = 16 \\ \tfrac{5}{2}x + y + 10z = 40 \end{array} \Rightarrow \begin{array}{r} 2x + 2y + 2z = 32 \\ 5x + 2y + 20z = 80 \\ \hline 3x \quad\quad + 18z = 48 \end{array}$$

$$\begin{array}{r} x + y + z = 16 \\ 4x + y + 8z = 42 \\ \hline 3x \quad\quad + 7z = 26 \end{array}$$

$$\begin{array}{r} 3x + 18z = 48 \\ 3x + 7z = 26 \\ \hline 11z = 22 \end{array} \quad \Rightarrow \quad z = 2$$

$3x + 7z = 26 \Rightarrow 3x + 7(2) = 26 \Rightarrow 3x = 12 \Rightarrow x = 4$

$x + y + z = 16 \Rightarrow 4 + y + 2 = 16 \Rightarrow y = 10$

Question 3 **(25 marks)**

Scientists can estimate the age of certain ancient items by measuring the proportion of carbon−14, relative to the total carbon content in the item. The formula used is $Q = e^{-\frac{0 \cdot 693t}{5730}}$, where Q is the proportion of carbon−14 remaining and t is the age, in years, of the item.

(a) An item is 2000 years old. Use the formula to find the proportion of carbon−14 in the item.

$$Q = e^{-\frac{0\cdot693t}{5730}} = e^{-\frac{0\cdot693\times2000}{5730}} = 0\cdot7851$$

(b) The proportion of carbon−14 in an item found at Lough Boora, County Offaly, was 0·3402. Estimate, correct to two significant figures, the age of the item.

$$Q = e^{-\frac{0\cdot693t}{5730}} = 0\cdot3402$$

$$\Rightarrow -\frac{0\cdot693t}{5730} = \ln 0\cdot3402$$

$$\Rightarrow t = -\frac{5730 \times \ln 0\cdot3402}{0\cdot693} \approx 8915 \approx 8900 \text{ years}$$

Question 4 **(25 marks)**

(a) Niamh has saved to buy a car. She saved an equal amount at the beginning of each month in an account that earned an annual equivalent rate (AER) of 4%.

(i) Show that the rate of interest, compounded monthly, which is equivalent to an AER of 4% is 0·327%, correct to 3 decimal places.

$$(1+i)^{12} = 1\cdot 04 \Rightarrow 1+i = \sqrt[12]{1\cdot 04} = 1\cdot 003273 \Rightarrow i = 0\cdot 003274$$

Hence, $r = 0{\cdot}327\%$

OR

$$(1.00327)^{12} = 1{\cdot}039953481$$
$$= 1{\cdot}0400$$

$$r = 4\%$$

(ii) Niamh has €15 000 in the account at the end of 36 months. How much has she saved each month, correct to the nearest euro?

$$15000 = P\left(1\cdot 00327^{36} + 1\cdot 00327^{35} + \cdots + 1\cdot 00327^{2} + 1\cdot 00327\right)$$

$$\Rightarrow P\left[\frac{1\cdot 00327\left(1\cdot 00327^{36} - 1\right)}{1\cdot 00327 - 1}\right] = 15000$$

$$\Rightarrow P[38\cdot 26326387] = 15000$$

$$\Rightarrow P = 392\cdot 02 \;=\; €392$$

OR

- Amortisation:

Step 1: Present Value

$$P = \frac{F}{(1+i)^t}$$

$$P = \frac{15000}{(1\cdot 04)^3} = 13334\cdot 95 \quad \text{OR} \quad P = \frac{15000}{(1\cdot 00327)^{36}} = 13336\cdot 73$$

Step 2:

$$A = \frac{(13334\cdot 95)(0\cdot 00327)(1\cdot 00327)^{36}}{1\cdot 00327^{36} - 1}$$

$$= €393\cdot 25$$

$$= €393$$

OR

- Present Value

$$P = \frac{F}{(1+i)^t}$$

$$P = \frac{15000}{(1\cdot 04)^3} = 13334\cdot 95$$

$$13334\cdot 95 = A\left(\frac{1}{1\cdot 00327} + \frac{1}{(1\cdot 00327)^2} + \dots\dots\dots\dots \frac{1}{(1\cdot 00327)^{36}}\right)$$

$$13334\cdot 95 = A\left[\frac{\frac{1}{1\cdot 00327}\left(1 - \left(\frac{1}{1\cdot 00327}\right)^{36}\right)}{1 - \frac{1}{1\cdot 00327}}\right]$$

$$A = €393\cdot 25$$

$$A = €393$$

(b) Conall borrowed to buy a car. He borrowed €15 000 at a monthly interest rate of 0·866%. He made 36 equal monthly payments to repay the entire loan. How much, to the nearest euro, was each of his monthly payments?

$$A = P\frac{i(1+i)^t}{(1+i)^t - 1}$$

$$= 15000\left[\frac{0\cdot 00866(1+0\cdot 00866)^{36}}{1\cdot 00866^{36} - 1}\right]$$

$$= 486\cdot 77$$

Monthly payment €487

OR

$$15000 = P\left(\frac{1}{1\cdot 00866} + \frac{1}{1\cdot 00866^{2}} + \cdots + \frac{1}{1\cdot 00866^{36}}\right)$$

$$\Rightarrow P\left[\frac{\frac{1}{1\cdot 00866}\left(1 - \frac{1}{1\cdot 00866^{36}}\right)}{1 - \frac{1}{1\cdot 00866}}\right] = 15000$$

$$\Rightarrow P[30\cdot 8151777] = 15000$$

$$\Rightarrow P = 486\cdot 77$$

Monthly payment €487

Section B	Contexts and Applications	100 marks

Answer **both** question 5 **and** question 6 from this section.

Question 5 **(50 marks)**

A stadium can hold 25 000 people. People attending a regular event at the stadium must purchase a ticket in advance. When the ticket price is €20, the expected attendance at an event is 12 000 people. The results of a survey carried out by the owners suggest that for every €1reduction, from €20, in the ticket price, the expected attendance would increase by 1000 people.

(a) If the ticket price was €18, how many people would be expected to attend?

$$12000 + (20 - 18)1000 = 14000$$

(b) Let x be the ticket price, where $x \leq 20$. Write down, in terms of x, the expected attendance at such an event.

$$12000 + (20 - x)1000 = 32000 - 1000x$$

(c) Write down a function f that gives the expected income from the sale of tickets for such an event.

$$f(x) = (32000 - 1000x)x$$

(d) Find the price at which tickets should be sold to give the maximum expected income.

$$f(x) = (32000 - 1000x)x$$

$$f'(x) = 32000 - 2000x = 0 \Rightarrow x = €16$$

(e) Find this maximum expected income.

$$f(x) = (32000 - 1000x)x$$

$$f(16) = (32000 - 16000)16 = €256\,000$$

(f) Suppose that tickets are instead priced at a value that is expected to give a full attendance at the stadium. Find the difference between the income from the sale of tickets at this price and the maximum income calculated at **(e)** above.

$32000 - 1000x = 25000 \Rightarrow 1000x = 7000 \Rightarrow x = 7$

$f(x) = (32000 - 1000x)x \Rightarrow f(7) = (32000 - 7000)7 = 175\,000$

Difference: €256 000 − €175 000 = €81 000

(g) The stadium was full for a recent special event. Two types of tickets were sold, a single ticket for €16 and a family ticket (2 adults and 2 children) for a certain amount. The income from this event was €365000. If 1000 more family tickets had been sold, the income from the event would have been reduced by €14000. How many family tickets were sold?

Single ticket: €16; Family ticket €y
Number of single tickets: p; Number of family tickets: $\frac{25000-p}{4}$

$16p + \frac{25000-p}{4}y = 365000$
$16(p-4000) + (\frac{25000-p}{4} + 1000)y = 351000 \Rightarrow 16p + \frac{29000-p}{4}y = 415000$

$\frac{29000-p}{4}y - \frac{25000-p}{4}y = 50000 \Rightarrow 4000y = 200000 \Rightarrow y = 50$

$16p + \frac{25000-p}{4}50 = 365000 \Rightarrow 7p = 105000 \Rightarrow p = 15000$

Number of family tickets: $\frac{25000-p}{4} = \frac{25000-15000}{4} = 2500$

OR

x = number of single tickets
f = number of family tickets
y = cost of family ticket

$x + 4f = 25000$
$16x + fy = 365000$
$16(x-4000) + (f+1000)y = 351000$

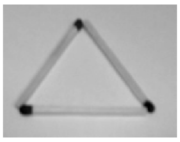
$$\begin{aligned} 16x - 64000 + fy + 1000y &= 351000 \\ 16x \quad\quad + fy \quad\quad &= 365000 \\ \hline 1000y &= 50000 \\ y &= 50 \end{aligned}$$

$$
\begin{aligned}
x + 4f &= 25000 \\
16x + 50f &= 365000 \\
16x + 64f &= 400000 \\
\hline
14f &= 35000 \\
f &= 2500
\end{aligned}
$$

Question 6 **(50 marks)**

Shapes in the form of small equilateral triangles can be made using matchsticks of equal length. These shapes can be put together into patterns. The beginning of a sequence of these patterns is shown below.

1



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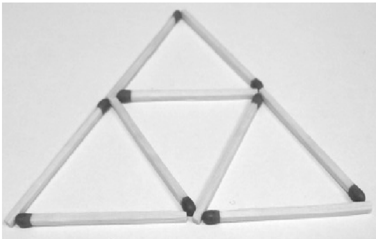

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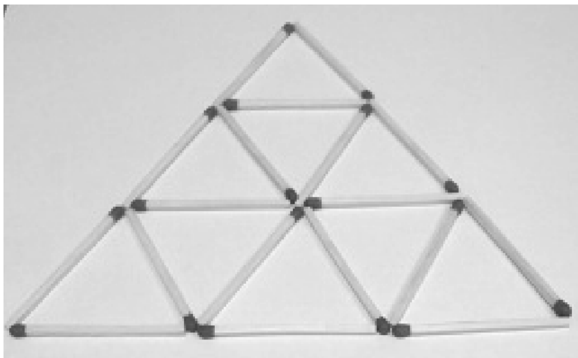
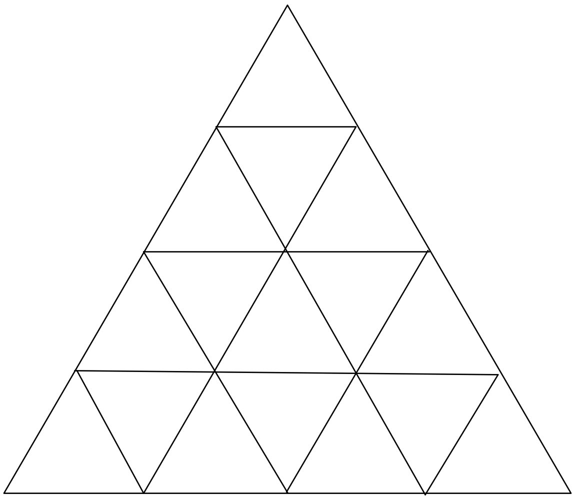


(a) **(i)** Draw the fourth pattern in the sequence.



(ii) The table below shows the number of small triangles in each pattern and the number of matchsticks needed to create each pattern. Complete the table.

Pattern	1^{st}	2^{nd}	3^{rd}	4^{th}
Number of small triangles	1	4	9	16
Number of matchsticks	3	9	18	30

(b) Write an expression in n for the number of triangles in the n^{th} pattern in the sequence.

n^2

OR

1 4 9 16 25

1st Diff: 3 5 7 9

2nd Diff: 2 2 2 $\leftarrow$ Constant $\Rightarrow$ quadratic

$T_n = an^2 + bn + c$
$2a = 2 \Rightarrow a = 1$

$T_n = n^2 + bn + c$
$T_1 = 1 + b + c = 1$
$T_2 = 4 + 2b + c = 4$
$\Rightarrow b = 0$
$c = 0$

$\Rightarrow T_n = n^2$

(c) Find an expression, in n, for the number of matchsticks needed to turn the $(n-1)^{\text{th}}$ pattern into the n^{th} pattern.

$3n$

OR

3 9 18 30

1st Diff: 6 9 12

2nd Diff: 3 3

Second difference constant $\Rightarrow$ quadratic pattern

$$T_n = an^2 + bn + c$$
$$2a = 3 \Rightarrow a = \frac{3}{2}$$

$$T_n = \frac{3}{2}n^2 + bn + c$$
$$T_1 = \frac{3}{2}(1)^2 + b(1) + c = 3 \quad \Rightarrow b + c = \frac{3}{2}$$
$$T_2 = \frac{3}{2}(2)^2 + b(2) + c = 9 \quad \Rightarrow 2b + c = 3$$

$$b + c = \frac{3}{2}$$
$$\underline{2b + c = 3}$$
$$b = \frac{3}{2}$$
$$c = 0$$

$$T_n = \frac{3}{2}n^2 + \frac{3}{2}n$$
$$T_{n-1} = \frac{3}{2}(n-1)^2 + \frac{3}{2}(n-1)$$
$$T_n - T_{n-1} = -\frac{3}{2}(n^2 - 2n + 1) - \frac{3}{2}(n-1) + \frac{3}{2}n^2 + \frac{3}{2}n$$
$$= -\frac{3}{2}n^2 + 3n - \frac{3}{2} - \frac{3}{2}n + \frac{3}{2} + \frac{3}{2}n^2 + \frac{3}{2}n$$
$$= 3n$$

(d) The number of matchsticks in the n^{th} pattern in the sequence can be represented by the function $u_n = an^2 + bn$ where $a, b \in \mathbb{Q}$ and $n \in \mathbb{N}$. Find the value of a and the value of b.

$$u_n = an^2 + bn$$
$$u_1 = a(1)^2 + b(1) = 3$$
$$u_2 = a(2)^2 + b(2) = 9$$

$$\begin{array}{c} a + b = 3 \\ 4a + 2b = 9 \end{array} \Rightarrow \begin{array}{l} 2a + 2b = 6 \\ \underline{4a + 2b = 9} \\ 2a \qquad = 3 \quad \Rightarrow \quad a = \tfrac{3}{2} \end{array}$$

$$a + b = 3 \Rightarrow \tfrac{3}{2} + b = 3 \Rightarrow b = \tfrac{3}{2}$$

OR

3, 9, 18, 30,.......
$= 3,\ 3+6,\ 3+6+9,\ 3+6+9+12,........$
$= S_1,\ S_2,\ S_3,\ S_4,.........$ of $3+6+9+12+........$

$S_n = \frac{n}{2}[2a+(n-1)d]$, where $a = d = 3$

$\Rightarrow S_n = \frac{n}{2}[6+(n-1)3]$

$\Rightarrow S_n = \frac{3n}{2}(n+1)$

Equating $an^2 + bn$ and $\frac{3n}{2}(n+1)$,

$\Rightarrow a = b = \frac{3}{2}$

(e) One of the patterns in the sequence has 4134 matchsticks. How many small triangles are in that pattern?

$u_n = \frac{3}{2}n^2 + \frac{3}{2}n = 4134$

$\Rightarrow n^2 + n - 2756$
$\Rightarrow (n+53)(n-52) = 0$
$\Rightarrow n = -53$ or $n = 52$.

$n^2 = 52^2 = 2704$ triangles

OR

From (d): $u_n = \frac{3}{2}n^2 + \frac{3}{2}n = 4134$

$3n^2 + 3n = 8268$

$n^2 + n - 2756 = 0$

$n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-1 \pm \sqrt{1-4(1)(-2756)}}{2(1)}$

$= \frac{-1 \pm \sqrt{11025}}{2}$

$= \frac{-1 \pm 105}{2}$

$= -53$ or 52

$\Rightarrow n = 52$

Number of triangles $n^2 = 52^2 = 2704$

Section C Functions and Calculus (old syllabus) 100 marks

Answer **both** Question 7 **and** Question 8 from this section.

Question 7 **(50 marks)**

(a) Differentiate $\frac{5x}{x + 4}$, with respect to x for $x \neq -4$.

$$y = \frac{5x}{x+4}$$

Let $u = 5x \Rightarrow \frac{du}{dx} = 5$

Let $v = x+4 \Rightarrow \frac{dv}{dx} = 1$

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2} = \frac{(x+4)(5)-(5x)(1)}{(x+4)^2} = \frac{5x+20-5x}{(x+4)^2} = \frac{20}{(x+4)^2}$$

OR

$$y = \frac{5x}{x+4}$$

$$y = (5x).(x+4)^{-1} = u.v$$

$u = 5x$ $\qquad$ $v = (x+4)^{-1}$

$\frac{du}{dx} = 5$ $\qquad$ $\frac{dv}{dx} = (-1)(x+4)^{-2}.(1) = \frac{-1}{(x+4)^2}$

$$\frac{dy}{dx} = u.\frac{dv}{dx} + v.\frac{du}{dx}$$

$$= (5x)\left(\frac{-1}{(x+4)^2}\right) + (x+4)^{-1}.(5)$$

$$= \frac{-5x}{(x+4)^2} + \frac{5}{x+4}$$

$$= \frac{-5x+5(x+4)}{(x+4)^2}$$

$$= \frac{-5x+5x+20}{(x+4)^2}$$

$$= \frac{20}{(x+4)^2}$$

(b) A curve is defined by the parametric equations

$$x = 1 + e^{-t}, \; y = t^2 + 2e^t.$$

(i) Show that $\frac{dy}{dx} = -2e^t\left(t + e^t\right)$.

$$x = 1 + e^{-t} \quad \Rightarrow \quad \frac{dx}{dt} = -e^{-t}$$

$$y = t^2 + 2e^t \quad \Rightarrow \quad \frac{dy}{dt} = 2t + 2e^t$$

$$\frac{dy}{dx} = \frac{dy}{dt}\frac{dt}{dx} = \frac{2t + 2e^t}{-e^{-t}} = -2e^t\left(t + e^t\right)$$

(ii) Hence, find the equation of the tangent to the curve at the point $x = 2$.

$$x = 2 \quad \Rightarrow \quad 2 = 1 + e^{-t} \quad \Rightarrow \quad e^{-t} = 1 \quad \Rightarrow \quad t = 0$$

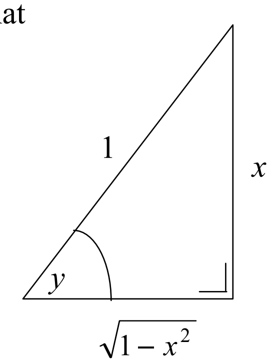
$$t = 0 \quad \Rightarrow \quad y = t^2 + 2e^t = 0 + 2e^0 = 2$$

$$\frac{dy}{dx} = -2e^t\left(t + e^t\right) = -2e^0\left(0 + e^0\right) = -2$$

$$y - y_1 = m(x - x_1) \quad \Rightarrow \quad y - 2 = -2(x - 2) \quad \Rightarrow \quad 2x + y - 6 = 0$$

(c) **(i)** Write x in terms of $\sin y$, using the diagram. Hence, show that

$$\frac{dy}{dx}=\frac{1}{\sqrt{1-x^2}}.$$



$$x=\sin y$$

$$\frac{dx}{dy}=\cos y=\frac{\sqrt{1-x^2}}{1} \quad \Rightarrow \quad \frac{dy}{dx}=\frac{1}{\sqrt{1-x^2}}$$

(ii) If $y=x+\sin^{-1}x$, prove that $\left(1-x^2\right)\frac{d^2y}{dx^2}-x\frac{dy}{dx}+x=0.$

$$y=x+\sin^{-1}x$$

$$\frac{dy}{dx}=1+\frac{1}{\sqrt{1-x^2}}=1+\left(1-x^2\right)^{-\frac{1}{2}}$$

$$\frac{d^2y}{dx^2}=0-\frac{1}{2}\left(1-x^2\right)^{-\frac{3}{2}}(-2x)=\frac{x}{\left(1-x^2\right)^{\frac{3}{2}}}$$

$$\left(1-x^2\right)\frac{d^2y}{dx^2}-x\frac{dy}{dx}+x=\left(1-x^2\right)\left(\frac{x}{\left(1-x^2\right)^{\frac{3}{2}}}\right)-x\left(1+\frac{1}{(1-x)^{\frac{1}{2}}}\right)+x$$

$$=\frac{x}{\sqrt{1-x^2}}-x-\frac{x}{\sqrt{1-x^2}}+x$$

$$=0$$

Question 8 **(50 marks)**

(a) Evaluate $\int_0^2 12e^{3x}dx$ and give your answer in the form $a(e^b - 1)$.

$$\int_0^2 12e^{3x}dx = \left[4e^{3x}\right]_0^2 = 4e^6 - 4e^0 = 4\left(e^6 - 1\right)$$

(b) The function $f(x) = x^3 + ax^2 + bx$ has turning points at $x = 2$ and $x = -\frac{4}{3}$.

(i) Find the value of a and the value of b.

$f(x) = x^3 + ax^2 + bx$

$f'(x) = 3x^2 + 2ax + b$

$f'(2) = 3(2)^2 + 2a(2) + b = 4a + b + 12 = 0 \quad \Rightarrow \quad 4a + b = -12$

$f'(-\frac{4}{3}) = 3(-\frac{4}{3})^2 + 2a(-\frac{4}{3}) + b = -\frac{8}{3}a + b + \frac{16}{3} = 0 \quad \Rightarrow \quad -8a + 3b = -16$

$8a + 2b = -24$

$-8a + 3b = -16$

$5b = -40 \quad \Rightarrow \quad b = -8$

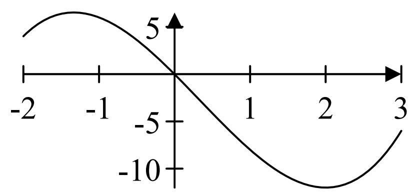
$4a + b = -12 \quad \Rightarrow \quad 4a - 8 = -12 \quad \Rightarrow \quad a = -1$

(ii) Find the co-ordinates of the turning points and hence draw a sketch of the curve $y = f(x)$.

$f(x) = x^3 - x^2 - 8x$

$f(2) = 2^3 - 2^2 - 8(2) = -12$ $(2, -12)$

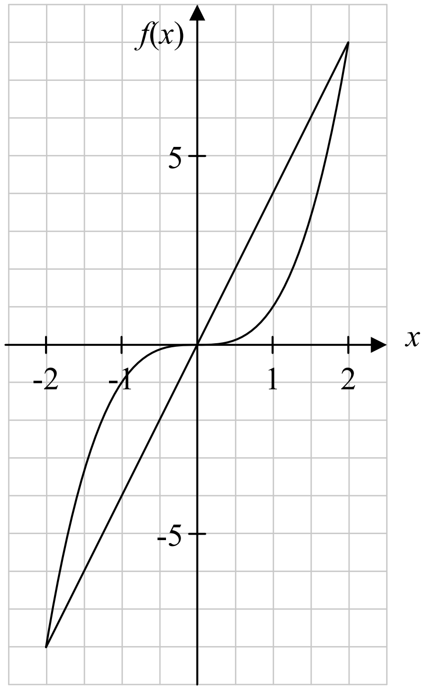
$f(-\frac{4}{3}) = (-\frac{4}{3})^3 - (-\frac{4}{3})^2 - 8(-\frac{4}{3}) = \frac{176}{27}$ .... $\left(-\frac{4}{3}, \frac{176}{27}\right)$



(c) **(i)** Draw the graphs of

$y = 4x$ and $y = x^3$

in the domain $-2 \le x \le 2$, $x \in \mathbb{R}$.



$y = 4x$

$x = -2 \Rightarrow y = -8$ $(-2, -8)$

$x = 2 \Rightarrow y = 8$ $(2, 8)$

$y = x^3$

$x = -2 \Rightarrow y = -8$ $(-2, -8)$

$x = -1 \Rightarrow y = -1$ $(-1, -1)$

$x = 0 \Rightarrow y = 0$ $(0, 0)$

$x = 1 \Rightarrow y = 1$ $(1, 1)$

$x = 2 \Rightarrow y = 8$ $(2, 8)$

(ii) Find the area of the region in the first quadrant enclosed by the two graphs.

$$\int_0^2 y\,dx = \int_0^2 x^3 dx = \left[\frac{x^4}{4}\right]_0^2 = \frac{16}{4} - 0 = 4$$

$$\int_0^2 y\,dx = \int_0^2 4x\,dx = \left[2x^2\right]_0^2 = 2(2)^2 - 0 = 8$$

Area enclosed by graphs: $8 - 4 = 4$

OR

$$A = \int_0^2 (y_1 - y_2)\,dx$$

$$= \int (4x - x^3)\,dx$$

$$= \left[2x^2 - \frac{x^4}{4}\right]_0^2$$

$$= \left[2(2)^2 - \frac{(2)^4}{4}\right] - [0 - 0]$$

$$= 8 - 4$$

$$= 4$$

(iii) Write down the total area enclosed between the two graphs and give a reason for your answer.

Total area: 8 The graphs are symmetrical in the origin.

Marking Scheme – Paper 1, Section A, Section B and Section C

Structure of the marking scheme

Candidate responses are marked according to different scales, depending on the types of response anticipated. Scales labelled A divide candidate responses into two categories (correct and incorrect). Scales labelled B divide responses into three categories (correct, partially correct, and incorrect), and so on. The scales and the marks that they generate are summarised in this table:

Scale label	B	C	D
No of categories	3	4	5
5 mark scales	0, 2, 5	0, 2, 4, 5	0, 2, 3, 4, 5
10 mark scales	0, 5, 10	0, 3, 7, 10	0, 3, 5, 8, 10
15 mark scales	0, 7, 15	0, 5, 10, 15	0, 4, 7, 11, 15

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the scheme, where necessary.

Marking scales – level descriptors

B-scales (three categories)

- response of no substantial merit
- partially correct response
- correct response

C-scales (four categories)

- response of no substantial merit
- response with some merit
- almost correct response
- correct response

D-scales (five categories)

- response of no substantial merit
- response with some merit
- response about half-right
- almost correct response
- correct response

In certain cases, typically involving incorrect rounding or omission of units, a mark that is one mark below the full-credit mark may also be awarded. Such cases are flagged with an asterisk. Thus, for example, *scale 10C** indicates that 9 marks may be awarded.

Summary of mark allocations and scales to be applied

Section A

Question 1

(a)	10C
(b)	10C
(c)	5C

Question 2

(a)	10C
(b)	15D

Question 3

(a)	15B
(b)	10C*

Question 4

(a)(i)	5C*
(a)(ii)	10D*
(b)	10C*

Section B

Question 5

(a)	10B
(b)	5B
(c)	5B
(d)	10B
(e)	5B
(f)	10C
(g)	5D

Question 6

(a)(i)	5B
(a)(ii)	5C
(b)	10B
(c)	10B
(d)	10C
(e)	10C

Section C

Question 7

(a)	15C
(b)(i)	10C
(b)(ii)	5D
(c)(i)	5C
(c)(ii)	15D

Question 8

(a)	10C
(b)(i)	10D
(b)(ii)	10C
(c)(i)	5B
(c)(ii)	10C
(c)(iii)	5B

Detailed marking notes

Section A

Question 1

(a) Scale 10C (0, 3, 7, 10)

Low Partial Credit:
- Does not multiply by conjugate
- Drops *i,* or $i^2 \neq -1$
- Incomplete cross-multiplication

High Partial Credit:
- Work not simplified

(b) Scale 10C (0, 3, 7, 10).

Low Partial Credit:
- Work with α
- Work with θ
- Work with modulus
- Plotting z

High Partial Credit:
- Correct z but incorrect or no plotting

Note: Accept r, θ and plot for full marks.

(c) Scale 5C (0, 2, 4, 5).

Low Partial Credit:
- Some work with De Moivre
- De Moivre not used correctly

High Partial Credit:
- Answer not simplified
- n included in answer

Note: Allow for full marks candidates incorrect angle from (b), with correct conclusion.

0: no use of De Moivre.

Question 2

(a) Scale 10C (0, 3, 7, 10)

Low Partial Credit:
- Factorises
- Gets roots
- Some use of quadratic root formula

High Partial Credit:
- Wrong shape of graph, but otherwise correct
- Incorrect deduction for correct values of x
- Correct shading on x-axis
- Using $x >$ only

(b) Scale 15D (0, 4, 7, 11, 15)

Low Partial Credit:
- Any relevant step to solution

Mid Partial Credit:
- Reduces to two unknowns correctly in one equation

High Partial Credit:
- Evaluates one unknown

Question 3

(a) Scale 15B (0, 7, 15)

Partial Credit:
- Value correctly substituted into e

(b) Scale 10C* (0, 3, 7, 10)

Low Partial Credit:
- Correct statement
- Uses logs (correctly or incorrectly)
- Makes an effort to isolate t

High Partial Credit:
- Gets correct linear equation for t

Note: Incomplete rounding or incorrect rounding or no rounding gets 9 marks.

Question 4

(a)(i) Scale 5C* (0, 2, 4, 5)

Low Partial Credit:
- Any relevant first step
- Correct statement with no work

High Partial Credit:
- *r* not as a %

Note: Incomplete rounding or incorrect rounding or no rounding gets 4 marks.

(a)(ii) Scale 10D* (0, 3, 5, 8, 10)

Low Partial Credit:
- Any relevant step
- Reference to 1·00327

Mid Partial Credit:
- Recognises G.P.

High Partial Credit:
- Expression for sum of G.P.

Note: Incomplete rounding or incorrect rounding or no rounding gets 9 marks.

OR

(a)(ii) Scale 10D* (0, 3, 5, 8, 10)

Low Partial Credit:
- No present value or incorrect present value

Mid Partial Credit:
- Present value correct

High Partial Credit:
- Correct substitution of all values in formula

Note: Incomplete rounding or incorrect rounding or no rounding gets 9 marks.

(b) Scale 10C* (0, 3, 7, 10)

Low Partial Credit:
- Any relevant step

High Partial Credit:
- Correct substitution of all values in formula
- Expression for sum of G.P. in solution

Note: Incomplete rounding or incorrect rounding or no rounding gets 9 marks.

Section B

Question 5

(a) Scale 10B (0, 5, 10)
Partial Credit:
- (20–18)1000 or equivalent

(b) Scale 5B (0, 2, 5)
Partial Credit:
- Expression $(20 - x)$

Note: Accept for 5 marks $12000 + (20 - x)1000$ or equivalent.

(c) Scale 5B (0, 2, 5)
Partial Credit:
- Correct number of people and/or correct rate in terms of x

(d) Scale 10B (0, 5, 10)
Partial Credit:
- Some correct differentiation of a quadratic function
- $(32000 - 2000x) = 0$ or equivalent
- Correct testing with incorrect deduction or no deduction
- Possible to get full marks without use of calculus
- Correct answer and no work

(e) Scale 5B (0, 2, 5)
Partial Credit:
- Some effort at substitution of 16 or equivalent

(f) Scale 10C (0, 3, 7, 10)
Low Partial Credit:
- Use of expression
- Use of 25000
- Some use of tables
- Equation solved
- Price of ticket found

High Partial Credit:
- Total income from sales

(g) Scale 5D (0, 2, 3, 4, 5)

Low Partial Credit:

- Correct number of family tickets
- One equation only
- Income from single tickets
- Income from family tickets

Mid Partial Credit:

- Two correct linear equations for income in two unknowns
- $y = 50$ without work, or $p = 15000$ without work

High Partial Credit:

- Correct value for p (single ticket)
- Correct value for y (family ticket)

OR

(g) Scale 5D (0, 2, 3, 4, 5)

Low Partial Credit:

- One correct equation

Mid Partial Credit:

- Three correct equations

High Partial Credit:

- One unknown calculated

Question 6

(a)(i) Scale 5B (0, 2, 5)
Partial Credit:
- Incomplete 4th line

(a)(ii) Scale 5C (0, 2, 4, 5)
Low Partial Credit:
- One or two correct entries in empty boxes

High Partial Credit:
- Three correct entries in empty boxes

(b) Scale 10B (0, 5, 10)
Partial Credit:
- 1^2, 2^2, 3^2 etc – recognising the natural numbers squared

OR

(b) Scale 10B (0, 5, 10)
Partial Credit:
- Second differences calculated

(c) Scale 10B (0, 5, 10)
Partial Credit:
- Recognition of series 6, 9, 12,.... or similar

OR

(c) Scale 10B (0, 5, 10)
Partial Credit:
- Second differences calculated

(d) Scale 10C (0, 3, 7, 10)

Low Partial Credit:

- One linear equation in a and b, e.g. u_1: $a+b=3$

High Partial Credit:

- Two correct linear equations

OR

(d) Scale 10C (0, 3, 7, 10)

Low Partial Credit:

- Recognition of A.P.

High Partial Credit:

- $a = d = 3$ and some use of S_n formula

(e) Scale 10C (0, 3, 7, 10)

Low Partial Credit:

- Expression of u_n in one variable only
- Quadratic equation

High Partial Credit:

- Values of n

Section C

Question 7

(a) Scale 15C (0, 5, 10, 15)

Low Partial Credit:
- Either $\frac{du}{dx}$ or $\frac{dv}{dx}$ correct

High Partial Credit:
- Correct formula and full differentiation

(b)(i) Scale 10C (0, 3, 7, 10)

Low Partial Credit:
- Either $\frac{dx}{dt}$ or $\frac{dy}{dt}$ correct

High Partial Credit:
- $\frac{dy}{dx}$ unsimplified

(b)(ii) Scale 5D (0, 2, 3, 4, 5)

Low Partial Credit:
- Value of t
- Equation of tangent only with no substitution

Mid Partial Credit:
- Value of y
- Value of slope

High Partial Credit:
- All values necessary for substitution

(c)(i) Scale 5C (0, 2, 4, 5)

Low Partial Credit:
- Correct value of $\sin y$
- Correct value of $\cos y$
- Getting $\sqrt{1-x^2}$

High Partial Credit:
- Correct differentiation of correct function

(c)(ii) Scale 15D (0, 4, 7, 11, 15)

Low Partial Credit:

- Some correct differentiation

Mid Partial Credit:

- $\frac{dy}{dx}$ correct

High Partial Credit:

- $\frac{d^2y}{dx^2}$ correct

Question 8

(a) Scale 10C (0, 3, 7, 10)

Low Partial Credit:
- Some correct integration
- Integrand does not contain e^{3x}

High Partial Credit:
- Correct integration

(b)(i) Scale 10D (0, 3, 5, 8, 10)

Low Partial Credit:
- Some differentiation correct
- Any relevant step, e.g. $f'(x) = 0$ at turning point

Mid Partial Credit:
- One correct expression in *a* and *b*

High Partial Credit:
- Two correct equations in *a* and *b*

(b)(ii) Scale 10C (0, 3, 7, 10)

Low Partial Credit:
- $f(2)$ attempted
- $f(-\frac{4}{3})$ attempted
- (0, 0) shown on graph or calculated
- Correct $f(x)$
- Curve of correct shape

High Partial Credit:
- No sketch but turning points calculated correctly

(c)(i) Scale 5B (0, 2, 5)

Partial Credit:
- One correct graph, e.g. $y = 4x$ correct but $y = x^3$ incorrect, or vice versa
- Some values calculated

(c)(ii) Scale 10C (0, 3, 7, 10)

Low Partial Credit:
- Points of intersection calculated or indicated (from Tables)
- Correct limits
- Area under one curve only found
- Correct area indicated

High Partial Credit:
- Area under each curve calculated correctly, but enclosed area not found

(c)(iii) Scale 5B (0, 2, 5)

Partial Credit:
- Total area correct, but no reason given

Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate Examination, 2013

Mathematics (Project Maths – Phase 2)

Paper 2

Higher Level

Monday 10 June Morning 9:30 – 12:00

300 marks

Model Solutions – Paper 2

Note: The model solutions for each question are not intended to be exhaustive – there may be other correct solutions. Any examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his / her advising examiner.

Instructions

There are **two** sections in this examination paper.

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	3 questions

Answer all nine questions, as follows:

In Section A, answer:

Questions 1 to 5 and

either Question 6A **or** Question 6B.

In Section B, answer Questions 7 to 9.

Write your answers in the spaces provided in this booklet. You will lose marks if you do not do so. There is space for extra work at the back of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

Marks will be lost if all necessary work is not clearly shown.

Answers should include the appropriate units of measurement, where relevant.

Answers should be given in simplest form, where relevant.

Write the make and model of your calculator(s) here:

Section A	Concepts and Skills	150 marks

Answer all six questions from this section.

Question 1 **(25 marks)**

(a) Explain each of the following terms:

(i) Sample space

> The set of all possible outcomes of an experiment.

(ii) Mutually exclusive events

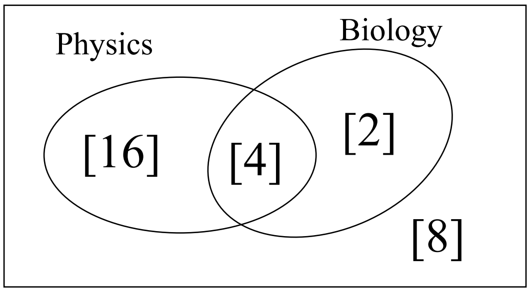
> Events E and F are mutually exclusive if they have no outcomes in common.
> i.e $P(E \cup F) = P(E)+P(F)$

(iii) Independent events

> Two events are independent if the outcome of one does not depend on the outcome of the other.
> i.e $P(E \cap F) = P(E).P(F)$ or $P(E|F) = P(E)$ or $P(F|E) = P(F)$

(b) In a class of 30 students, 20 study Physics, 6 study Biology and 4 study both Physics and Biology.

(i) Represent the information on the Venn Diagram.



A student is selected at random from this class.
The events E and F are:

E: The student studies Physics
F: The student studies Biology.

(ii) By calculating probabilities, investigate if the events E and F are independent.

> $P(E \cap F) = \frac{4}{30}$
> $P(E) \times P(F) = \frac{20}{30} \times \frac{6}{30} = \frac{4}{30}$
> $P(E \cap F) = P(E) \times P(F) \quad \Rightarrow \quad$ E and F are independent events

Question 2 **(25 marks)**

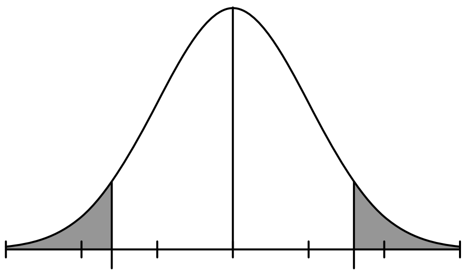
(a) A random variable X follows a normal distribution with mean 60 and standard deviation 5.

(i) Find $P(X \leq 68)$.

$$P(X \leq 68) = P\left(Z \leq \frac{68-60}{5}\right) = P(Z \leq 1 \cdot 6) = 0 \cdot 9452$$

(ii) Find $P(52 \leq X \leq 68)$.

$$P(52 \leq X \leq 68) = P\left(\frac{52-60}{5} \leq Z \leq \frac{68-60}{5}\right)$$
$$= P(-1 \cdot 6 \leq Z \leq 1 \cdot 6)$$



$$P(Z \leq -1 \cdot 6) = P(Z \geq 1 \cdot 6)$$
$$= 1 - P(Z \leq 1 \cdot 6)$$
$$= 1 - 0 \cdot 9452 = 0 \cdot 0548$$

$$P(-1 \cdot 6 \leq Z \leq 1 \cdot 6) = P(Z \leq 1 \cdot 6) - P(Z \leq -1 \cdot 6)$$
$$= 0 \cdot 9452 - 0 \cdot 0548 = 0 \cdot 8904$$

OR

$$P(52 \leq X \leq 68) = P\left(\frac{52-60}{5} \leq Z \leq \frac{68-60}{5}\right)$$
$$= P(-1 \cdot 6 \leq Z \leq 1 \cdot 6)$$
$$= 1 - 2P(Z \geq 1 \cdot 6)$$
$$= 1 - 2(1 - P(Z \leq 1 \cdot 6))$$
$$= 1 - 2(1 - 0 \cdot 9452) = 1 - 2(0 \cdot 0548) = 1 - 0 \cdot 1096 = 0 \cdot 8904$$

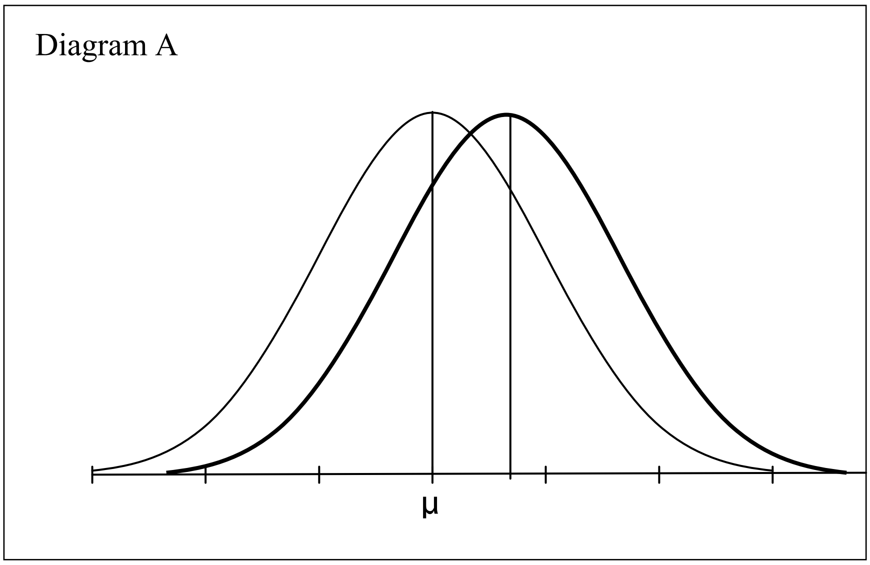
(b) The heights of a certain type of plant, when ready to harvest, are known to be normally distributed, with a mean of μ. A company tests the effects of three different types of growth hormone on this type of plant. The three hormones were used on different large samples of the crop. After applying each hormone, it was found that the heights of the plants in the samples were still normally distributed at harvest time.

The diagrams A, B and C show the expected distribution of the heights of the plants, at harvest time, without the use of the hormones.

The effect, on plant growth, of each of the hormones is described. Sketch, on each diagram, a new distribution to show the effect of the hormone.

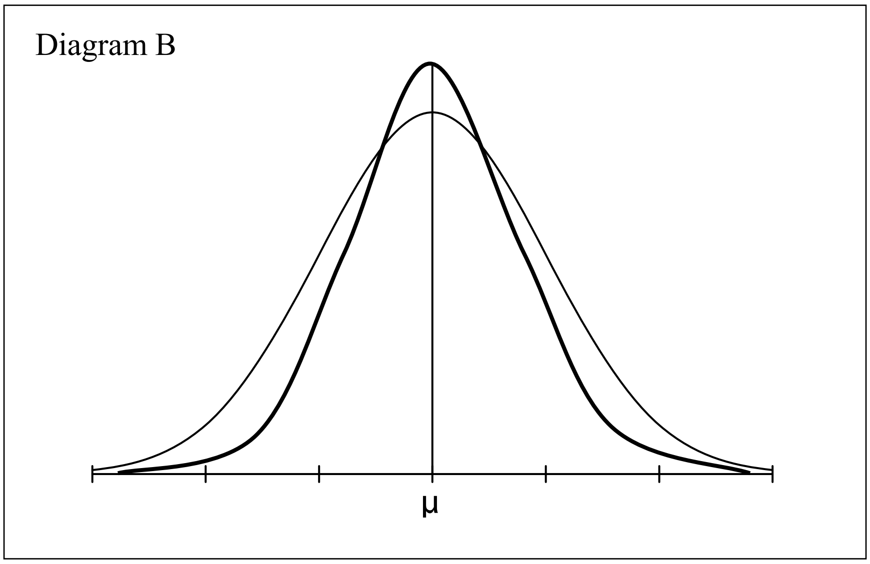
Hormone A

The effect of hormone A was to increase the height of all of the plants.



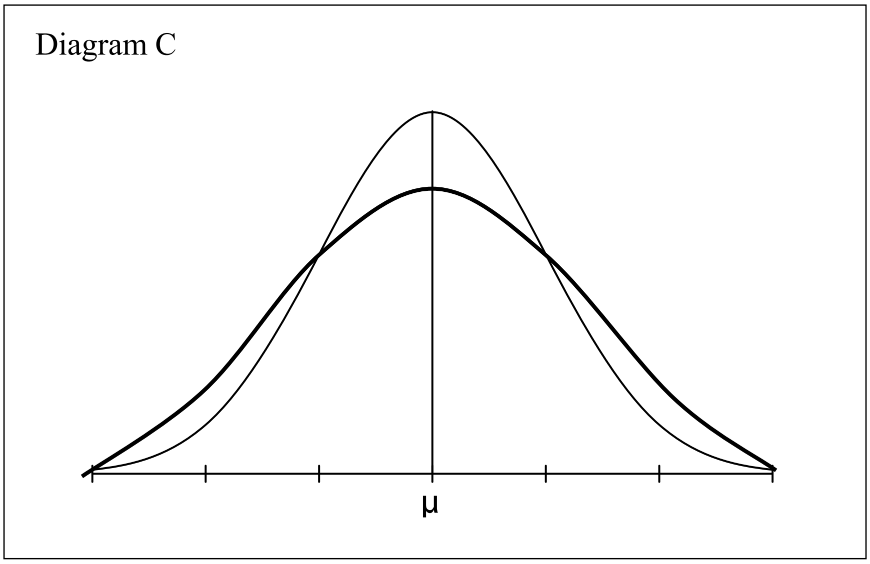
Hormone B

The effect of hormone B was to reduce the number of really small plants and the number of really tall plants. The mean was unchanged.



Hormone C

The effect of hormone C was to increase the number of small plants and the number of tall plants. The mean was unchanged.



Question 3 **(25 marks)**

The equations of six lines are given:

Line	Equation
h	$x = 3 - y$
i	$2x - 4y = 3$
k	$y = -\frac{1}{4}(2x - 7)$
l	$4x - 2y - 5 = 0$
m	$x + \sqrt{3}y - 10 = 0$
n	$\sqrt{3}x + y - 10 = 0$

(a) Complete the table below by matching each description given to one or more of the lines.

Description	Line(s)
A line with a slope of 2.	l
A line which intersects the y-axis at $(0, -2\frac{1}{2})$.	l
A line which makes equal intercepts on the axes.	h
A line which makes an angle of $150°$ with the positive sense of the x-axis.	m
Two lines which are perpendicular to each other.	l and k

(b) Find the acute angle between the lines m and n.

Slope of m: $m_1 = -\frac{1}{\sqrt{3}}$

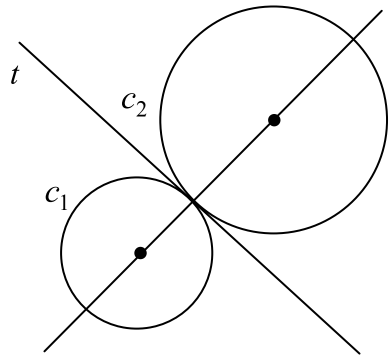
Slope of n: $m_2 = -\sqrt{3}$

$$\tan\theta = \pm\frac{m_1 - m_2}{1 + m_1 m_2} = \pm\frac{-\frac{1}{\sqrt{3}} + \sqrt{3}}{1 - \frac{1}{\sqrt{3}}\left(-\sqrt{3}\right)} = \pm\frac{\frac{-1+3}{\sqrt{3}}}{1+1} = \pm\frac{1}{\sqrt{3}}$$

$$\tan\theta = \frac{1}{\sqrt{3}} \quad \Rightarrow \quad \theta = 30°$$

Question 4 **(25 marks)**

The circles c_1 and c_2 touch externally as shown.



$$\sqrt{g^2 + f^2 - c} = \sqrt{1+1+7} = 3$$

(a) Complete the following table:

Circle	Centre	Radius	Equation
c_1	$(-3, -2)$	2	$(x+3)^2 + (y+2)^2 = 4$ **OR** $x^2 + y^2 + 6x + 4y + 9 = 0$
c_2	$(1, 1)$	3	$x^2 + y^2 - 2x - 2y - 7 = 0$

(b) **(i)** Find the co-ordinates of the point of contact of c_1 and c_2.

Divide line segment joining $(-3, -2)$ and $(1, 1)$ in ratio 2 : 3

$$\left(\frac{2(1)+3(-3)}{2+3}, \frac{2(1)+3(-2)}{2+3}\right) = \left(-\frac{7}{5}, -\frac{4}{5}\right)$$

OR

Slope line of centres = $\frac{3}{4}$.

Equation line of centres: $y - 1 = \frac{3}{4}(x-1) \Rightarrow 3x - 4y + 1 = 0$

$c_1 - c_2 = 4x + 3y + 8 = 0$

$4x + 3y + 8 = 0 \cap 3x - 4y + 1 = 0 \Rightarrow x = -\frac{7}{5}, y = -\frac{4}{5}$

(ii) Hence, or otherwise, find the equation of the tangent, *t*, common to c_1 and c_2.

Slope of line of centres: $\dfrac{1+2}{1+3} = \dfrac{3}{4}$

Slope of tangent: $m = -\dfrac{4}{3}$

Equation of tangent: $y + \frac{4}{5} = -\frac{4}{3}\left(x + \frac{7}{5}\right)$

$\Rightarrow \quad 3y + \frac{12}{5} = -4x - \frac{28}{5}$

$\Rightarrow \quad 4x + 3y + 8 = 0$

OR

$c_1 - c_2 = x^2 + y^2 + 6x + 4y + 9 - (x^2 + y^2 - 2x - 2y - 7) = 0$

$\Rightarrow 6x + 4y + 9 - (-2x - 2y - 7) = 0$

$\Rightarrow 8x + 6y + 16 = 0 \Rightarrow 4x + 3y + 8 = 0$

OR

$xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$

$x(-\dfrac{7}{5}) + y(-\dfrac{4}{5}) + 3(x + (-\dfrac{7}{5})) + 2(y + (-\dfrac{4}{5}) + 9 = 0$

$\Rightarrow 4x + 3y + 8 = 0$

Question 5 **(25 marks)**

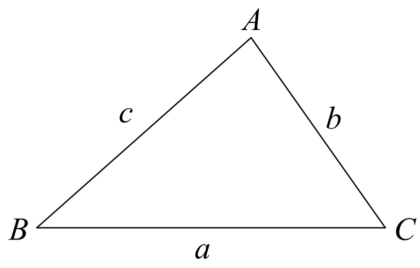
(a) In a triangle *ABC,* the lengths of the sides are *a*, *b* and *c*.
Using a formula for the area of a triangle, or otherwise,
prove that

$$\frac{a}{\sin\angle A}=\frac{b}{\sin\angle B}=\frac{c}{\sin\angle C}.$$

$$\frac{1}{2}ac\sin\angle B=\frac{1}{2}ab\sin\angle C$$

Divide by $\frac{1}{2}abc$

$$\frac{\sin\angle B}{b}=\frac{\sin\angle C}{c}\Rightarrow\frac{b}{\sin\angle B}=\frac{c}{\sin\angle C}$$



OR

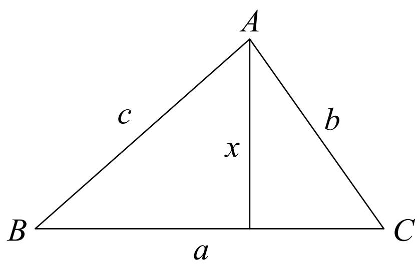
Case 1

$\sin\angle B=\frac{x}{c}$ $\quad$ $\sin\angle C=\frac{x}{b}$

$x=c\sin\angle B$ $\quad$ $x=b\sin\angle C$

$b\sin\angle C=c\sin\angle B$

$$\frac{b}{\sin\angle B}=\frac{c}{\sin\angle C}$$



Case 2

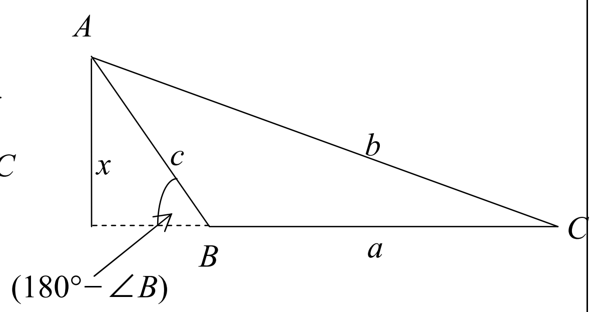
$\sin(180^\circ-\angle B)=\frac{x}{c}$ $\quad$ $\sin\angle C=\frac{x}{b}$

$x=c\sin(180^\circ-\angle B)$ $\quad$ $x=b\sin\angle C$

$x=c\sin\angle B$

$b\sin\angle C=c\sin\angle B$

$$\frac{b}{\sin\angle B}=\frac{c}{\sin\angle C}$$



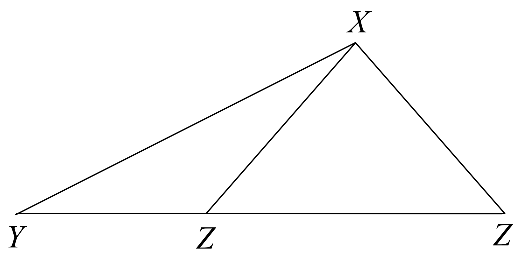
(b) In a triangle *XYZ*, $|XY|=5$ cm, $|XZ|=3$ cm and $|\angle XYZ|=27°$.

(i) Find the two possible values of $|\angle XZY|$. Give your answers correct to the nearest degree.

$$\frac{3}{\sin 27°}=\frac{5}{\sin\angle Z}\quad\Rightarrow\quad\sin\angle Z=\frac{5\sin 27°}{3}=0\cdot756$$

$$\Rightarrow\quad|\angle Z|=49°\quad\text{or}\quad|\angle Z|=131°$$

(ii) Draw a sketch of the triangle *XYZ*, showing the two possible positions of the point *Z*.



(c) In the case that $|\angle XZY| < 90^\circ$, write down $|\angle ZXY|$, and hence find the area of the triangle *XYZ*, correct to the nearest integer.

$$|\angle ZXY| = 180^\circ - (27^\circ + 49^\circ) = 104^\circ$$

$$\Delta = \tfrac{1}{2}ab\sin C = \tfrac{1}{2}(5)(3)\sin 104^\circ = 7 \cdot 27 = 7\text{ cm}^2$$

Question 6 **(25 marks)**

Answer **either** 6A **or** 6B.

Question 6A

(a) Complete each of the following statements.

(i) The circumcentre of a triangle is the point of intersection of

the perpendicular bisectors of the sides of the triangle

(ii) The incentre of a triangle is the point of intersection of

the bisectors of the angles of the triangle

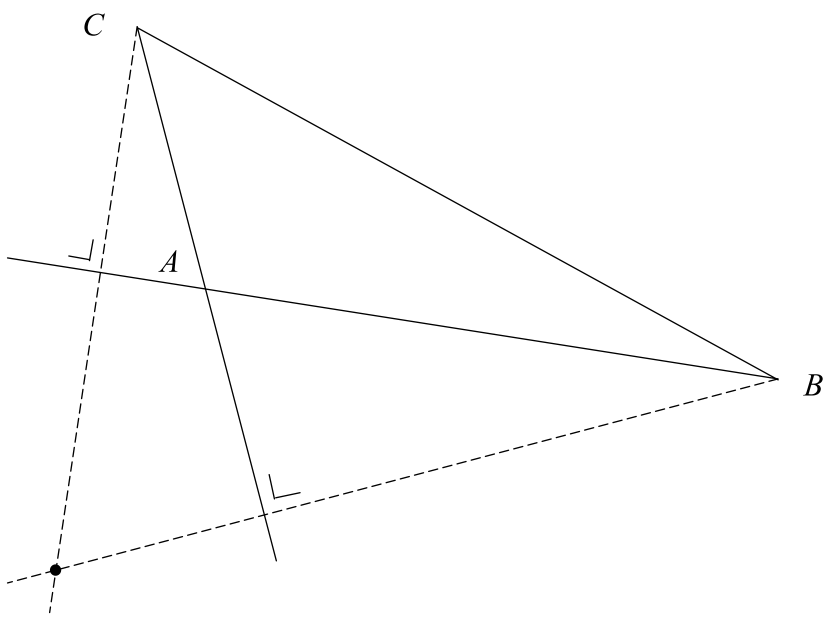
(iii) The centroid is the point of intersection of

the medians of the triangle

(b) In an equilateral triangle, the circumcentre, the incentre and the centroid are all in the same place. Explain why this is the case.

In an equilateral triangle the medians are perpendicular to the opposite sides and bisect the angles. Therefore, the perpendicular bisectors of the sides, the bisectors of the angles and the median are the same line and intersect at one point.

(c) Construct the orthocentre of the triangle *ABC* below. Show all construction lines clearly.



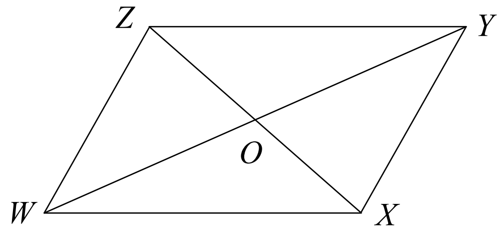
OR

Question 6B

(a) A quadrilateral (four sided figure) has two sides which are parallel and equal in length. Prove that the quadrilateral is a parallelogram.

In the quadrilateral $WXYZ$, $WX \parallel ZY$ and $|WX| = |ZY|$

To Prove: $WXYZ$ is a parallelogram.



Join Z to X and Y to W

Proof:

In ΔZOY and ΔOWX ,

$|ZY| = |WX|$

$|\angle ZYO| = |\angle OWX| \ldots ZY \parallel WX$

$|\angle YZO| = |\angle OXW| \ldots ZY \parallel WX$

Hence, ΔZOY and ΔOWX are congruent since AAS

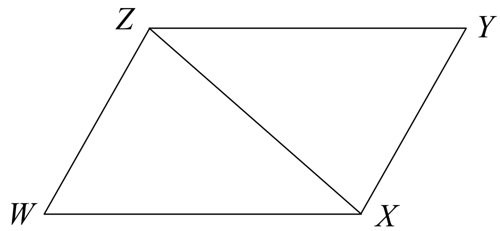
Hence, $|ZO| = |OX|$ and $|YO| = |OW|$

Hence, the diagonals of $WXYZ$ bisect each other $\Rightarrow$ $WXYZ$ is a parallelogram.

OR

In the quadrilateral $WXYZ$, $WX \parallel ZY$ and $|WX| = |ZY|$

To Prove: $WXYZ$ is a parallelogram.



Join Z to X

Proof:

In ΔWXZ and ΔYZX ,

|WX| = |ZY|

$|\angle YZX| = |\angle WXZ| \ldots ZY \parallel WX$

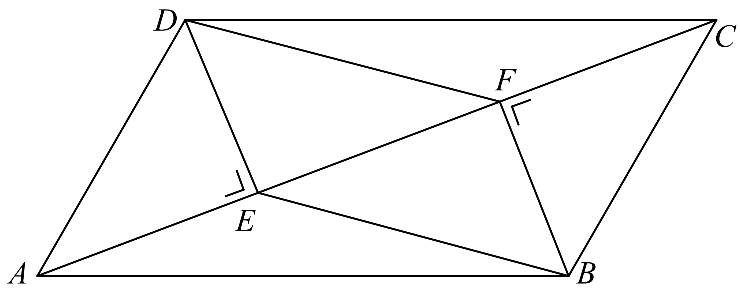
$|ZX| = |ZX| \ldots$ common to both

Hence, ΔWXZ and ΔZXY are congruent since SAS

$\Rightarrow WZ$ and XY parallel

$\Rightarrow$ $WXYZ$ is a parallelogram.

(b) In the parallelogram *ABCD,*
DE is perpendicular to *AC.*
BF is perpendicular to *AC*.
Prove that *EBFD* is a parallelogram.



In the parallelogram *ABCD*,
$DE \perp AC$ and $AC \perp BF \Rightarrow DE \parallel BF$.

In the parallelogram *ABCD*,
area of ΔDAC = area of $\Delta ABC \Rightarrow |DE| = |BF|$.

$DE \parallel BF$ and $|DE| = |BF| \Rightarrow$ *EBFD* is a parallelogram.

Section B Contexts and Applications 150 marks

Answer **all three** questions from this section.

Question 7 **(75 marks)**

Go Fast Airlines provides internal flights in Ireland, short haul flights to Europe and long haul flights to America and Asia. On long haul flights the company sells economy class, business class and executive class tickets. All passengers have a baggage allowance of 20 kg and must pay a cost per kg for any weight over the 20 kg allowance.
Each month the company carries out a survey among 1000 passengers. Some of the results of the survey for May are shown below.

Gender	Male: 479	Female: 521

Previously flown with *Go Fast Airlines*	Yes: 682	No: 318
Would fly again with *Go Fast Airlines*	Yes: 913	No: 87

Passenger Age	Mean age: 42 Median age: 31

Spend on in-flight facilities	Mean spend: €18·65 Median spend: €32·18

Was flight delayed	Yes	No	Don't Know
	231	748	21

Passenger satisfaction with overall service	Satisfied	Not satisfied	Don't Know
	664	238	98

(a) *Go Fast Airlines* used a **stratified random sample** to conduct the survey.

(i) Explain what is meant by a **stratified random sample**.

> The population is divided into different subgroups which have common characteristics. Random samples are drawn from each subgroup according to their proportion of the population.

(ii) Write down 4 different passenger groups that the company might have included in their sample.

One solution:

Long haul economy class passengers.
Long haul business class passengers.
Long haul executive class passengers.
Short haul passengers

(b) **(i)** What is the probability that a passenger selected at random from this sample

- had their flight delayed

$$\frac{231}{1000} = 0\cdot 231 \text{ or } \frac{231}{979} = 0\cdot 236$$

- Was not satisfied with the overall service

$$\frac{238}{1000} = 0\cdot 238 \text{ or } \frac{238}{902} = 0\cdot 264$$

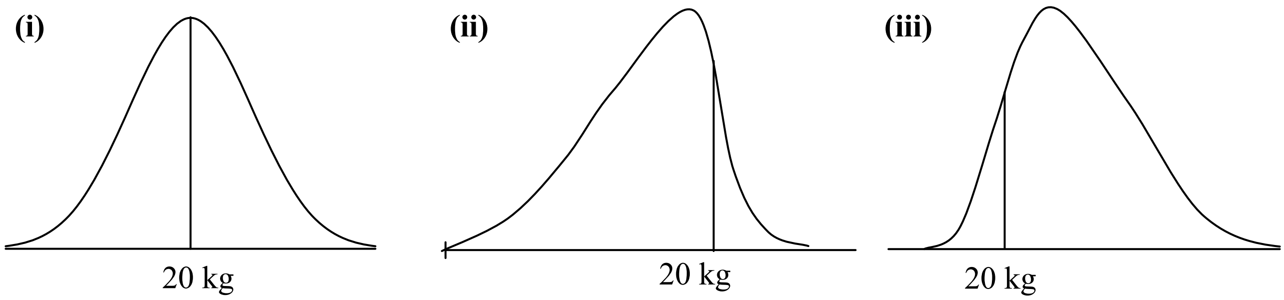
(ii) An employee suggests that the probability of selecting a passenger whose flight was delayed and who was not satisfied with the overall service should be equal to the product of the two probabilities in **(i)** above. Do you agree with the employee?

Answer: No.

Reason:

If it was this would imply that the events were independent but this is not likely since a passenger who had his flight delayed is likely to be not satisfied with the service.

(c) Which of the graphs below do you think is most likely to represent the distribution of the weights of passenger baggage?



Answer: Graph (ii)

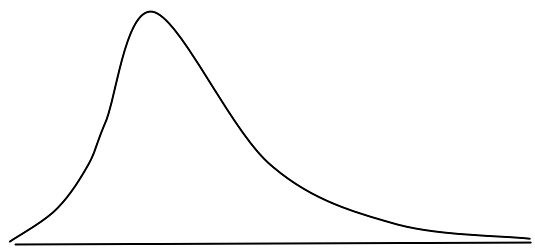
Reason:

A lot of the passengers are likely to have baggage with a weight of less than the maximum 20 kg.

(d) **(i)** Draw a sketch of the possible distribution of the ages of the passengers based on the data in the survey.

(ii) Explain your answer.

The median is less than the mean so the graph is skewed to the right.



(e) **(i)** The company repeatedly asserts that 70% of their customers are satisfied with their overall service Use an hypothesis test at the 5% level of significance to decide whether there is sufficient evidence to conclude that their claim is valid in May. State the null hypothesis and state your conclusion clearly.

Null Hypothesis: The satisfaction level is unchanged. $p = 0 \cdot 7$
The 95% margin of error for a sample of size 1000 is $\frac{1}{\sqrt{1000}} = 0 \cdot 0316$.
The recorded satisfaction level for May is $0 \cdot 664$.
This is outside the range $[0 \cdot 7 - 0 \cdot 0316, 0 \cdot 7 + 0 \cdot 0316] = [0 \cdot 7316, 0 \cdot 6684]$.
Reject the null hypothesis.
There is evidence to conclude that the company claim is not valid in May.

OR

Null Hypothesis: The satisfaction level is unchanged. $p = 0 \cdot 7$
The 95% margin of error for a sample of size 1000 is $\frac{1}{\sqrt{1000}} = 0 \cdot 0316$.
The 95% confidence interval for the population proportion is

$$0 \cdot 664 - 0 \cdot 0316 < p < 0 \cdot 664 + 0 \cdot 0316 = 0 \cdot 6324 < p < 0 \cdot 6956$$

$p = 0 \cdot 7$ is outside this range.
Reject the null hypothesis.
There is evidence to conclude that the company claim is not valid in May.

(ii) A manager of the airline says: "If we survey 2000 passengers from June on, we will half the margin of error in our surveys." Is the manager correct?

Answer: No.

Reason:
For a sample of size n, the margin of error is

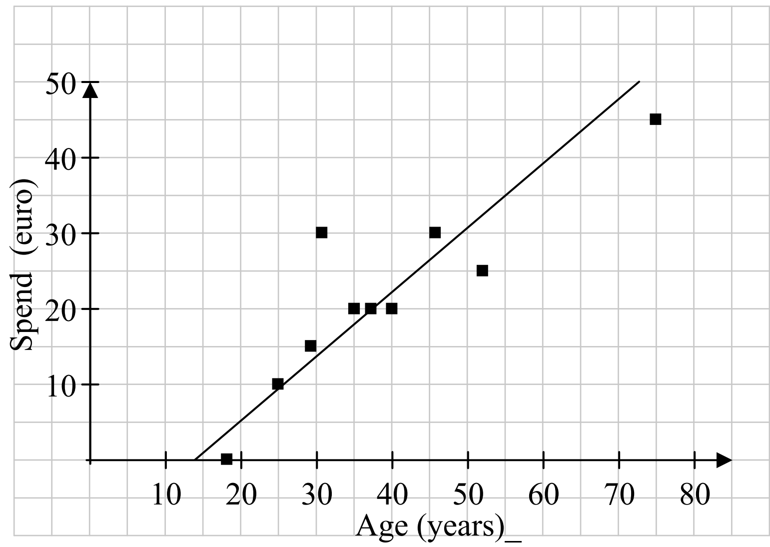
$$\frac{1}{\sqrt{n}}. \quad \frac{1}{2}\frac{1}{\sqrt{1000}} \neq \frac{1}{\sqrt{2000}}$$

or $0{\cdot}0158 \neq \cdot 022$

(f) The responses of ten individual passengers to the questions on age and in-flight spend are given below.

Age (years)	46	29	37	18	25	75	52	35	40	31
In-flight spend (euro)	30	15	20	0	10	45	25	20	20	30

(i) Draw a scatter plot of the data.



(ii) Calculate the correlation coefficient between passenger age and in-flight spend.

$0 \cdot 88$

(iii) What can you conclude from the completed scatter plot and the correlation coefficient?

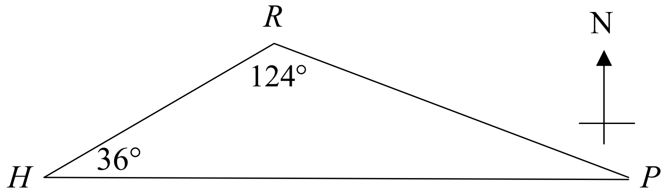
Older passengers tend to spend more.

(iv) Sketch the line of best fit in the completed scatter plot above.

Question 8 **(30 marks)**

Errors in this question mean that different valid approaches can result in different values for certain quantities. This will have caused difficulty for some candidates. See marking notes.

(a) A port P is directly East of a port H. To sail from H to P, a ship first sails 80 km, in the direction shown in the diagram, to the point R before turning through an angle of $124°$ and sailing 110 km directly to P.



(i) Find the distance from R to HP.

Some possible solutions:

$$\sin 36° = \frac{d}{80} \quad \Rightarrow \quad d = 80\sin 36° = 47\cdot 02\text{ km}$$

OR

$$\sin 20° = \frac{d}{110} \quad \Rightarrow \quad d = 110\sin 20° = 37\cdot 62\text{ km}$$

(ii) Calculate $|HP|$.

Some possible solutions:

$$|HP|^2 = 80^2 + 110^2 - 2(80)(110)\cos 124°$$
$$= 6400 + 12100 + 9841\cdot 79 = 28341\cdot 79$$
$$|HP| = 168\cdot 35\text{ km}$$

OR

$$\frac{|HP|}{\sin 124^0} = \frac{110}{\sin 36^0} \Rightarrow |HP| = 155\cdot 148\text{ km}$$

OR

$$\frac{|HP|}{\sin 124°} = \frac{80}{\sin 20°} \Rightarrow |HP| = 193\cdot 915\text{ km}$$

OR

$$|HP| = 80\cos 36° + 110\cos 20° = 64\cdot 72 + 103\cdot 366 = 168\cdot 087\text{ km}$$

OR

$$\frac{1}{2}|HP|(47\cdot 02)=\frac{1}{2}(80)(110)\sin 124^\circ$$
$$\Rightarrow\quad |HP|=\frac{(80)(110)(0\cdot 8290)}{47\cdot 02}=155\cdot 15\quad \text{km}$$

OR

$$|HP|=\frac{47\cdot 02}{\tan 36^\circ}+\frac{47\cdot 02}{\tan 20^\circ}=64\cdot 721+129\cdot 186=193\cdot 9\quad\text{km}$$

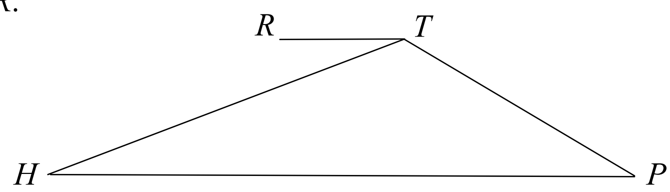
OR

$$|HP|=47\cdot 02\tan 54^\circ+47\cdot 02\tan 70^\circ=193\cdot 9\quad\text{km}$$

(b) The point T is directly East of the point R.

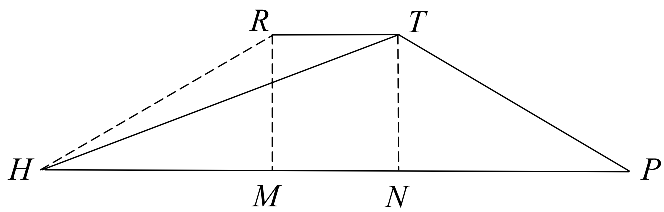
$|HT|=110$ km and $|TP|=80$ km.

Find $|RT|$.



Some possible solutions:

$$\cos 36^\circ=\frac{|HM|}{80}$$
$$\Rightarrow\ |HM|=80\cos 36^\circ$$
$$=64\cdot 72\ \text{km}$$



$|NP|=64\cdot 72$

$|RT|=|MN|=168-2(64\cdot 72)=38\cdot 56\,\text{km}$

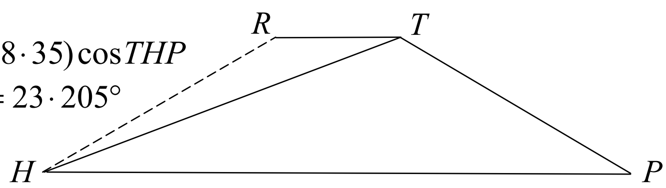
OR

Taking ΔHTP

$80^2=110^2+168\cdot 35^2-2(110)(168\cdot 35)\cos THP$

$\Rightarrow \cos\angle THP=\cdot 9191\Rightarrow \angle THP=23\cdot 205^\circ$

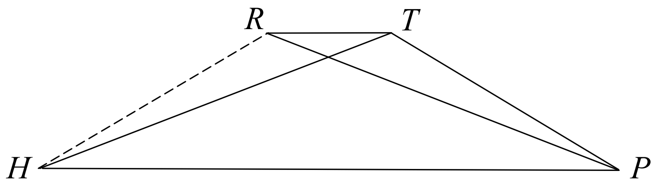
$\Rightarrow \angle RHT=12\cdot 795^\circ$



Taking ΔHRT

$$|RT|^2=110^2+80^2-2(110)(80)\cos 12\cdot 795^\circ$$
$$|RT|^2=1337\cdot 0\Rightarrow |RT|=36\cdot 56\quad\text{km}$$

OR



$|\angle RPH| = |\angle THP| = 20^0$

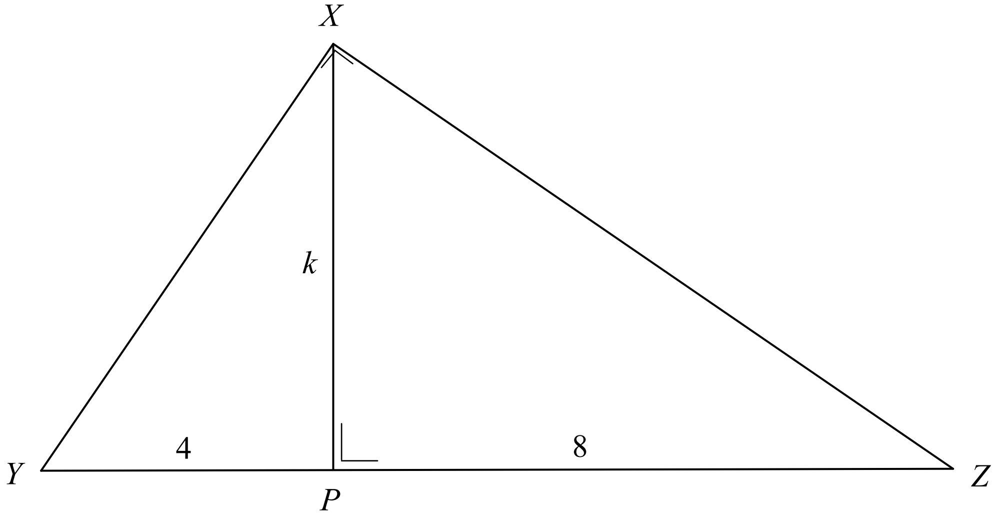
$\Rightarrow |\angle RHT| = 16^0$

$|RT|^2 = 110^2 + 80^2 - 2(110)(80)\cos 16°$

$|RT|^2 = 1581\cdot 824 \Rightarrow |RT| = 39\cdot 77$ km

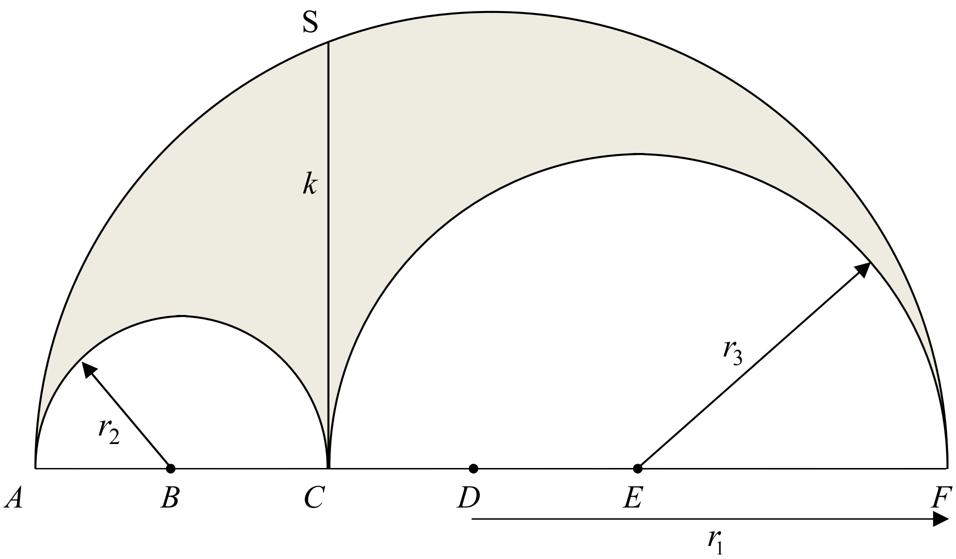
Question 9 **(45 marks)**

(a) The triangle XYZ is right-angled at X and XP is perpendicular to YZ.
$|YP| = 4$, $|PZ| = 8$ and $|PX| = k$. Find the value of k.



$$|XY|^2 = 4^2 + k^2 \Rightarrow k^2 = |XY|^2 - 16$$
$$|XZ|^2 = 8^2 + k^2 \Rightarrow k^2 = |XZ|^2 - 64$$
$$2k^2 = \left(|XY|^2 + |XZ|^2\right) - 80 = 144 - 80 = 64$$
$$\Rightarrow \quad k = \sqrt{32} = 4\sqrt{2}$$

(b) The shaded region in the diagram below is called an **arbelos.** It is a plane semicircular region of radius r_1 from which semicircles of radius r_2 and r_3 are removed, as shown. In the diagram $SC \perp AF$ and $|SC| = k$.



(i) Show that, for fixed r_1, the perimeter of the arbelos is independent of the values of r_2 and r_3.

Perimeter $= \pi r_1 + (\pi r_2 + \pi r_3) = \pi(r_1 + (r_2 + r_3)) = \pi(r_1 + r_1) = 2\pi r_1$

which is independent of r_2 and r_3

(ii) If $r_2 = 2$ and $r_3 = 4$, show that the area of the arbelos is the same as the area of the circle of diameter *k*.

$$\begin{aligned}\text{Area of arbelos} &= \tfrac{1}{2}\pi r_1^2 - \tfrac{1}{2}\pi(r_2^2 + r_3^2)\\ &= \tfrac{1}{2}\pi(6^2) - \tfrac{1}{2}\pi(2^2 + 4^2)\\ &= \tfrac{1}{2}\pi(36 - 20)\\ &= 8\pi\end{aligned}$$

$k^2 + 4 = 36$

$k = \sqrt{32}$

$$\text{Area of circle} = \pi\left(\frac{k}{2}\right)^2 = \pi\left(\frac{\sqrt{32}}{2}\right)^2 = \frac{\pi(\sqrt{32})^2}{4} = 8\pi$$

(c) To investigate the area of an arbelos, a student fixed the value of r_1 at 6 cm and completed the following table for different values of r_2 and r_3.

(i) Complete the table.

r_1	r_2	r_3	Area of arbelos
6	1	5	$\frac{1}{2}\pi\left(6^2-(1^2+5^2)\right)=5\pi \text{ cm}^2$
6	2	4	$\frac{1}{2}\pi\left(6^2-(2^2+4^2)\right)=8\pi \text{ cm}^2$
6	3	3	$\frac{1}{2}\pi\left(6^2-(3^2+3^2)\right)=9\pi \text{ cm}^2$
6	4	2	$\frac{1}{2}\pi\left(6^2-(4^2+1^2)\right)=8\pi \text{ cm}^2$
6	5	1	$\frac{1}{2}\pi\left(6^2-(5^2+1^2)\right)=5\pi \text{ cm}^2$

(ii) In general for $r_1 = 6$ cm and $r_2 = x,\ 0 < x < 6, x \in \mathbb{R}$, find an expression in x for the area of the arbelos.

$$\begin{aligned}\text{Area of arbelos } &= \tfrac{1}{2}\pi r_1^2 - \tfrac{1}{2}\pi\left(r_2^2 + r_3^2\right)\\ &= \tfrac{1}{2}\pi\left(r_1^2 - (r_2^2 + r_3^2\right)\\ &= \tfrac{1}{2}\pi\left(36 - \left(x^2 + (6-x)^2\right)\right)\\ &= \pi\left(6x - x^2\right)cm^2\end{aligned}$$

(iii) Hence, or otherwise, find the maximum area of an arbelos that can be formed in a semi circle of radius 6 cm.

$$A = \pi\left(6x - x^2\right) \Rightarrow \frac{dA}{dx} = \pi(6-2x)$$

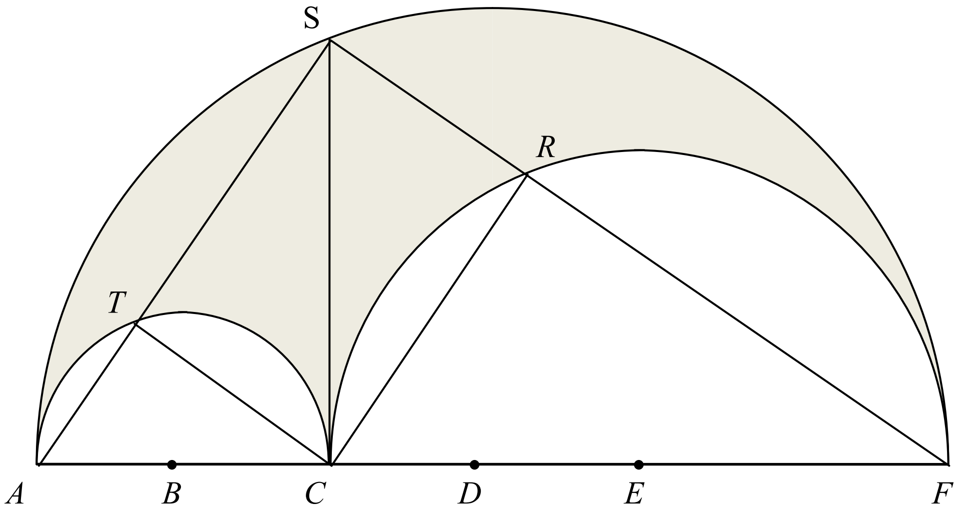
$$\pi(6-2x) = 0 \Rightarrow x = 3$$

$$\frac{dA}{dx} = \pi(6-2x) \Rightarrow \frac{d^2A}{dx^2} = -2\pi < 0 \Rightarrow \text{maximum}$$

Maximum area when $x = 3$, giving area = 9π cm^2

(d) *AS* and *FS* cut the two smaller semicircles at *T* and *R* respectively.
Prove that *RSTC* is a rectangle.

/



$|\angle TSR| = 90°$ ..... Angle in a semicircle
$|\angle CTA| = 90°$ ..... Angle in a semicircle
Hence, $|\angle STC| = 90°$
$|\angle FRC| = 90°$ ..... Angle in a semicircle
Hence, $|\angle CRS| = 90°$

Hence, the angles in *RSTC* are right angles and so *RSTC* is a rectangle.

Marking Scheme – Paper 2, Section A and Section B

Structure of the marking scheme

Candidate responses are marked according to different scales, depending on the types of response anticipated. Scales labelled A divide candidate responses into two categories (correct and incorrect). Scales labelled B divide responses into three categories (correct, partially correct, and incorrect), and so on. The scales and the marks that they generate are summarised in this table:

Scale label	A	B	C	D
No of categories	2	3	4	5
5 mark scales	0, 5	0, 3, 5	0, 3, 4, 5	
10 mark scales		0, 5, 10	0, 3, 8, 10	0, 3, 7, 9, 10
15 mark scales			0, 5, 10, 15	0, 4, 7, 11, 15

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the scheme, where necessary.

Marking scales – level descriptors

A-scales (two categories)
- incorrect response
- correct response

B-scales (three categories)
- response of no substantial merit
- partially correct response
- correct response

C-scales (four categories)
- response of no substantial merit
- response with some merit
- almost correct response
- correct response

D-scales (five categories)
- response of no substantial merit
- response with some merit
- response about half-right
- almost correct response
- correct response

In certain cases, typically involving incorrect rounding or omission of units, a mark that is one mark below the full-credit mark may also be awarded. Such cases are flagged with an asterisk. Thus, for example, *scale 10C** indicates that 9 marks may be awarded.

Summary of mark allocations and scales to be applied

Section A

Question 1

(a) (i)	5B
(a) (ii)	5B
(a) (iii)	5B
(b)	10D

Question 2

(a) (i)	10C
(a) (ii)	5C
(b)	10C

Question 3

(a)	10D
(b)	15D

Question 4

(a)	10C
(b) (i)	10D
(b) (ii)	5C

Question 5

(a)	5C
(b) (i)	10C*
(b) (ii)	5B
(c)	5B*

Question 6A

(a)	10D
(b)	5B
(c)	10D

Question 6B

(a)	10C
(b)	15C

Section B

Question 7

(a) (i)	5B
(a) (ii)	5B
(b) (i)	5B
(b) (ii)	5B
(c)	5B
(d) (i)	5B
(d) (ii)	5B
(e) (i)	10D
(e) (ii)	5B
(f) (i)	10C
(f) (ii)	5A
(f) (iii)	5B
(f) (iv)	5B

Question 8

(a) (i)	10C*
(a) (ii)	10C*
(b)	10B*

Question 9

(a)	10D
(b) (i)	5B
(b) (ii)	5B
(c) (i)	10B
(c) (ii)	5B*
(c) (iii)	5C*
(d)	5B

Detailed marking notes

Section A

Question 1

(a)(i) Scale 5B (0, 3, 5)

Partial Credit:
- Incomplete statement but with some merit
- Reference to outcomes
- Use of # symbol

(a)(ii) Scale 5B (0, 3, 5)

Partial Credit:
- Incomplete statement but with some merit
- Some reference to 'common' or 'intersection' of sets
- Venn diagram with E and F (or equivalent) shown but no reference to outcomes in common
- Reference to P(EUF)

(a)(iii) Scale 5B (0, 3, 5)

Partial Credit:
- Incomplete statement but with some merit
- Reference to comparing/contrasting the outcomes of two events
- Reference to $P(E \cap F)$ or $P(E|F)$ or $P(F|E)$

(b) Scale 10D (0, 3, 7, 9, 10)

Low Partial Credit:
- Venn Diagram with two or more entries correct
- Reference to $P(E \cap F)$ only
- Reference to P(E) or P(F) only

Mid Partial Credit:
- Venn diagram with either $P(E \cap F)$ or P(E) or P(F) or P((E|F) or P(F|E) calculated

High Partial Credit:
- $P(E \cap F)$ and $P(E) \times P(F)$ found but correct conclusion not stated or implied
- Error in sample space but correct conclusion from candidate's work

Question 2

(a)(i) Scale 10C (0, 3, 8, 10)

Low Partial Credit:
- Any relevant step
- Formula written

High Partial Credit:
- Reference to 1·6
- Incorrect reading of tables

(a)(ii) Scale 5C (0, 3, 4, 5)

Low Partial Credit:
- Any relevant step other than in a(i)
- Diagram clearly indicating a 'new' area
- Reference to – 1·6
- $P(Z \leq -1 \cdot 6) = 0 \cdot 0548$ and stops

High Partial Credit:
- Probability of both situations calculated but fails to finish correctly
- Correct method with some error

(b) Scale 10C (0, 3, 8, 10)

Low Partial Credit:
- One correct
- Bell shape in one or more

High Partial Credit:
- Two correct

Question 3

(a) Scale 10D (0, 3, 7, 9, 10)

Low Partial Credit:
- Any one correct
- Any line sketched correctly

Mid Partial Credit:
- Any two correct

High Partial Credit:
- Any four correct

(b) Scale 15D (0, 4, 7, 11, 15)

Low Partial Credit:
- Any reasonable step e.g. correct relevant formula
- Slope *m* or *n*

Mid Partial Credit:
- Both slopes

High Partial Credit:
- $\tan\theta = \frac{1}{\sqrt{3}}$ and stops

Question 4

(a) Scale 10C (0, 3, 8, 10)

Low Partial Credit:

- Any one correct
- Any reasonable step

High Partial Credit:

- Any two correct
- Correct approach but error in work

(b)(i) Scale 10D (0, 3, 7, 9, 10)

Low Partial Credit:

- Some reference to ratio
- $c_1 \cap c_2$ and stops
- Any reasonable step

Mid Partial Credit:

- Some correct substitution into correct ratio formula
- Substitution resulting in a quadratic equation in one variable

High Partial Credit:

- Error in ratio formula but finishes
- Finds one of the ordinates only

(b)(ii) Scale 5C (0, 3, 4, 5)

Low Partial Credit:

- Slope of line of centres
- Any reasonable step

High Partial Credit:

- Uses point of contact with slope of diameter for equation
- $c_1 - c_2 = 0$ but not in linear format

Question 5

(a) Scale 5C (0, 3, 4, 5)

Low Partial Credit:

- Relevant diagram
- One statement of area in trigonometric format
- Sine of a relevant angle in right angled triangle written in terms of sides
- Any reasonable step

High Partial Credit:

- Correct approach but one error in work

(b)(i) Scale 10C* (0, 3, 8, 10)

Low Partial Credit:

- Relevant formula
- Any reasonable step

High Partial Credit:

- Error in substitution into formula but continues
- One value only
- Correct method but one error in work

(b)(ii) Scale 5B (0, 3, 5)

Partial Credit:

- One position only shown
- Triangle(s) sketched but Z not indicated

(c) Scale 5B* (0, 3, 5)

Partial Credit:

- $|\angle ZXY|$ only
- Error in substitution into area formula
- Any reasonable step

Question 6A

(a) Scale 10D (0, 3, 7, 9, 10)

Low Partial Credit:
- One partially correct statement
- One partially correct sketch
- Any reasonable step

Mid Partial Credit:
- One fully correct statement
- One fully correct sketch

High Partial Credit:
- Two correct statements
- Two correct sketches

(b) Scale 5B (0, 3, 5)

Partial Credit:
- Some relevant reference to side(s) of triangle
- Some relevant reference to angles of triangle
- An incomplete 'rough' sketch

(c) Scale 10D (0, 3, 7, 9, 10)

Low Partial Credit:
- Some correct elements of construction
- Some evidence of understanding of term orthocentre
- Any reasonable step

Mid Partial Credit:
- One correct altitude

High Partial Credit:
- Two altitudes but not intersecting

Question 6B

(a) Scale 10C (0, 3, 8, 10)

Low Partial Credit:

- Any correct step e.g.,
    - Identifies two equal sides
    - Identifies two equal angles

High Partial Credit:

- Proof with correct steps but without justification of steps
- Congruent triangles established but fails to complete
- No conclusion stated or implied

(b) Scale 15C (0, 5, 10, 15)

Low Partial Credit:

- Any correct step e.g.,
    - Establishes parallel sides
    - Identifies two equal sides

High Partial Credit:

- Proof with one step not fully established
- No conclusion stated or implied

Section B

Question 7

(a)(i) Scale 5B (0, 3, 5)
Partial Credit:
- Reference to subgroups
- Reference to sampling
- Definition of random sample

(a)(ii) Scale 5B (0, 3, 5)
Partial Credit:
- Not clearly disjoint groups
- Incomplete number of groups (at least two)

(b)(i) Scale 5B (0, 3, 5)
Partial Credit:
- One correct probability
- Some evidence of relevant understanding

Note: Accept any other answer in the range [0·231, 0·252], provided a suitable rationale is given

(b)(ii) Scale 5B (0, 3, 5)
Partial Credit:
- Correct answer without explanation
- Correct answer with incorrect or incomplete explanation

(c) Scale 5B (0, 3, 5)
Partial Credit:
- Correct answer without explanation
- Correct explanation but omits to nominate correct graph

(d)(i) Scale 5B (0, 3, 5)
Partial Credit:
- A partially correct bell shape curve

(d)(ii) Scale 5B (0, 3, 5)
Partial Credit:
- Mention of mean and median but interpretation not related to candidate’s curve

(e)(i) Scale 10D (0, 3, 7, 9, 10)

Low Partial Credit:
- One relevant step e.g. null hypothesis stated only
- Some work towards margin of error

Mid Partial Credit:
- Substantive work with one or more critical omissions
- Margin of error and range found but fails to continue

High partial Credit
- Failure to state null hypothesis correctly
- Failure to cotextualise answer (e.g. stops at 'Reject null hypothesis')

Note: Accept candidate work based on disregarding *don't knows*, yielding an observed satisfaction rating of 664/902 and a corresponding $n = 902$.

(e)(ii) Scale 5B (0, 3, 5)

Partial Credit:
- Correct answer but no explanation
- Partially correct explanation

(f)(i) Scale 10C (0, 3, 8, 10)

Low Partial Credit:
- Correct scale with at least two points plotted

High Partial Credit:
- Correct scales but not all points plotted (one or two omissions)
- All points plotted but scales incorrect

(f)(ii) Scale 5A (0, 5)

(f)(iii) Scale 5B (0, 3, 5)

Partial Credit:
- Partially correct answer e.g. not in context
- Positive or strong positive correlation and stops

(f)(iv) Scale 5B (0, 3, 5)

Low Partial Credit:
- Straight line but clearly not best fit

Question 8

(a)(i) Scale 10C* (0, 3, 8, 10)

Low Partial Credit:
- Identifies a relevant right angle
- $\sin 36° = \frac{\text{opp}}{\text{hypt}}$ or equivalent

High Partial Credit:
- $\sin 36° = \frac{d}{80}$ or equivalent (e.g. $\sin 20° = \frac{d}{110}$)

Note:(i) Accept candidate answer from this section if and when used in later sections.
(ii) Units - apply penalty once only in question
(iii) Do not penalise candidates for rounding off answers in **(a)(i)** and **(a)(ii)**.

(a)(ii) Scale 10C*(0, 3, 8, 10)

Low Partial Credit:
- Identifies Cosine Rule
- $\cos 36° = \frac{\text{adj}}{\text{hypt}}$ or $\cos 20° = \frac{\text{adj}}{110}$ or $\tan 36° = \frac{d}{\text{adj}}$ or equivalent statements

High Partial Credit:
- $|HP|^2$ calculated and stops
- Substantially correct work with one error

(b) Scale 10B*(0, 5, 10)

Partial Credit:
- Some relevant work

Note: Where there is no evidence of impact of error, mark according to candidate's work. There are several correct approaches. Track candidate's data throughout, accepting any valid approach to each part.

Where there is evidence of impact of the error, award full marks in the part in which the impact occurs and subsequent part(s). Ensure that all such scripts are forwarded for review.

Criteria for identifying impact of error:

- Attempts a part more than once and gets different values for the same length.
  For example:
  - In part **(a)(i)**, calculates d as $80\sin 36° = 47\cdot 02$ **and as** $110\sin 20° = 37\cdot 62$.
  - Calculates |HP| using the cosine rule and the sine rule (or other method), getting different values.
- Calculates |HR| as $110\frac{\sin 20^{o}}{\sin 36^{o}} = 64$, contradicting an identified value of 80 for |HR|.

- Correctly generates a value for any angle that conflicts with an already calculated or identified value for the same angle.
- Values correctly obtained lead to a value of cosine or sine that is outside the range [−1, 1].
- Attempts to construct an accurate scaled diagram and encounters difficulty.
- Any explicit statement suggesting awareness of conflicting data.

If you encounter any other evidence of potential impact, not covered by the above, please notify your Advising Examiner immediately.

Question 9

(a) Scale 10D (0, 3, 7, 9, 10)

Low Partial Credit:

- One correct statement in k^2
- One correct ratio involving k
- Some relevant use of Pythagoras
- Any reasonable step e.g. establishing similar triangles

Mid Partial Credit:

- Two correct statements in k^2
- Two correct ratios involving k

High Partial Credit:

- k^2 evaluated, but k not found

(b)(i) Scale 5B (0, 3, 5)

Partial Credit:

- One relevant perimeter
- $r_2 + r_3 = r_1$ written or implied
- Any reasonable step

(b)(ii) Scale 5B (0, 3, 5)

Partial Credit:

- Area of one semi circle
- Area of circle with diameter k
- Area of arbelos only

(c)(i) Scale 5B (0, 5, 10)

Partial Credit:

- Two correct entries

(c)(ii) Scale 5B* (0, 3, 5)

Partial Credit:

- Identifies $r_3 = 6 - x$
- Area of either semicircle in terms of x

(c)(iii) Scale 5C* (0, 3, 4, 5)

Low Partial Credit:

- Some correct differentiation
- Incomplete quadratic graph sketched

High Partial Credit:

- $x = 3$ established as maximum but area not calculated
- Graphical result not interpreted

(d) Scale 5B (0, 3, 5)

Partial Credit:

- One relevant step

Marcanna breise as ucht freagairt trí Ghaeilge

(Bonus marks for answering through Irish)

Ba chóir marcanna de réir an ghnáthráta a bhronnadh ar iarrthóirí nach ngnóthaíonn níos mó ná 75% d'iomlán na marcanna don pháipéar. Ba chóir freisin an marc bónais sin a shlánú **síos**.

Déantar an cinneadh agus an ríomhaireacht faoin marc bónais i gcás gach páipéir ar leithligh.

Is é 5% an gnáthráta agus is é 300 iomlán na marcanna don pháipéar. Mar sin, bain úsáid as an ngnáthráta 5% i gcás iarrthóirí a ghnóthaíonn 225 marc nó níos lú, e.g. 198 marc $\times$ 5% = 9·9 $\Rightarrow$ bónas = 9 marc.

Má ghnóthaíonn an t-iarrthóir níos mó ná 225 marc, ríomhtar an bónas de réir na foirmle [300 – bunmharc] $\times$ 15%, agus an marc bónais sin a shlánú **síos**. In ionad an ríomhaireacht sin a dhéanamh, is féidir úsáid a bhaint as an tábla thíos.

Bunmharc	Marc Bónais
226	11
227 – 233	10
234 – 240	9
241 – 246	8
247 – 253	7
254 – 260	6
261 – 266	5
267 – 273	4
274 – 280	3
281 – 286	2
287 – 293	1
294 – 300	0

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