M27

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LEAVING CERTIFICATE EXAMINATION, 2002

MATHEMATICS — ORDINARY LEVEL

PAPER 1 (300 marks)

THURSDAY, 6 JUNE - MORNING, 9.30 to 12.00

Attempt **SIX QUESTIONS** (50 marks each).

WARNING: Marks will be lost if all necessary work is not clearly shown.

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1. **(a)** Copper and zinc are mixed in the ratio 19 : 6.
The amount of copper used is 133 kg.
How many kilogrammes of zinc are used?

(b) Four telephone calls cost €3.85, €7.45, €8.40 and €11.55.

(i) John estimates the total cost of the four calls by ignoring the cent part in the cost of each call. Calculate the percentage error in his estimate.

(ii) Anne estimates the total cost of the four calls by rounding the cost of each call to the nearest euro. Calculate the percentage error in her estimate.

(c) A raffle to raise money for a charity is being held.

The first prize is €100, the second is €85, the third is €65 and the fourth is €50.

The cost of printing tickets is €42 for the first 500 tickets and €6 for each additional 100 tickets. The smallest number of tickets that can be printed is 500.

Tickets are being sold at €1.50 each.

(i) What is the minimum possible cost of holding the raffle?

(ii) If 500 tickets are printed, how many tickets must be sold in order to avoid a loss?

(iii) If 1000 tickets are printed and 65% of the tickets are sold, how much money will be raised for the charity?

2. **(a)** Solve for x

$$\frac{x-7}{2} = \frac{x+3}{6}.$$

(b) **(i)** Show that $x+2$ is a factor of $2x^3 + 7x^2 + x - 10$.

(ii) Hence, or otherwise, find the three roots of $2x^3 + 7x^2 + x - 10 = 0$.

(c) **(i)** Express b in terms of a and c where $\dfrac{8a-5b}{b} = c$.

(ii) Hence, or otherwise, evaluate b when $a = 2^{\frac{5}{2}}$ and $c = 3^3$.

3. **(a)** Solve the inequality $5x + 1 \geq 4x - 3$ for $x \in \mathbf{R}$ and illustrate the solution set on a number line.

(b) **(i)** Solve for x and y

$$y = 10 - 2x$$
$$x^2 + y^2 = 25.$$

(ii) Hence, find the two possible values of $x^3 + y^3$.

(c) Let $f(x) = x^2 + ax + t$ where $a,\ t \in \mathbf{R}$.

(i) Find the value of a, given that $f(-5) = f(-1)$.

(ii) Given that there is only one value of x for which $f(x) = 0$, find the value of t.

4. **(a)** Given that $i^2 = -1$, simplify

$$2(3 - i) + i(4 + 5i)$$

and write your answer in the form $x + yi$ where $x, y \in \mathbf{R}$.

(b) Let $z = 5 + 4i$.

(i) Plot z and $\bar{z}$ on an Argand diagram, where $\bar{z}$ is the complex conjugate of z.

(ii) Calculate $z\bar{z}$.

(iii) Express $\frac{z}{\bar{z}}$ in the form $u + vi$ where $u,\ v \in \mathbf{R}$.

(c) p and k are real numbers such that $p(2 + i) + 8 - ki = 5k - 3 - i$.

(i) Find the value of p and the value of k.

(ii) Investigate if $p + ki$ is a root of the equation $z^2 - 4z + 13 = 0$.

5. **(a)** Write down the next three terms in each of the following arithmetic sequences

(i) $-10, -8, -6, \ldots\ldots\ldots\ldots$

(ii) $4.1,\ 4.7,\ 5.3,\ \ldots\ldots\ldots\ldots$

(b) The sum of the first n terms of an arithmetic series is given by

$$S_n = \frac{3n}{2}(n+3).$$

(i) Calculate the first term of the series.

(ii) By calculating S_9 and S_{10}, find T_{10} (the tenth term of the series).

(c) The first three terms of a geometric sequence are

$$k-3,\ 2k-4,\ 4k-3, \ldots\ldots\ldots$$

where k is a real number.

(i) Find the value of k.

(ii) Hence, write down the value of each of the first four terms of the sequence.

6. **(a)** Let $f(x) = \frac{1}{3}(x-8)$ for $x \in \mathbf{R}$.

Evaluate $f(5)$.

(b) **(i)** Find $\frac{dy}{dx}$ where $y = (x-1)^7$ and evaluate your answer at $x = 2$.

(ii) Find $\frac{dy}{dx}$ where $y = (x^3 - 3)(x^2 - 4)$ and simplify your answer.

(c) Let $f(x) = x^3 - ax + 7$ for all $x \in \mathbf{R}$ and for $a \in \mathbf{R}$.

(i) The slope of the tangent to the curve $y = f(x)$ at $x = 1$ is -9.
Find the value of a.

(ii) Hence, find the co-ordinates of the local maximum point and the local minimum point on the curve $y = f(x)$.

7. **(a)** Differentiate $7x^3 - 3x^2 + 9x$ with respect to x.

(b) **(i)** Differentiate $x^5 - 17 + \frac{1}{x^5}$ with respect to x.

(ii) Differentiate $\frac{2x}{x-1}$ with respect to x and simplify your answer.

(c) A marble rolls along the top of a table. It starts to move at $t = 0$ seconds.
The distance that it has travelled at t seconds is given by

$$s = 14t - t^2$$

where s is in centimetres.

(i) What distance has the marble travelled when $t = 2$ seconds?

(ii) What is the speed of the marble when $t = 5$ seconds?

(iii) When is the speed of the marble equal to zero?

(iv) What is the acceleration of the marble?

8. Let $f(x) = \frac{1}{x+2}$.

(i) Find $f(-6)$, $f(-3)$, $f(-1)$, $f(0)$ and $f(2)$.

(ii) For what real value of x is $f(x)$ not defined?

(iii) Draw the graph of $f(x) = \frac{1}{x+2}$ for $-6 \le x \le 2$.

(iv) Find $f'(x)$, the derivative of $f(x)$.

(v) Find the two values of x at which the slope of the tangent to the graph is $-\frac{1}{9}$.

(vi) Show that there is no tangent to the graph of f that is parallel to the x-axis.