Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate 2019

Marking Scheme

Applied Mathematics

Ordinary Level

Note to teachers and students on the use of published marking schemes

Marking schemes published by the State Examinations Commission are not intended to be standalone documents. They are an essential resource for examiners who receive training in the correct interpretation and application of the scheme. This training involves, among other things, marking samples of student work and discussing the marks awarded, so as to clarify the correct application of the scheme. The work of examiners is subsequently monitored by Advising Examiners to ensure consistent and accurate application of the marking scheme. This process is overseen by the Chief Examiner, usually assisted by a Chief Advising Examiner. The Chief Examiner is the final authority regarding whether or not the marking scheme has been correctly applied to any piece of candidate work.

Marking schemes are working documents. While a draft marking scheme is prepared in advance of the examination, the scheme is not finalised until examiners have applied it to candidates' work and the feedback from all examiners has been collated and considered in light of the full range of responses of candidates, the overall level of difficulty of the examination and the need to maintain consistency in standards from year to year. This published document contains the finalised scheme, as it was applied to all candidates' work.

In the case of marking schemes that include model solutions or answers, it should be noted that these are not intended to be exhaustive. Variations and alternatives may also be acceptable. Examiners must consider all answers on their merits, and will have consulted with their Advising Examiners when in doubt.

Future Marking Schemes

Assumptions about future marking schemes on the basis of past schemes should be avoided. While the underlying assessment principles remain the same, the details of the marking of a particular type of question may change in the context of the contribution of that question to the overall examination in a given year. The Chief Examiner in any given year has the responsibility to determine how best to ensure the fair and accurate assessment of candidates' work and to ensure consistency in the standard of the assessment from year to year. Accordingly, aspects of the structure, detail and application of the marking scheme for a particular examination are subject to change from one year to the next without notice.

General Guidelines

1. Penalties of three types are applied to candidates' work as follows:

Slips	- numerical slips	S(-1)
Blunders	- mathematical errors	B(-3)
Misreading	- if not serious	M(-1)

Serious blunder or omission or misreading which oversimplifies:
- award the attempt mark only.

Attempt marks are awarded as follows: 5 (att 2), 10 (att 3).

2. The marking scheme shows one correct solution to each question. In many cases there are other equally valid methods.

1. The points P and Q lie on a straight level road.
A car passes P with a speed of 7 m s^{-1} and accelerates uniformly, with acceleration a, for 3·5 seconds to a speed of 21 m s^{-1}.
It then travels at a constant speed of 21 m s^{-1} for 9·5 seconds.
Finally, the car decelerates uniformly to a speed of 7 m s^{-1} at Q.
The car travels 98 metres while decelerating.
Find
(i) the acceleration, a
(ii) $|PQ|$, the distance from P to Q
(iii) the average speed of the car as it travels from P to Q.
A van travels from P to Q and takes the same amount of time as the car.
The van passes P with a speed of 7 m s^{-1} and accelerates uniformly to a maximum speed of k m s^{-1}.
It then decelerates uniformly to a speed of 7 m s^{-1} at Q.
(iv) Draw a speed-time graph of the motion of the van from P to Q.
(v) Find the value of k.

(i) $v = u + at$

$21 = 7 + 3.5a$

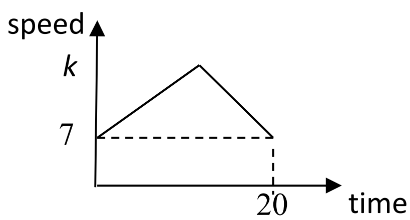
$a = 4 \text{ m s}^{-2}$ (10)

(ii) $|PQ| = 3.5(7) + \frac{1}{2}(3.5)(14) + 9.5(21) + 98 = 346.5$ m. (10)

(iii) $7^2 = 21^2 + 2a(98) \Rightarrow a = -2$ (5)
$7 = 21 - 2t \Rightarrow t = 7$ (5)
Time $= 3.5 + 9.5 + 7 = 20$

Average speed $= \frac{346.5}{20} = 17.325 \text{ m s}^{-1}$ (5)

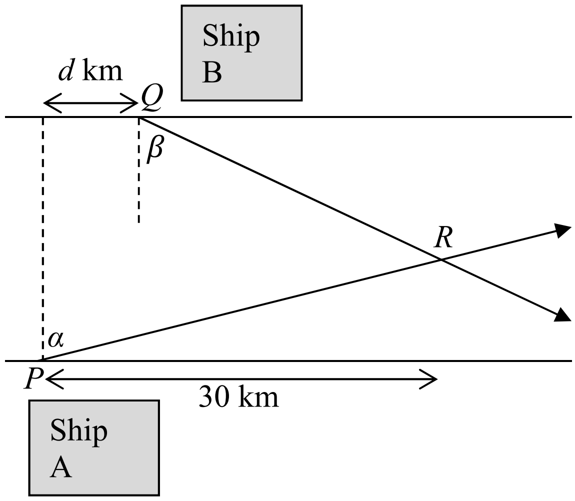
(iv)



(5)

(v) $\frac{1}{2}(20)(k-7) + 7(20) = 346.5$
$k = 27.65$ (10) (50)

2. P is a point on the southern bank of a river. Q is a point on the northern bank of the river, d km downstream from P.



Ship A departs from P at a constant speed of 68 km h^{-1} in the direction north α east, where $\tan\alpha = \frac{15}{8}$.

At the same time, ship B departs from Q at a constant speed of 58 km h^{-1} in the direction south β east, where $\tan\beta = \frac{20}{21}$.

Find (i) the velocity of ship A in terms of $\vec{i}$ and $\vec{j}$

(ii) the velocity of ship B in terms of $\vec{i}$ and $\vec{j}$

(iii) the velocity of A relative to B in terms of $\vec{i}$ and $\vec{j}$.

The paths of A and B intersect at point R, which is 30 km downstream from P.

Find (iv) the time it takes ship A to reach point R

(v) the value of d if ship B reaches point R twelve minutes after ship A

(vi) the width of the river, assuming its banks are parallel.

(i) $\overrightarrow{V_A} = 68\sin\alpha\,\vec{i} + 68\cos\alpha\,\vec{j}$ (5)

$= 60\,\vec{i} + 32\,\vec{j}$ (5)

(ii) $\overrightarrow{V_B} = 58\sin\beta\,\vec{i} - 58\cos\beta\,\vec{j}$ (5)

$= 40\,\vec{i} - 42\,\vec{j}$ (5)

(iii) $\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$ (5)

$\overrightarrow{V_{AB}} = 20\,\vec{i} + 74\,\vec{j}$ (5)

(iv) $t = \frac{30}{60} = 0.5$ hr (5)

(v) $30 = d + 40(0.5 + 0.2)$ (5)

$d = 2$ km.

(vi) $w = 32(0.5) + 42(0.7)$ (5)

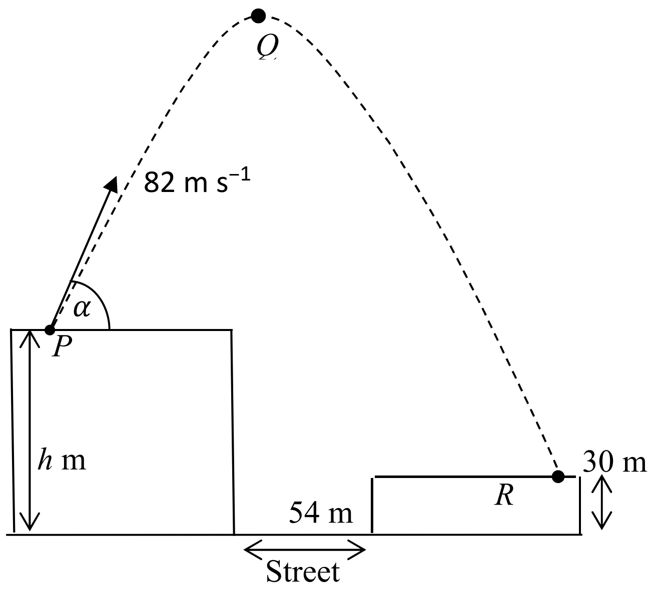
$w = 45.4$ km. (5) (50)

3. A particle is projected from point *P*, as shown in the diagram, with initial speed 82 m s^{-1} at an angle of α to the horizontal, where $\tan\alpha = \frac{40}{9}$.

P is a point on the horizontal roof of a tall building of height *h* m.

Q is the highest point reached by the particle.

The particle lands at *R*, a point on the horizontal roof of another building of height 30 m, as shown.



Find **(i)** the initial velocity of the particle in terms of $\vec{\imath}$ and $\vec{\jmath}$

(ii) the velocity of the particle after 5 seconds of motion in terms of $\vec{\imath}$ and $\vec{\jmath}$

(iii) the displacement of *Q* from *P* in terms of $\vec{\imath}$ and $\vec{\jmath}$

(iv) the value of *h*, given that the particle lands at *R* after 17 seconds of motion.

The street between the buildings is horizontal and is 54 m wide.

(v) Find the time for which the particle is **not** passing over a building.

(i)

$$\vec{u} = 82\cos\alpha\ \vec{\imath} + 82\sin\alpha\ \vec{\jmath} \qquad (5)$$

$$\vec{u} = 82 \times \frac{9}{41}\ \vec{\imath} + 82 \times \frac{40}{41}\ \vec{\jmath}$$

$$\vec{u} = 18\ \vec{\imath} + 80\ \vec{\jmath} \qquad (5)$$

(ii)

$$\vec{v} = 18\ \vec{\imath} + (80 - 50)\ \vec{\jmath}$$

$$\vec{v} = 18\ \vec{\imath} + 30\ \vec{\jmath} \qquad (10)$$

(iii)

$$\vec{v_j} = 0 \quad \Rightarrow \quad 80 - 10t = 0$$

$$t = 8 \qquad (5)$$

$\vec{r} = 18t\ \ \vec{\imath} + (80t - 5t^2)\ \vec{\jmath}$
$\vec{r} = 144\ \ \vec{\imath} + 320\ \vec{\jmath}$ (5)

(iv) $\vec{r_j} = -(h - 30)$

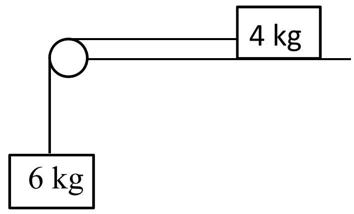
$80(17) - 5(17^2) = 30 - h$ (5)

$-85 = 30 - h$

$h = 115$ (5)

(v) $t = \frac{54}{18} = 3$ (10) (50)

4. (a) A particle of mass 4 kg is connected to another particle of mass 6 kg by a taut light inelastic string which passes over a smooth light pulley at the edge of a rough horizontal table.

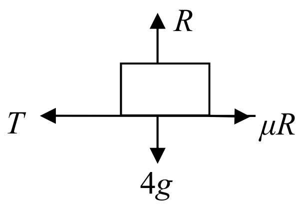


The coefficient of friction between the 4 kg mass and the table is μ.

The system is released from rest and both masses move 1·5 metres in their first second of motion.

(i) Show on separate diagrams the forces acting on each particle.
(ii) Show that the common acceleration of the particles is 3 m s^{-2}.
(iii) Find the tension in the string.
(iv) Find the value of μ.

(i)



(10)

(ii)

$$s = ut + \frac{1}{2}at^2$$

$$1.5 = 0 + \frac{1}{2}a(1)$$

$$a = 3 \ \text{m s}^{-2}$$ (5)

(iii)

$$6g - T = 6a$$

$$60 - T = 18$$

$$T = 42 \text{ N}$$ (10)

(iv)

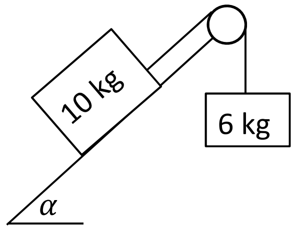
$$T - \mu R = 4a$$

$$42 - \mu(40) = 12$$

$$\mu = \frac{3}{4}$$ (5) (30)

4 (b) Masses of 10 kg and 6 kg are connected by a taut light inelastic string which passes over a light smooth pulley, as shown in the diagram.

The 10 kg mass lies on a smooth plane inclined at α to the horizontal, where $\tan\alpha = \frac{4}{3}$.



The 6 kg mass hangs vertically.
The system is released from rest.

Find **(i)** the common acceleration of the masses
(ii) the tension in the string.

(i) $10g\sin\alpha - T = 10a$ (5)

$8g - T = 10a$

$T - 6g = 6a$ (5)

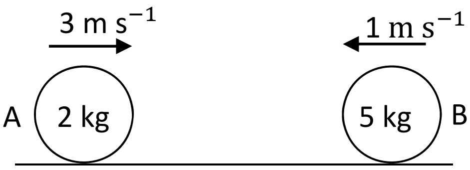
$2g = 16a$

$a = \frac{g}{8} = 1.25 \text{ m s}^{-2}$ (5)

(ii) $T - 60 = 7.5$

$T = 67.5 \text{ N}$ (5) (20)

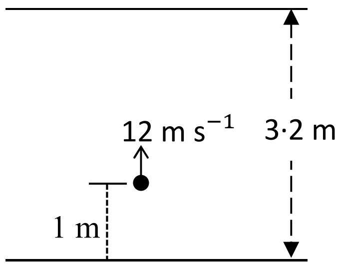
5. (a) Two smooth spheres A and B are sliding towards each other on a smooth horizontal table, with speeds of 3 m s^{-1} and 1 m s^{-1}, respectively.
Sphere A, of mass 2 kg, collides directly with sphere B, of mass 5 kg.
The coefficient of restitution for the collision is $\frac{2}{5}$.



Find **(i)** the speeds of A and immediately after the collision
(ii) the loss of kinetic energy due to the collision
(iii) the magnitude of the impulse imparted to B due to the collision.

(b) A ball is fired vertically upward in a room with a smooth horizontal floor and a smooth horizontal ceiling. The height of the room is 3·2 metres.

The ball is fired upwards at a speed of 12 m s^{-1} from a height of 1 metre above the floor.
The coefficient of restitution for all collisions between the ball and the ceiling and between the ball and the floor is $\frac{3}{5}$.



(i) Find the speed of the ball immediately after striking the ceiling.
(ii) Investigate whether the ball strikes the ceiling again after rebounding from the floor.

(a) (i)

PCM $\quad 2(3) + 5(-1) = 2v_1 + 5v_2$ (5)

$$2v_1 + 5v_2 = 1$$

NEL $\quad v_1 - v_2 = -\frac{2}{5}(3+1)$ (5)

$$v_1 - v_2 = -\frac{8}{5}$$

$v_1 = -1 \qquad v_2 = 0.6$ (5)

(ii)

$$\text{KE}_\text{B} = \frac{1}{2}(2)(3)^2 + \frac{1}{2}(5)(-1)^2 = 11.5$$
$$\text{KE}_\text{A} = \frac{1}{2}(2)(-1)^2 + \frac{1}{2}(5)(0.6)^2 = 1.9$$

$\text{KE}_\text{B} - \text{KE}_\text{A} = 11.5 - 1.9 = 9.6\ \text{J}$ (10)

(iii) $\quad I = |5 \times (0.6) - 5 \times (-1)| = 8$ (5) (30)

(b) (i)

$v^2 = u^2 + 2as$
$v^2 = 144 + 2 \times (-10) \times 2.2$
$v = 10$ (5)
$ev = 6$ (5)

(ii)

$v^2 = u^2 + 2as$
$v^2 = 36 + 2 \times (10) \times 3.2$
$v = 10$
$ev = 6$ (5)

$v^2 = u^2 + 2as$
$0 = 36 - 20s$
. $s = 1.8 < 3.2$ $\Longrightarrow$ does not strike ceiling (5) (20)

6. (a) Particles of weight 4 N, 10 N, *p* N, and 7 N are placed at the points (3, 8), (*p*, −6), (4, *q*), and (*p*, *p*) respectively.
The co-ordinates of the centre of gravity of the system are (1·5, *q*).

Find **(i)** the value of *p*
(ii) the value of *q*.

(i) $1.5 = \frac{4(3)+10p+4p+7p}{21+p}$ (5)

$31.5 + 1.5p = 12 + 21p$

$p = 1$ (5)

(ii) $q = \frac{4(8)+10(-6)+pq+7p}{21+p}$ (5)

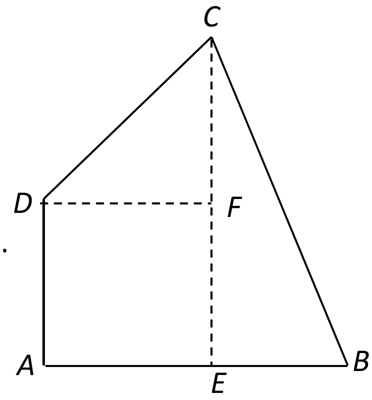
$22q = -21 + q$

$q = -1$ (5) (20)

(b) A uniform quadrilateral lamina has vertices *A*, *B*, *C* and *D.*

The co-ordinates of the points are *A*(0, 0), *B*(15, 0), *C*(9, 12) and *D*(0, 6).

Find the co-ordinates of the centre of gravity of the lamina.



	area	c.g.	
AEFD	$6 \times 9 = 54$	$(4.5, 3)$	(5)
DFC	$\frac{1}{2}(9)(6) = 27$	$(6, 8)$	(5)
EBC	$\frac{1}{2}(6)(12) = 36$	$(11, 4)$	(5)
ABCD	117	(x, y)	(5)

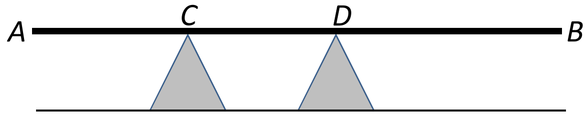
$117x = 54 \times 4.5 + 27 \times 6 + 36 \times 11$

$x = 6.85$ (5)

$117y = 54 \times 3 + 27 \times 8 + 36 \times 4$

$y = 4.46$ (5) (30)

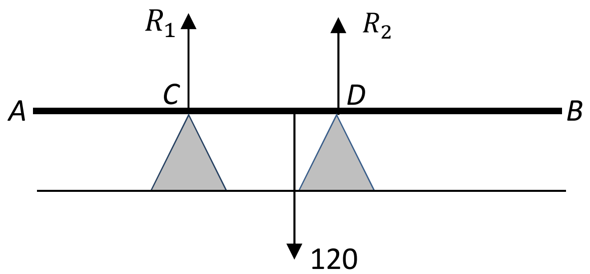
7. **(a)** A uniform beam, [AB], lies horizontally and in equilibrium on supports at points C and D of the beam, as shown in the diagram.



The mass of the beam is 12 kg and the length of the beam is 70 cm.
$|AC| = |CD| = 20$ cm.

(i) Find the reaction forces at C and D.

(ii) A mass of m kg is now placed at B.
This causes the beam to be on the point of lifting off the support at C.
Find the value of m.



(i) ↻ C

$R_2 \times 20 = 120 \times (35 - 20)$ (5)

$R_2 = 90$ (5)

$R_1 + R_2 = 120$ (5)

$R_1 + 90 = 120$

$R_1 = 30$ (5)

(ii) $R_1 = 0$ (5)

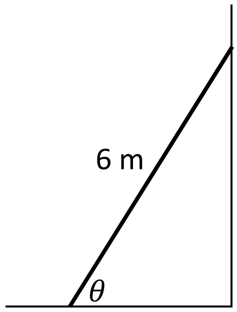
$mg \times 30 = R_2 \times 5$

$10m \times 30 = 120 \times 5$

$m = 2$ kg. (5) (30)

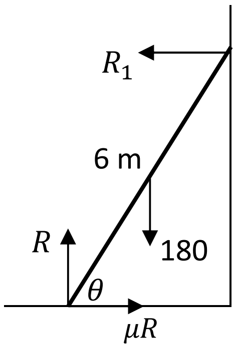
7. (b) A uniform ladder, of weight 180 N, rests on rough horizontal ground and leans against a smooth vertical wall.

The length of the ladder is 6 m.
The ladder makes an angle θ with the ground, where $\tan\theta = \frac{24}{7}$.



The ladder is in equilibrium and is on the point of slipping.

Find the coefficient of friction between the ladder and the ground.



$R_1 \times 6\sin\theta = 180 \times 3\cos\theta$ (5)

$R_1 \tan\theta = 90$

$R_1 \frac{24}{7} = 90$

$R_1 = 26.25$ (5)

$R = 180$ (5)

$\mu R = R_1$

$\mu = \frac{R_1}{R} = \frac{26.25}{180} = \frac{7}{48}$ (5) (20)

8. (a) A particle describes a horizontal circle of radius 0·8 metres with uniform angular velocity ω radians per second. The mass of the particle is 0·4 kg. The particle completes 12 revolutions every minute.

Find **(i)** the value of ω
(ii) the speed of the particle
(iii) the acceleration of the particle
(iv) the centripetal force on the particle.

(i)

$$T = \frac{60}{12} = 5$$

$$\frac{2\pi}{\omega} = 5$$

$$\omega = \frac{2\pi}{5} = 1.26 \ \text{rad s}^{-1}$$ (5)

(ii)

$$v = r\omega$$

$$v = 0.8 \times \frac{2\pi}{5}$$

$$v = 0.32\pi = 1.006 \ \text{m s}^{-1}$$ (5)

(iii)

$$a = r\omega^2$$

$$= 0.8 \times \left(\frac{2\pi}{5}\right)^2$$

$$= 1.26 \ \text{m s}^{-2}$$ (5)

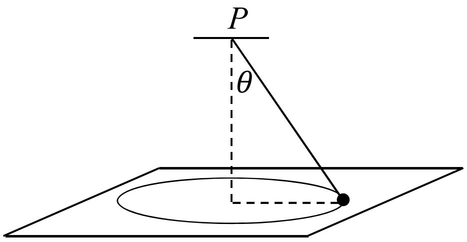
(iv)

$$F = mr\omega^2$$

$$= 0.4 \times 1.26$$

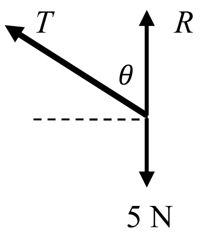
$$= 0.5 \ \text{N}$$ (5) (20)

8 (b) A smooth particle of mass 0·5 kg is attached by a light inelastic string to a fixed point *P*. The particle describes a horizontal circle, of radius 0·2 m, on the smooth surface of a horizontal table.



The centre of the circle is vertically below *P*.
The string makes an angle θ with the vertical, where $\tan\theta = \frac{4}{3}$.
The speed of the particle is $1\ \text{m s}^{-1}$.

Find **(i)** the tension in the string
(ii) the reaction force between the particle and the table.



(i) $T\sin\theta = \frac{mv^2}{r}$ (5)

$T \times 0.8 = \frac{0.5\times 1}{0.2}$ (5)

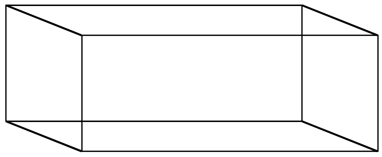
$T = 3.125\ \text{N}$ (5)

(ii) $R + T\cos\theta = 0.5g$ (5)

$R + 3.125 \times 0.6 = 5$ (5)

$R = 3.125\ \text{N}$ (5) (30)

9. **(a)** A solid rectangular block is used as a floating platform. It has length 6 m, width 5 m and height 2 m.

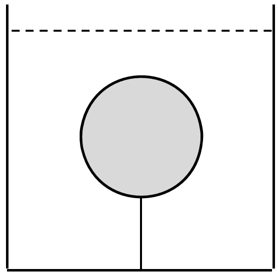


The platform floats at rest in water with its upper surface (6 m by 5 m) horizontal.
80% of the platform lies below the surface of the water.

Find **(i)** the weight of the platform
(ii) the mass of the platform in tonnes.

[Density of water = 1000 kg m^{-3}]

(b) A solid sphere has radius 12 cm.
The relative density of the sphere is 0·7 and it is completely immersed in a liquid of relative density 1·5.
The sphere is held at rest by a light inelastic vertical string which is attached to the base of the tank.



Find the tension in the string.

(a) **(i)** $W = B$ (5)

$= \rho V g$ (5)

$= 1000 \times (1.6 \times 6 \times 5) \times 10$ (5)

$= 480\ 000 \ \text{ N}$ (5)

(ii) $\text{mass} = \frac{W}{g} = 48\ 000 = 48 \ \text{ tonnes}$ (5) (25)

(b) $T + W = B$ (5)

$T + 700Vg = 1500Vg$ (10)

$T = 800Vg$

$T = 800 \times \left\{\frac{4\pi}{3} \times (0.12)^3\right\} \times 10$ (5)

$T = 57.9 \ \text{ N.}$ (5) (25)

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