Leaving Certificate 2016

Marking Scheme

Mathematics

Higher Level
Note to teachers and students on the use of published marking schemes

Marking schemes published by the State Examinations Commission are not intended to be standalone documents. They are an essential resource for examiners who receive training in the correct interpretation and application of the scheme. This training involves, among other things, marking samples of student work and discussing the marks awarded, so as to clarify the correct application of the scheme. The work of examiners is subsequently monitored by Advising Examiners to ensure consistent and accurate application of the marking scheme. This process is overseen by the Chief Examiner, usually assisted by a Chief Advising Examiner. The Chief Examiner is the final authority regarding whether or not the marking scheme has been correctly applied to any piece of candidate work.

Marking schemes are working documents. While a draft marking scheme is prepared in advance of the examination, the scheme is not finalised until examiners have applied it to candidates’ work and the feedback from all examiners has been collated and considered in light of the full range of responses of candidates, the overall level of difficulty of the examination and the need to maintain consistency in standards from year to year. This published document contains the finalised scheme, as it was applied to all candidates’ work.

In the case of marking schemes that include model solutions or answers, it should be noted that these are not intended to be exhaustive. Variations and alternatives may also be acceptable. Examiners must consider all answers on their merits, and will have consulted with their Advising Examiners when in doubt.

Future Marking Schemes

Assumptions about future marking schemes on the basis of past schemes should be avoided. While the underlying assessment principles remain the same, the details of the marking of a particular type of question may change in the context of the contribution of that question to the overall examination in a given year. The Chief Examiner in any given year has the responsibility to determine how best to ensure the fair and accurate assessment of candidates’ work and to ensure consistency in the standard of the assessment from year to year. Accordingly, aspects of the structure, detail and application of the marking scheme for a particular examination are subject to change from one year to the next without notice.
Contents

Paper 1

Solutions and marking scheme ................................................................. 2
  Structure of the marking scheme ....................................................... 3
  Summary of mark allocations and scales to be applied ...................... 4
  Model solutions and detailed marking notes ...................................... 5

Paper 2

Solutions and marking scheme ................................................................. 23
  Structure of the marking scheme ....................................................... 24
  Summary of mark allocations and scales to be applied ...................... 25
  Model solutions and detailed marking notes ...................................... 26

Marcanna breise as ucht freagairt trí Ghaeilge ........................................ 44
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Leaving Certificate 2016

Model Solutions and Marking Scheme

Mathematics

Higher Level

Paper 1
Marking Scheme – Paper 1, Section A and Section B

Structure of the marking scheme

Candidate responses are marked according to different scales, depending on the types of response anticipated. Scales labelled A divide candidate responses into two categories (correct and incorrect). Scales labelled B divide responses into three categories (correct, partially correct, and incorrect), and so on. The scales and the marks that they generate are summarised in this table:

<table>
<thead>
<tr>
<th>Scale label</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>No of categories</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>5 mark scales</td>
<td>0, 5</td>
<td>0, 2, 5</td>
<td>0, 2, 4, 5</td>
<td>0, 2, 3, 4, 5</td>
<td></td>
</tr>
<tr>
<td>10 mark scales</td>
<td>0, 10</td>
<td>0, 5, 10</td>
<td>0, 3, 7, 10</td>
<td>0, 2, 5, 8, 10</td>
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<tr>
<td>15 mark scales</td>
<td>0, 15</td>
<td>0, 7, 15</td>
<td>0, 5, 10, 15</td>
<td>0, 4, 7, 11, 15</td>
<td></td>
</tr>
<tr>
<td>20 mark scales</td>
<td>0, 20</td>
<td>0, 10, 20</td>
<td>0, 7, 13, 20</td>
<td>0, 5, 10, 15, 20</td>
<td></td>
</tr>
<tr>
<td>25 mark scales</td>
<td>0, 25</td>
<td>0, 12, 25</td>
<td>0, 8, 17, 25</td>
<td>0, 6, 12, 19, 25</td>
<td>0, 5, 10, 15, 20, 25</td>
</tr>
</tbody>
</table>

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the scheme, where necessary.

Marking scales – level descriptors

**A-scales (two categories)**
- incorrect response
- correct response

**B-scales (three categories)**
- response of no substantial merit
- partially correct response
- correct response

**C-scales (four categories)**
- response of no substantial merit
- response with some merit
- almost correct response
- correct response

**D-scales (five categories)**
- response of no substantial merit
- response with some merit
- response about half-right
- almost correct response
- correct response

**E-scales (six categories)**
- response of no substantial merit
- response with some merit
- response almost half-right
- response more than half-right
- almost correct response
- correct response

In certain cases, typically involving incorrect rounding, omission of units, a misreading that does not oversimplify the work or an arithmetical error that does not oversimplify the work, a mark that is one mark below the full-credit mark may also be awarded. Thus, for example, in scale 10C, 9 marks may be awarded. Throughout the scheme indicate by use of * where an arithmetic error occurs.
### Summary of mark allocations and scales to be applied

#### Section A

<table>
<thead>
<tr>
<th>Question 1</th>
<th></th>
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</thead>
<tbody>
<tr>
<td>(a)</td>
<td>5B</td>
<td></td>
</tr>
<tr>
<td>(b)</td>
<td>10C</td>
<td></td>
</tr>
<tr>
<td>(c)</td>
<td>10C</td>
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<table>
<thead>
<tr>
<th>Question 2</th>
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<tbody>
<tr>
<td>(a)</td>
<td>10C</td>
<td></td>
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<tr>
<td>(b)</td>
<td>15C</td>
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<thead>
<tr>
<th>Question 3</th>
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<tbody>
<tr>
<td>(a)(i)</td>
<td>5C</td>
<td></td>
</tr>
<tr>
<td>(ii)</td>
<td>5C</td>
<td></td>
</tr>
<tr>
<td>(iii)</td>
<td>5B</td>
<td></td>
</tr>
<tr>
<td>(b)</td>
<td>10C</td>
<td></td>
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<table>
<thead>
<tr>
<th>Question 4</th>
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<tbody>
<tr>
<td>(a)</td>
<td>15D</td>
<td></td>
</tr>
<tr>
<td>(b)(i)</td>
<td>5C</td>
<td></td>
</tr>
<tr>
<td>(ii)</td>
<td>5D</td>
<td></td>
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</tbody>
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<thead>
<tr>
<th>Question 5</th>
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<tbody>
<tr>
<td>(a)(i)</td>
<td>10D</td>
<td></td>
</tr>
<tr>
<td>(ii)</td>
<td>5B</td>
<td></td>
</tr>
<tr>
<td>(b)(i)</td>
<td>5B</td>
<td></td>
</tr>
<tr>
<td>(ii)</td>
<td>5B</td>
<td></td>
</tr>
</tbody>
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<table>
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<th>Question 6</th>
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<tbody>
<tr>
<td>(a)</td>
<td>10D</td>
<td></td>
</tr>
<tr>
<td>(b)(i)+(ii)</td>
<td>15D</td>
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</tbody>
</table>

#### Section B

<table>
<thead>
<tr>
<th>Question 7</th>
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<tbody>
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<td>(a)(i)</td>
<td>10C</td>
<td></td>
</tr>
<tr>
<td>(a)(ii)</td>
<td>10C</td>
<td></td>
</tr>
<tr>
<td>(b)</td>
<td>10C</td>
<td></td>
</tr>
<tr>
<td>(b)(ii)</td>
<td>10C</td>
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<table>
<thead>
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</thead>
<tbody>
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<td>(a)(i)</td>
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<td></td>
</tr>
<tr>
<td>(a)(ii)</td>
<td>5B</td>
<td></td>
</tr>
<tr>
<td>(a)(iii)</td>
<td>5B</td>
<td></td>
</tr>
<tr>
<td>(a)(iv)</td>
<td>10D</td>
<td></td>
</tr>
<tr>
<td>(b)(i)</td>
<td>10D</td>
<td></td>
</tr>
<tr>
<td>(b)(ii)</td>
<td>5B</td>
<td></td>
</tr>
<tr>
<td>(b)(iii)</td>
<td>10C</td>
<td></td>
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<table>
<thead>
<tr>
<th>Question 9</th>
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<th></th>
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</thead>
<tbody>
<tr>
<td>(a)(i)</td>
<td>10C</td>
<td></td>
</tr>
<tr>
<td>(a)(ii)</td>
<td>10C</td>
<td></td>
</tr>
<tr>
<td>(a)(iii)</td>
<td>15D</td>
<td></td>
</tr>
<tr>
<td>(b)(i)</td>
<td>5B</td>
<td></td>
</tr>
<tr>
<td>(b)(ii)</td>
<td>10C</td>
<td></td>
</tr>
<tr>
<td>(b)(iii)</td>
<td>5B</td>
<td></td>
</tr>
</tbody>
</table>

[4]
Model Solutions & Marking Notes

Note: The model solutions for each question are not intended to be exhaustive – there may be other correct solutions. Any Examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his / her Advising Examiner.

<table>
<thead>
<tr>
<th>Q1</th>
<th>Model Solution – 25 Marks</th>
<th>Marking Notes</th>
</tr>
</thead>
</table>
| (a) | $-4 - 3i$ | Scale 5B (0, 2, 5)  
  Partial Credit:  
  • real or imaginary part correct |
| (b) | $r = \sqrt{1^2 + 1^2} = \sqrt{2}$  
  $\theta = \frac{\pi}{4}$  
  $(1+i)^8 = \left(\sqrt{2} \left(\cos \frac{\pi}{4} + i\sin \frac{\pi}{4}\right)\right)^8$  
  $(1+i)^8 = 16(\cos 2\pi + i\sin 2\pi)$  
  $(1+i)^8 = 16(1) = 16$ | Scale 10C (0, 3, 7, 10)  
  Low Partial Credit:  
  • correct answer without use of De Moivre’s  
  • modulus or argument correct  
  • formula  
  • statement of De Moivre’s  
  High Partial Credit:  
  • $16(\cos 2\pi + i\sin 2\pi)$  
  Note: not De Moivre and incorrect answer merits 0 marks |
| (c) | $z = \frac{(2-i) \pm \sqrt{(-2+i)^2 - 4(3-i)}}{2}$  
  $= \frac{(2-i) \pm \sqrt{-9}}{2}$  
  $= \frac{2 \pm i \sqrt{-9}}{2}$  
  $= 1 - 2i$ or $1 + i$ | Scale 10C (0, 3, 7, 10)  
  Low Partial Credit:  
  • root formula with some substitution  
  High Partial Credit  
  • formula fully substituted  
  Or  
  Scale 10C (0, 3, 7, 10)  
  Low Partial Credit:  
  • equation rearranged  
  • $-\frac{b}{a}$  
  High Partial Credit  
  • correct substitution |
Or

\[(z - 1 - i)(z - z_1)\]
\[= z^2 - z - zi - z_1 + z_1 + z_1 i\]
\[= z^2 - (1 + i + z_1)z + z_1(1 + i)\]
\[= z^2 + (-2 + i)z + (3 - i)\]
\[\Rightarrow z_1(1 + i) = 3 - i\]
\[z_1 = \frac{3 - i}{1 + i} \cdot \frac{1 - i}{1 - i} = 1 - 2i\]

Or

\[\frac{z - 1 + 2i}{z - 1 - i}\]
\[\frac{z^2 - z - iz}{z^2 - 2z + iz + 3 - i}\]
\[- z + 2iz + 3 - i\]
\[- z + 1 + i\]
\[2iz + 2 - 2i\]
\[2iz + 2 - 2i\]

\[z - 1 + 2i = 0\]
\[z = 1 - 2i\]

Or

\[(1 + i)(m + ni) = 3 - i\]
\[(m - n) + (m + n)i = 3 + (-1)i\]
\[m - n = 3 \quad \text{and} \quad m + n = -1\]
Solving \[m = 1 \quad \text{and} \quad n = -2\]

Or

Scale 10C (0, 3, 7, 10)

Low Partial Credit:
- correct factor(s)
- identification of equal terms

High Partial Credit
- long division formulated correctly
- two correct lines in division

Note: substitution of \((1 + i)\) merits 0 marks
Q2  Model Solution – 25 Marks

(a)  
\[ x^2 - 8x + 16 \geq 4 \]
\[ x^2 - 8x + 12 \geq 0 \]
\[ (x - 2)(x - 6) \geq 0 \]
\[ x = 2 \quad x = 6 \]
\[ \{x|x \leq 2\} \cup \{x|x \geq 6\} \]

Or

\[ x - 4 \geq 2 \cup x - 4 \leq -2 \]
\[ x \geq 6 \cup x \leq 2 \]

Or

Graphical method (must indicate range on X-axis somehow)

\[ y = |x - 4| \]

\[ x \leq 2 \cup x \geq 6 \]

Marking Notes

Scale 10C (0, 3, 7, 10)

Low Partial Credit:
- either side squared
- one correct linear inequality written
- stating range of natural numbers only

High Partial Credit:
- correct solutions to quadratic

Full Credit:
- correct answer without work

Note: use of natural numbers in range merits High Partial Credit at most
(b)  

\[ x = \frac{-3y - 1}{2} \]  

\[ \left(\frac{-3y - 1}{2}\right)^2 + \left(\frac{-3y - 1}{2}\right)(y) + 2y^2 = 4 \]

\[ 11y^2 + 4y - 15 = 0 \]

\[ (11y + 15)(y - 1) = 0 \]

\[ y = \frac{-15}{11} \text{ or } y = 1 \]

\[ x = \frac{-3\left(\frac{-15}{11}\right) - 1}{2} \text{ or } x = \frac{-3(1) - 1}{2} \]

\[ x = \frac{17}{11} \text{ or } x = -2 \]

or

\[ y = \frac{-2x - 1}{3} \]

\[ x^2 + x\left(\frac{-2x - 1}{3}\right) + 2\left(\frac{-2x - 1}{3}\right)^2 = 4 \]

\[ 11x^2 + 5x - 34 = 0 \]

\[ (11x - 17)(x + 2) = 0 \]

\[ x = \frac{17}{11} \text{ or } x = -2 \]

\[ y = \frac{-15}{11} \text{ or } y = 1 \]
Q3 Model Solution – 25 Marks

(a)
(i)  
<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>0.5</th>
<th>1</th>
<th>ln(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x) = \frac{2}{e^x}$</td>
<td>2</td>
<td>1.21</td>
<td>0.74</td>
<td>0.5</td>
</tr>
<tr>
<td>$g(x) = e^x - 1$</td>
<td>0</td>
<td>0.65</td>
<td>1.72</td>
<td>3</td>
</tr>
</tbody>
</table>

Scale 5C (0, 2, 4, 5)  
**Low Partial Credit**  
- one entry correct  

**High Partial Credit**  
- 5 entries correct

(ii)  
![Graph of f(x) and g(x)]

Scale 5C (0, 2, 4, 5)  
**Low Partial Credit**  
- one plot correct  

**High Partial Credit**  
- 5 plots correct  
- one correct graph  
- no labelling

**Notes:**  
- straight lines NOT acceptable  
- one clear label merits full credit  
- one ambiguous label merits High Partial Credit at most

(iii)  
$f(x) = g(x)$ when $x = 0.7$

Scale 5B (0, 2, 5)  
**Partial Credit**  
- point of intersection clearly indicated on graph, but value of $x$ not stated
### Q3 | Model Solution – Continued

(b)  

\[
\frac{e^x - 1}{e^x} = \frac{2}{e^x} \\
e^{2x} - e^x = 2 \\
(e^x)^2 - e^x - 2 = 0 \\
(e^x - 2)(e^x + 1) = 0 \\
e^x = 2 \text{ or } e^x = -1 \\
x = \ln 2 \\
\text{or } x = 0.693
\]

Or

\[(e^x)^2 - e^x - 2 = 0\]

Let \( y = e^x \Rightarrow y^2 - y - 2 = 0 \)

\[
y = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-2)}}{2(1)} \\
= \frac{1 \pm \sqrt{1 + 8}}{2} \\
= \frac{1 \pm 3}{2} \\
\Rightarrow y = 2 \text{ or } y = -1 \text{ (not possible)} \\
y = e^x \Rightarrow e^x = 2 \\
x = \ln 2 \text{ or } x = 0.693
\]

### Marking Notes

Scale 10C (0, 3, 7, 10)

**Low Partial Credit**
- substitution correct

**High Partial Credit**
- correct factors of quadratic
- root formula correctly substituted

\[
e^x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-2)}}{2(1)}
\]

**Note:** oversimplification of equation (i.e. not treating as quadratic) merits Low Partial Credit at most

Or

Scale 10C (0, 3, 7, 10)

**Low Partial Credit**
- substitution correct

**High Partial Credit**
- root formula correctly substituted

\[
y = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-2)}}{2(1)}
\]

**Note:** oversimplification of equation (i.e. not treating as quadratic) merits Low Partial Credit at most
<table>
<thead>
<tr>
<th>Q4</th>
<th>Model Solution – 25 Marks</th>
<th>Marking Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td></td>
<td>Scale 15D (0, 4, 7, 11, 15)</td>
</tr>
<tr>
<td></td>
<td>( P_1: \ 8^1 - 1 = 7 ) (divisible by 7)</td>
<td>Low Partial Credit</td>
</tr>
<tr>
<td></td>
<td>( P_k: \ \text{Assume} \ 8^k - 1 \text{ is divisible by 7} )</td>
<td>• ( P_1 ) step</td>
</tr>
<tr>
<td></td>
<td>( 8^k - 1 = 7M )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( 8^k = 7M + 1 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( P_{k+1}: \ 8^{k+1} - 1 = 8(8^k) - 1 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( = 8(7M + 1) - 1 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( = 56M + 7 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( = 7(8M + 1) )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( P_{k+1} ) is divisible by 7</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( P_1 ) is true</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( P_k ) true  ( \Rightarrow ) ( P_{k+1} ) is true</td>
<td></td>
</tr>
<tr>
<td></td>
<td>So, ( P_{k+1} ) true whenever ( P_k ) true.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Since ( P_1 ) true, then, by induction, ( P_n ) is true for all natural numbers ( \geq 1 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Or</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( P_{k+1} = 8^{k+1} - 1 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( = 8.8^k - 1 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( = (7 + 1).8^k - 1 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( = 7(8^k) + (8^k - 1) )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Obviously divisible by 7  From ( P_k )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>So, ( P_{k+1} ) true whenever ( P_k ) true.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Since ( P_1 ) true, then, by induction, ( P_n ) is true for all natural numbers ( \geq 1 )</td>
<td></td>
</tr>
</tbody>
</table>

\*Note: accept \( P_1 \) step, \( P_k \) step and \( P_{k+1} \) step in any order*
(b)

(i) 
\[ p = \log_a 2, \quad q = \log_a 3 \]
\[ \log_a \frac{8}{3} = \log_a 8 - \log_a 3 \]
\[ = \log_a (2^3) - \log_a 3 \]
\[ = 3 \log_a 2 - \log_a 3 \]
\[ = 3p - q \]

Scale 5C (0, 2, 4, 5)

*low Partial Credit*

- \[ \log_a 8 - \log_a 3 \]

*High Partial Credit*

- \[ \log_a 8 = 3 \log_a 2 \] (and/or \[ = 3p \])

(ii) 
\[ \log_a \frac{9a^2}{16} = \log_a (3a)^2 - \log_a (2)^4 \]
\[ = 2 \log_a 3 + 2 \log_a a - 4 \log_a 2 \]
\[ = 2q + 2(1) - 4p \]
\[ = 2q + 2 - 4p \]

Scale 5D (0, 2, 3, 4, 5)

*Low Partial Credit*

- \[ \log_a 9a^2 - \log_a 16 \]

*Mid Partial Credit*

- \[ 2 \log_a 3 \]
- \[ 2 \log_a a \]
- \[ 4 \log_a 2 \]
- \[ 4p \] or \[ 2q \] or \[ 2 \]

*High Partial Credit*

- \[ 2(\log_a 3 + \log_a a) - 4 \log_a 2 \] or equivalent
**Q5 Model Solution – 25 Marks**

<table>
<thead>
<tr>
<th>(a)</th>
<th>Model Solution – 25 Marks</th>
<th>Marking Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i)</td>
<td>[(5x - 9)^2 = (x - 1)^2 + (4x)^2]</td>
<td>Scale 10D (0, 2, 5, 8, 10)</td>
</tr>
<tr>
<td></td>
<td>[8x^2 - 88x + 80 = 0]</td>
<td>\textit{Low Partial Credit}</td>
</tr>
<tr>
<td></td>
<td>[x^2 - 11x + 10 = 0]</td>
<td>• any use of Pythagoras</td>
</tr>
<tr>
<td></td>
<td>[(x - 1)(x - 10) = 0]</td>
<td>\textit{Mid Partial Credit}</td>
</tr>
<tr>
<td></td>
<td>[x = 1 \text{ or } x = 10]</td>
<td>• fully correct substitution</td>
</tr>
<tr>
<td></td>
<td>[x = 10]</td>
<td>\textit{High Partial Credit}</td>
</tr>
<tr>
<td></td>
<td>Scale 10D (0, 2, 5, 8, 10)</td>
<td>• both roots correct</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(a)</th>
<th>Model Solution – 25 Marks</th>
<th>Marking Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>(ii)</td>
<td>Sides=9, 40, 41</td>
<td>Scale 5B (0, 2, 5)</td>
</tr>
<tr>
<td></td>
<td>[9^2 + 40^2 = 41^2]</td>
<td>\textit{Partial Credit}</td>
</tr>
<tr>
<td></td>
<td>[81 + 1600 = 1681]</td>
<td>• 9 or 40 or 41</td>
</tr>
<tr>
<td></td>
<td>[1681 = 1681]</td>
<td>• using 1 or −10 from candidates work</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(b)</th>
<th>Model Solution – 25 Marks</th>
<th>Marking Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i)</td>
<td>Function is bijective if inverse exists</td>
<td>Scale 5B (0, 2, 5)</td>
</tr>
<tr>
<td></td>
<td>[f^{-1}(x) = \frac{x + 2}{3}]</td>
<td>\textit{Partial Credit}</td>
</tr>
<tr>
<td></td>
<td>⇒ Function is injective.</td>
<td>• (f^{-1}(x)) written</td>
</tr>
<tr>
<td></td>
<td>\textit{or}</td>
<td>• (f(x)) drawn</td>
</tr>
<tr>
<td></td>
<td>Horizontal line test.</td>
<td>• (f(a) = f(b))</td>
</tr>
<tr>
<td></td>
<td>\textit{or}</td>
<td>• (f(a) = f(b))</td>
</tr>
<tr>
<td></td>
<td>[f(a) = f(b)]</td>
<td>• (f(a) = f(b))</td>
</tr>
<tr>
<td></td>
<td>[3a - 2 = 3b - 2]</td>
<td>• (a = b)</td>
</tr>
<tr>
<td></td>
<td>[\Rightarrow a = b]</td>
<td>• (a = b)</td>
</tr>
<tr>
<td></td>
<td>\textit{or}</td>
<td>• (a = b)</td>
</tr>
<tr>
<td></td>
<td>[\forall a, b \in A, f(a) = f(b) \Rightarrow a = b]</td>
<td>• (a = b)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(b)</th>
<th>Model Solution – 25 Marks</th>
<th>Marking Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>(ii)</td>
<td>[f(x) = 3x - 2]</td>
<td>Scale 5B (0, 2, 5)</td>
</tr>
<tr>
<td></td>
<td>[f^{-1}(x) = \frac{x + 2}{3}]</td>
<td>\textit{Partial Credit}</td>
</tr>
<tr>
<td></td>
<td>any relevant transpose</td>
<td>• any relevant transpose</td>
</tr>
<tr>
<td>Q6</td>
<td>Model Solution – 25 Marks</td>
<td>Marking Notes</td>
</tr>
<tr>
<td>----</td>
<td>---------------------------</td>
<td>---------------</td>
</tr>
<tr>
<td>(a)</td>
<td>[ f(x + h) - f(x) = (2x + 2h + 4)^2 - (2x + 4)^2 ]</td>
<td>Scale 10D (0, 2, 5, 8, 10)</td>
</tr>
<tr>
<td></td>
<td>[ \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} = ]</td>
<td>Low Partial Credit</td>
</tr>
<tr>
<td></td>
<td>[ \lim_{h \to 0} \frac{(2x + 2h + 4)^2 - (2x + 4)^2}{h} ]</td>
<td>• any ( f(x + h) )</td>
</tr>
<tr>
<td></td>
<td>[ = \lim_{h \to 0} \left( \frac{(4x^2 + 8hx + 4h^2 + 16x + 16h + 16) - (4x^2 + 16x + 16)}{h} \right) ]</td>
<td>Mid Partial Credit</td>
</tr>
<tr>
<td></td>
<td>[ = \lim_{h \to 0} \frac{8hx + 4h^2 + 16h}{h} = 8x + 16 ]</td>
<td>• limit of ( \frac{f(x + h) - f(x)}{h} )</td>
</tr>
<tr>
<td></td>
<td>or [ f(x) = (2x + 4)^2 = 4x^2 + 16x + 16 ]</td>
<td>High Partial Credit</td>
</tr>
<tr>
<td></td>
<td>[ f(x + h) = 4(x + h)^2 + 16(x + h) + 16 ]</td>
<td>• limit of ( \frac{(2x + 2h + 4)^2 - (2x + 4)^2}{h} )</td>
</tr>
<tr>
<td></td>
<td>[ = 4x^2 + 8hx + 4h^2 + 16x + 16h + 16 ]</td>
<td>Notes:</td>
</tr>
<tr>
<td></td>
<td>[ \lim_{h \to 0} \frac{8hx + 4h^2 + 16h}{h} ]</td>
<td>- omission of limit sign penalised once only</td>
</tr>
<tr>
<td></td>
<td>[ = 8x + 16 ]</td>
<td>- answer not from 1st Principles merits 0 marks</td>
</tr>
</tbody>
</table>
(b)
(i)+
(ii)

\[ y = x \cdot \sin \frac{1}{x} \]
\[
\frac{dy}{dx} = \sin \frac{1}{x} + x \left( \cos \frac{1}{x} \right) \left( -\frac{1}{x^2} \right)
\]
\[
\frac{dy}{dx} = \sin \frac{1}{x} - \frac{1}{x} \cos \frac{1}{x}
\]
\[
\frac{dy}{dx} = \sin \frac{\pi}{4} - \frac{\pi}{4} \cos \frac{\pi}{4}
\]
\[
= 0.15
\]

Scale 15D (0, 4, 7, 11, 15)

*Low Partial Credit*
- any correct differentiation

*Mid Partial Credit*
- product rule applied

*High Partial Credit*
- correct differentiation

**Note:** one penalty for calculator in wrong mode
<table>
<thead>
<tr>
<th>Q7</th>
<th>Model Solution – 40 Marks</th>
<th>Marking Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td></td>
<td>Scale 10C (0, 3, 7, 10)</td>
</tr>
</tbody>
</table>
| (i) | \( v = \frac{4}{3} \pi r^3 \Rightarrow \frac{dv}{dr} = 4\pi r^2 \) | **Low Partial Credit**  
\( \frac{dv}{dt} = 250 \text{ cm}^2/\text{s} \)  
\( \frac{dr}{dt} = \frac{dr}{dv} \cdot \frac{dv}{dt} = \frac{1}{4\pi r^2} \cdot 250 \)  
\( \frac{dr}{dt} = \frac{250}{4\pi 400} = \frac{5}{32\pi} \text{ cm/s} \)  
**High Partial Credit**  
\( \frac{dr}{dt} \)

| (ii) | \( a = 4\pi r^2 \Rightarrow \frac{da}{dr} = 8\pi r \) | Scale 10C (0, 3, 7, 10)  
**Low Partial Credit**  
\( \frac{da}{dt} = \frac{da}{dr} \cdot \frac{dr}{dt} = 8\pi r \cdot \frac{5}{32\pi} \)  
\( \frac{5(20)}{4} \)  
\( = 25 \text{ cm}^2/\text{s} \)  
**High Partial Credit**  
\( \frac{da}{dt} \)

| (b) |                         | Scale 10C (0, 3, 7, 10) |
| (i) | \(-x^2 + 10x = 0\) \(x(-x + 10) = 0\) \(x = 0\) or \(x = 10\) | **Low Partial Credit**  
\( \text{gets } x = 0 \) only  
**High Partial Credit**  
\( \text{quadratic factorised} \)  
**Note:** \( f'(x) = 0 \Rightarrow 2x - 10 = 0 \Rightarrow x = 5 \) merits 0 marks

| (ii) | \( \frac{1}{10} - 0 \int_0^{10} (-x^2 + 10x) \, dx \) | Scale 10C (0, 3, 7, 10)  
**Low Partial Credit**  
\( \text{integration set up} \)  
**High Partial Credit**  
\( \text{correct integration with some substitution} \)
<table>
<thead>
<tr>
<th>Q8</th>
<th>Model Solution – 55 Marks</th>
<th>Marking Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
| (i) | \( f(x) = -0.274x^2 + 1.193x + 3.23 \)  
\( f'(x) = -0.548x + 1.193 = 0 \)  
\( x = 2.177 \) m  
\( f(2.177) = -0.274(2.177)^2 \)  
\( + 1.193(2.177) + 3.23 \)  
\( = -1.2986 + 2.5972 + 3.23 \)  
\( = 4.529 \) m  
\( \text{or} \)  
\( -0.274(x^2 - \frac{1193}{274} x - \frac{1615}{137}) \)  
\( -0.274(x - \frac{1193}{548})^2 + 4.5285 \)  
Max Height = 4.529 m | Scale 10C (0, 3, 7, 10)  
Low Partial Credit  
• any correct differentiation  
• effort made at completing square  
• trial and error with more than one value of \( x \) tested  
High Partial Credit  
• \( x \) value correct  
Note: if correct answer by trial and error, must show points on each side of max point to be lower to earn full credit |
| (ii) | \( \tan \theta = -0.548(4.5) + 1.193 \)  
\( \tan \theta = -1.273 \)  
\( \theta = 51.8^\circ = 52^\circ \) | Scale 5B (0, 2, 5)  
Partial Credit  
• \( \tan \)  
Note: right angled triangles may appear in diagram given in equation |
| (iii) | Map \( A \rightarrow C \)  
\( (-0.5, 2.565) \rightarrow (0, 2) \)  
\( 2.177 - (-0.5) = 2.677 \)  
\( 4.529 - 0.565 = 3.964 \)  
\( (2.177, 4.529) \rightarrow (2.677, 3.964) \) | Scale 5B (0, 2, 5)  
Partial Credit  
• \((-0.5, 2.565) \rightarrow (0, 2) \)
\[ g(x) = ax^2 + bx + c \]
\[ C(0, 2) \in g(x) \implies c = 2 \]

\[ B(4.5, 3.05) \in g(x) \]
\[ 3.05 = a(4.5)^2 + b(4.5) + 2 \]
\[ \implies 20.25a + 4.5b = 1.05 \quad \ldots \text{(i)} \]

\[ g'(x) = 2ax + b = 0 \]
\[ \implies 2a(2.677) + b = 0 \]
\[ 5.354a + b = 0 \quad \ldots \text{(ii)} \]

From (i) and (ii)
\[ a = -0.273 \]
\[ b = 1.462 \]

\[ g(x) = -0.273x^2 + 1.462x + 2 \]

(Note: a third equation that could be used is
\[ 3.964 = a(2.677)^2 + b(2.677) + 2 \quad \ldots \text{(iii)} \]

Or

Equation of parabola with vertex \((h, k)\):
\[ g(x) = a(x - h)^2 + k \]
\[ C(0, 2) \text{ on curve: } (h, k) = (2.677, 3.964) \]
\[ 2 = a(-2.677)^2 + 3.964 \]
\[ -1.964 = a(7.166329) \]
\[ a = -0.27405 = -0.274 \]

Parabola:
\[ g(x) = -0.274[(x - 2.677)^2] + 3.964 \]

or
\[ g(x) = f(x - 0.5) - 0.565 \]
\[ g(x) = -0.274(x - 0.5)^2 + 1.193(x - 0.5) + 3.23 - 0.565 \]
\[ g(x) = -0.274x^2 + 1.467x + 2 \]
### (b)

#### (i) 200 m Race:

\[
y = a(b - x)^c
\]

\[
y = 4.99087(42.5 - 23.8)^{1.01} = 1000
\]

Javelin:

\[
y = a(x - b)^c
\]

\[
y = 15.9803(58.2 - 3.8)^{1.04} = 1020
\]

#### Scale 10D (0, 2, 5, 8, 10)

**Low Partial Credit**
- some relevant substitution into one formula

**Mid Partial Credit**
- one value of \( y \) found
- some relevant substitution into both formulas

**High Partial Credit**
- one value correct and some relevant substitution into second formula
- uses incorrect formula (once only)

#### (ii)

\[
y = a(x - b)^c
\]

\[
1295 = 15.9803(x - 3.8)^{1.04}
\]

\[
81.0373 = (x - 3.8)^{1.04} = z^{1.04}
\]

\[
\log z = \log 81.0373
\]

\[
z = 68.4343 = (x - 3.8)
\]

\[
x = 72.2343 = 72.23 \text{ m}
\]

#### Scale 5B (0, 2, 5)

**Partial Credit**
- some relevant substitution into formula

#### (iii)

\[
y = a(b - x)^c
\]

\[
1087 = 0.11193(254 - 121.84)^c
\]

\[
\frac{1087}{0.11193} = (132.16)^c
\]

\[
\log 9711.426 = c \log 132.16
\]

\[
c = \frac{\log 9711.426}{\log 132.16} = 1.88
\]

#### Scale 10C (0, 3, 7, 10)

**Low Partial Credit**
- some relevant substitution into formula

**High Partial Credit**
- fully correct substitution into formula
<table>
<thead>
<tr>
<th>Q9</th>
<th>Model Solution – 55 Marks</th>
<th>Marking Notes</th>
</tr>
</thead>
</table>
| (a)(i) | 4, 2, 1, \( \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16} \) | Scale 10C (0, 3, 7, 10)  
Low Partial Credit  
- some listing of terms  
- \( S_n \) formula  
High Partial Credit  
- listing of exactly 7 correct terms  
- formula fully substituted |
|  | \( S_n = \frac{a(1 - r^n)}{1 - r} \) |  
\( S_n = \frac{4 \left( 1 - \left( \frac{1}{2} \right)^n \right)}{1 - \frac{1}{2}} = 7.9375 \)  
\( - \frac{1}{2^n} = - \frac{1}{128} \)  
\( n = 7 \) | |
| (a) (ii) |  |  
Scale 10C (0, 3, 7, 10)  
Low Partial Credit  
- \( S_\infty \) formula  
High Partial Credit  
- formula fully substituted |
<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chg x</td>
<td>+4</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>1/4</td>
<td>0</td>
<td>-1/16</td>
<td>0</td>
<td>1/64</td>
</tr>
<tr>
<td>Chg y</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>-1/2</td>
<td>0</td>
<td>1/8</td>
<td>0</td>
<td>-1/32</td>
<td>0</td>
</tr>
</tbody>
</table>

(a)
(iii)

\[
S_{\infty} = \frac{4}{1 - \left( -\frac{1}{4} \right)} = 3.2 = \frac{16}{5}
\]

\[
S_{\infty} = \frac{2}{1 - \left( -\frac{1}{4} \right)} = 1.6 = \frac{8}{5}
\]

\[
(\frac{16}{5}, \frac{8}{5}) \text{ or } (3.2, 1.6)
\]

Scale 15D (0, 4, 7, 11, 15)

Low Partial Credit
- 2 extra entries correct in either row

Mid Partial Credit
- either row fully correct

High Partial Credit
- one co-ordinate correct

Notes:
- need to see \( S_{\infty} \) correctly used to move beyond Mid Partial Credit
- no \( S_{\infty} \) merits Mid Partial Credit at most

(b)
(i)

\[ G_5 = \text{Female, Male, Female, Female, Male} \]

Scale 5B (0, 2, 5)

Partial Credit
- one correct entry

(b)
(ii)

\[ G_6 = G_5 + G_4 = 5 + 3 = 8 \]

\[ G_7 = G_6 + G_5 = 8 + 5 = 13 \]

Scale 10C (0, 3, 7, 10)

Low Partial Credit
- \( G_6 = G_5 + G_4 \)
- \( G_7 = G_6 + G_5 \)
- \( G_7 \) or \( G_6 \) correct
- 8 and/or 13 without work

High Partial Credit
- correct substitution in both
\( G_3 = \frac{(1 + \sqrt{5})^3 - (1 - \sqrt{5})^3}{2^3 \sqrt{5}} = 2 \)

\[
(1 + \sqrt{5})^3 = (1 + 3\sqrt{5} + 3\sqrt{5}^2 + \sqrt{5}^3) = 16 + 8\sqrt{5}
\]

\[
(1 - \sqrt{5})^3 = (1 - 3\sqrt{5} + 3\sqrt{5}^2 - \sqrt{5}^3) = 16 - 8\sqrt{5}
\]

\[
G_3 = \frac{6\sqrt{5} + 2\sqrt{5}^3}{8\sqrt{5}} = \frac{6 + 2\sqrt{5}^2}{8} = \frac{16}{8} = 2 \quad \text{Q.E.D.}
\]

Scale 5B (0, 2, 5)

**Partial Credit**
- some correct substitution
- using approximate value for \( \sqrt{5} \)
- \( G_3 = 2 \)
- some effort at cubing

**Note:** use of \( \sqrt{5} \) as approximation, even if rounded off to 2 at end of work merits at most Partial Credit
Marking Scheme – Paper 1, Section A and Section B
Structure of the marking scheme
Candidate responses are marked according to different scales, depending on the types of response anticipated. Scales labelled A divide candidate responses into two categories (correct and incorrect). Scales labelled B divide responses into three categories (correct, partially correct, and incorrect), and so on. The scales and the marks that they generate are summarised in this table:

<table>
<thead>
<tr>
<th>Scale label</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>No of categories</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>5 mark scales</td>
<td>0, 2, 5</td>
<td>0, 2, 4, 5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10 mark scales</td>
<td>0, 5, 10</td>
<td>0, 3, 7, 10</td>
<td>0, 3, 5, 8, 10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>15 mark scales</td>
<td></td>
<td>0, 5, 10, 15</td>
<td></td>
<td>0, 4, 7, 11, 15</td>
<td></td>
</tr>
<tr>
<td>20 mark scales</td>
<td></td>
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<td></td>
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</tr>
<tr>
<td>25 mark scales</td>
<td></td>
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</tbody>
</table>

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the scheme, where necessary.

Marking scales – level descriptors

A-scales (two categories)
- incorrect response
- correct response

B-scales (three categories)
- response of no substantial merit
- partially correct response
- correct response

C-scales (four categories)
- response of no substantial merit
- response with some merit
- almost correct response
- correct response

D-scales (five categories)
- response of no substantial merit
- response with some merit
- response about half-right
- almost correct response
- correct response

E-scales (six categories)
- response of no substantial merit
- response with some merit
- response almost half-right
- response more than half-right
- almost correct response
- correct response

In certain cases, typically involving incorrect rounding, omission of units, a misreading that does not oversimplify the work or an arithmetical error that does not oversimplify the work, a mark that is one mark below the full-credit mark may also be awarded. Thus, for example, in scale 10C, 9 marks may be awarded. Throughout the scheme indicate by use of * where an arithmetical error occurs.
Summary of mark allocations and scales to be applied

### Section A

<table>
<thead>
<tr>
<th>Question 1</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) 10C</td>
<td></td>
</tr>
<tr>
<td>(b) 15D</td>
<td></td>
</tr>
</tbody>
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<table>
<thead>
<tr>
<th>Question 2</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) 10C</td>
<td></td>
</tr>
<tr>
<td>(b) 15D</td>
<td></td>
</tr>
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<table>
<thead>
<tr>
<th>Question 3</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) 15C</td>
<td></td>
</tr>
<tr>
<td>(b) 10D</td>
<td></td>
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<table>
<thead>
<tr>
<th>Question 4</th>
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</thead>
<tbody>
<tr>
<td>(a)(i) 15C</td>
<td></td>
</tr>
<tr>
<td>(a)(ii) 5C</td>
<td></td>
</tr>
<tr>
<td>(b) 5C</td>
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<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>(a)(i) 5B</td>
<td></td>
</tr>
<tr>
<td>(ii) 10C</td>
<td></td>
</tr>
<tr>
<td>(b) 10C</td>
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<table>
<thead>
<tr>
<th>Question 6</th>
<th></th>
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</thead>
<tbody>
<tr>
<td>(a) 10C</td>
<td></td>
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<tr>
<td>(b) 10C</td>
<td></td>
</tr>
<tr>
<td>(c) 5C</td>
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### Section B

<table>
<thead>
<tr>
<th>Question 7</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)(i) 10C</td>
<td></td>
</tr>
<tr>
<td>(a)(ii) 10B</td>
<td></td>
</tr>
<tr>
<td>(a)(iii) 10C</td>
<td></td>
</tr>
<tr>
<td>(a)(iv) 10C</td>
<td></td>
</tr>
<tr>
<td>(a)(v) 10D</td>
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</tr>
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<td>(b) 5C</td>
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<table>
<thead>
<tr>
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</thead>
<tbody>
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<td></td>
</tr>
<tr>
<td>(b) 5B</td>
<td></td>
</tr>
<tr>
<td>(c) 5C</td>
<td></td>
</tr>
<tr>
<td>(d)(i) 10C</td>
<td></td>
</tr>
<tr>
<td>(d)(ii) 10C</td>
<td></td>
</tr>
<tr>
<td>(e) 5B</td>
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<tr>
<td>(f) 5B</td>
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</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Question 9</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)(i) 10D</td>
<td></td>
</tr>
<tr>
<td>(a)(ii) 5C</td>
<td></td>
</tr>
<tr>
<td>(a)(iii) 15D</td>
<td></td>
</tr>
<tr>
<td>(b) 10C</td>
<td></td>
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<tr>
<td>(c) 5B</td>
<td></td>
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<tr>
<td>(d) 5C</td>
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</tr>
</tbody>
</table>
**Model Solutions & Detailed Marking Notes**

*Note: The model solutions for each question are not intended to be exhaustive – there may be other correct solutions. Any Examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his/her Advising Examiner.*

<table>
<thead>
<tr>
<th>Q1</th>
<th>Model Solution – 25 Marks</th>
<th>Marking Notes</th>
</tr>
</thead>
</table>
| (a) | Slope $AC = -\frac{2}{3}$  
perp. slope $= \frac{3}{2}$  
$y - 3 = \frac{3}{2}(x - 5)$  
$3x - 2y = 9$ | Scale 10C (0, 3, 7, 10)  
*Low Partial Credit*  
- slope formula with some relevant substitution  
- $3 = 5m+c$  
- $y - y_1 = m(x - x_1)$ with $x_1$ or $y_1$ or both substituted  
*High Partial Credit*  
- perpendicular slope  
- equation of line through $B$ parallel to $AC$ |
| (b) | Point of intersection of the altitudes  
Slope $AB = \frac{3 + 2}{5 - 6} = -\frac{5}{1}$  
perp. slope $= \frac{1}{5}$  
$y - 4 = \frac{1}{5}(x + 3)$  
$x - 5y + 23 = 0$  
Orthocentre:  
$3x - 2y = 9 \cap x - 5y = -23$  
$\Rightarrow y = 6 \quad x = 7$  
$(7,6)$  
*or*  
If $BC$ chosen:  
Slope $BC = \frac{3 - 4}{5 + 3} = -\frac{1}{8}$  
perp. slope $= 8$  
Equation of altitude:  
$y + 2 = 8(x - 6)$  
Equation: $8x - y = 50$  
Orthocentre:  
$3x - 2y = 9 \cap 8x - y = 50$  
$\Rightarrow y = 6 \quad x = 7$  
$(7,6)$ | Scale 15D (0, 4, 7,11,1.5)  
*Low Partial Credit*  
- demonstration of understanding of orthocentre (e.g. mentions altitude)  
- slope formula with some relevant substitution  
- altitude from part (a)  
*Mid Partial Credit*  
- equation of an altitude other than (a)  
- some relevant substitution towards finding a second altitude and altitude from (a)  
- correct construction  
*High Partial Credit*  
- two correct altitudes  
- correct construction with orthocentre (7, 6) |
### Q2 Model Solution – 25 Marks

#### (a)

\[
y - 6 = \frac{1}{7}(x + 1) \\
x - 7y + 43 = 0
\]

**Marking Notes**

- Scale 10C (0, 3, 7, 10)
  - Low Partial Credit: equation of line formula with some relevant substitution
  - High Partial Credit: equation of line not in required form

#### (b)

\[
D = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}} \\
D = \frac{|3(-g) + 4(-f) - 21|}{\sqrt{3^2 + 4^2}} \\
25 = | -3g - 4f - 21| \\
-3g - 4f - 21 = \pm 25 \\
\Rightarrow 3g + 4f = -46 \quad \text{(i)} \\
\text{and } 3g + 4f = 4 \quad \text{(ii)}
\]

But \((-g, -f) \in x - 7y + 43 = 0 \Rightarrow -g + 7f + 43 = 0 \quad \text{(iii)} \Rightarrow g = -7f + 43

Solving: \(g = 7f + 43 \text{ and } 3g + 4f = -46 \Rightarrow f = -7 \text{ and } g = -6\)
Centre \((6, 7)\)
\((x - 6)^2 + (y - 7)^2 = 25\)

**Marking Notes**

- Scale 15D (0, 4, 7, 11, 15)
  - Low Partial Credit: some correct substitution into relevant formula (line, circle, perpendicular distance).
  - Mid Partial Credit: one relevant equation in \(g\) and \(f\)
  - High Partial Credit: two relevant equations (either (i) and (iii) or (ii) and (iii))
Q3 | Model Solution – 25 Marks | Marking Notes
--- | --- | ---
(a) | \[
\frac{2 \cos \frac{7A + A}{2} \cos \frac{7A - A}{2}}{2 \cos \frac{7A + A}{2} \sin \frac{7A - A}{2}} = \frac{2 \cos 4A \cos 3A}{2 \cos 4A \sin 3A} = \frac{\cos 3A}{\sin 3A} = \cot 3A
\] | Scale 15C (0, 5, 10, 15)
\textit{Low Partial Credit}  
• sum to product formula with some substitution  
\textit{High Partial Credit}  
• sum to product formula fully substituted

(b) | Method 1:  
\[
\cos^2 \theta = \frac{1}{2} (1 + \cos 2\theta)
\]
\[
= \frac{1}{2} (1 + \frac{1}{9}) = \frac{5}{9}
\]
\[
\cos \theta = \pm \frac{\sqrt{5}}{3}
\]
\textbf{or}

Method 2:  
\[
\cos 2\theta = 1 - 2 \sin \theta = \frac{1}{9}
\]
\[
9 - 18 \sin^2 \theta = 1
\]
\[
\sin^2 \theta = \frac{4}{9} \Rightarrow \sin \theta = \pm \frac{2}{3} \Rightarrow \cos \theta = \pm \frac{\sqrt{5}}{3}
\]
\textbf{or}

Method 3:  
\[
\cos 2\theta = \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} = \frac{1}{9}
\]
\[
9 - 9 \tan^2 \theta = 1 + \tan^2 \theta
\]
\[
\tan^2 \theta = \frac{4}{5}
\]
\[
\Rightarrow \tan \theta = \pm \frac{2}{\sqrt{5}} \Rightarrow \cos \theta = \pm \frac{\sqrt{5}}{3}
\] | Scale 10D (0, 3, 5, 8, 10)
\textit{Low Partial Credit}  
• Use of a relevant formula in \(\cos 2\theta\)
  • \(\cos^{-1} \left(\frac{1}{3}\right) = 83.62^\circ\)
  • \(\theta = 41.8^\circ\)
\textit{Mid Partial Credit}  
• correct substitution (method 1)
  • expression in \(\sin^2 \theta\) (method 2)
  • expression in \(\tan^2 \theta\) (method 3)
  • expression in \(\cos^2 \theta\) (method 4)
  • \(\theta = 41.8^\circ\) and \(\theta = 132.2^\circ\) or \(\theta = 221.8^\circ\)
\textit{High Partial Credit}  
• one value only (e.g. \(+\frac{\sqrt{5}}{3}\))
  • values found for \(\cos 41.8^\circ\) and \(\cos 138.2^\circ\)
  or \(\cos 221.8^\circ\)
Method 4:

\[
\sin^2 \theta = \frac{1}{2} (1 - \cos 2\theta)
\]

\[
1 - \cos^2 \theta = \frac{1}{2} (1 - \cos 2\theta)
\]

\[
2 - 2\cos^2 \theta = 1 - \cos 2\theta
\]

\[
\cos^2 \theta = \frac{1 + \cos 2\theta}{2} = \frac{1 + \frac{1}{9}}{2}
\]

\[
\cos^2 \theta = \frac{5}{9}
\]

\[
\cos \theta = \pm \frac{\sqrt{5}}{3}
\]
<table>
<thead>
<tr>
<th>Q4</th>
<th>Model Solution – 25 Marks</th>
<th>Marking Notes</th>
</tr>
</thead>
</table>
| (a) | (i) \[ \angle ABD = \angle CBD = 90^\circ \quad \text{...(i)} \] \[ \angle BDC + \angle BCD = 90^\circ \quad \text{...angles in triangle sum to 180°} \] \[ \angle ADB + \angle BDC = \angle BDC + \angle BCD \] \[ \angle ADB = \angle BCD \quad \text{....(ii)} \] \[ \therefore \text{Triangles are equiangular (or similar)} \] or \[ \angle ABD = \angle CBD = 90^\circ \quad \text{...(i)} \] \[ \angle DAB = \angle DAC \quad \text{same angle} \Rightarrow \angle ADB = \angle DCA \quad \text{(reasons as above) which is also } \triangle DCB \quad \text{....(ii)} \] | Scale 15C (0, 5, 10, 15)  
*Low Partial Credit*  
- identifies one angle of same size in each triangle  
*High Partial Credit*  
- identifies second angle of same size in each triangle  
- implies triangles are similar without justifying (ii) in model solution or equivalent |
| (a) | (ii) \[ y = x \] \[ \Rightarrow y^2 = x \] \[ y = \sqrt{x} \] or \[ |AD|^2 + |DC|^2 = |AC|^2 \] \[ |AD| = \sqrt{x^2 + y^2} \] \[ |DC| = \sqrt{y^2 + 1} \] \[ x^2 + y^2 + y^2 + 1 = (x + 1)^2 \] \[ 2y^2 = 2x \] \[ y = \sqrt{x} \] \[ \text{Or} \] \[ \frac{\sqrt{x^2 + y^2}}{\sqrt{y^2 + 1}} = \frac{y}{1} \Rightarrow x^2 + y^2 = y^2(y^2 + 1) \] \[ y^4 = x^2 \Rightarrow y^2 = x \Rightarrow y = \sqrt{x} \] | Scale 5C (0, 2, 4, 5)  
*Low Partial Credit*  
- one set of corresponding sides identified  
- indicates relevant use of Pythagoras  
*High Partial Credit*  
- corresponding sides fully substituted  
- expression in \( y^2 \) or \( y^4 \), i.e. fails to finish |
Construction

Scale 5C (0, 2, 4, 5)

Low Partial Credit
- perpendicular line drawn at U or T
- relevant use of 1 cm length
- mid point of incorrect extended segment constructed

High Partial Credit
- correct mid-point constructed
<table>
<thead>
<tr>
<th>Q5</th>
<th>Model Solution – 25 Marks</th>
<th>Marking Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(i)</td>
<td>John: ✓ ✓ x ✓ ✓</td>
<td>Scale 5B (0, 2, 5)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Partial Credit</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• 1 correct column</td>
</tr>
<tr>
<td></td>
<td>David: ✓ x ✓ ✓ ✓</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Mike: x ✓ ✓ ✓ ✓</td>
<td></td>
</tr>
<tr>
<td>(a)</td>
<td></td>
<td>Scale 10C (0, 3, 7, 10)</td>
</tr>
<tr>
<td>(ii)</td>
<td>$P(\text{win}) = \left( \frac{1}{5} \times \frac{1}{6} \times \frac{3}{4} \right) + \left( \frac{1}{5} \times \frac{5}{6} \times \frac{1}{4} \right)$</td>
<td>Low Partial Credit</td>
</tr>
<tr>
<td></td>
<td>$+ \left( \frac{4}{5} \times \frac{1}{6} \times \frac{1}{4} \right) + \left( \frac{1}{5} \times \frac{1}{6} \times \frac{1}{4} \right)$</td>
<td>• one correct triple (numerical or descriptive)</td>
</tr>
<tr>
<td></td>
<td>$= \frac{13}{120}$</td>
<td>• probability of any one Miss</td>
</tr>
<tr>
<td>(b)</td>
<td></td>
<td>High Partial Credit</td>
</tr>
<tr>
<td></td>
<td>$P(A \cap B) = P(A) \times P(B)$</td>
<td>• 4 correct triples (numerical)</td>
</tr>
<tr>
<td></td>
<td>$0.1 = (x + 0.1) \times 0.4$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$0.4x = 0.06$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$x = 0.15$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>or</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$P(A</td>
<td>B) = P(A)$</td>
</tr>
<tr>
<td></td>
<td>$\frac{0.1}{0.4} = x + 0.1$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$x = 0.15$</td>
<td></td>
</tr>
</tbody>
</table>

[32]
(a)\[ P(M, 3, 3) = \frac{1}{26} \times \frac{1}{10} \times \frac{1}{10} = \frac{1}{2600} \]

Scale 10C (0, 3, 7, 10)

**Low Partial Credit**
- any correct relevant probability

**High Partial credit**
- correct probabilities but not expressed as single fraction or equivalent

**Note:** Accept correct answer without supporting work

(b)\[
\begin{array}{|c|c|c|c|}
\hline
\text{Event} & \text{Payout} & \text{Prob} (P(x)) & x.P(x) \\
\hline
\text{Win} & 1000 & \frac{1}{2600} & \frac{1000}{2600} \\
\text{let. 1 No.} & 50 & \frac{9}{2600} & \frac{450}{2600} \\
\text{let. 2 No.} & 50 & \frac{9}{2600} & \frac{450}{2600} \\
\text{let. only} & 50 & \frac{81}{2600} & \frac{4050}{2600} \\
\text{Fail to win} & 0 & & 0 \\
\hline
\end{array}
\]

\[
\sum x.P(x) = \frac{5950}{2600} = 2.29
\]

Club loses 29 cent per play

Or

\[
\begin{array}{|c|c|c|c|}
\hline
\text{Event} & \text{Payout} & \text{Prob} (P(x)) & x.P(x) \\
\hline
\text{Win} & -998 & \frac{1}{2600} & \frac{-998}{2600} \\
\text{let. 1 No.} & -48 & \frac{9}{2600} & \frac{-432}{2600} \\
\text{let. 2 No.} & -48 & \frac{9}{2600} & \frac{-432}{2600} \\
\text{let. only} & -48 & \frac{81}{2600} & \frac{-3888}{2600} \\
\text{Fail to win} & +2 & \frac{2500}{2600} & \frac{5000}{2600} \\
\hline
\end{array}
\]

\[
\sum x.P(x) = \frac{-750}{2600} = -29 \text{ cent}
\]
<table>
<thead>
<tr>
<th>(c)</th>
</tr>
</thead>
</table>
| Profit = Revenue – Pay-out  
| 600 = 845(x – 2.29) |
| \[ x = \frac{600 + 845(2.29)}{845} \]  
| \[ x = 3 \]  
| or  
| \[ \frac{600}{845} = 0.71 \]  
| \[ 0.71 + 2.29 = 3 \]  

Scale 5C (0, 2, 4, 5)

Low Partial Credit
- links profit, revenue and payout

High partial Credit
- formula fully substituted
<table>
<thead>
<tr>
<th>Q7</th>
<th>Model Solution – 55 Marks</th>
<th>Marking Notes</th>
</tr>
</thead>
</table>
| (a) | **(i)**  

\[ |EC|^2 = 3^2 + 2.5^2 = 15.25 \]

\[ |EC| = \sqrt{15.25} \]

\[ |EC| = 3.905 \]

\[ \Rightarrow |AC| = 1.9525 \]

\[ = 1.95 \]

| **(ii)**  

\[ \tan 50^\circ \frac{|AB|}{1.95} \]

\[ |AB| = 1.95(1.19175) = 2.3239 \]

\[ |AB| = 2.3 \]

| **(iii)**  

\[ |BC|^2 = 1.95^2 + 2.3^2 \]

\[ |BC| = 3.015377 \]

\[ |BC| = 3 \]

Also: \[ \sin 40^\circ = \frac{1.95}{|BC|} \text{ or } \cos 40^\circ = \frac{2.3}{|BC|} \text{ or } \]

\[ \cos 50^\circ = \frac{1.95}{|BC|} \text{ or } \sin 50^\circ = \frac{2.3}{|BC|} \]

| **(iv)**  

\[ 3^2 = 3^2 + 2.5^2 - 2(3)(2.5)\cos \alpha \]

\[ 15 \cos \alpha = 6.25 \]

\[ \alpha = 65^\circ \]

\[ \text{or} \]

\[ \cos \alpha = \frac{1.25}{3} \]

\[ \alpha = 65^\circ \]

Scale 10C (0, 3, 7, 10)

*Low Partial Credit*

- Pythagoras with relevant substitution

*High Partial Credit*

- \[ |EC| \text{ correct} \]

- \[ |AC| = \frac{1}{2}\sqrt{15.25} \]

Scale 10B (0, 5, 10)

*Partial Credit*

- \[ \tan \text{ formulated correctly} \]

Scale 10C (0, 3, 7, 10)

*Low Partial Credit*

- Pythagoras with relevant substitution

*High Partial Credit*

- \[ |BC| \text{ fully substituted} \]

- \[ |BC| = \frac{1.95}{\sin 40^\circ} \text{ (i.e. } |BC| \text{ isolated}) \]

Scale 10C (0, 3, 7, 10)

*Low Partial Credit*

- cosine rule with some relevant substitution

- \[ \cos \alpha \text{ with some relevant substitutions} \]

- identifies three sides of triangle \( BCD \)

*High Partial Credit*

- cosine rule with full relevant substitutions

- cosine ratio with full relevant substitutions

[35]
### (a)

\[ A = 2 \times \text{isosceles triangle} + 2 \times \text{equilateral triangle} \]

\[
= 2 \times \left[ \frac{1}{2} (2.5)(3) \sin 65^\circ \right] + 2 \times \left[ \frac{1}{2} (3)(3) \sin 60^\circ \right]
\]

\[
= 14.59
\]

\[ A = 15 \]

**Scale 10D (0, 3, 5, 8, 10)**

- **Low Partial Credit**
  - recognises area of 4 triangles

- **Mid Partial Credit**
  - Area of 1 triangle correct

- **High Partial Credit**
  - area of isosceles triangle and equilateral triangle

**Note:** Area = 4 isosceles or 4 equilateral triangles merit HPC at most

### (b)

\[
\tan 60^\circ = \frac{3}{|CA|}
\]

\[ \Rightarrow |CA| = \sqrt{3} \]

\[ |CE| = 2\sqrt{3} \]

\[ x^2 + x^2 = (2\sqrt{3})^2 \]

\[ x = \sqrt{6} \]

**Scale 5C (0, 2, 4, 5)**

- **Low Partial Credit**
  - effort at Pythagoras but without \(|CA|\) (or \(|CE|\))
  - \(|CA|\) found

- **High Partial Credit**
  - \(|CE| = 2\sqrt{3}\)
<table>
<thead>
<tr>
<th>Q8</th>
<th>Model Solution – 45 Marks</th>
<th>Marking Notes</th>
</tr>
</thead>
</table>
| (a) | Period = \( \frac{2\pi}{\frac{\pi}{6}} \) = 12 hours | Scale SC (0, 2, 4, 5)  
Low Partial Credit  
- some use of \( 2\pi \) or \( \frac{\pi}{6} \)  
- range of \( \cos \) function  
High partial credit  
- period or range correct  
Note: Accept correct period and/or range without work |
| Range = \( [1.6 - 1.5, 1.6 + 1.5] \) = [0.1 m, 3.1 m] | |
| (b) | Max = 1.6 + 1.5(1) = 3.1 m.  
or  
3.1 m from range | Scale SC (0, 2, 5)  
Partial Credit  
- max occurs when \( \cos A = 1 \) or \( t = 0 \)  
- effort at \( h'(t) \)  
Note: Accept correct answer without work |
| (c) | \[ h'(t) = 1.5(- \sin \left( \frac{\pi t}{6} \right) \frac{\pi}{6} \]  
\[ h'(2) = 1.5(- \sin \left( \frac{2\pi}{6} \right) \frac{\pi}{6} \]  
\[ = -0.68017 = -0.68 \text{ m/h} \]  
Tide is going out at a rate of 0.68 m per hour at 2 am | Scale SC (0, 2, 4, 5)  
Low Partial Credit  
- effort at differentiation  
High Partial Credit  
- correct numerical answer but not in context |
(d)(i)  

\[ h(t) = 1 \cdot 6 + 1 \cdot 5 \cos \left( \frac{\pi}{6} t \right) \]

<table>
<thead>
<tr>
<th>Time</th>
<th>12 am</th>
<th>3 am</th>
<th>6 am</th>
<th>9 am</th>
<th>12 pm</th>
<th>3 pm</th>
<th>6 pm</th>
<th>9 pm</th>
<th>12 am</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t )</td>
<td>0</td>
<td>3</td>
<td>6</td>
<td>9</td>
<td>12</td>
<td>15</td>
<td>18</td>
<td>21</td>
<td>24</td>
</tr>
<tr>
<td>Height</td>
<td>3·1</td>
<td>1·6</td>
<td>1·1</td>
<td>1·6</td>
<td>3·1</td>
<td>1·6</td>
<td>1·1</td>
<td>1·6</td>
<td>3·1</td>
</tr>
</tbody>
</table>

(d)(i)  

Scale 10C (0, 3, 7, 10)  

*Low Partial Credit*  
- one correct height  

*High Partial Credit*  
- five correct heights
Graph

<table>
<thead>
<tr>
<th>Scale</th>
<th>10C (0, 3, 7, 10)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low Partial Credit</td>
<td>one correct plot</td>
</tr>
<tr>
<td>High Partial Credit</td>
<td>at least 7 correct plots</td>
</tr>
<tr>
<td></td>
<td>plots correct but graph not sketched or sketched incorrectly</td>
</tr>
</tbody>
</table>
| (e) | | Scale 5B (0, 2, 5)  
Partial Credit  
• height of Low tide or High tide correctly identified  
Notes:  
(i) candidates may show work for this section on graph  
(ii) accept values from candidate’s graph  
(iii) accept correct answer from graph without work |
|---|---|---|
| **Low tide** = 0.1 m  
**High tide** = 3.1 m  
**Difference** = 3.1 − 0.1 = 3 m | | |

| (f) | | Scale 5B (0, 2, 5)  
Partial Credit  
• time of entry to port or leave port correctly identified  
• value(s) for \(h = 2\) and/or \(h = 1.5\) on sketch  
• time estimated using relevant values other than those required for the maximum time.  
Notes:  
(i) candidates may show relevant work for this section on graph  
(ii) accept values from candidate’s graph |
|---|---|---|
| **Enter port at 9:30 approx**  
**Leave port before 15:15 approx**  
**Time** = 15:15 − 9:30 = 5 hr 45 min approx. | | |
<table>
<thead>
<tr>
<th>Q9</th>
<th>Model Solution – 50 Marks</th>
<th>Marking Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(i)</td>
<td>$\mu = 39400, \sigma = 12920$</td>
<td>Scale 10D (0, 3, 5, 8, 10)</td>
</tr>
<tr>
<td></td>
<td>$z = \frac{x - \mu}{\sigma}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$z = 1.59$</td>
<td>Mid Partial Credit</td>
</tr>
<tr>
<td></td>
<td>$P(z &gt; 1.59) = 1 - P(z &lt; 1.59)$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$= 1 - 0.9441 = 0.0559$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$= 5.59%$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$= 5.6%$</td>
<td></td>
</tr>
<tr>
<td>(a)</td>
<td></td>
<td>Scale 5C (0, 2, 4, 5)</td>
</tr>
<tr>
<td>(ii)</td>
<td>$P(z \leq z_1) = 0.9$</td>
<td>Low Partial Credit</td>
</tr>
<tr>
<td></td>
<td>$z_1 = 1.28$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\Rightarrow z_2 = -1.28$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\Rightarrow \frac{x - 39400}{12920} = -1.28$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$x = 22862.40$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$= \€22 862$</td>
<td></td>
</tr>
<tr>
<td>(a)</td>
<td></td>
<td>Scale 15D (0, 4, 7, 11,15)</td>
</tr>
<tr>
<td>(iii)</td>
<td>$\mu = 39400, \sigma = 12920$, $\bar{x} = 38280, \ n = 1000$</td>
<td>Low Partial Credit</td>
</tr>
<tr>
<td></td>
<td>$H_0 \Rightarrow \mu = 39400$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$H_1 \Rightarrow \mu \neq 39400$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$z = \frac{38280 - 39400}{12920} = -2.74$</td>
<td>Mid Partial Credit</td>
</tr>
<tr>
<td></td>
<td>$\frac{12920}{\sqrt{1000}}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$-2.74 &lt; -1.96$</td>
<td>High Partial Credit</td>
</tr>
<tr>
<td></td>
<td>Result is significant. There is evidence to reject the null hypothesis</td>
<td></td>
</tr>
<tr>
<td></td>
<td>The mean income has changed.</td>
<td></td>
</tr>
</tbody>
</table>
Confidence Interval:
\[
\bar{x} \pm 1.96 \frac{\sigma}{\sqrt{n}}
\]

\[
39200 \pm 1.96 \frac{12920}{\sqrt{1000}}
\]

[38599.2, 40200.8]

38280 outside range
Result is significant. There is evidence to reject the null hypothesis
The mean income has changed.

Confidence Interval:
\[
\bar{x} \pm 1.96 \frac{\sigma}{\sqrt{n}}
\]

\[
38280 \pm 1.96 \frac{12920}{\sqrt{1000}}
\]

[37479.2, 39080.8]

39400 outside range
Result is significant. There is evidence to reject the null hypothesis
The mean income has changed.
<table>
<thead>
<tr>
<th>Q9</th>
<th>Marking Notes</th>
</tr>
</thead>
</table>
| (b)  | Scale 10C (0, 3, 7, 10)  
Low Partial Credit  
- interval formulated with some correct substitution  
High Partial Credit  
- interval formulated with fully correct substitution |
|      |              |
|      | \[26974 - 1.96 \left( \frac{5120}{\sqrt{400}} \right) \leq \mu \leq 26974 + 1.96 \left( \frac{5120}{\sqrt{400}} \right) \leq 26472.24 \leq \mu \leq 27475.76\] |
| (c)  | Scale 5B (0, 2, 5)  
Partial Credit  
- mentions 30 (or more) but not contextualised |
|      |              |
|      | The distribution of sample means will be normally distributed |
| (d)  | Scale 5C (0, 2, 4, 5)  
Low Partial Credit  
- \( \frac{1}{\sqrt{n}} \)  
High Partial Credit  
- \( n \) formulated with fully correct substitution |
|      |              |
|      | \( \frac{1}{\sqrt{n}} = 0.045 \)  
\( \frac{1}{0.045} = \sqrt{n} \)  
\( n = \left( \frac{1}{0.045} \right)^2 = 493.827 \) |
|      | Note: Accept 493 farmers or 494 farmers |
Marcanna breise as ucht freagairt trí Ghaeilge

(Bonus marks for answering through Irish)

Ba chóir marcanna de réir an ghnáthráta a bhronadh ar iarrthóirí nach ng nóthaíonn níos mó ná 75% d’iomlán na marcanna don pháipéar. Ba chóir freisin an marc bónaísin a shlánú síos.

Déantar an cinneadh agus an ríomhaireacht faoin marc bónaísin i gcás gach páipéar ar leithligh.

Is é 5% an gnáthráta agus is é 300 iomlán na marcanna don pháipéar. Mar sin, bain úsáid as an gnáthráta 5% i gcás iarrthóirí a ghnóthaíonn 225 marc nó níos lú, e.g. 198 marc × 5% = 9.9 ⇒ bónaí = 9 marc.

Má ghnóthaíonn an t-iarrthóir níos mó ná 225 marc, ríomhtar an bónaísin de réir na foirmle [300 – bunmharc] × 15%, agus an marc bónaísin a shlánú síos. In ionad an ríomhaireachta sin a dhéanamh, is féidir úsáid a bhaint as an tábla thíos.

<table>
<thead>
<tr>
<th>Bunmharc</th>
<th>Marc Bónais</th>
</tr>
</thead>
<tbody>
<tr>
<td>226</td>
<td>11</td>
</tr>
<tr>
<td>227 – 233</td>
<td>10</td>
</tr>
<tr>
<td>234 – 240</td>
<td>9</td>
</tr>
<tr>
<td>241 – 246</td>
<td>8</td>
</tr>
<tr>
<td>247 – 253</td>
<td>7</td>
</tr>
<tr>
<td>254 – 260</td>
<td>6</td>
</tr>
<tr>
<td>261 – 266</td>
<td>5</td>
</tr>
<tr>
<td>267 – 273</td>
<td>4</td>
</tr>
<tr>
<td>274 – 280</td>
<td>3</td>
</tr>
<tr>
<td>281 – 286</td>
<td>2</td>
</tr>
<tr>
<td>287 – 293</td>
<td>1</td>
</tr>
<tr>
<td>294 – 300</td>
<td>0</td>
</tr>
</tbody>
</table>
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