Leaving Certificate Examination

Mathematics (Project Maths)

Paper 2

Ordinary Level

Monday 14 June  Morning 9:30 – 12:00

300 marks

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<td>Centre stamp</td>
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Instructions

There are three sections in this examination paper:

Section 0  Area and Volume (old syllabus)  50 marks  1 question
Section A  Concepts and Skills        125 marks  5 questions
Section B  Contexts and Applications  125 marks  3 questions

Answer all nine questions, as follows:

In Section 0, answer Question 1.
In Section A, answer Questions 2, 3, 4, 5 and 6.
In Section B, answer:
  Question 7
  Question 8
  either Question 9A or Question 9B.

Write your answers in the spaces provided in this booklet. There is space for extra work at the back of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the booklet of Formulae and Tables. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

Marks will be lost if all necessary work is not clearly shown.

Answers should include the appropriate units of measurement, where relevant.

Answers should be given in simplest form, where relevant.
Question 1

(a) A circle is inscribed in a square as shown. 
   The radius of the circle is 9 cm.
   (i) Find the perimeter of the square.
   (ii) Calculate the area of the square.

(b) The diagram shows a sketch of a field $ABCD$ that has one uneven edge. At equal intervals of 5 m along $[BC]$, perpendicular measurements are made to the uneven edge, as shown on the sketch.

   (i) Use Simpson’s rule to estimate the area of the field.
(ii) The actual area of the field is 200 m$^2$. Find the percentage error in the estimate.

(c) (i) The diameter of a solid metal sphere is 9 cm. Find the volume of the sphere in terms of $\pi$.

(ii) The sphere is melted down. All of the metal is used to make a solid shape which consists of a cone on top of a cylinder, as shown in the diagram.

The cone and the cylinder both have height 8 cm. The cylinder and the base of the cone both have radius $r$ cm.

Calculate $r$, correct to one decimal place.
Answer all five questions from this section.

Question 2

(a) A line crosses the x-axis at \( x = 3 \) and the y-axis at \( y = 2 \).

Find the equation of the line.

(b) The equations of two lines \( l_1 \) and \( l_2 \) are:

\[
\begin{align*}
l_1 & : x + 3y = 8 \\
l_2 & : 6x - 2y = 15.
\end{align*}
\]

Determine whether these lines are perpendicular. Justify your answer clearly.
Question 3  
(25 marks)

(a) A circle has centre (0, 0) and passes through the point (3, 4).

(i) Find the equation of the circle.

(ii) Find the co-ordinates of the two points at which the circle crosses the y-axis.

(b) A circle has centre (2, 4) and touches the y-axis. Find the equation of the circle.
Question 4  (25 marks)

(a) Using a calculator, or otherwise, find the mean and standard deviation of the data in the following frequency table.

<table>
<thead>
<tr>
<th>x</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>f</td>
<td>16</td>
<td>38</td>
<td>26</td>
<td>20</td>
</tr>
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</table>

Mean = __________  
Standard deviation = __________

(b) Below is a stem-and-leaf plot of the heights of a group of students, in centimetres.

```
13 | 3
13 | 5  6
14 | 0  0  1
14 | 6  6  7  8
15 | 0  1  2  2  3  3
15 | 5  5  6  7
```

Key: 13 | 3 means 133 cm.

(i) How many students are in the group?

Answer: __________

(ii) What is the range of heights in the group?

(iii) What percentage of the students are between 145 cm and 154 cm in height?
Question 5 (25 marks)

(a) Helen has enough credit to download three songs from the internet. There are seven songs that she wants.

(i) How many different possible selections of three songs can she make?

(ii) If there is one particular song that she definitely wants, how many different selections can she now make?

(b) (i) Two fair coins are tossed. What is the probability of getting two heads?

(ii) Two fair coins are tossed 1000 times. How often would you expect to get two heads?

(c) Síle hands Pádraig a fair coin and tells him to toss it ten times. She says that if he gets ten heads then she will give him a prize. The first nine tosses are all heads. How likely is it that the last toss will also be a head? Tick the correct answer, and give a reason.

- Extremely unlikely
- Fairly unlikely
- 50-50 chance
- Fairly likely
- Almost certain

Reason: 

Question 6  (25 marks)

The diagram shows a triangle $ABC$ in which $|AB| = 6\, \text{cm}$, $|CB| = 10\, \text{cm}$, and $\angle ABC = 50^\circ$.

(a) Calculate the area of triangle $ABC$, correct to the nearest cm$^2$.

(b) Calculate the length of $[AC]$, correct to one decimal place.

(c) The triangle $A'BC'$ is the image of triangle $ABC$ under the enlargement with centre $B$ and scale factor 3. Find the area of $A'BC'$, correct to the nearest cm$^2$. 
Answer Question 7, Question 8, and either Question 9A or Question 9B.

Question 7  Probability and Statistics  (40 marks)

The table below gives motor insurance information for fully licensed, 17 to 20-year-old drivers in Ireland in 2007. All drivers who had their own insurance policy are included.

<table>
<thead>
<tr>
<th></th>
<th>Number of drivers</th>
<th>Number of claims</th>
<th>Average cost per claim</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>9634</td>
<td>977</td>
<td>€6108</td>
</tr>
<tr>
<td>Female</td>
<td>6743</td>
<td>581</td>
<td>€6051</td>
</tr>
</tbody>
</table>


Questions (a) to (e) below refer to drivers in the table above only.

(a) What is the probability that a randomly selected male driver made a claim during the year?
   Give your answer correct to three decimal places.

(b) What is the probability that a randomly selected female driver made a claim during the year?
   Give your answer correct to three decimal places.

(c) What is the expected value of the cost of claims on a male driver’s policy?

(d) What is the expected value of the cost of claims on a female driver’s policy?
(e) The male drivers were paying an average of €1688 for insurance in 2007 and the female drivers were paying an average of €1024. Calculate the average surplus for each group, and comment on your answer.

(Note: the *surplus* is the amount paid for the policy minus the expected cost of claims.)

<table>
<thead>
<tr>
<th></th>
<th>Male</th>
<th>Female</th>
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<tbody>
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<td></td>
<td></td>
<td></td>
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Comment:

(f) A 40-year-old female driver with a full license has a probability of 0.07 of making a claim during the year. The average cost of such claims is €3900. How much should a company charge such drivers for insurance in order to show a surplus of €175 per policy?

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Windows are sometimes in the shape of a pointed arch, like the one shown in the picture.

A person is designing such an arched window. The outline is shown in the diagram below the picture.

The centre for the arc $AB$ is $C$ and the centre for the arc $AC$ is $B$. $|BD| = 2.4$ metres and $|DE| = 1.8$ metres.

(a) Show that $\angle ABC = 60^\circ$.

(b) Find the length of the arc $AB$.
Give your answer in metres, correct to three decimal places.

(c) Find the length of the perimeter of the window.
Give your answer in metres, correct to two decimal places.
(d) Find the height of the window.
Give your answer in metres, correct to two decimal places.

(e) Make an accurate scaled drawing below of the outline of the window, using the scale 1:30.
That is, 1 cm on your diagram should represent 30 cm in reality.
Students in two schools – one in County Kerry and the other in County Offaly – were arguing about which county had the nicest weather in the summer. They agreed to record the highest temperature at each school on ten randomly selected days during the summer of 2009. The results were as follows:

<table>
<thead>
<tr>
<th>Temperature at Kerry school (°C)</th>
<th>Temperature at Offaly school (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>18.5</td>
<td>22.1</td>
</tr>
<tr>
<td>17.6</td>
<td>17.2</td>
</tr>
<tr>
<td>17.1</td>
<td>17.0</td>
</tr>
<tr>
<td>17.1</td>
<td>17.0</td>
</tr>
<tr>
<td>17.8</td>
<td>19.1</td>
</tr>
<tr>
<td>18.0</td>
<td>18.4</td>
</tr>
<tr>
<td>16.9</td>
<td>17.6</td>
</tr>
<tr>
<td>19.8</td>
<td>17.0</td>
</tr>
</tbody>
</table>

(a) Construct a back-to-back stem-and-leaf plot of the above data.

(b) State two differences between the two distributions.

Difference 1:

Difference 2:

(c) Perform a Tukey Quick Test on the data, stating clearly what can be concluded.
(d) The students in Offaly looked also at the amount of sunshine. They recorded the number of hours of sunshine each day in July 2009. The data are summarised in the table below.

<table>
<thead>
<tr>
<th>Hours of sunshine</th>
<th>≤ 2</th>
<th>≤ 4</th>
<th>≤ 6</th>
<th>≤ 8</th>
<th>≤ 10</th>
<th>≤ 12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of days</td>
<td>11</td>
<td>12</td>
<td>20</td>
<td>29</td>
<td>30</td>
<td>31</td>
</tr>
</tbody>
</table>

Draw a cumulative frequency curve to represent this data, using the scale indicated.

(e) Use your cumulative frequency curve to estimate:

(i) the median number of hours of sunshine

(ii) the number of days with more than 7 hours of sunshine.

(f) The mean amount of sunshine per day in Offaly in July generally is 4·24 hours. A day is chosen at random from the days in July 2009, as described in part (d) above. What is the probability that the amount of sunshine on that day was less than the mean?

(Data in this question adapted from *Monthly Weather Bulletin, July 2009*, at www.met.ie.)
(a) The photograph shows the Dockland building in Hamburg, Germany.

The diagram below is a side view of the building. It is a parallelogram.

The parallelogram is 29 metres high.
The top and bottom edges are 88 metres long.

(i) Find the area of this side of the building.

(ii) If $|BD| = |AD|$, find $|BC|$.
(iii) The lines $BC$ and $AD$ are parallel. Find the distance between these parallel lines.

(b) There is a theorem on your geometry course that can be used to construct the tangent to a circle at a given point on the circle. State this theorem and use it to construct the tangent to the circle shown at the point $P$.

Theorem:

(c) In the diagram, the line $l$ is a tangent to the circle. Find the values of $x$, $y$ and $z$.

$x = \underline{\hphantom{00000}}$

$y = \underline{\hphantom{00000}}$

$z = \underline{\hphantom{00000}}$