Coimisiún na Scrúduithe Stáit
State Examinations Commission

LEAVING CERTIFICATE EXAMINATION, 2007

MATHEMATICS – ORDINARY LEVEL

PAPER 1 (300 marks)

THURSDAY, 7 JUNE – MORNING, 9:30 to 12:00

Attempt SIX QUESTIONS (50 marks each).

WARNING: Marks will be lost if all necessary work is not clearly shown.

Answers should include the appropriate units of measurement, where relevant.
1. (a) Convert 164 miles to kilometres, taking 5 miles to be equal to 8 kilometres.

(b) €8500 was invested for 2 years at compound interest.

(i) The rate of interest for the first year was 4%.
    Find the amount of the investment at the end of the first year.

(ii) The amount of the investment at the end of the second year was €9237·80.
    Find the rate of interest for the second year.

(c) The table shows the hours Alan worked over four days.

<table>
<thead>
<tr>
<th>Day</th>
<th>Thursday</th>
<th>Friday</th>
<th>Saturday</th>
<th>Sunday</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hours worked</td>
<td>9</td>
<td>9</td>
<td>9·5</td>
<td>$h$</td>
</tr>
</tbody>
</table>

Alan’s basic rate of pay is €15·60 per hour.  
He is paid one and a half times the basic rate for work on Saturday and Sunday.

(i) Calculate Alan’s total pay for Thursday, Friday and Saturday.

(ii) Alan was paid a total of €702 for the four days’ work.  
    Find $h$, the number of hours Alan worked on Sunday.

2. (a) Find the solution set of $4x - 15 < 1$, $x \in \mathbb{N}$.

(b) (i) Find the value of $\frac{x+3y+5}{2x+2y}$ when $x = \frac{5}{2}$ and $y = \frac{1}{3}$.

(ii) Find the value of $x$ for which $2^{x+3} = 4^x$.

(c) (i) Solve the equation $x - \frac{1}{x} = 2$ and write your solutions in the form $a \pm \sqrt{b}$, where $a, b \in \mathbb{N}$.

(ii) Verify one of your solutions.
3. (a) Solve \( 2x = 3(5-x) \).

(b) Solve the simultaneous equations
\[
\begin{align*}
\frac{x}{4} - \frac{y}{3} &= \frac{5}{6} \\
2x - 6 &= 3y.
\end{align*}
\]

(c) Let \( f(x) = 2x^3 + 11x^2 + 4x - 5 \).

(i) Verify that \( f(-1) = 0 \).

(ii) Solve the equation \( 2x^3 + 11x^2 + 4x - 5 = 0 \).

4. (a) Given that \( i^2 = -1 \), simplify
\[
3(2 - 4i) + i(5 - 6i)
\]
and write your answer in the form \( x + yi \), where \( x, y \in \mathbb{R} \).

(b) Let \( z = 5 - 3i \).

(i) Plot \( z \) and \( -z \) on an Argand diagram.

(ii) Calculate \( |z - 1| \).

(iii) Find the value of the real number \( k \) such that \( k + 4z = 20 \).

(c) Let \( u = 3 + 2i \).

(i) Find the value of \( u^2 + \bar{u}^2 \), where \( \bar{u} \) is the complex conjugate of \( u \).

(ii) Investigate whether \( \frac{13}{u} = \bar{u} \).
5. (a) The $n$th term of a sequence is given by $T_n = 1 - n$.

(i) Find $T_5$, the fifth term.

(ii) Find $T_5 - T_{10}$ where $T_{10}$ is the tenth term.

(b) The first term of an arithmetic series is 3 and the common difference is 4.

(i) Find, in terms of $n$, an expression for $T_n$, the $n$th term.

(ii) How many terms of the series are less than 200?

(iii) Find the sum of these terms.

(c) The first two terms of a geometric series are $\frac{1}{3} + \frac{1}{9} + ...$

(i) Find $r$, the common ratio.

(ii) Find an expression for $S_n$, the sum of the first $n$ terms.

Write your answer in the form $\frac{1}{k}\left(1 - \frac{1}{3^n}\right)$ where $k \in \mathbb{N}$.

(iii) The sum of the first $n$ terms of the geometric series $\frac{p}{3} + \frac{p}{9} + ...$ is $1 - \frac{1}{3^n}$.

Find the value of $p$. 

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6. (a) Let $g(x) = x^2 - 6x, \ x \in \mathbb{R}$.

(i) Write down $g'(x)$, the derivative of $g(x)$.

(ii) For what value of $x$ is $g'(x) = 0$?

(b) A cold object is placed in a warm room. Its temperature $C$ degrees after time $t$ minutes is shown in the following graph.

(i) After what time interval is the temperature of the object 0 degrees?

(ii) What is the rise in temperature of the object in the first 10 minutes?

(iii) The relationship between the temperature $C$ and the time $t$ is given by

$$C = \frac{1}{2}(t + k).$$

Find the value of $k$.

(c) Let $f(x) = (5x - 2)^4$ for $x \in \mathbb{R}$.

(i) Find $f'(x)$, the derivative of $f(x)$.

(ii) Find the co-ordinates of the point on the curve $y = f(x)$ at which the slope of the tangent is 20.
7. (a) Differentiate \( 6x^4 - 3x^2 + 7x \) with respect to \( x \).

(b) (i) Differentiate \( (x^2 + 9)(4x^3 + 5) \) with respect to \( x \).

(ii) Given that \( y = \frac{3x}{2x + 5} \), find \( \frac{dy}{dx} \).

Write your answer in the form \( \frac{k}{(2x + 3)^n} \), where \( k, n \in \mathbb{N} \).

(c) A car starts from rest at the point \( a \).

\[ b \quad \quad a \]

The distance of the car from \( a \), after \( t \) seconds, is given by

\[ s = 2t^2 + 2t \]

where \( s \) is in metres.

(i) Find the speed of the car after 2 seconds.

(ii) Find the acceleration of the car.

(iii) The distance from \( a \) to the point \( b \) is 24 metres. After how many seconds does the car reach the point \( b \)?

8. (a) Let \( f(x) = \frac{1}{4}(6 - 2x) \) for \( x \in \mathbb{R} \). Evaluate \( f(5) \).

(b) Differentiate \( x^2 - 3x \) with respect to \( x \) from first principles.

(c) Let \( f(x) = \frac{1}{x+7}, \quad x \in \mathbb{R}, \quad x \neq -7 \).

(i) Given that \( f(k) = 1 \), find \( k \).

(ii) Find \( f'(x) \), the derivative of \( f(x) \).

(iii) Show that the curve \( y = f(x) \) has no turning points.