Coimisiún na Scrúduithe Stáit
State Examinations Commission

LEAVING CERTIFICATE EXAMINATION, 2010

MATHEMATICS – ORDINARY LEVEL

PAPER 1 (300 marks)

FRIDAY, 11 JUNE – AFTERNOON, 2:00 to 4:30

Attempt SIX QUESTIONS (50 marks each)

WARNING: Marks will be lost if all necessary work is not clearly shown.

Answers should include the appropriate units of measurement, where relevant.
1. (a) Express 40 metres as a fraction of 1 kilometre. Give your answer in its simplest form.

(b) (i) Calculate the value of
\[
\frac{57.6 + 80.44}{1.3 \times 10^4}
\]
and write your answer correct to three decimal places.

(ii) An importer buys an item for £221 sterling when the rate of exchange is €1 = £0.85 sterling.

He sells it at a profit of 14% of the cost price.

Calculate, in euro, the price for which he sells the item.

(c) (i) What sum of money invested at 5% per annum compound interest will amount to €8682 in 3 years?

Give your answer correct to the nearest euro.

(ii) A sum of €P was invested at r % per annum compound interest.

The interest for the first year was €220.

The interest for the second year was €228.80.

Calculate r and P.

2. (a) Find the values of x which satisfy
\[
2(3 + 4x) \leq 22, \quad \text{where } x \in \mathbb{N}.
\]

(b) Solve for x and y
\[
2x - y = 1
\]
\[
x^2 - xy = -6.
\]

(c) (i) Show, by division, that 3x + 1 is a factor of \(3x^3 + 4x^2 - 89x - 30\).

(ii) Hence, or otherwise, solve the equation
\[
3x^3 + 4x^2 - 89x - 30 = 0.
\]
3. (a) Given that \(3(b + a) = t(6 - a)\),
calculate the value of \(a\) when \(t = 3\) and \(b = -4\).

(b) Solve for \(x\)
\[
5(x + 1)^2 = 2(x + 1) + 5.
\]
Give your answer correct to two decimal places.

(c) (i) \(2 + \sqrt{3}\) is a root of the equation \(x^2 - 4x + c = 0\), where \(c\) is a real number.
Find the value of \(c\) and write down the other root.

(ii) The equation \(x^2 + 10x + k = 0\) has equal roots.
Find the value of the real number \(k\) and write down the value of each root.

4. (a) Given that \(i^2 = -1\), simplify
\[
(4 + 2i)(3 - i)
\]
and write your answer in the form \(x + yi\), where \(x, y \in \mathbb{R}\).

(b) Let \(u = 4 + 3i\) and \(w = 6 - 8i\).

(i) Find the value of the real number \(k\) such that \(|u| = k \cdot |w|\).

(ii) Express \(\frac{w}{u}\) in the form \(x + yi\).

(c) Let \(z = a + bi\), where \(a, b \in \mathbb{R}\).
Find the value of \(a\) and the value of \(b\) for which
\[
3z - 10i = (2 - 3i)z.
\]
5. (a) The first term of a geometric sequence is 4 and the common ratio is 0.5. Write down the first five terms of the sequence.

(b) In an arithmetic series, the first term is 6 and the fifth term is 22.

(i) Find \( d \), the common difference.

(ii) Find \( T_{14} \), the fourteenth term.

(iii) Find \( S_{20} \), the sum of the first twenty terms.

(c) In a geometric series, the fourth term is 9 and the seventh term is 243.

(i) Find \( r \), the common ratio.

(ii) Find \( a \), the first term.

(iii) Find \( S_{8} \), the sum of the first eight terms.

6. (a) Let \( h(x) = x^2 + 1 \), where \( x \in \mathbb{R} \).

Write down a value of \( x \) for which \( h(x) = 50 \).

(b) Let \( g(x) = \frac{1}{x - 2} \), where \( x \in \mathbb{R} \) and \( x \neq 2 \).

(i) Copy and complete the following table:

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>1</th>
<th>1.5</th>
<th>1.75</th>
<th>2.25</th>
<th>2.5</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( g(x) )</td>
<td></td>
<td>-1</td>
<td>-4</td>
<td></td>
<td></td>
<td>2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(ii) Draw the graph of the function \( g \) in the domain \( 0 \leq x \leq 4 \).

(c) Let \( f(x) = x - \frac{5}{x} \), where \( x \in \mathbb{R} \) and \( x \neq 0 \).

(i) Find \( f'(x) \), the derivative of \( f(x) \).

(ii) Find the co-ordinates of the two points at which the tangent to the curve \( y = f(x) \) is parallel to the line \( y = 6x \).
7. (a) Differentiate \( x^2 - 6x + 1 \) with respect to \( x \).

(b) (i) Differentiate \( 5 - 3x \) with respect to \( x \) from first principles.

(ii) Given that \( y = (x^2 - 4)(3x - 1) \), find the value of \( \frac{dy}{dx} \) when \( x = 2 \).

(c) The speed, \( v \), of an object at time \( t \) is given by
\[
v = 96 + 40t - 4t^2
\]
where \( t \) is in seconds and \( v \) is in metres per second.

(i) At what times will the speed of the object be 96 metres per second?

(ii) What will the acceleration of the object be at \( t = 2.5 \) seconds?

(iii) At what value of \( t \) will the acceleration become negative?

8. Let \( f(x) = x^3 - 3x + 1 \), where \( x \in \mathbb{R} \).

(i) Find \( f(-3), f(-2), f(0), f(2) \) and \( f(3) \).

(ii) Find \( f'(x) \), the derivative of \( f(x) \).

(iii) Find the co-ordinates of the local maximum point and of the local minimum point of the curve \( y = f(x) \).

(iv) Draw the graph of the function \( f \) in the domain \(-3 \leq x \leq 3\).

(v) Find the range of values of \( k \) for which the equation
\[
x^3 - 3x + 1 = k
\]
has three real solutions (roots).
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