# Contents

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>GENERAL GUIDELINES FOR EXAMINERS – PAPER 1</td>
<td>2</td>
</tr>
<tr>
<td>QUESTION 1</td>
<td>3</td>
</tr>
<tr>
<td>QUESTION 2</td>
<td>7</td>
</tr>
<tr>
<td>QUESTION 3</td>
<td>11</td>
</tr>
<tr>
<td>QUESTION 4</td>
<td>14</td>
</tr>
<tr>
<td>QUESTION 5</td>
<td>19</td>
</tr>
<tr>
<td>QUESTION 6</td>
<td>23</td>
</tr>
<tr>
<td>QUESTION 7</td>
<td>28</td>
</tr>
<tr>
<td>QUESTION 8</td>
<td>32</td>
</tr>
<tr>
<td>GENERAL GUIDELINES FOR EXAMINERS – PAPER 2</td>
<td>36</td>
</tr>
<tr>
<td>QUESTION 1</td>
<td>37</td>
</tr>
<tr>
<td>QUESTION 2</td>
<td>41</td>
</tr>
<tr>
<td>QUESTION 3</td>
<td>44</td>
</tr>
<tr>
<td>QUESTION 4</td>
<td>49</td>
</tr>
<tr>
<td>QUESTION 5</td>
<td>53</td>
</tr>
<tr>
<td>QUESTION 6</td>
<td>56</td>
</tr>
<tr>
<td>QUESTION 7</td>
<td>61</td>
</tr>
<tr>
<td>QUESTION 8</td>
<td>65</td>
</tr>
<tr>
<td>QUESTION 9</td>
<td>69</td>
</tr>
<tr>
<td>QUESTION 10</td>
<td>72</td>
</tr>
<tr>
<td>QUESTION 11</td>
<td>76</td>
</tr>
<tr>
<td>BONUS MARKS FOR ANSWERING THROUGH IRISH</td>
<td>80</td>
</tr>
</tbody>
</table>
MARKING SCHEME

LEAVING CERTIFICATE EXAMINATION 2005

MATHEMATICS – HIGHER LEVEL – PAPER 1

GENERAL GUIDELINES FOR EXAMINERS – PAPER 1

1. Penalties of three types are applied to candidates’ work as follows:
   - Blunders - mathematical errors/omissions (-3)
   - Slips - numerical errors (-1)
   - Misreadings (provided task is not oversimplified) (-1).

   Frequently occurring errors to which these penalties must be applied are listed in the scheme. They are labelled: B1, B2, B3,…, S1, S2,…, M1, M2,…etc. These lists are not exhaustive.

2. When awarding attempt marks, e.g. Att(3), note that
   - any correct, relevant step in a part of a question merits at least the attempt mark for that part
   - if deductions result in a mark which is lower than the attempt mark, then the attempt mark must be awarded
   - a mark between zero and the attempt mark is never awarded.

3. Worthless work is awarded zero marks. Some examples of such work are listed in the scheme and they are labelled as W1, W2,…etc.

4. The phrase “hit or miss” means that partial marks are not awarded – the candidate receives all of the relevant marks or none.

5. The phrase “and stops” means that no more work is shown by the candidate.

6. Special notes relating to the marking of a particular part of a question are indicated by an asterisk. These notes immediately follow the box containing the relevant solution.

7. The sample solutions for each question are not intended to be exhaustive lists – there may be other correct solutions. Any examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his/her advising examiner.

8. Unless otherwise indicated in the scheme, accept the best of two or more attempts – even when attempts have been cancelled.

9. The same error in the same section of a question is penalised once only.

10. Particular cases, verifications and answers derived from diagrams (unless requested) qualify for attempt marks at most.

11. A serious blunder, omission or misreading results in the attempt mark at most.

12. Do not penalise the use of a comma for a decimal point, e.g. €5.50 may be written as €5,50.
QUESTION 1

Part (a) 10 marks Att 3
Part (b) 20 (5, 15) marks Att (2, 5)
Part (c) 20 (5, 5, 5, 5) marks Att (2, 2, 2, 2)

Part (a) 10 marks Att 3

1(a) Solve the simultaneous equations:

\[ \frac{x}{5} - \frac{y}{4} = 0 \]

\[ 3x + \frac{y}{2} = 17 \]

1(a)  

(i) \[ \frac{x}{5} - \frac{y}{4} = 0 \times 20 \Rightarrow 4x - 5y = 0 \]

(ii) \[ 3x + \frac{y}{2} = 17 \times 10 \Rightarrow 30x + 5y = 170 \]

\[ 34x \]

\[ = 170 \]

\[ x = 5 \]

(i) \[ 4x - 5y = 0 \]

\[ 4(5) \]

\[ = 5y \]

\[ \Rightarrow y = 4 \]

\[ x = 5 \]

\[ y = 4 \]

Blunders (-3)
B1 second variable not found.

Slips (-1)
S1 numerical.
S2 not changing sign in subtraction.

Attempts
A1 no solution.
A2 correct solution by trial and error.

Worthless
W1 values for \( x \) and \( y \).

Part (b)(i) 5 marks Att 2

1(b)(i)
Express \( 2^{\frac{1}{2}} + 2^{\frac{1}{2}} + 2^{\frac{1}{2}} + 2^{\frac{1}{2}} \) in the form \( 2^{\frac{p}{q}} \), where \( p, q \in \mathbb{Z} \).

1(b)(i)

\[ 2^{\frac{1}{2}} + 2^{\frac{1}{2}} + 2^{\frac{1}{2}} + 2^{\frac{1}{2}} = 4 \left( 2^{\frac{1}{2}} \right) = \left( 2^{\frac{1}{2}} \right)^2 = 2^{\frac{2}{2}} \]

Blunders (-3)
B1 indices.

Slips (-1)
S1 not elements of \( \mathbb{Z} \).
Part (b)(ii) 15 marks

Let \( f(x) = ax^3 + bx^2 + cx + d \).
Show that \((x-t)\) is a factor of \( f(x) - f(t) \)

\[ f(x) = ax^3 + bx^2 + cx + d \]
\[ f(t) = at^3 + bt^2 + ct + d \]
\[ f(x) - f(t) = a(x^3 - t^3) + b(x^2 - t^2) + c(x - t) \]
\[ = a(x - t)(x^2 + tx + t^2) + b(x - t)(x + t) + c(x - t) \]
\[ = (x - t)[a(x^2 + tx + t^2) + b(x + t) + c] \]

or

\[ f(x) = ax^3 + bx^2 + cx + d \]
\[ f(t) = at^3 + bt^2 + ct + d \]
\[ f(x) - f(t) = ax^3 + bx^2 + cx - at^3 - bt^2 - ct \]
\[ \frac{ax^2 + (at + b)x + (at^2 + bt + c)}{(x - t)} \]
\[ \frac{ax^3 - atx^2}{(at + b)x^2 - (at + b)x} \]
\[ \frac{at^2 + bt + c}{at^2 + bt + c} - \frac{x - at^3 - bt^2 - ct}{x - at^3 - bt^2 - ct} \]

* Accept solution by division by \((x-t)\) for full marks.

**Blunders (-3)**
B1 indices.
B2 factors

**Slips (-1)**
S1 numerical.
S2 not changing sign when subtracting in division.
Part (c) 20(5, 5, 5, 5) marks  

1(c) $(x - p)^2$ is a factor of $x^3 + qx + r$

Show that $27r^2 + 4q^3 = 0$

Express the roots of $3x^2 + q = 0$ in terms of $p$.

<table>
<thead>
<tr>
<th>Factor</th>
<th>5 marks</th>
<th>Att 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Values</td>
<td>5 marks</td>
<td>Att 2</td>
</tr>
<tr>
<td>Show</td>
<td>5 marks</td>
<td>Att 2</td>
</tr>
<tr>
<td>Express</td>
<td>5 marks</td>
<td>Att 2</td>
</tr>
</tbody>
</table>

1(c) (Show)

$$x^2 - 2px + p^2 \frac{x + 2p}{x^3 + qx + r}$$

$$x^3 - 2px^2 + p^2x$$

$$\frac{2px^2 - p^2x + qx + r}{2px^2 - 4p^2x + 2p^3}$$

$$\frac{3p^2x + qx + r - 2p^3}{0} = 0$$

Remainder must = 0 since $(x - p)^2$ is a factor

$$\Rightarrow (3p^2 + q)x + (r - 2p^3) = (0)x + (0)$$

$$\Rightarrow (i) : 3p^2 + q = 0 \quad \Rightarrow q = -3p^2$$

$$\Rightarrow (ii) : r - 2p^3 = 0 \quad \Rightarrow r = 2p^3$$

$$\therefore 27r^2 + 4q^3 = 27(2p^3)^2 + 4(-3p^2)^3$$

$$= 108p^6 - 108p^6$$

$$= 0$$

or

If $(x - p)^2$ is a factor of $f(x)$, then let $(x + a)$ be other factor.

$$\therefore (x^2 - 2px + p^2)(x + a) = x^3 + qx + r$$

$$x^3 + (-2p + a)x^2 + (p^2 - 2pa)x + p^2(a) = x^3 + (0)x^2 + (q)x + r$$

Equating like to like

(i) $-2p + a = 0$  

(ii) $p^2 - 2pa = q$  

(iii) $p^2a = r$

(i) $a = 2p$  

$q = p^2 - 2p(2p) = -3p^2$  

$r = p^2(2p) = 2p^3$

$$27r^2 + 4q^3 = 27(2p^3)^2 + 4(-3p^2)^3$$

$$= 108p^6 - 108p^6$$

$$= 0$$
\( (Express) \quad 3x^2 + q = 0 \)
\[
\begin{align*}
3x^2 &= -q \\
3x^2 &= -(-3p^2) \\
3x^2 &= 3p^2 \\
x^2 &= p^2 \\
x &= \pm p
\end{align*}
\]

**Blunders (-3)**
- B1 indices.
- B2 not like to like.
- B3 root from equation.
- B4 \( r \) not found, having found \( q \).
- B5 roots from equation (in “express” part).

**Slips (-1)**
- S1 numerical.
- S2 not changing sign in subtraction (division).

**Attempts**
- A1 remainder \( \neq 0 \) in division.
- A2 any attempt at division.
QUESTION 2

Part (a) 10 marks  Att 3

Part (b) 20 (5, 5, 5, 5) marks  Att (2, 2, 2, 2)

Part (c) 20 (5, 5, 5, 5) marks  Att (2, 2, 2, 2)

Part (a) 10 marks  Att 3

2(a) Solve for $x$: $|x - 1| < 7$, where $x \in \mathbb{R}$

2(a)

$|x - 1| < 7$

$\Rightarrow -7 < x - 1 < 7$

$\therefore -7 < x - 1 \text{ and } x - 1 < 7$

$-6 < x \text{ and } x < 8$

$\therefore -6 < x < 8$

or

2(a)

$|x - 1| < 7$

$(x - 1)^2 < 49$

$x^2 - 2x + 1 < 49$

$x^2 - 2x - 48 < 0$

Solve:

$x^2 - 2x - 48 = 0$

$(x + 6)(x - 8) = 0$

$x + 6 = 0 \text{ or } x - 8 = 0$

$x = -6 \text{ or } x = 8$

$f(x) < 0$ when $-6 < x < 8$

or

2(a)

$|x - 1| < 7$

$(x - 1)^2 < 49$

$x^2 - 2x - 48 < 0$

$(x - 8)(x + 6) < 0$

case I:

$(x - 8) > 0 \text{ and } (x + 6) < 0$

$x > 8 \text{ and } x < -6$

not possible

case II:

$(x - 8) < 0 \text{ and } (x + 6) > 0$

$x < 8 \text{ and } x > -6$

$\therefore -6 < x < 8$

Blunders (-3)

B1 upper limit.

B2 lower limit.

B3 expansion of $(x - 1)^2$, once only.
The cubic equation $4x^3 + 10x^2 - 7x - 3 = 0$ has one integer root and two irrational roots. Express the irrational roots in simplest surd form.

\[
\begin{align*}
\Rightarrow x = -3 & \text{ is a root} \quad \Rightarrow (x + 3) \text{ is a factor} \\
& \\
& \\
4x^2 - 2x - 1 \\
\therefore x + 3 \bigg| 4x^3 + 10x^2 - 7x - 3 \\
& \\
4x^3 + 12x^2 \\
& \\
2x^2 - 7x \\
& \\
-2x^2 - 6x \\
& \\
-x - 3 \\
& \\
-x - 3 \\
& \\
& \\
\therefore x = \frac{2 \pm \sqrt{4 + 16}}{2(4)} = \frac{2 \pm \sqrt{20}}{8} = \frac{2 \pm 2\sqrt{5}}{8} = \frac{1 \pm \sqrt{5}}{4} \\
{\text{Irrational roots:}} & \\
\frac{1 + \sqrt{5}}{4}, \quad \frac{1 - \sqrt{5}}{4}
\end{align*}
\]
Finds root $x = -3$ as above, and continues as follows:

$$x = -3 \text{ is a root } \Rightarrow (x + 3) \text{ is a factor}$$

$$\therefore \text{ other factor } = (4x^2 + ax - 1)$$

$$\therefore (x + 3)(4x^2 + ax - 1) = 4x^3 + 10x^2 - 7x - 3$$

$$4x^3 + 12x^2 + ax^2 + 3ax - x - 3 = 4x^3 + 10x^2 - 7x - 3$$

$$4x^3 + (a + 12)x^2 + (3a - 1)x - 3 = 4x^3 + 10x^2 - 7x - 3$$

Equating coefficients:

(i) $a + 12 = 10$ and/or (ii) $(3a - 1) = -7$

$$a = -2 \quad \quad \quad \quad \quad \quad 3a = -6$$

$$a = -2 \quad \quad \quad \quad \quad \quad a = -2$$

$$f(x) = (x + 3)(4x^2 - 2x - 1) = 0$$

Irrational roots: $\frac{1 \pm \sqrt{5}}{4}$, as above.

**Blunders (-3)**

B1 indices.
B2 root formula, once only.
B3 not like to like..
B4 deduction factor from root or no factor.

**Slips (-1)**

S1 numerical.
S2 not changing sign in subtraction (Division).
S3 roots not in simplest form, once only.

<table>
<thead>
<tr>
<th>Part (c)</th>
<th>20 (5, 5, 5, 5) marks</th>
<th>Att (2, 2, 2, 2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2(c)(i)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Let $f(x) = \frac{x^2 + k^2}{mx}$, where $k$ and $m$ are constants and $m \neq 0$

(i) Show that $f(km) = f\left(\frac{k}{m}\right)$

(ii) $a$ and $b$ are real numbers such that $a \neq 0, b \neq 0$ and $a \neq b$.

Show that if $f(a) = f(b)$, then $ab = k^2$. 
\( f(x) = \frac{x^2 + k^2}{mx}, \ [k, m \text{ constants}] \)

\[(i)\] show that \( f(km) = f\left(\frac{k}{m}\right) \)

\[
f(km) = \frac{(km)^2 + k^2}{m(km)} = \frac{k^2(m^2 + 1)}{k^2(m^2)} = \frac{k}{m^2}(m^2 + 1)
\]

\[
f\left(\frac{k}{m}\right) = \frac{\left(\frac{k}{m}\right)^2 + k^2}{m\left(\frac{k}{m}\right)} = \frac{\frac{k^2}{m^2} + k^2}{k} = \frac{k^2 + m^2k^2}{m^2k}
\]

\[
= \frac{k^2(1 + m^2)}{k^2(m^2)}
\]

\[
= \frac{k}{m^2}(m^2 + 1)
\]

\( \Rightarrow f(km) = f\left(\frac{k}{m}\right) \)

\[(ii)\] \( f(a) = \frac{a^2 + k^2}{ma} \)

\( f(b) = \frac{b^2 + k^2}{mb} \)

\[f(a) = f(b) \Rightarrow \frac{a^2 + k^2}{ma} = \frac{b^2 + k^2}{mb}\]

multiply across by \( mab \):

\[
b(a^2 + k^2) = a(b^2 + k^2)
\]

\[
a^2b + bk^2 = ab^2 + ak^2
\]

\[
a^2b - ab^2 = ak^2 - bh^2
\]

\[
ab(a - b) = k^2(a - b)
\]

\[(a - b) \neq 0 \Rightarrow ab = k^2\]

**Blunders (-3)**

B1 indices
QUESTION 3

Part (a) 10 (5, 5) marks Att (2, 2)
Part (b) 20 (5, 10, 5) marks Att (2, 3, 2)
Part (c) 20 (5, 5, 5, 5) marks Att (2, 2, 2, 2)

Part (a) \( A^3 \) 5 marks Att 2
\( A^{-1} \) 5 marks Att 2

3(a) Given that \( A = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \), show that \( A^3 = A^{-1} \).

\[
A = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \\
A^{-1} = \frac{1}{-1-0} \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} = A
\]

\[
A^3 = A.A.A = A^2.A \\
A^2 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I
\]

\[
A^3 = A^2.A = I.A = A = A^{-1}
\]

or

\[
A^{-1} = A \text{ as above, and:} \\
A^3 = A.A.A \\
= A^{-1}.A.A \\
= IA \\
= A^{-1}
\]

or

\[
A^{-1} = A \text{ as above, and:} \\
A^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}^3 \\
= \begin{pmatrix} (1)^3 & (0)^3 \\ (0)^3 & (-1)^3 \end{pmatrix} \\
= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \\
= A
\]

Blunders (-3)
B1 formula inverse.

Slips (-1)
S1 each incorrect element.
S2 numerical.
Part (b) Solve the quadratic equation:

\[2iz^2 + (6 + 2i)z + (3 - 6i) = 0, \text{ where } i^2 = -1\]

### Part (b) Values in formula

<table>
<thead>
<tr>
<th>Evaluate (\sqrt{b^2 - 4ac})</th>
<th>10 marks</th>
<th>Att 3</th>
</tr>
</thead>
</table>

Roots 5 marks Att 2

### 3(b) Solve:

\[2iz^2 + (6 + 2i)z + (3 - 6i) = 0\]

\[
z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

\[
z = \frac{-6 - 2i \pm \sqrt{(6 + 2i)^2 - 4(2i)(3 - 6i)}}{2(2i)}
\]

\[
z = \frac{-6 - 2i \pm \sqrt{36 + 24i^2 + 4i^2 - 24i + 48i^2}}{4i}
\]

\[
z = \frac{-6 - 2i \pm \sqrt{36 - 52}}{4i}
\]

\[
z = \frac{-6 - 2i \pm \sqrt{-16}}{4i}
\]

\[
z = \frac{-6 - 2i \pm 4i}{4i}
\]

\[
z = \frac{-6 - 2i + 4i}{4i} \quad \text{or} \quad \frac{-6 - 2i - 4i}{4i}
\]

\[
z = \frac{-6 + 2i}{4i} \quad \text{or} \quad \frac{-6 - 6i}{4i}
\]

\[
z = \frac{-3 + i}{2i} \quad \text{or} \quad \frac{-3 - 3i}{2i}
\]

\[
z_1 = \frac{-3 + i}{2i} \cdot \frac{i}{i} = \frac{-3i + i^2}{2i^2} = \frac{-3i - 1}{-2} = \frac{1 + 3i}{2}
\]

\[
z_2 = \frac{-3 - 3i}{2i} \cdot \frac{i}{i} = \frac{-3i - 3i^2}{2i^2} = \frac{3 - 3i}{-2} = \frac{-3 + 3i}{2}
\]

**Blunders (-3)**

B1 indices.

B2 \(i\).

B3 expansion \((6 + 2i)^2\) once only.

B4 root formula, once only.

**Slips (-1)**

S1 numerical.

S2 \(i\) in denominator.

**Attempts**

A1 3 marks for \(\sqrt{a + bi}\) and stops.

A2 2 marks for \(z = a + bi\) and stops.
Part (c) 20(5, 5, 5, 5) marks Att (2, 2, 2, 2)

3(c) (i) \( z = \cos \theta + i \sin \theta \). Use De Moivre’s theorem to show that
\[ z^n + \frac{1}{z^n} = 2 \cos n \theta \] for \( n \in \mathbb{N} \).

(ii) Expand \( \left( z + \frac{1}{z} \right)^4 \) and hence express \( \cos^4 \theta \) in terms of \( \cos 4 \theta \) and \( \cos 2 \theta \).

<table>
<thead>
<tr>
<th>Part (c) (i)</th>
<th>5 marks</th>
<th>Part (c) (ii)</th>
<th>5 marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/(z^n)</td>
<td></td>
<td>( z + \frac{1}{z} = (\cos \theta + i \sin \theta) + (\cos \theta + i \sin \theta)^{-1} )</td>
<td></td>
</tr>
<tr>
<td>Value</td>
<td>5 marks</td>
<td>= ( \cos \theta + i \sin \theta + \cos \theta - i \sin \theta )</td>
<td></td>
</tr>
<tr>
<td>(ii) Expansion</td>
<td>5 marks</td>
<td>= 2 \cos \theta</td>
<td></td>
</tr>
<tr>
<td>Express</td>
<td>5 marks</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[
\left( z + \frac{1}{z} \right)^4 = z^4 + 4 z^2 + 4 \left( \frac{1}{z^2} \right) + 1 \left( \frac{1}{z^4} \right) \\
= \frac{1}{16} \left[ 2 \cos 4 \theta + 8 \cos 2 \theta + 6 \right] \\
= \frac{1}{8} \left[ 2 \cos 4 \theta + 4 \cos 2 \theta + 3 \right] \\
= \frac{1}{16} [2 \cos 4 \theta + 8 \cos 2 \theta + 6] \\
= \frac{1}{8} [2 \cos 4 \theta + 4 \cos 2 \theta + 3] \\
\]

**Blunders (-3)**

B1 statement De Moivre, once only.  
B2 application De Moivre.  
B3 binomial expansion.  
B4 \( i \)  
B5 answer not in required form.  
B6 indices.

**Slips (-1)**

S1 numerical  
W1 not using De Moivre.  
W2 not using “hence” in part (ii).
### QUESTION 4

<table>
<thead>
<tr>
<th>Part (a)</th>
<th>10 marks</th>
<th>Att 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Part (b)</td>
<td>20 (5, 5, 5, 5) marks</td>
<td>Att (2, 2, 2, 2)</td>
</tr>
<tr>
<td>Part (c)</td>
<td>20 (15, 5) marks</td>
<td>Att (5, 2)</td>
</tr>
</tbody>
</table>

#### 4(a) Write the recurring decimal $0.6\overline{3}$ as an infinite geometric series and hence as a fraction.

\[
\begin{align*}
0.\overline{63} & = 0.636363\ldots \\
& = 0.63 + 0.0063 + 0.000063 + \ldots \\
& = \frac{63}{100} + \frac{63}{10000} + \frac{63}{1000000} + \ldots \\
\therefore \quad a &= \frac{63}{100} \quad r = \frac{1}{100} \\
S_{\infty} &= \frac{a}{1-r} \\
&= \frac{\frac{63}{100}}{1-\frac{1}{100}} = \frac{\frac{63}{100}}{\frac{99}{100}} = \frac{63}{99} = \frac{7}{11}
\end{align*}
\]

**Blunders (-3)**
- B1 indices.
- B2 formula for infinite series.
- B3 incorrect $a$.
- B4 incorrect $r$.
- B5 not as infinite series.

**Slips (-1)**
- S1 numerical.

**Attempts**
- A1 correct answer with no work or by other method (i.e. not using geometric series).
Part (b)(i)  The first three terms in the binomial expansion of \((1 + kx)^n\) are \(1 - 21x + 189x^2\). Find the value of \(n\) and the value of \(k\).

<table>
<thead>
<tr>
<th>Part(b)(i) equations</th>
<th>5 marks</th>
<th>Att 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>((1 + kx)^n = 1 + \binom{n}{1}(kx) + \binom{n}{2}(kx)^2 + \ldots)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(= 1 + (nk)x + \frac{n(n-1)}{2!} \cdot k^2 x^2 + \ldots)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(= 1 + (nk)x + \left[\frac{n(n-1)k^2}{2}\right]x^2 + \ldots)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(= 1 - 21x + 189x^2)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

[i]: \(nk = -21\)  
[ii]: \(\frac{n(n-1)k^2}{2} = 189\)

\[i\] \(\Rightarrow k = \frac{-21}{n}\)

\[ii\] \(\Rightarrow n(n-1)k^2 = 378\)

sub. in: \(n(n-1)\left(\frac{-21}{n}\right)^2 = 378\)

\((n^2 - n)(441) = 378n^2\)

\(441n^2 - 441n - 378n^2 = 0\)

\(63n^2 - 441n = 0\)

\(63(n - 7) = 0\)

\(\Rightarrow n = 0 \text{ or } n = 7\)

\(\therefore n = 7 \Rightarrow k = \frac{-21}{7} = -3\)

\(n = 7; \ k = -3\)

* Since must be integers, accept correct values by observation from \(nk = -21\), with verification.

**Blunders** (-3)
B1 errors in binomial expansion, once only.
B2 \(\binom{n}{r}\)
B3 indices.
B4 not like to like
B5 factors
B6 value from factor.
B7 second value not found, having found first.

**Slips** (-1)
S1 numerical.
A sequence is defined by \( u_n = (2 - n)2^{n-1} \).

Show that \( u_{n+2} - 4u_{n+1} + 4u_n = 0 \), for all \( n \in \mathbb{N} \).

\[
\begin{align*}
4 (b)(ii) & \quad u_n = (2 - n)2^{n-1} \\
& \quad u_{n+1} = [2 - (n + 1)]2^{(n+1)-1} = (1 - n)2^n \\
& \quad u_{n+2} = [2 - (n + 2)]2^{(n+2)-1} = (-n)2^{n+1} \\
& \quad u_{n+2} - 4u_{n+1} + 4u_n = \left(-n.2^{n+1}\right) - 4\left[(1-n)2^n\right] + 4\left[(2-n)2^{n-1}\right] \\
& \quad = -n.2^{n+1} - \left(2^2\right)\left(2^n\right)\left(1-n\right) + 2^2\left(2^{n-1}\right)\left(2-n\right) \\
& \quad = -n.2^{n+1} - 2^{n+2} + 2n.2^{n+1} + 2.2^{n+1} - n.2^{n+1} \\
& \quad = 2.2^{n+1} - 2^{n+2} \\
& \quad = 2.2^{n+1} - 2.2^{n+1} = 0
\end{align*}
\]

or

\[
\begin{align*}
4 (b)(ii) & \quad u_n = (2 - n)2^{n-1} = 2^n - n.2^{n-1} = 2^n - \frac{n}{2}(2^n) \\
& \quad u_{n+1} = [2 - (n + 1)]2^{(n+1)-1} = (1 - n)2^n \\
& \quad u_{n+2} = [2 - (n + 2)]2^{(n+2)-1} = (-n)2^{n+1} = -2n.2^n \\
\text{Let} \quad a = 2^n \\
\therefore \quad u_{n+2} - 4u_{n+1} + 4u_n = -2na - 4(1-n)a + 4\left[a - \frac{na}{2}\right] \\
& \quad = -2na - 4a + 4na + 4a - 2na \\
& \quad = 0
\end{align*}
\]

Blunders (-3)
B1 indices.
B2 factors.

Slips (-1)
S1 numerical.

Attempts
A1 must do some correct relevant work with indices in “show”.
4 (c) (i) Show that \( \frac{a+b}{2} \leq \sqrt{\frac{a^2+b^2}{2}} \), where \( a \) and \( b \) are real numbers.

(ii) The lengths of the sides of a right-angled triangle are \( a, b \) and \( c \), where \( c \) is the length of the hypotenuse.

Using the result from part (i), or otherwise, show that \( a + b \leq c\sqrt{2} \).

\[
\begin{align*}
\text{Part (c)(i)} & \quad 20(15,5) \text{ marks} & \quad \text{Att (5,2)} \\
4(c)(i) & \quad \frac{a+b}{2} \leq \sqrt{\frac{a^2+b^2}{2}} \\
\text{case: } (a+b) \text{ positive:} & \quad \Leftrightarrow \left( \frac{a+b}{2} \right)^2 \leq \frac{a^2+b^2}{2} \\
& \quad \frac{a^2+2ab+b^2}{4} \leq \frac{a^2+b^2}{2} \\
& \quad a^2+2ab+b^2 \leq 2a^2+2b^2 \\
& \quad 0 \leq a^2-2ab+b^2 \\
& \quad 0 \leq (a-b)^2 \\
\Rightarrow & \quad \text{True when } (a+b) \text{ positive.}
\end{align*}
\]

\[
\begin{align*}
\text{case: } (a+b) \text{ negative:} & \quad (a+b)<0 \Rightarrow \frac{(a+b)}{2} < 0 \\
& \quad \Rightarrow \frac{a+b}{2} \leq \sqrt{\frac{a^2+b^2}{2}}, \text{ since } \sqrt{x} > 0 \text{ always.} \\
\Rightarrow & \quad \text{True when } (a+b) \text{ negative.}
\end{align*}
\]

or

\[
\begin{align*}
4(c)(i) & \quad (a-b)^2 \geq 0 \\
& \quad \text{for all } a, b \in \mathbb{R}.
\end{align*}
\]

\[
\begin{align*}
& \quad a^2-2ab+b^2 \geq 0 \\
& \quad (a^2-2ab+b^2)+(a^2+b^2) \geq (a+b)^2 \\
& \quad 2a^2+2b^2 \geq a^2+2ab+b^2 \\
& \quad 2(a^2+b^2) \geq (a+b)^2 \\
\text{divide across by 4:} & \quad \frac{a^2+b^2}{2} \geq \frac{(a+b)^2}{4} \\
& \quad \frac{a^2+b^2}{2} \geq \left( \frac{a+b}{2} \right)^2 \\
& \quad \sqrt{\frac{a^2+b^2}{2}} \geq \sqrt{\left( \frac{a+b}{2} \right)^2} \geq \frac{a+b}{2}
\end{align*}
\]
From (i) above, \[ \frac{a + b}{2} \leq \sqrt{\frac{a^2 + b^2}{2}} = \sqrt{\frac{c^2}{2}} = \frac{c}{\sqrt{2}}. \]

\[ a + b \leq \frac{c}{\sqrt{2}} \]
\[ a + b \leq \frac{2c}{\sqrt{2}} \]
\[ a + b \leq c\sqrt{2} \]

**Blunders** (-3)

B1 indices
B2 inequality sign.
B3 deduction.
B4 \( a \) and \( b \) both positive.
B5 expansion \( (a - b)^2 \).
B6 right angled triangle.

**Slips** (-1)

S1 numerical.

**Worthless**

W1 particular values for \( a \) and \( b \).
QUESTION 5

<table>
<thead>
<tr>
<th>Part (a)</th>
<th>15 (5, 5, 5) marks</th>
<th>Att (2, 2, 2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Part (b)</td>
<td>15 (5, 5, 5) marks</td>
<td>Att (2, 2, 2)</td>
</tr>
<tr>
<td>Part (c)</td>
<td>20 (5, 5, 5, 5) marks</td>
<td>Att (2, 2, 2, 2)</td>
</tr>
</tbody>
</table>

| Part (a) | 15(5, 5, 5) marks | Att 2 |

<table>
<thead>
<tr>
<th>5(a) Solv for: $\sqrt{10-x} = 4 - x$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Part 5(a) Quadratic</strong></td>
</tr>
<tr>
<td><strong>Factors</strong></td>
</tr>
<tr>
<td><strong>Solution</strong></td>
</tr>
</tbody>
</table>

5(a) $\sqrt{10-x} = 4 - x$

$10 - x = (4 - x)^2$

$10 - x = 16 - 8x + x^2$

$0 = x^2 - 7x + 6$

$0 = (x - 1)(x - 6)$

$\Rightarrow x - 1 = 0$ or $x - 6 = 0$

$x = 1 \quad x = 6$

Test:

$x = 1$ L.H.S.: $\sqrt{10-x} = \sqrt{10-1} = \sqrt{9} = 3$

R.H.S.: $4 - x = 4 - 1 = 3$

$\therefore$ L.H.S. = R.H.S.

$x = 6$ L.H.S.: $\sqrt{10-x} = \sqrt{10-6} = \sqrt{4} = 2$

R.H.S.: $4 - x = 4 - 6 = -2$

$\therefore$ L.H.S. $\neq$ R.H.S.

6 is not a solution.

Ans: $x = 1$.

**Blunders (-3)**

B1 indices.

B2 expansion $(4 - x)^2$ once only.

B3 factors.

B4 root formula once only.

B5 deduction values from factors.

**Slips (-1)**

S1 numerical.

S2 excess value.

**Attempts**

A1 $x = 1$ and no other works merits 2 marks.
5(b) Prove by induction that \[ \sum_{r=1}^{n} (3r - 2) = \frac{n}{2}(3n - 1) \]

Part (b) \( P(1) \) 5 marks
Assume \( 5 \) marks
\[ P(k + 1) \] 5 marks

\[ \sum_{r=1}^{n} (3r - 2) = \frac{n}{2}(3n - 1) \]

Test \( n = 1: u_1 = 3(1) - 2 = 1 \)
\[ \frac{n}{2}(3n - 1) = \frac{1}{2}(3 - 1) = \frac{1}{2}(2) = 1 \]
\[ \therefore \text{True for } n = 1 \]

Assume true for \( n = k \)
\[ S_k = \frac{k}{2}(3k - 1) \]

To prove:
\[ S_{k+1} = \frac{(k+1)}{2}[3(k+1) - 1] \]
\[ = \frac{k+1}{2}[3k + 2] \]
\[ = \frac{1}{2}(k+1)(3k + 2) \]

\[ \text{Proof: } \]
\[ S_{k+1} = S_k + U_{k+1} \]
\[ = \frac{k}{2}(3k - 1) + [3(k + 1) - 2] \]
\[ = \frac{k}{2}(3k - 1) + (3k + 1) \]
\[ = \frac{3k^2 - k + 6k + 2}{2} \]
\[ = \frac{1}{2}[3k^2 + 5k + 2] \]
\[ = \frac{1}{2}[(k+1)(3k + 2)] \]

\[ P(k+1) \]

So, \( P(k+1) \) true whenever \( P(k) \) true. Since \( P(1) \) true, then by induction \( P(n) \) true for all positive integers \( n \) \((n \in \mathbb{N}, n \geq 1)\).

Blunders (-3)
B1 indices.
B2 \( n \neq 1 \)(must prove \( n = 1 \) not enough to say true for \( n = 1 \))
B3 factors.

Slips (-1)
S1 numerical.
Part (c) 20(5, 5, 5, 5)marks Att (2, 2, 2, 2)

5(c) (i) Show that \( \frac{1}{\log_a b} = \log_b a \), where \( a, b > 0 \) and \( a, b \neq 1 \).

(ii) Show that \( \frac{1}{\log_a c} + \frac{1}{\log_b c} + \frac{1}{\log_c d} + \ldots + \frac{1}{\log_r c} = \frac{1}{\log_{r+1} c} \), where \( c > 0, c \neq 1 \).

Part (c) (i) 5 marks Att 2

(ii) log, c to a new base 5 marks Att 2

log(2.3.4...r) 5 marks Att 2

completion 5 marks Att 2

5(c)(i) \[ \log_b a = \frac{\log_a a}{\log_a b} = \frac{1}{\log_a b} \]

5(c)(ii) From (i): \[ \log_c 2 = \frac{1}{\log_2 c} \]

Similarly \[ \log_c 3 = \frac{1}{\log_3 c}, \ldots, \log_c r = \frac{1}{\log_r c} \]

\[ \therefore \frac{1}{\log_2 c} + \frac{1}{\log_3 c} + \frac{1}{\log_4 c} + \ldots + \frac{1}{\log_r c} = \log_c 2 + \log_c 3 + \log_c 4 + \ldots + \log_c r \]

= \log_c (2.3.4...........r)

= \log_c (r!)

= \frac{1}{\log_{r+1} c}.

or

5(c)(ii) \[ \log_2 c = \frac{\log_{r+1} c}{\log_r 2} \Rightarrow \frac{1}{\log_2 c} = \frac{\log_r 2}{\log_r c} \]

Similarly, \[ \frac{1}{\log_3 c} = \frac{\log_r 3}{\log_r c} \], etc.

\[ \therefore \frac{1}{\log_2 c} + \frac{1}{\log_3 c} + \frac{1}{\log_4 c} + \ldots + \frac{1}{\log_r c} = \frac{\log_2 r}{\log_r c} + \frac{\log_3 r}{\log_r c} + \frac{\log_4 r}{\log_r c} + \ldots + \frac{\log_r r}{\log_r c} \]

= \frac{\log_2 r}{\log_r c} + \frac{\log_3 r}{\log_r c} + \frac{\log_4 r}{\log_r c} + \ldots + \frac{\log_r r}{\log_r c} = \frac{\log_r (2.3.4...........r)}{\log_r c} = \frac{\log_r (r!)}{\log_r c} = \frac{1}{\log_{r+1} c}

or
5(c)(ii) \[ \log_2 c = \frac{\log_{10} c}{\log_{10} 2}, \quad \log_3 c = \frac{\log_{10} c}{\log_{10} 3}, \quad \text{etc.} \]

\[ \therefore \quad \frac{1}{\log_2 c} + \frac{1}{\log_3 c} + \frac{1}{\log_4 c} + \cdots + \frac{1}{\log_r c} \]

\[ = \frac{\log_{10} 2}{\log_{10} c} + \frac{\log_{10} 3}{\log_{10} c} + \frac{\log_{10} 4}{\log_{10} c} + \cdots + \frac{\log_{10} r}{\log_{10} c} \]

\[ = \frac{\log_{10} 2 + \log_{10} 3 + \log_{10} 4 + \cdots + \log_{10} r}{\log_{10} c} \]

\[ = \frac{\log_{10} (2.3.4\cdots r)}{\log_{10} c} \]

\[ = \frac{\log_{10} (r!)}{\log_{10} (c)} \]

\[ = \log_c (r!) \]

\[ = \frac{1}{\log_{r!} c} \]

- **Blunders (-3)**
  - B1 log laws.
  - B2 factorial.
  - B3 change of base.

- **Worthless**
  - W1 no change of base.
QUESTION 6

Part (a) 10 (5, 5) marks
Part (b) 20 marks
Part (c) 20 (10, 5, 5) marks

Note: The marking of Question 6 is not based on slips, blunders and attempts. In the case of each part, descriptions or typical examples of work meriting particular numbers of marks are given. The mark awarded must be one of the marks indicated. For example, in part (a)(i), descriptions are given for work meriting 0, 2 or 5 marks. It is therefore not permissible to award, 1, 3 or 4 marks for this part.

Part (a) 10 (5, 5) marks

6 (a) Differentiate with respect to \( x \)

(i) \((1 + 7x)^3\)  

(ii) \(\sin^{-1}\left(\frac{x}{5}\right)\)

Part (a) (i) 5 marks

6(a)(i) \[ \frac{dy}{dx} = 3(1 + 7x)^2 \cdot 7 = 21(1 + 7x)^2. \]

5 marks: correct derivative in any form. (e.g. middle step above is acceptable, as is expansion followed by correct differentiation, unsimplified).

2 marks: differentiates with one or more errors, provided at least some aspect correct.

0 marks: no correct differentiation done. (e.g. integrates or expands the given expression).

Part (a) (ii) 5 marks

6(a)(ii) \[ y = \sin^{-1}\left(\frac{x}{5}\right) = \sin^{-1}\left(\frac{x}{a}\right) \Rightarrow a = 5 \]

\[ \frac{dy}{dx} = \frac{1}{\sqrt{a^2 - x^2}} = \frac{1}{\sqrt{25 - x^2}} \]

or

\[ \frac{dy}{dx} = \frac{1}{\sqrt{1 - f(x)^2}} \cdot f'(x) = \frac{1}{\sqrt{1 - \left(\frac{x}{5}\right)^2}} \cdot \left(\frac{1}{5}\right) \]

\[ = \frac{1}{\sqrt{\frac{25-x^2}{25}}} \]

\[ = \frac{1}{\sqrt{25 - x^2}} \]

or

\[ y = \sin^{-1}\left(\frac{x}{5}\right) \Rightarrow \sin y = \frac{x}{5} \]

\[ \therefore \cos y \frac{dy}{dx} = \frac{1}{5} \]

\[ \therefore \frac{dy}{dx} = \frac{1}{\cos y} \cdot \frac{1}{5} \]

\[ = \frac{1}{\frac{\sqrt{25-x^2}}{5}} \cdot \frac{1}{5} \]

\[ = \frac{1}{\sqrt{25 - x^2}} \]

\[ \sin y = \frac{x}{5} \Rightarrow \cos y = \frac{\sqrt{25 - x^2}}{5} \]
Part (b) 20 marks

6 (b)

Let $y = \frac{1 - \cos x}{1 + \cos x}$.

Show that $\frac{dy}{dx} = t + t^3$, where $t = \tan \frac{x}{2}$.

5 marks: correct derivative in terms of $x$, simplified or otherwise.
2 marks: differentiates with at least some aspect correct; fails to give answer in terms of $x$.
0 marks: no correct differentiation done. (e.g. integrates or rearranges the given expression, or gives only the first step in the second method above)

or

Part (b) 20 marks

6(b)(ii)

$y = \frac{1 - \cos x}{1 + \cos x} = \frac{u}{v}$

$\frac{dy}{dx} = \frac{(1 + \cos x)(\sin x) - (1 - \cos x)(-\sin x)}{(1 + \cos x)^2}$

$= \frac{\sin x + \sin x \cos x + \sin x - \sin x \cos x}{(1 + \cos x)^2}$

$= \frac{2 \sin x}{(1 + \cos x)^2}$

$= \frac{2(2 \sin \frac{x}{2} \cos \frac{x}{2})}{(2 \cos^2 \frac{x}{2})^2}$

$= \frac{4 \sin \frac{x}{2} \cos \frac{x}{2}}{4 \cos^4 \frac{x}{2}}$

$= \sin \frac{x}{2} \cdot \frac{1}{\cos \frac{x}{2} \cos^2 \frac{x}{2}}$

$= \tan \frac{x}{2} \left(\sec^2 \frac{x}{2}\right)$

$= \tan \frac{x}{2} (1 + \tan^2 \frac{x}{2})$

$= t(1 + t^2)$

$= t + t^3$

or

$\frac{dy}{dx} = \frac{2 \sin x}{(1 + \cos x)^2} = \left[\frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}\right]^2$

$= \left[\frac{\sec^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}\right]^2$

$= \frac{4t}{(1 + t^2)(1 + t^2)}$

$= \frac{4t}{(1 + t^2)\left(\frac{2}{1 + t^2}\right)}$

$= t(1 + t^2)$

$= t + t^3$
6(b)(ii)

\[
\frac{dy}{dx} = \frac{2 \sin x}{(1 + \cos x)^2} = \frac{2}{\left[2 \cos^2 \frac{x}{2}\right]^2} \\
= \frac{4 \tan \frac{x}{2}}{(1 + \tan^2 \frac{x}{2})4 \cos^2 \frac{x}{2} \cdot \cos^2 \frac{x}{2}} \\
= \frac{4t}{\sec^2 \frac{x}{2} \cdot 4 \frac{1}{\sec^2 \frac{x}{2} \cdot \sec^2 \frac{x}{2}}} \\
= t \left(\sec^2 \frac{x}{2}\right) \\
= t \left(1 + \tan^2 \frac{x}{2}\right) \\
= t \left(1 + t^2\right) \\
= t + t^3
\]

or

6(b)(ii)

\[
y = \frac{1 - \cos x}{1 + \cos x} = \frac{1 - \frac{1}{1+t^2}}{1 + \frac{1}{1+t^2}} = \frac{\left(1 + t^2\right) - (1 - t^2)}{(1 + t^2) + (1 - t^2)} \\
y = \frac{2t^2}{2} = t^2 \\
y = \left(\tan \frac{x}{2}\right)^2
\]

\[
\frac{dy}{dx} = 2(tan \frac{x}{2})^3 \cdot \left(\sec^2 \frac{x}{2}\right)^{\frac{1}{2}} \\
= (\tan \frac{x}{2}) \left(1 + \tan^2 \frac{x}{2}\right) \\
= t \left(1 + t^2\right) \\
= t + t^3
\]

\[
\frac{dt}{dx} = 2t \frac{dt}{dx}
\]

\[
= 2t \left[\frac{1}{2} \sec^2 \frac{x}{2}\right] \\
= 2t \left[\frac{1}{2} \left(1 + \tan^2 \frac{x}{2}\right)\right] \\
= t \left(1 + t^2\right) \\
= t + t^3
\]

20 marks: fully correct solution.

17 marks: correct expression for \(\frac{dy}{dx}\) in terms of \(t\) alone, but not simplified to required form or solution with one or two non-critical errors, simplified fully. [critical error = one that significantly alters the nature or complexity of the task].

14 marks: correct expression for \(\frac{dy}{dx}\) in terms of \(x\), simplified or correctly establishes that \(y = t^2\) or that \(\frac{dt}{dx} = \frac{1}{2} \left(1 + t^2\right)\)

7 marks: correct or almost-correct expression for \(\frac{dy}{dx}\) in terms of \(x\) or correct expression for \(\frac{dt}{dx}\) in terms of \(x\) or correct expression for \(\frac{dy}{dx}\) in terms of \(x\) or correct but unsimplified expression for \(y\) in terms of \(t\) or \(\tan \frac{x}{2}\)

0 marks: no relevant work.
6 (c) The equation of a curve is \( y = \frac{x}{x-1} \), where \( x \neq 1 \).

(i) Show that the curve has no local maximum or local minimum point.

\[
\begin{align*}
y &= \frac{x}{x-1} \\
\frac{dy}{dx} &= \frac{(x-1)(1)-(x)(1)}{(x-1)^2} \\
&= \frac{x-1-x}{(x-1)^2} \\
&= \frac{-1}{(x-1)^2} \\
&\neq 0
\end{align*}
\]

No local max/local min

10 marks: Correct solution, including assertion that derivative \( \neq 0 \) or <0 or similar conclusion.
7 marks: Correct derivative.
3 marks: Substantial error(s) in differentiation.
0 marks: No relevant work

6 (c) (ii) Write down the equations of the asymptotes and hence sketch the curve.

Vertical asymptote: \( x-1 = 0 \implies x = 1 \)
Horizontal asymptote:
\[
y = \frac{x}{x-1} = \frac{1}{1-\frac{1}{x}} \rightarrow 1 \text{ as } x \rightarrow \pm\infty \implies y = 1
\]

5 marks: Correct solution, (equations of both asymptotes, and sketch).
2 marks: One or two equations correct, or sketch of correct form.
0 marks: No significant work of merit.
Show that the curve is its own image under the symmetry in the point of intersection of the asymptotes.

\[ S_p(a) = b \]

\[ a = (x, y) \Rightarrow b = (2 - x, 2 - y) \]

Test to see if \( b(2 - x, 2 - y) \) is on curve \( y = \frac{x}{x - 1} \):

\[
(2 - y) = \frac{(2 - x)}{(2 - x) - 1} \\
2 - y = \frac{2 - x}{1 - x} \\
2 - \frac{2 - x}{1 - x} = y \\
\Leftrightarrow y = \frac{2(1 - x) - (2 - x)}{1 - x} = \frac{-x}{1 - x} \\
\Leftrightarrow y = \frac{x}{x - 1} \quad \text{(i.e. } b \text{ is on the curve if and only if } a \text{ is.)}
\]

or

\[ p(1, 1): \text{point of intersection of asymptotes} \]

\( a \left( x, \frac{x}{x - 1} \right) \) on curve \( y = \frac{x}{x - 1} \):

\[ S_p(a) = b \Rightarrow b \left[ 2 - x, 2 - \frac{x}{x - 1} \right] \]

\[ b \left( 2 - x, \frac{2(x - 1) - x}{x - 1} \right) \]

\[ b \left( 2 - x, \frac{x - 2}{x - 2} \right) \]

Symmetry if \( b(2 - x, \frac{x - 2}{x - 2}) \) is \( y = \frac{2 - x}{1 - x} \):

\[
\frac{(2 - x)}{(2 - x) - 1} = \frac{2 - x}{1 - x} = \frac{x - 2}{x - 1}.
\]

5 marks: Fully correct solution.

2 marks: Correctly finds image of general point on the curve, or

Identifies general point on the curve in terms of one variable, or

Fully or partially works a particular case, or

Identifies \( (1, 1) \) as the point of intersection of the asymptotes.

0 marks: no relevant work.
QUESTION 7

Part (a) 10 marks  Att 3

Part (b) 20 (5, 5, 5, 5) marks  Att (2, 2, 2, 2)

Part (c) 20 (5, 5, 5, 5) marks  Att (2, 2, 2, 2)

Part (a) 10 marks  Att 3

7(a) Find from first principles the derivative of \( x^2 \) with respect to \( x \).

\[
\begin{align*}
\text{Part (a) } & \quad 10 \text{ marks } \quad \text{Att } 3 \\
7(a) & \\
& f(x) = x^2 \\
& f(x + h) = (x + h)^2 \\
& f(x + h) - f(x) = (x^2 + 2hx + h^2) - x^2 \\
& f(x + h) - f(x) = 2hx + h^2 \\
& \frac{f(x + h) - f(x)}{h} = 2x + h \\
& \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} = 2x \\
\end{align*}
\]

or

\[
\begin{align*}
\text{Part (a) } & \quad 10 \text{ marks } \quad \text{Att } 3 \\
7(a) & \\
& y = x^2 \\
& y + \Delta y = (x + \Delta x)^2 \\
& \Delta y = (x + \Delta x)^2 - x^2 \\
& = x^2 + 2x\Delta x + \Delta x^2 - x^2 \\
& = 2x\Delta x + \Delta x^2 \\
& \frac{\Delta y}{\Delta x} = 2x + \Delta x \\
& \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = 2x \\
\end{align*}
\]

Blunders (-3)

B1 expansion of \((a + b)^2\) once only.

B2 indices.

B3 no limit shown or implied, or no indication \( \to 0 \).

B4 \( x\Delta x = \Delta x^2 \)

Worthless

W1 not from 1st principles.
Part (b) (i)  

The parametric equations of a curve are:

\[ x = 8 + \ln t^2 \]
\[ y = \ln(2 + t^2), \text{ where } t > 0. \]

Find \( \frac{dy}{dx} \) in terms of \( t \) and calculate its value at \( t = \sqrt{2} \).

---

Part (b)(i) \[ \frac{dx}{dt}, \frac{dy}{dt} \]

5 marks  Att 2

**value** 5 marks  Att 2

\[ x = 8 + \ln t^2 \]
\[ x = 8 + 2 \ln t \]
\[ \frac{dx}{dt} = 2 \left( \frac{1}{t} \right) = \frac{2}{t} \]
\[ \frac{dy}{dt} = \frac{1}{2 + t^2} \cdot 2t \]
\[ \frac{dy}{dx} = \left( \frac{dy}{dt} \right) \left( \frac{dt}{dx} \right) = \left( \frac{2t}{2 + t^2} \right) \left( \frac{t}{2} \right) = \frac{t^2}{2 + t^2} \]

At \( t = \sqrt{2} \): \( t^2 = 2 \)

\[ \Rightarrow \frac{dy}{dx} = \frac{t^2}{2 + t^2} = \frac{2}{2 + 2} = \frac{1}{2} \]

* \( f''(x) \) must be expressed as a function of \( t \) for second 5 marks.

**Blunders** (-3)

B1 differentiation.
B2 logs.
B3 indices
B4 definition of \( \frac{dy}{dx} \)
B5 incorrect value or no value.

**Attempts**

A1 error in differentiation formula.

**Worthless**

W1 integration.
W2 no differentiation.
Part (b) (ii) 10 (5, 5) marks  
Att (2, 2)

7 (b) (ii) Find the slope of the tangent to the curve \( xy^2 + y = 6 \) at the point (1, 2).

<table>
<thead>
<tr>
<th>Part (b)(ii) Differentiation</th>
<th>5 marks</th>
<th>Att 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slope</td>
<td>5 marks</td>
<td>Att 2</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
7 (b) (ii) & \quad xy^2 + y = 6 \\
& \quad \left( 2xy \frac{dy}{dx} + y^2 \right) + \frac{dy}{dx} = 0 \\
& \quad \frac{dy}{dx} (2xy + 1) = -y^2 \\
& \quad \frac{dy}{dx} = -\frac{y^2}{2xy + 1} \\
\end{align*}
\]

At \( p(1, 2) \)
\[
\begin{align*}
x = 1 & \quad \text{and} \quad y = 2 \\
m = \frac{dy}{dx} & = -\frac{(2)^2}{2(1)(2) + 1} = -\frac{4}{5} \\
\end{align*}
\]

Blunders (-3)
B1 differentiation.
B2 indices.
B3 incorrect value of \( x \) or no value of \( x \).
B4 incorrect value of \( y \) or no value of \( y \).

Slips (-1)
S1 numerical.

Attempts
A1 error in differentiation formula.
A2 \[
\frac{dy}{dx} = 2xy \frac{dy}{dx} + y^2 + \frac{dy}{dx} \quad \text{and uses all three} \quad \left( \frac{dy}{dx} \right) \text{terms.}
\]

Part (c) 20 (5, 5, 5, 5) marks  
Att (2, 2, 2, 2)

7 (c) (i) Write down a quadratic equation whose roots are \( \pm \sqrt{k} \).

(ii) Hence use the Newton-Raphson method to show that the rule
\[
u_{n+1} = \frac{(u_n)^2 + k}{2u_n}
\]
can be used to find increasingly accurate approximations for \( \sqrt{k} \).

(iii) Using the above rule and taking \( \frac{3}{2} \) as the first approximation for \( \sqrt{3} \), find the third approximation, as a fraction.
7(c) (i) Roots \( \pm \sqrt{k} \) \( \Rightarrow \) Equation: \( x^2 - k = 0 \).

7(c)(ii) Equation: \( x^2 = k \) or \( x^2 - k = 0 \), so let \( f(x) = x^2 - k \).

\[
\therefore \quad f(u_n) = u_n^2 - k \\
\frac{df}{du} = 2u_n
\]

Newton-Raphson:

\[
u_{n+1} = u_n - \frac{f(u_n)}{\frac{df}{du}}
= u_n - \frac{u_n^2 - k}{2u_n}
= \frac{2u_n^2 - (u_n^2 - k)}{2u_n}
= \frac{u_n^2 + k}{2u_n}
\]

Hence the given rule is the Newton-Raphson method applied to \( f(x) = x^2 - k \). Thus it can be used with a suitable initial value to find increasingly accurate approximations for \( \sqrt{k} \).

7(c)(iii) \( u_2 = \frac{u_1^2 + k}{2u_1} \)

\( k = 3; \quad u_1 = \frac{3}{2} \)

\[
\begin{align*}
u_2 &= \frac{\left(\frac{3}{2}\right)^2 + 3}{2\left(\frac{3}{2}\right)} = \frac{\frac{9}{4} + 3}{3} = \frac{21}{12} = \frac{7}{4} \\
u_3 &= \frac{(u_2)^2 + k}{2u_2} = \frac{\left(\frac{7}{4}\right)^2 + 3}{2\left(\frac{7}{4}\right)} = \frac{\frac{49}{16} + 3}{\frac{7}{2}} = \frac{\frac{97}{16}}{\frac{7}{2}} = \frac{97}{56}
\end{align*}
\]

Blunders (-3)

B1 equation
B2 Newton-Raphson formula; apply once only to second 5 marks in (ii) or to 5 marks in (iii).
B3 differentiation.
B4 indices.
B5 \( k \neq 3 \).
B6 \( U_1 \neq \frac{3}{2} \), once only
B7 \( U_3 \) not found.

Slips (-1)

S1 numerical.
S2 not as fraction.

Misreadings (-1)

M1 takes “above rule” in c(iii) to mean “Newton-Raphson method” and uses this in (iii).
QUESTION 8

Part (a) 10 (5, 5) marks  Att (2, 2)
Part (b) 20 (10, 10) marks  Att (3, 3)
Part (c) 20 (10, 10) marks  Att (3, 3)

Part (a) 10 (5, 5) marks  Att (2, 2)

8 (a) Find (i) \( \int (2 + x^3) \, dx \)  
(ii) \( \int e^{3x} \, dx. \)

Part (i) 5 marks  Att 2
(ii) 5 marks  Att 2

8 (a) (i) \( \int (2 + x^3) \, dx = 2x + \frac{x^4}{4} + c \)
(ii) \( \int e^{3x} \, dx = \frac{e^{3x}}{3} + c \)

* If \( c \) shown once, then no penalty

Blunders (-3)
B1 integration.
B2 no ‘c’ (Penalise 1st integration)
B3 indices.

Attempts
A1 anything + c.

Worthless
W1 differentiation instead of integration.

Part (b) 20 (10, 10) marks  Att (3, 3)

8 (b) (i) Evaluate \( \int_{\frac{\pi}{8}}^{\pi} \frac{2x+1}{x^2+x+1} \, dx. \)
(ii) Evaluate \( \int_0^{\frac{\pi}{2}} \sin^2 2\theta \, d\theta. \)

Part (b) (i) 10 marks  Att 3
(ii) 10 marks  Att 3

8(b)(i) \[
\int_1^4 \frac{2x+1}{x^2+x+1} \, dx = \int \frac{(2x+1)dx}{x^2+x+1} = \int \frac{du}{u} = \ln u = \ln(x^2+x+1) \bigg|_1^4 = \ln(16+4+1) - \ln(1+1+1) = \ln \frac{21}{3} = \ln 7
\]
8(b)(ii) \[
\int_0^{\pi/8} \sin^2 2\theta \, d\theta. = \frac{1}{2} \left[ \theta - \frac{\sin 4\theta}{4} \right]_0^{\pi/8}
\]

\[
= \frac{1}{2} \left( \frac{\pi}{8} - \frac{\sin 4\pi}{8} \right) - (0 - 0)
\]

\[
= \frac{1}{2} \left( \frac{\pi}{8} - \frac{1}{4} \right)
\]

\[
= \frac{\pi}{16} - \frac{1}{8}
\]

or

8(b)(ii) \[
\int_0^{\pi/8} \sin^2 2\theta \, d\theta
\]

= \frac{1}{2} \left[ (1 - \cos 4\theta) \right]_0^{\pi/8}

\[
= \frac{1}{2} \left( \theta - \frac{\sin 4\theta}{4} \right)\]

\[
= \frac{1}{2} \left( \frac{\pi}{8} - \frac{\sin 4\pi}{8} \right) - (0 - 0)
\]

\[
= \frac{1}{2} \left( \frac{\pi}{8} - \frac{1}{4} \right)
\]

\[
= \frac{\pi}{16} - \frac{1}{8}
\]

Blunders (-3)
B1 integration.
B2 indices.
B3 limits.
B4 no limits.
B5 incorrect order in applying limits.
B6 not calculating substituted limits.
B7 not changing limits.
B8 differentiation.
B9 trig formula.

Slips (-1)
S1 numerical.
S2 trig value.

Worthless
W1 differentiation instead of integration except where other work merits attempt.

Note: Incorrect substitution and unable to finish yields attempt at most.
Note: (-3) is maximum deduction when evaluating limits
Note: In 8(b)(ii), do not penalise \( \frac{\pi}{16} = 11.25^\circ \), etc.
Part (c)(i) 10 marks

Evaluate \[ \int_{1}^{2} \frac{1}{\sqrt{3+2x-x^2}} \, dx \, . \]

\[
\int_{1}^{2} \frac{1}{\sqrt{3+2x-x^2}} \, dx = \int_{\sqrt{2}-(x-1)^2}^{1} \frac{dx}{\sqrt{2^2 - (x-1)^2}}
\]

\[
\int \frac{du}{\sqrt{2^2-u^2}} = \sin^{-1}\left( \frac{u}{2} \right)
\]

\[
= \sin^{-1}\left( \frac{1-x}{2} \right)
\]

\[
= \sin^{-1}\left( \frac{1}{2} \right) - \sin^{-1}(0) = \frac{\pi}{6} - 0 = \frac{\pi}{6}
\]

or

\[
\int_{1}^{2} \frac{1}{\sqrt{3+2x-x^2}} \, dx = \int_{1}^{1} \frac{-dw}{\sqrt{2^2-w^2}}
\]

\[
= -\sin^{-1}\left( \frac{w}{2} \right)
\]

\[
= -\sin^{-1}\left( \frac{1-x}{2} \right)
\]

\[
= -\left[ \sin^{-1}\left( \frac{-1}{2} \right) - \sin^{-1}(0) \right] = -\left[ -\frac{\pi}{6} - 0 \right] = \frac{\pi}{6}
\]

**Blunders (-3)**

B1 integration
B2 completing square once only.
B3 limits
B4 no limits
B5 incorrect order in applying limits
B6 not calculating substituted limits
B7 not changing limits.
B8 differentiation.

**Slips (-1)**

S1 numerical
S2 trig value.
Worthless:
W1 no effort at completing square
W2 differentiation instead of integration except where other work merits attempt.
W3 puts \( u = 3 + 2x - x^2 \)

Note: Incorrect substitution and unable to finish yields attempt at most.
Note: (-3) is maximum deduction when evaluating limits

<table>
<thead>
<tr>
<th>Part (c) (ii)</th>
<th>10 marks</th>
<th>Att 3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>8 (c) (ii)</strong> Use integration methods to derive a formula for the volume of a cone.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Vol of cone, with height = \( h \), and base-radius = \( r \)

Equation \( op \): slope = \( \frac{r}{h} \); through (0, 0) \( \Rightarrow y = \frac{r}{h}x \)

\[
V = \int_0^h \pi x^2 \, dx = \pi \int_0^h \left( \frac{rx}{h} \right)^2 \, dx = \frac{\pi r^2}{h^2} \int_0^h x^2 \, dx
\]

\[
= \frac{\pi r^2}{3h^2} \left[ x^3 \right]^h_0 = \frac{\pi r^2}{3h^2} \left[ h^3 - 0 \right] = \frac{1}{3} \pi r^2 h
\]

Blunders (-3)
B1 integration
B2 slope of line.
B3 equation of line.
B4 volume formula provided it is quadratic
B5 limits
B6 no limits.
B7 incorrect order in applying limits.
B8 indices.

Slips (-1)
S1 numerical

Attempts
A1 uses \( v = \pi y \)

Worthless
W1 differentiation instead of integration.

Note: (-3) is maximum deduction when evaluating limits.
MARKING SCHEME

LEAVING CERTIFICATE EXAMINATION 2005

MATHEMATICS – HIGHER LEVEL – PAPER 2

GENERAL GUIDELINES FOR EXAMINERS – PAPER 2

1. Penalties of three types are applied to candidates’ work as follows:
   • Blunders - mathematical errors/omissions (-3)
   • Slips - numerical errors (-1)
   • Misreadings (provided task is not oversimplified) (-1).

   Frequently occurring errors to which these penalties must be applied are listed in the scheme. They are labelled: B1, B2, B3,…, S1, S2,…, M1, M2,…etc. These lists are not exhaustive.

2. When awarding attempt marks, e.g. Att(3), note that
   • any correct, relevant step in a part of a question merits at least the attempt mark for that part
   • if deductions result in a mark which is lower than the attempt mark, then the attempt mark must be awarded
   • a mark between zero and the attempt mark is never awarded.

3. Worthless work is awarded zero marks. Some examples of such work are listed in the scheme and they are labelled as W1, W2,…etc.

4. The phrase “hit or miss” means that partial marks are not awarded – the candidate receives all of the relevant marks or none.

5. The phrase “and stops” means that no more work is shown by the candidate.

6. Special notes relating to the marking of a particular part of a question are indicated by an asterisk. These notes immediately follow the box containing the relevant solution.

7. The sample solutions for each question are not intended to be exhaustive lists – there may be other correct solutions. Any examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his/her advising examiner.

8. Unless otherwise indicated in the scheme, accept the best of two or more attempts – even when attempts have been cancelled.

9. The same error in the same section of a question is penalised once only.

10. Particular cases, verifications and answers derived from diagrams (unless requested) qualify for attempt marks at most.

11. A serious blunder, omission or misreading results in the attempt mark at most.

12. Do not penalise the use of a comma for a decimal point, e.g. €5.50 may be written as €5,50.
QUESTION 1

Part (a) 15 marks  Att 5
Circles $S$ and $K$ touch externally.
Circle $S$ has centre $(8, 5)$ and radius 6.
Circle $K$ has centre $(2, -3)$.
Calculate the radius of $K$.

Radius of $K$  15 marks  Att 5

\[
ab = \sqrt{(8-2)^{2} + (5+3)^{2}} = \sqrt{36 + 64} = 10.
\]
But $r + 6 = 10$. \(\therefore r \text{ (radius } K\text{)} = 4.\)

Blunders (−3)
B1 Error in distance formula.

Slips (−1)
S1 Arithmetic error.

Attempts (5 marks)
A1 Distance between centres.
A2 Correct condition for circles touching externally.

Part (b) 20 (5, 5, 10) marks  Att (2, 2, 3)
Part (c) 15 (5, 5, 5) marks  Att (2, 2, 2)

1(b) (i) Prove that the equation of the tangent to the circle $x^{2} + y^{2} = r^{2}$
at the point \((x_{1}, y_{1})\) is $xx_{1} + yy_{1} = r^{2}$.

Slope of tangent  5 marks  Att 2
Finish  5 marks  Att 2

1(b) (i)
Equation of tangent $T$: \(y - y_{1} = m(x - x_{1})\).
Slope of normal $op = \frac{y_{1} - 0}{x_{1} - 0} = \frac{y_{1}}{x_{1}}$.
\(\therefore\) Slope of $T$ at point $p(x_{1}, y_{1}) = -\frac{x_{1}}{y_{1}}$.
Equation of $T$: \(y - y_{1} = \frac{-x_{1}}{y_{1}} (x - x_{1}) \Rightarrow yy_{1} - y_{1}^{2} = -xx_{1} + x_{1}^{2}\)
\(xx_{1} + yy_{1} = x_{1}^{2} + y_{1}^{2}\).
But \((x_{1}, y_{1})\in x^{2} + y^{2} = r^{2} \Rightarrow x_{1}^{2} + y_{1}^{2} = r^{2}\).
\(\therefore\) Equation of tangent $T$: \(xx_{1} + yy_{1} = r^{2}\).

or
1(b) (i) \[ x^2 + y^2 = r^2 \Rightarrow 2x + 2y \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{x}{y}. \]

Slope of tangent \( T \) at point \( p(x_1, y_1) \) is \( \frac{-x_1}{y_1} \).

Equation of \( T \) : \[ y - y_1 = \frac{-x_1}{y_1} (x - x_1) \Rightarrow yy_1 - y_1^2 = -xx_1 + x_1^2 \]

\[ xx_1 + yy_1 = x_1^2 + y_1^2. \]

But \( (x_1 , y_1) \in x^2 + y^2 = r^2 \Rightarrow x_1^2 + y_1^2 = r^2. \)

\[ \therefore \text{Equation of tangent } T: xx_1 + yy_1 = r^2. \]

**Blunders (−3)**
B1 Incorrect sign in slope formula.
B2 Slope formula inverted.
B3 Incorrect perpendicular slope.
B4 Error in differentiation.
B5 Fails to show that \( x_1^2 + y_1^2 = r^2. \)

**Slips (−1)**
S1 Arithmetic error.

**Attempts (2, 2 marks)**
A1 Correct slope of normal.
A2 Correct differentiation.
A3 Correct substitution into tangent formula and stops.
A4 Stops at \( xx_1 + yy_1 = x_1^2 + y_1^2. \)

**Part (b) (ii) 10 marks Att 3**

1 (b) (ii) Hence, or otherwise, find the two values of \( b \) such that the line \( 5x + by = 169 \) is a tangent to the circle \( x^2 + y^2 = 169. \)

**Values of \( b \) 10 marks Att 3**

1 (b) (ii) By part (i) the line \( 5x + by = 169 \) is a tangent to the circle \( x^2 + y^2 = 169 \) at the point \( (5, b) \).

But \( (5, b) \in x^2 + y^2 = 169 \Rightarrow 25 + b^2 = 169. \)

\[ b^2 = 144 \Rightarrow b = \pm 12. \]

or

1 (b) (ii) Perpendicular distance from centre of circle to tangent \( 5x + by = 169 \) equals radius.

\[ \frac{|5(0) + b(0) - 169|}{\sqrt{25 + b^2}} = 13 \Rightarrow | -169 | = 13 \sqrt{25 + b^2} \]

\[ \sqrt{25 + b^2} = 13 \Rightarrow 25 + b^2 = 169 \Rightarrow b^2 = 144. \therefore b = \pm 12. \]
**Blunders (-3)**
B1 Error in solving for $b$ other than slip.
B2 Only one correct value of $b$ given.
B3 Incorrect radius.

**Slips (-1)**
S1 Arithmetic error.

**Attempts (3 marks)**
A1 (5, $b$) point of tangency.
A2 Perpendicular distance formula with substitution.

<table>
<thead>
<tr>
<th>Part (c)</th>
<th>15 marks (5, 5, 5)</th>
<th>Att (2, 2, 2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (c)</td>
<td>A circle passes through the points (7, 2) and (7, 10). The line $x = -1$ is a tangent to the circle. Find the equation of the circle.</td>
<td></td>
</tr>
</tbody>
</table>

**Two equations in $g, f$ and $c$**

| Value of $f$ | 5 marks | Att 2 |
| Finish | 5 marks | Att 2 |

1 (c) Circle: $x^2 + y^2 + 2gx + 2fy + c = 0$. 
(7, 2) $\in C \Rightarrow 49 + 4 + 14g + 4f + c = 0 \Rightarrow 14g + 4f + c = -53$ 
(7, 10) $\in C \Rightarrow 49 + 100 + 14g + 20f + c = 0 \Rightarrow 14g + 20f + c = -149$ 
$. \Rightarrow 16f = -96 \Rightarrow f = -6$.
$x + 1 = 0$ is a tangent. 
$. \Rightarrow$ Perpendicular distance from $(-g, -f)$ to $x + 1 = 0$ equals radius. 
$. \Rightarrow \left| \frac{-g + 1}{1} \right| = \sqrt{g^2 + 36 - c} \Rightarrow g^2 - 2g + 1 = g^2 + 36 - c \Rightarrow 2g - c = -35.$
But $14g + 4f + c = -53 \Rightarrow 14g + c = -29$. But $2g - c = -35$.
$. \Rightarrow 16g = -64. \Rightarrow g = -4$ and $c = 27$. 
$. \Rightarrow$ Circle: $x^2 + y^2 - 8x - 12y + 27 = 0$.

| or |  |
| y value of centre | 5 marks | Att 2 |
| 'Quadratic' in $x$ | 5 marks | Att 2 |
| Finish | 5 marks | Att 2 |

1 (c) $a(7, 2)$ and $b(7, 10)$. $\Rightarrow$ Mid-point of $[ab]$ is $(7, 6)$. 
Equation of mediator of chord $[ab]$ is $y = 6$.
Centre point of circle is $c(x, 6)$.
As $x = -1$ is a tangent then point of tangency is $d(-1, 6)$.
$. \Rightarrow |cd|^2 = |cd|^2 \Rightarrow (x + 1)^2 = (x - 7)^2 + 16.$
$. \Rightarrow x^2 + 2x + 1 = x^2 - 14x + 49 + 16 \Rightarrow 16x = 64 \Rightarrow x = 4.$
$. \Rightarrow$ Centre is $(4, 6)$ and radius $= 5$
$. \Rightarrow$ Equation of circle is $(x - 4)^2 + (y - 6)^2 = 25.$
Blunders (−3)
B1  Error in mid-point formula.
B2  Error in perpendicular distance formula.
B3  Error in radius formula.
B4  Circle equation formula error.

Slides (−1)
S1  Arithmetic error.

Attempts (2, 2, 2 marks)
A1  One equation in f, g and c.
A2  Mid-point of [ab].
A3  Attempt at solving simultaneous equations.
A4  |ca|, |cb| or |cd| found.
A5  Distance from centre to tangent with substitution.
A6  Attempt at solving quadratic for x.
A7  Value of third unknown.
A8  Length of radius.
QUESTION 2

Part (a)  5 marks
Part (b)  25 (10, 5, 10) marks
Part (c)  20 (15, 5) marks

Note: The marking of Question 2 is not based on slips, blunders and attempts. In the case of each part, descriptions or typical examples of work meriting particular numbers of marks are given. The mark awarded must be one of the marks indicated. For example, in part (a)(i), descriptions are given for work meriting 2, 4 or 5 marks. It is therefore not permissible to award, 1 or 3 marks for this part.

Part (a)  5 marks

2 (a) Copy the parallelogram oabc into your answerbook.

Showing your work, construct the point d such that
\[ \overrightarrow{d} = \frac{1}{2} \overrightarrow{a} + \frac{1}{2} \overrightarrow{b} - \overrightarrow{c}, \]
where o is the origin.

Point d  5 marks

* Accept any labelled parallelogram with vertices o, a, b, c.

5 marks: point d shown in correct position in diagram. Point d need not be joined to origin.

4 marks: Correct work with one error or omission e.g. \[ \frac{1}{2} (\overrightarrow{a} + \overrightarrow{b}) \] or \[ \frac{1}{2} \overrightarrow{a} - \overrightarrow{c} \] or \[ \frac{1}{2} \overrightarrow{b} - \overrightarrow{c} \] correctly on diagram.

2 marks: One correct significant step e.g. \[ \frac{1}{2} \overrightarrow{a} \] or \[ \frac{1}{2} \overrightarrow{b} \] or \[ - \overrightarrow{c} \] or \[ (\overrightarrow{a} + \overrightarrow{b}) \] correctly shown on diagram.

0 marks: No significant work of merit.

Part (b)  25 (10, 5, 10) marks

Part (b) (i)  15 (10, 5) marks

2 (b) (i) \[ \overrightarrow{p} = 3 \overrightarrow{i} + 4 \overrightarrow{j}. \] \( \overrightarrow{q} \) is the unit vector in the direction of \( \overrightarrow{p}. \)

(i) Express \( \overrightarrow{q} \) and \( \overrightarrow{q}^\perp \) in terms of \( \overrightarrow{i} \) and \( \overrightarrow{j}. \)

Express \( \overrightarrow{q} \)  10 marks

2 (b)(i) \[ \overrightarrow{q} = \overrightarrow{p} \cdot \frac{3 \overrightarrow{i} + 4 \overrightarrow{j}}{\sqrt{9 + 16}} = \frac{3 \overrightarrow{i} + 4 \overrightarrow{j}}{5}. \]
10 marks: Correct solution for \( \vec{q} \), simplified or otherwise.

7 marks: Calculates \( \vec{p} \) correctly but does not give unit vector or writes \( \vec{p} \) and stops or divides \( 3 \vec{i} + 4 \vec{j} \) by any number.

3 marks: Unit vector expressed as \( \frac{a \vec{i} + b \vec{j}}{\sqrt{a^2 + b^2}} \).

0 marks: No significant work of merit.

Express \( \vec{q} \) 5 marks

2 (b) (i) \[
\vec{q} = -\frac{4}{5} \vec{i} + \frac{3}{5} \vec{j}, \text{ or equivalent from candidates } \vec{q}.
\]

5 marks: Fully correct answer.

2 marks: Gives \( \vec{q} = \frac{4}{5} \vec{i} - \frac{3}{5} \vec{j} \) as solution, or equivalent from candidates \( \vec{q} \).

0 marks: Any other answer.

Part (b) (ii) 10 marks

Express \( 11 \vec{i} - 2 \vec{j} \) in the form \( k \vec{q} + l \vec{q} \perp \), where \( k, l \in \mathbb{R} \).

Express \( 10 \) marks

2 (b) (ii) \[
\begin{align*}
\vec{kq} + l \vec{q} \perp &= 11 \vec{i} - 2 \vec{j}, \\
\left(\frac{3}{5} \vec{i} + \frac{4}{5} \vec{j}\right) + l \left(-\frac{4}{5} \vec{i} + \frac{3}{5} \vec{j}\right) &= 11 \vec{i} - 2 \vec{j}, \\
\left(\frac{3}{5}k - \frac{4}{5}l\right) \vec{i} + \left(\frac{4}{5}k + \frac{3}{5}l\right) \vec{j} &= 11 \vec{i} - 2 \vec{j}, \\
\therefore 3k - 4l &= 55 \quad \text{and} \quad 4k + 3l = -10, \\
k - 12l &= 165, \\
16k + 12l &= -40, \\
25k &= 125 \therefore k = 5. \quad \text{But } 3(5) - 4l = 55 \Rightarrow l = -10.
\end{align*}
\]

\( \therefore 11 \vec{i} - 2 \vec{j} = 5 \vec{q} - 10 \vec{q} \perp \).

10 marks: Correct \( k \) and \( l \) found.

7 marks: Solves for \( k \) and/or for \( l \) with minor error(s).

3 marks: One equation in \( k \) and \( l \) allowing for minor error(s).

0 marks: No significant work of merit.
Part (c)  20 marks (15, 5)  -  -
Part (c) (i)  15 marks  -  -

2 (c)  \[ \overrightarrow{u} = \overrightarrow{i} + 5 \overrightarrow{j} \text{ and } \overrightarrow{v} = 4 \overrightarrow{i} + 4 \overrightarrow{j}. \]

(i) Find \( \cos \angle uov \), where \( o \) is the origin.

\[
\cos \angle uov = \frac{(\overrightarrow{i} + 5 \overrightarrow{j}) \cdot (4 \overrightarrow{i} + 4 \overrightarrow{j})}{\| \overrightarrow{i} + 5 \overrightarrow{j} \| \| 4 \overrightarrow{i} + 4 \overrightarrow{j} \|} = \frac{4 + 20}{\sqrt{26} \sqrt{32}} = \frac{24}{8 \sqrt{13}} = \frac{3}{\sqrt{13}}.
\]

15 marks: \( \cos \angle uov \) expressed as fraction of real numbers, simplified or otherwise.

10 marks: Correctly evaluates \( \overrightarrow{u} \cdot \overrightarrow{v} \) and either \( \| \overrightarrow{u} \| \) or \( \| \overrightarrow{v} \| \) allowing for minor error(s).

5 marks: Correctly evaluates \( \| \overrightarrow{u} \| \) or \( \| \overrightarrow{v} \| \) or \( \overrightarrow{u} \cdot \overrightarrow{v} \).

0 marks: No significant work of merit.

Part (c) (ii)  5 marks  -  -

2 (c) (ii)  \[ \overrightarrow{r} = (1 - k) \overrightarrow{u} + k \overrightarrow{v}, \text{ where } k \in \mathbb{R} \text{ and } k \neq 0. \]

Find the value of \( k \) for which \( \angle uov = \angle vor \).

\[
\cos \angle vor = \frac{(4 \overrightarrow{i} + 4 \overrightarrow{j}) \cdot ((1 + 3k) \overrightarrow{i} + (5 - k) \overrightarrow{j})}{\sqrt{32} \sqrt{(1 + 3k)^2 + (5 - k)^2}} = \frac{3}{\sqrt{13}}.
\]

\[
\therefore \frac{4 + 12k + 20 - 4k}{4 \sqrt{2} \sqrt{26 - 4k + 10k^2}} = \frac{3}{\sqrt{13}}.
\]

\[
\sqrt{13}(24 + 8k) = 12 \sqrt{2} \sqrt{26 - 4k + 10k^2}
\]

\[
\sqrt{13}(6 + 2k) = 3 \sqrt{2} \sqrt{26 - 4k + 10k^2}
\]

\[
568 + 312k + 52k^2 = 468 - 72k + 180k^2 \quad \Rightarrow \quad 128k^2 - 384 = 0
\]

\[
\therefore k^2 - 3k = 0 \quad \Rightarrow \quad k - 3 = 0 \quad \text{as } k \neq 0. \quad \therefore \quad k = 3.
\]

5 marks: Fully correct solution.

4 marks: Complete solution with minor error(s).

3 marks: Correct or substantially correct equation in \( k \) (without \( \overrightarrow{i} \) and \( \overrightarrow{j} \)).

2 marks: \( \overrightarrow{r} \) expressed in the form \( a \overrightarrow{i} + b \overrightarrow{j} \), allowing for minor error(s).

0 marks: No significant work of merit.
QUESTION 3

| Part (a) | 15 marks | Att 5 |
| Part (b) | 20 (10, 5, 5) marks | Att (3, 2, 2) |
| Part (c) | 15 (10, 5) marks | Att (3, 2) |

### Part (a) 15 marks Att 5

**3 (a)** The line $L_1: 3x - 2y + 7 = 0$ and the line $L_2 : 5x + y + 3 = 0$ intersect at the point $p$.

Find the equation of the line through $p$ perpendicular to $L_2$.

#### Equation of line 15 marks Att 5

**3 (a)**

$$3x - 2y + 7 = 0 \Rightarrow 3x - 2y = -7$$

$$5x + y + 3 = 0 \Rightarrow 10x + 2y = -6$$

$$13x = -13 \Rightarrow x = -1. \quad \therefore y = 2. \quad p(-1, 2).$$

$L_2 : y = -5x - 3 \Rightarrow$ slope $L_2 = -5$. $\therefore$ perpendicular slope $= m = \frac{1}{5}$.

Equation of line $: y - 2 = \frac{1}{5}(x + 1) \Rightarrow x - 5y + 11 = 0$.

**or**

**3 (a)**

Required line: $3x - 2y + 7 + \lambda(5x + y + 3) = 0$.

$\therefore x(3 + 5\lambda) + y(\lambda - 2) + (7 + 3\lambda) = 0$

Slope $= \frac{3 + 5\lambda}{2 - \lambda}$.

$L_2 : y = -5x - 3 \Rightarrow$ slope $L_2 = -5$. $\therefore$ Slope of required line $= \frac{1}{5}$.

$$\frac{3 + 5\lambda}{2 - \lambda} = \frac{1}{5} \Rightarrow 15 + 25\lambda = 2 - \lambda \Rightarrow 26\lambda = -13. \quad \therefore \lambda = \frac{1}{2}.$$

$$\therefore \frac{1}{2}x - \frac{5}{2}y + \frac{11}{2} = 0 \Rightarrow \text{Required line}: x - 5y + 11 = 0.$$

---

**Blunders (-3)**

B1 Error in slope of $L_2$ other than slip.

B2 Incorrect perpendicular slope.

**Slips (-1)**

S1 Arithmetic error.

**Attempts (5 marks)**

A1 $x$ or $y$ coordinate of point $p$.

A2 Correct slope of $L_2$.

A3 Correct perpendicular slope.
Part (b) 20 (10, 5, 5) marks  Att (3, 2, 2)
Part (b) (i) 10 marks  Att 3

3 (b) (i) The line $K$ passes through the point $(-4, 6)$ and has slope $m$, where $m > 0$.

Write down the equation of $K$ in terms of $m$.

Equation of $K$ 10 marks  Att 3

3 (b) (i) \[ y - 6 = m(x + 4). \]

Blunders (-3)
B1 Error in equation line formula.

Slips (-1)
S1 Arithmetic error.

Attempts (3 marks)
A1 Equation of line with some substitution.

Part (b) (ii) 5 marks  Att 2

3 (b) (ii) Find, in terms of $m$, the co-ordinates of the points where $K$ intersects the axes.

Co-ordinates 5 marks  Att 2

3 (b) (ii) \[ y - 6 = m(x + 4) \Rightarrow mx - y + 6 + 4m = 0. \]

Cuts x-axis at \[ p(x, 0), \quad mx = -6 - 4m \Rightarrow x = \frac{-6 - 4m}{m}. \quad p \left( \frac{-6 - 4m}{m}, 0 \right). \]

Cuts y-axis at \[ q(0, y), \quad y = 6 + 4m. \quad q(0, 6 + 4m). \]

Blunders (-3)
B1 Equation of axes incorrect.

Slips (-1)
S1 Arithmetic error.

Attempts (2 marks)
A1 One correct coordinate.
The area of the triangle formed by $K$, the $x$-axis and the $y$-axis is 54 square units. Find the possible values of $m$.

Values of $m$

Area triangle $opq = 54$ square units.

Area triangle $opq = \frac{1}{2}|x_1y_2 - x_2y_1|$

$\therefore \frac{1}{2}(0)(0) - \frac{-6 - 4m}{m}(6 + 4m) = 54.$

$(6 + 4m)(6 + 4m) = 108m.$

$\therefore 16m^2 + 48m + 36 = 108m \Rightarrow 16m^2 - 60m + 36 = 0$

$4m^2 - 15m + 9 = 0 \Rightarrow (4m - 3)(m - 3) = 0.$

$\therefore m = \frac{3}{4}$ or $m = 3.$

Blunders (-3)

B1 Error in triangle area formula.

B2 Error in factors or quadratic formula.

B3 Misuse of modulus in formula.

Slips (-1)

S1 Arithmetic error.

Attempts (2 marks)

A1 Triangle area formula with some substitution.

A2 Quadratic in $m$. 
Part (c)  15 (10, 5) marks  Att (3, 2)

### Part (c) (i)  10 marks  Att 3

**3 (c) (i)**  
\[ f \] is the transformation \((x, y) \rightarrow (x', y')\), where \(x' = 3x - y\) and \(y' = x + 2y\).

(i) Prove that \(f\) maps every pair of parallel lines to a pair of parallel lines.  
You may assume that \(f\) maps every line to a line.

---

#### Prove  10 marks  Att 3

### 3(c)(i)

\[
\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 3 & -1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \Rightarrow \begin{pmatrix} x' \\ y' \end{pmatrix} = \frac{1}{7} \begin{pmatrix} 2 & 1 \\ 3 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \]

Let \(L\) have equation: \(ax + by + c = 0\), and \(M\): \(ax + by + d = 0\).

\[ \therefore f(L): \frac{a}{7}(-x' + 3y') + \frac{b}{7}(2x' + y') + c = 0 \quad \Rightarrow \quad f(L): (-a + 2b)x' + (3a + b)y' + 7c = 0 \]

and \(f(M): \frac{a}{7}(-x' + 3y') + \frac{b}{7}(2x' + y') + d = 0 \quad \Rightarrow \quad f(L): (-a + 2b)x' + (3a + b)y' + 7d = 0 \)

So, \(f(L) \parallel f(M)\), since \((-a + 2b)(3a + b) = (3a + b)(-a + 2b)\), \(\text{[i.e. } a_1b_2 = a_2b_1]\)

---

### or

3 (c) (i)  
\[ L: y = mx + c \text{ and } M: y = mx + k \] are two parallel lines.  
\(x' = 3x - y \quad \Rightarrow \quad 2x' = 6x - 2y\)

\(y' = x + 2y \quad \Rightarrow \quad y' = x + 2y. \quad \therefore \quad 2x' + y' = 7x \quad \Rightarrow \quad x = \frac{1}{7}(2x' + y').\)

But \(y' = x + 2y \quad \Rightarrow \quad y' = \frac{1}{7}(2x' + y') + 2y \quad \Rightarrow \quad y = \frac{1}{7}(-x' + 3y').\)

\[ \therefore f(L): \frac{1}{7}(-x' + 3y') = \frac{m}{7}(2x' + y') + c \quad \Rightarrow \quad f(L): -x' + 3y' = 2mx' + my' + 7c. \]

\[ \therefore f(L): (3-m)y' = (1+2m)x' + 7c \quad \Rightarrow \quad f(L): y' = \left(\frac{1+2m}{3-m}\right)x' + \frac{7c}{3-m}. \]

Similarly \(f(M): y' = \left(\frac{1+2m}{3-m}\right)x' + \frac{7k}{3-m}.\)

Both lines have same slope, \(\frac{1+2m}{3-m}\), \(\therefore \) parallel.

---

### or

Let \(L\) and \(M\) pass through \(p\) and \(q\) respectively and both be in the direction \(\vec{m}\).

\[ \therefore L = \vec{p} + tm \quad \text{and} \quad M = \vec{q} + tm \quad \text{, where } t \in \mathbb{R} \]

\[ \therefore f(L) = f(\vec{p} + tm) = f(\vec{p}) + tf(\vec{m}) \quad \text{and} \quad f(M) = f(\vec{q} + tm) = f(\vec{q}) + tf(\vec{m}) \]

\[ \therefore f(L) \text{ and } f(M) \text{ are both lines in the direction of } f(\vec{m}), \text{ and hence are parallel.} \]

* Note: second method above fails to deal with the case where \(L\) and \(M\) are vertical, or where they have slope 3. Do not penalise this.
Blunders (-3)
B1 Error in determining slope other than slip.
B2 Incorrect matrix or matrix multiplication.
B3 Failure to establish image lines parallel.

Slips (-1)
S1 Arithmetic error.

Attempts (3 marks)
A1 Expressing \(x\) or \(y\) in term of primes.
A2 Correct matrix for \(f\).
A3 Finds image of one line and stops.

Part (c) (ii) 5 marks Att 2
3 (c) (ii) \(oabc\) is a parallelogram, where \([ob]\) is a diagonal and \(o\) is the origin.
Given that \(f(c) = (-1, 9)\), find the slope of \(ab\).

Slope \(ab\) 5 marks Att 2
3 (c) (ii) \(f(c) = (-1, 9)\). \(x = \frac{1}{7}(2x' + y')\) and \(y = \frac{1}{7}(-x' + 3y')\).
\[\therefore x = 1 \text{ and } y = 4 \Rightarrow c(1, 4).\]
Slope \(oc = 4 \Rightarrow \text{slope } ab = 4\) as \(ab\) is parallel to \(oc\).

or

3 (c) (ii) Matrix \(f = \begin{pmatrix} 3 & -1 \\ 1 & 2 \end{pmatrix}\) \(\Rightarrow\) \(\begin{pmatrix} 3 & -1 \\ 1 & 2 \end{pmatrix}^{-1} = \begin{pmatrix} -1 & 1 \\ -9 & 3 \end{pmatrix}\) \(=\) \(\begin{pmatrix} 2 & 1 \\ -1 & 3 \end{pmatrix}\) \(=\) \(\begin{pmatrix} 1/7 \end{pmatrix}\) \(=\) \(\begin{pmatrix} 7/28 \end{pmatrix}\) \(=\) \(\begin{pmatrix} 1/4 \end{pmatrix}\) \(=\) \(c\).
\[\therefore \text{Slope } oc = 4 \Rightarrow \text{slope } ab = 4.\]

or

3 (c) (ii) \(f(c) = (-1, 9)\). \(x' = 3x - y\) and \(y' = x + 2y\).
\(3x - y = -1 \Rightarrow 6x - 2y = -2\)
\(x + 2y = 9 \Rightarrow x + 2y = 9\)
\[\therefore 7x = 7 \Rightarrow x = 1 \text{ and hence } y = 4.\]
\[\therefore c(1, 4) \text{ and slope } oc = 4.\]
But \(ab\) is parallel to \(oc \Rightarrow \text{slope } ab = 4.\)

Blunders (-3)
B1 Slope \(oc\) and stops.
B2 Incorrect matrix.
B3 Incorrect matrix multiplication other than slip.

Slips (-1)
S1 Arithmetic error.

Attempts (2 marks)
A1 Two simultaneous equations.
A2 Correct point \(c\) and stops.
# QUESTION 4

<table>
<thead>
<tr>
<th>Part (a)</th>
<th>10 marks</th>
<th>Att 3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>4 (a)</strong></td>
<td>Evaluate $\lim_{\theta \to 0} \frac{\sin 4\theta}{3\theta}$.</td>
<td></td>
</tr>
</tbody>
</table>

* Accept correct answer without work. If candidate’s answer is correct, ignore the work.

<table>
<thead>
<tr>
<th>Evaluate</th>
<th>10 marks</th>
<th>Att 3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>4 (a)</strong></td>
<td>$\lim_{\theta \to 0} \frac{\sin 4\theta}{3\theta} = \lim_{\theta \to 0} \left( \frac{\sin 4\theta}{4\theta} \times \frac{4\theta}{3\theta} \right) = \lim_{\theta \to 0} \left( \frac{\sin 4\theta}{4\theta} \right) \times \frac{4}{3} = \frac{4}{3}$.</td>
<td></td>
</tr>
</tbody>
</table>

or $f(\theta) = \sin 4\theta$ and $g(\theta) = 3\theta$. ∴ $\lim_{\theta \to 0} \frac{f(\theta)}{g(\theta)} = \frac{f'(0)}{g'(0)} = \frac{4\cos(0)}{3} = \frac{4}{3}$.

### Blunders (−3)

B1 $\sin 4\theta = 4\sin \theta$.

B2 Error in differentiation.

### Slips (−1)

S1 Arithmetic error.

### Attempts (3 marks)

A1 Has $\frac{\sin 4\theta}{4\theta}$ in solution.

A2 Correct differentiation.

<table>
<thead>
<tr>
<th>Part (b)</th>
<th>20 (10, 5, 5) marks</th>
<th>Att (3, 2, 2)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Part (b) (i)</strong></td>
<td>10 marks</td>
<td>Att 3</td>
</tr>
<tr>
<td><strong>4 (b) (i)</strong></td>
<td>Using $\cos 2A = \cos^2 A - \sin^2 A$, or otherwise, prove $\cos^2 A = \frac{1}{2}(1 + \cos 2A)$.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Prove</th>
<th>10 marks</th>
<th>Att 3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>4 (b) (i)</strong></td>
<td>$\cos 2A = \cos^2 A - \sin^2 A = \cos^2 A - \left(1 - \cos^2 A\right)$</td>
<td></td>
</tr>
</tbody>
</table>

∴ $2\cos^2 A = 1 + \cos 2A \Rightarrow \cos^2 A = \frac{1}{2}(1 + \cos 2A)$. |
Blunders (−3)
B1 Error in cos2A formula.
B2 Error in sin^2A formula.

Slips (−1)
S1 Arithmetic error.

Attempts (3 marks)
A1 Correct substitution for cos2A.
A2 Sin^2A = 1 − cos^2A.

<table>
<thead>
<tr>
<th>Part (b) (ii)</th>
<th>10 (5, 5) marks</th>
<th>Att (2, 2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 (b) (ii)</td>
<td>Hence, or otherwise, solve the equation 1 + cos2x = cosx, where 0° ≤ x ≤ 360°.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Quadratic in Cosx</th>
<th>5 marks</th>
<th>Att 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solution for x</td>
<td>5 marks</td>
<td>Att 2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>4 (b) (ii)</th>
<th>1 + cos2x = cosx ⇒ 2cos^2x = cosx.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>cosx(2cosx − 1) = 0 ⇒ cosx = 0 or cosx = 1/2.</td>
</tr>
<tr>
<td></td>
<td>∴ x = 90°, 270° or x = 60°, 300°. ∴ solution = {60°, 90°, 270°, 300°}.</td>
</tr>
</tbody>
</table>

Blunders (−3)
B1 Incorrect substitution for 1+cos2x or cos2x.
B2 Error in factors.
B3 Each incorrect solution or missing solution.

Slips (−1)
S1 Arithmetic error.

Attempts (2, 2 marks)
A1 cos 2x = cos^2x − sin^2x.
A2 Correct factors.
A3 One correct solution.
Part (c) 20 (10, 5, 5) marks  
Att (3, 2, 2)  

Part (c) (i) 15 (10, 5) marks  
Att (3, 2)  

4 (c) (i) \[ S_1 \text{ is a circle of radius 9 cm and } S_2 \text{ is a circle of radius 3 cm.} \]  
\[ S_1 \text{ and } S_2 \text{ touch externally at } f. \]  
A common tangent touches \( S_1 \) at point \( a \) and \( S_2 \) at \( b \).  

(i) Find the area of the quadrilateral \( abcd \).  
Give your answer in surd form. 

\[
|ec|^2 = |dc|^2 - |de|^2 \quad \Rightarrow \quad |ec|^2 = 144 - 36 = 108. \quad \therefore \quad |ec| = \sqrt{108} = 6\sqrt{3}. \quad \text{But } |ec| = |ab|.  
\]

Area of the quadrilateral \( abcd \)  
\[ = \frac{1}{2} \left[ |ab| \parallel |ad| + |bc| \right] = \frac{1}{2} \left[ 6\sqrt{3} \left( 9 + 3 \right) \right] = 36\sqrt{3} \text{ cm}^2. \]

or

Area of quadrilateral \( abcd = \) triangle \( dce \) + rectangle \( ecba \)  
\[ = \frac{1}{2} \left( 6 \right) \left( 6\sqrt{3} \right) + 3 \left( 6\sqrt{3} \right) \quad = 36\sqrt{3}. \]

Blunders (−3)  
B1 Incorrect application of Pythagoras.  
B2 Error in area formula.  

Slips (−1)  
S1 Arithmetic error.  

Attempts (3, 2 marks)  
A1 Correct length of \( |dc| \) or \( |de| \).  
A2 Area of triangle \( dce \) or rectangle \( ecba \) correct.  
A3 Area formula for trapezium \( abcd \) with some substitution.
Find the area of the shaded region, which is bounded by $[ab]$ and the minor arcs $af$ and $bf$.  

**Area of shaded region**

\[
\cos \angle edc = \frac{6}{12} = \frac{1}{2} \quad \Rightarrow \quad \angle edc = 60^\circ. \quad \therefore \angle bcf = 30^\circ + 90^\circ = 120^\circ.
\]

Area of sector $adf = \frac{1}{2} r^2 \theta = \frac{1}{2} \left( 81 \right) \left( \frac{\pi}{3} \right) = \frac{27 \pi}{2}.$

Area of sector $bcf = \frac{1}{2} r^2 \theta = \frac{1}{2} \left( 9 \right) \left( \frac{2 \pi}{3} \right) = 3 \pi.$

\[\therefore \text{ Area of shaded region } = 36 \sqrt{3} - \frac{27 \pi}{2} - 3 \pi = 36 \sqrt{3} - \frac{33 \pi}{2}.\]

**Blunders (−3)**  
B1 Error in sector area formula.  
B2 Finds area of both sectors but fails to finish.  
B3 Incorrect conversion from degree to radians.  

**Slips (−1)**  
S1 Arithmetic error.  

**Attempts (2 marks)**

A1 $|\angle edc| = 60^\circ$ or $|\angle ecd| = 30^\circ$ or $|\angle bcf| = 120^\circ$.  
A2 $\cos \angle edc = \frac{6}{12}$ or $\sin \angle ecd = \frac{6}{12}$.  

---

Blunders (−3)

B1 Error in sector area formula.
B2 Finds area of both sectors but fails to finish.
B3 Incorrect conversion from degree to radians.

Slips (−1)

S1 Arithmetic error.

Attempts (2 marks)

A1 $|\angle edc| = 60^\circ$ or $|\angle ecd| = 30^\circ$ or $|\angle bcf| = 120^\circ$.
A2 $\cos \angle edc = \frac{6}{12}$ or $\sin \angle ecd = \frac{6}{12}$.  

---
QUESTION 5

| Part (a) | 15 marks | Att 5 |
| Part (b) | 20 (15, 5) marks | Att (5, 2) |
| Part (c) | 15 (5, 5, 5) marks | Att (2, 2, 2) |

**Part (a)**

The area of an equilateral triangle is $4\sqrt{3}$ cm$^2$.
Find the length of a side of the triangle.

**Length of side**

<table>
<thead>
<tr>
<th>15 marks</th>
<th>Att 5</th>
</tr>
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</table>

5(a) \hspace{1cm} \text{Area of triangle } = \frac{1}{2} ab \sin C, \text{ where } a = b \text{ and } |\angle C| = \frac{\pi}{3}.
\[
\therefore \frac{1}{2} a^2 \sin \frac{\pi}{3} = 4\sqrt{3} \Rightarrow \frac{1}{2} a^2 \frac{\sqrt{3}}{2} = 4\sqrt{3}.
\]
\[
\therefore a^2 = 16 \Rightarrow a = 4. \text{ Length of side } = 4 \text{ cm.}
\]

**Blunders (−3)**

B1 Error in triangle area formula.
B2 Incorrect evaluation of $\sin 60^\circ$.
B3 $\sin 60^\circ$ in decimal form.

**Slips (−1)**

S1 Arithmetic error.

**Attempts (5 marks)**

A1 Triangle area formula with substitution.

**Part (b)**

<table>
<thead>
<tr>
<th>20 (15, 5) marks</th>
<th>Att (5, 2)</th>
</tr>
</thead>
</table>

**Part (b) (i)**

In the triangle $xyz$, $|\angle xyz| = 2\beta$ and $|\angle xzy| = \beta$.
\[
|xy| = 3 \text{ and } |xz| = 5.
\]

(i) Use this information to express $\sin 2\beta$ in the form $\frac{a}{b} \sin \beta$, where $a, b \in \mathbb{N}$.

**Express**

<table>
<thead>
<tr>
<th>15 marks</th>
<th>Att 5</th>
</tr>
</thead>
</table>

\[
\frac{\sin 2\beta}{5} = \frac{\sin \beta}{3} \Rightarrow \sin 2\beta = \frac{5}{3} \sin \beta.
\]
Blunders (−3)
B1 Error in substitution into Sine rule.

Slips (−1)
S1 Arithmetic error.

Attempts (5 marks)
A1 \( \frac{3}{\sin \beta} \) or \( \frac{5}{\sin 2\beta} \).

Part (b) (ii) 5 marks Att 2

5 (b) (ii) Hence express \( \tan \beta \) in the form \( \frac{\sqrt{c}}{d} \), where \( c, d \in \mathbb{N} \).

Express \( \tan \beta \) 5 marks Att 2

\[
\begin{align*}
\sin 2\beta &= \frac{5}{3} \sin \beta \quad \Rightarrow \quad 2\sin \beta \cos \beta = \frac{5}{3} \sin \beta. \\
\therefore \cos \beta &= \frac{5}{6} \quad \Rightarrow \quad \tan \beta = \frac{\sqrt{11}}{5}.
\end{align*}
\]

Blunders (−3)
B1 Error in sin2 formula.
B2 Incorrect ratio of sides for \( \cos \beta \) or \( \tan \beta \).
B3 Incorrect application of Pythagoras.
B4 \( \cos \beta = \frac{5}{6} \) and stops.

Slips (−1)
S1 Arithmetic error.

Attempts (2 marks)
A1 Equation in \( \beta \).
Part (c) 15 (5, 5, 5) marks Att (2, 2, 2)

5 (c) \(pqrs\) is a vertical wall of height \(h\) on level ground. 
\(p\) is a point on the ground in front of the wall.
The angle of elevation of \(r\) from \(p\) is \(\theta\) and the angle of elevation of \(s\) from \(p\) is \(2\theta\).
\(|pq|=3|pt|\).
Find \(\theta\).

\[\tan \theta = \frac{h}{3x} \Rightarrow h = 3x \tan \theta. \text{ Also } \tan 2\theta = \frac{h}{x} \Rightarrow h = x \tan 2\theta.\]
\[\therefore 3 \tan \theta = x \tan 2\theta \Rightarrow 3 \tan \theta = \frac{2 \tan \theta}{1 - \tan^2 \theta} \Rightarrow 3t(1 - t^2) = 2t, \text{ where } t = \tan \theta.\]
\[\therefore 3t - 3t^3 = 2t \Rightarrow 3t^3 - t = 0. \quad t(3t^2 - 1) = 0 \Rightarrow t^2 = \frac{1}{3}, t \neq 0.\]
\[\therefore t = \tan \theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = \frac{\pi}{6}.\]

Blunders (−3)
B1 Incorrect ratio of sides for \(\tan\).
B2 Error in \(\tan 2\theta\) formula.
B3 Incorrect factors.
B4 Incorrect value for \(\theta\).

Slips (−1)
S1 Arithmetic error.

Attempts (2, 2, 2 marks)
A1 \(\tan \theta\) or \(\tan 2\theta\) expressed as ratio of sides.
A2 \(\tan 2\theta\) expressed in terms of \(\tan \theta\).
A3 Correct value for \(\tan^2 \theta\).
QUESTION 6

| Part (a) | 10 (5, 5) marks | Att (-, 2) |
| Part (b) | 25 (5, 5, 5, 5, 5) marks | Att (2, 2, 2, 2, 2) |
| Part (c) | 15 (5, 5, 5) marks | Att (2, 2, 2) |

Part (a) | 10 (5, 5) marks | Att (-, 2) |

Part (a) (i) | 5 marks | Hit/Miss |

6 (a) (i) How many three-digit numbers can be formed from the digits 1, 2, 3, 4, 5, if

(i) the three digits are all different

6 (a) (i) Answer \( = P_3^5 = 5 \times 4 \times 3 = 60 \).

Part (a) (ii) | 5 marks | Att 2 |

6 (a) (ii) How many three-digit numbers can be formed from the digits 1, 2, 3, 4, 5, if

(ii) the three digits are all the same?

6 (a) (ii) Answer \( = 5 \times 1 \times 1 = 5 \).

Blunders (−3)
B1 5×5×1.

Slips (−1)
S1 Arithmetic error.

Attempts (2 marks)
A1 5×5×5.
Part (b) 25 (5, 5, 5, 5) marks  
Att (2, 2, 2, 2)

Part (b) (i) 20 (5, 5, 5, 5) marks  
Att (2, 2, 2)

6 (b) (i) Solve the difference equation \( u_{n+2} - 4u_{n+1} - 8u_n = 0 \), where \( n \geq 0 \), given that \( u_0 = 0 \) and \( u_1 = 2 \).

---

**Characteristic equation** 5 marks 
**Characteristic roots** 5 marks 
**Simultaneous equations** 5 marks 
**Solution** 5 marks

6 (b) (i)

\[
\begin{align*}
    u_{n+2} - 4u_{n+1} - 8u_n &= 0 & \Rightarrow & & x^2 - 4x - 8 = 0. \\
    \therefore & & x &= \frac{4 \pm \sqrt{16+32}}{2} = \frac{4 \pm \sqrt{48}}{2} = \frac{4 \pm 4\sqrt{3}}{2} = 2 \pm 2\sqrt{3}. \\
    u_n &= k\left(2+2\sqrt{3}\right)^n + l\left(2-2\sqrt{3}\right)^n . \\
    u_0 &= 0 & \Rightarrow & & k + l = 0. \ l = -k. \\
    u_1 &= 2 & \Rightarrow & & k\left(2+2\sqrt{3}\right) + l\left(2-2\sqrt{3}\right) = 2 \\
    \therefore & & k\left(2+2\sqrt{3}\right) - k\left(2-2\sqrt{3}\right) &= 2 & \Rightarrow & & 2k + 2k\sqrt{3} - 2k + 2k\sqrt{3} = 2 \\
    \therefore & & 4k\sqrt{3} &= 2 & \Rightarrow & & k = \frac{1}{2\sqrt{3}} = \frac{\sqrt{3}}{6}. \ \therefore \ l &= -\frac{\sqrt{3}}{6}. \\
    \therefore & & u_n &= \frac{\sqrt{3}}{6}\left(2+2\sqrt{3}\right)^n - \frac{\sqrt{3}}{6}\left(2-2\sqrt{3}\right)^n .
\end{align*}
\]

---

**Blunders** (-3)
B1 Error in characteristic equation. 
B2 Error in quadratic formula. 
B3 Incorrect use of initial conditions.

**Slips** (-1)
S1 Arithmetic error.

**Attempts** (2, 2, 2, 2 marks)
A1 An equation in \( k \) and \( l \). 
A2 Correct value for \( k \) or \( l \).
Part (b) (ii)  

6 (b) (ii) Verify that your solution gives the correct value for \( u_2 \).

Verify  

6 (b) (ii)  

\[ u_2 - 4u_1 - 8u_0 = 0. \] But \( u_1 = 2 \) and \( u_0 = 0. \)  
\[ \therefore u_2 = 8 + 0 = 8. \]  
But \( u_2 = \frac{\sqrt{3}}{6} \left( 2 + 2\sqrt{3} \right)^2 - \frac{\sqrt{3}}{6} \left( 2 - 2\sqrt{3} \right)^2 = \frac{\sqrt{3}}{6} \left( 4 + 8\sqrt{3} + 12 - 4 + 8\sqrt{3} - 12 \right) \]  
\[ u_2 = \frac{\sqrt{3}}{6} \left( 16\sqrt{3} \right) = 8. \therefore \text{ Verified.} \]

or

6 (b) (ii)  

\[ u_n = \frac{\sqrt{3}}{6} \left( 2 + 2\sqrt{3} \right)^n - \frac{\sqrt{3}}{6} \left( 2 - 2\sqrt{3} \right)^n. \]  
\[ \therefore u_2 = \frac{\sqrt{3}}{6} \left( 2 + 2\sqrt{3} \right)^2 - \frac{\sqrt{3}}{6} \left( 2 - 2\sqrt{3} \right)^2. \]  
\[ u_2 = \frac{\sqrt{3}}{6} \left( 4 + 8\sqrt{3} + 12 - 4 + 8\sqrt{3} - 12 \right) \Rightarrow u_2 = \frac{\sqrt{3}}{6} \left( 16\sqrt{3} \right) \Rightarrow u_2 = 8. \]  
Substituting \( u_0 = 0, u_1 = 2 \) and \( u_2 = 8 \) into \( u_{n+2} - 4u_{n+1} - 8u_n \),  
gives \( 8 - 4(2) - 0 = 0. \therefore \text{ Verified.} \]

Blunders (-3)  
B1 Error in calculating \( u_2 \) other than slip.  
B2 Finds \( u_2 \) but fails to verify.

Slips (-1)  
S1 Arithmetic error.

Attempts (2 marks)  
A1 Correct value for \( u_2 \).
Part (c) 15 (5, 5, 5) marks Att (2, 2, 2)
Part (c) (i) 5 marks Att 2

6 (c) (i) Nine cards are numbered from 1 to 9. Three cards are drawn at random from the nine cards.

(i) Find the probability that the card numbered 8 is not drawn.

**Probability 5 marks Att 2**

6 (c) (i) Total outcomes (choose three cards from nine): \( ^9C_3 = 84 \).

Outcomes of interest (choose three from the eight allowed): \( ^8C_3 = 56 \).

\[ \therefore \text{Probability} = \frac{56}{84} = \frac{2}{3}. \]

or

6 (c) (i) (first card not 8) and (second card not 8) and (third card not 8)

\[ \Rightarrow \text{Probability} = \frac{8}{9} \times \frac{7}{8} \times \frac{6}{7} = \frac{2}{3}. \]

**Blunders (−3)**
B1 Incorrect number of possible outcomes.

**Slips (−1)**
S1 Arithmetic error.

**Attempts (2 marks)**
A1 Correct number of possible outcomes.
A2 Correct number of favourable outcomes.

Part (c) (ii) 5 marks Att 2

6 (c) (ii) Nine cards are numbered from 1 to 9. Three cards are drawn at random from the nine cards.

(ii) Find the probability that all three cards drawn have odd numbers.

**Probability 5 marks Att 2**

6 (c) (ii) Outcomes of interest (choose three from the five odd-numbered): \( ^5C_3 = 10 \).

\[ \therefore \text{Probability} = \frac{10}{84} = \frac{5}{42}. \]

or

6 (c) (ii) (first card odd) and (second card odd) and (third card odd)

\[ \therefore \text{Probability} = \frac{5}{9} \times \frac{4}{8} \times \frac{3}{7} = \frac{60}{504} = \frac{5}{42}. \]

**Blunders (−3)**
B1 Incorrect number of possible outcomes.

**Slips (−1)**
S1 Arithmetic error.

**Attempts (2 marks)**
A1 Correct number of possible outcomes.
A2 Correct number of favourable outcomes.
9 cards are numbered from 1 to 9. Three cards are drawn at random from the nine cards.

(iii) Find the probability that the sum of the numbers on the cards drawn is greater than the sum of the numbers on the cards not drawn.

Outcomes of interest:
- Sum of all the cards numbered 1 to 9 is 45.
- Sum of three drawn cards must be $\geq 23$, (i.e. more than half of total).
- Sum of cards 7, 8, 9 = 24
- Sum of cards 6, 8, 9 = 23
- No other possibilities.
- Only two possible favourable outcomes.

\[
\therefore \text{Probability} = \frac{2}{84} = \frac{1}{42}
\]

**Blunders (-3)**
B1 Incorrect number of possible outcomes.

**Slips (-1)**
S1 Arithmetic error.

**Attempts (2 marks)**
A1 Correct number of favourable outcomes.
A2 Correct number of possible outcomes.
A3 One correct element properly identified e.g. $9+8+7=24 > 21$. 
QUESTION 7

Part (a) 10 (5, 5) marks
Part (b) 20 (5, 5, 5, 5) marks
Part (c) 20 (5, 5, 5, 5) marks

Part (a) 10 (5, 5) marks
Part (a) (i) 5 marks

7 (a) (i) How many different groups of four can be selected from five boys and six girls?

7 (a) (i) Choose four from eleven ⇒ answer $= \binom{11}{4} = 330$.

7 (a) (ii) How many of these groups consist of two boys and two girls?

7 (a) (ii) Choose two from five and choose two from six ⇒ answer $= \binom{5}{2} \times \binom{6}{2} = 10 \times 15 = 150$.

Blunders (−3)
B1 $\binom{5}{2} + \binom{6}{2}$.

Slips (−1)
S1 Arithmetic error.

Attempts (2 marks)
A1 $\binom{5}{2}$ or $\binom{6}{2}$.

Part (b) 20 (5, 5, 5, 5) marks
Part (b) (i) 5 marks

7 (b) (i) There are sixteen discs in a board-game: five blue, three green, six red and two yellow.
Four discs are chosen at random. What is the probability that

(i) the four discs are blue

7 (b) (i) Total outcomes (choose four discs from sixteen): $\binom{16}{4} = 1820$.
Outcomes of interest (choose four of the five blue): $\binom{5}{4} = 5$.

∴ Probability $= \frac{5}{1820} = \frac{1}{364}$.

or

7 (b) (i) (first blue) and (second blue) and (third blue) and (fourth blue)

∴ Probability $= \frac{5}{16} \times \frac{4}{15} \times \frac{3}{14} \times \frac{2}{13} = \frac{120}{43680} = \frac{1}{364}$.

Blunders (−3)
B1 Incorrect number of possible outcomes.

Slips (−1)
S1 Arithmetic error.

Attempts (2 marks)
A1 Correct number of possible outcomes.
A2 Correct number of favourable outcomes.
There are sixteen discs in a board-game: five blue, three green, six red and two yellow. Four discs are chosen at random. What is the probability that

(ii) the four discs are the same colour

<table>
<thead>
<tr>
<th>Outcomes of interest: (four blue or four red): $^5C_4 + ^6C_4 = 5 + 15 = 20. $</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\therefore$ Probability $\frac{20}{1820} = \frac{1}{91}$.</td>
</tr>
</tbody>
</table>

Alternatively,

<table>
<thead>
<tr>
<th>Probability = $P(4$ blue $) + P(4$ red $)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$=\left(\frac{5}{16}\times\frac{4}{15}\times\frac{3}{14}\times\frac{2}{13}\right) + \left(\frac{6}{16}\times\frac{5}{15}\times\frac{4}{14}\times\frac{3}{13}\right) = \frac{120 + 360}{43680} = \frac{480}{43680} = \frac{1}{91}$.</td>
</tr>
</tbody>
</table>

**Blunders** (-3)

B1 Incorrect number of possible outcomes.

**Slips** (-1)

S1 Arithemetic error.

**Attempts** (2 marks)

A1 Correct number of possible outcomes.
A2 Correct number of favourable outcomes.
A3 $P(4$ red $)$ correct.

There are sixteen discs in a board-game: five blue, three green, six red and two yellow. Four discs are chosen at random. What is the probability that

(iii) all four discs are different in colour

<table>
<thead>
<tr>
<th>Outcomes of interest: one blue and one green and one red and one yellow: $^5C_1\times^3C_1\times^6C_1\times^2C_1 = 180. $</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\therefore$ Probability $\frac{180}{1820} = \frac{9}{91}$.</td>
</tr>
</tbody>
</table>

Alternatively,

<table>
<thead>
<tr>
<th>(first blue) and (second green) and (third red) and (fourth yellow) or any permutation;</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\therefore$ Probability $\frac{5}{16}\times\frac{3}{15}\times\frac{6}{14}\times\frac{2}{13}\times4! = \frac{4320}{43680} = \frac{9}{91}$.</td>
</tr>
</tbody>
</table>

**Blunders** (-3)

B1 Incorrect number of possible outcomes.

**Slips** (-1)

S1 Arithemetic error.

**Attempts** (2 marks)

A1 Correct number of possible outcomes.
A2 Correct number of favourable outcomes.
7 (b) (iv) There are sixteen discs in a board-game: five blue, three green, six red and two yellow. Four discs are chosen at random. What is the probability that

(iv) two of the discs are blue and two are not blue?

**Probability** 5 marks Att 2

7 (b) (iv) Of interest: (choose two of five blue and two of remaining eleven) \( \binom{5}{2} \times \binom{11}{2} = 550. \)

\[ \therefore \text{Probability} = \frac{550}{1820} = \frac{55}{182}. \]

or

7 (b) (iv) (first blue) and (second blue) and (third not blue) and (fourth not blue), or any permutation thereof;

\[ \therefore \text{Probability} = \frac{5}{16} \times \frac{4}{15} \times \frac{11}{14} \times \frac{10}{13} \times \frac{4!}{2! \times 2!} = \frac{52800}{174720} = \frac{55}{182}. \]

**Blunders (-3)**

B1 Incorrect number of possible outcomes.

**Slips (-1)**

S1 Arithmetic error.

**Attempts (2 marks)**

A1 Correct number of possible outcomes.

A2 Correct number of favourable outcomes.

A3 P (two blue) correct.

A4 P (two are not blue) correct.

Part (c) 20 (5, 5, 5, 5) marks Att (2, 2, -, 2)

Part (c) (i) 10 (5, 5) marks Att (2, 2)

7 (c) (i) On 1st September 2003 the mean age of the first-year students in a school is 12.4 years and the standard deviation is 0.6 years. One year later all of these students have moved into second year and no other students have joined them.

(i) State the mean and the standard deviation of the ages of these students on 1st September 2004. Give a reason for each answer.

**Mean** 5 marks Att 2

Mean = 13.4 years.

As all the students are one year older, the mean is increased by one.

**Standard deviation** 5 marks Att 2

Standard deviation = 0.6 years.

The spread of ages in the group is still the same.

or

As they are each one year older and the mean is increased by one, each deviation from the mean is unchanged, and hence so is the standard deviation.
Blunders (−3)
B1 Reason for new mean not given or incorrect reason.
B2 Reason for new standard deviation not given or incorrect reason.

Slips (−1)
S1 Arithmetic error.

Attempts (2, 2 marks)
A1 Correct new mean.
A2 Correct new standard deviation.

<table>
<thead>
<tr>
<th>Part (c) (ii)</th>
<th>5 marks</th>
<th>Hit/Miss</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>7 (c) (ii)</strong></td>
<td>A new group of first-year students begin on 1st September 2004. This group has a similar age distribution and is of a similar size to the first-year group of September 2003. (ii) State the mean age of the combined group of the first-year and second-year students on 1st September 2004.</td>
<td></td>
</tr>
</tbody>
</table>

Combined mean 5 marks Hit/Miss

<table>
<thead>
<tr>
<th>Part (c) (ii)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean</strong> = ( \frac{12.4 + 13.4}{2} ) = 12.9 years.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Part (c) (iii)</th>
<th>5 marks</th>
<th>Att 2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>7 (c) (iii)</strong></td>
<td>State whether the standard deviation of the ages of this combined group is less than, equal to, or greater than 0.6 years. Give a reason for your answer.</td>
<td></td>
</tr>
</tbody>
</table>

State & reason 5 marks Att 2

| **7 (c) (iii)** | Standard deviation > 0.6 years. There is a greater spread of ages in the combined group than in a single year group. [or: Data more spread out.] |

Blunders (−3)
B1 Incorrect reason given.
B2 No reason given.

Slips (−1)
S1 Arithmetic error.

Attempts (2 marks)
A1 States greater than 0.6 years.

[Aside: the actual value is approximately 0.8; this is not required.]
**QUESTION 8**

<table>
<thead>
<tr>
<th>Part (a)</th>
<th>15 marks</th>
<th>Att 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Part (b)</td>
<td>20 (10, 5, 5) marks</td>
<td>Att (3, 2, 2)</td>
</tr>
<tr>
<td>Part (c)</td>
<td>15 (5, 5, 5) marks</td>
<td>Att (2, 2, 2)</td>
</tr>
</tbody>
</table>

**Part (a)** 15 marks Att 5

8 (a) Use integration by parts to find \( \int x^2 \ln x \, dx \).

**Integration by parts** 15 marks Att 5

\[
\int x^2 \ln x \, dx = uv - \int vdu.
\]

\[
u = \ln x \implies du = \frac{1}{x} \, dx.
\]

\[
dv = x^2 \, dx \implies v = \int x^2 \, dx = \frac{1}{3} x^3.
\]

\[
\therefore \int x^2 \ln x \, dx = \frac{1}{3} x^3 \ln x - \int \frac{1}{3} x^3 \left( \frac{1}{x} \right) \, dx = \frac{1}{3} x^3 \ln x - \frac{1}{3} \int x^2 \, dx
\]

\[
= \frac{1}{3} x^3 \ln x - \frac{1}{9} x^3 + \text{constant}.
\]

**Blunders (−3)**

B1 Incorrect differentiation or integration.

B2 Constant of integration omitted.

B3 Incorrect ‘parts’ formula.

**Slips (−1)**

S1 Arithmetic error.

**Attempts (5 marks)**

A1 Correct assigning to parts formula.

A2 Correct differentiation or integration.
Part (b) (i)  Derive the Maclaurin series for \( f(x) = \ln(1 + x) \) up to and including the term containing \( x^3 \).

Maclaurin series

\[
f(x) = f(0) + \frac{f'(0)x}{1!} + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} + \ldots
\]

\[
f(x) = \ln(1 + x) \quad \Rightarrow \quad f(0) = \ln 1 = 0.
\]

\[
f'(x) = \frac{1}{1+x} = (1+x)^{-1} \quad \Rightarrow \quad f'(0) = 1.
\]

\[
f''(x) = -1(1+x)^{-2} \quad \Rightarrow \quad f''(0) = -1.
\]

\[
f'''(x) = 2(1+x)^{-3} \quad \Rightarrow \quad f'''(0) = 2.
\]

\[
\therefore \quad f(x) = \ln(1 + x) = 0 + x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \ldots = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \ldots
\]

Blunders (−3)

B1 Incorrect differentiation.
B2 Incorrect evaluation of \( f^{(n)}(0) \).
B3 Each term not derived.
B4 Error in Maclaurin series.

Attempts (3 marks)

A1 Correct expansion of \( \ln(1 + x) \) given but not derived.
A2 \( f(0) \) correct.
A3 Any one correct term derived.

Part (b) (ii)  Use those terms to find an approximation for \( \ln \frac{11}{10} \).

Find approximation

\[
\ln \frac{11}{10} = \ln \left(1 + \frac{1}{10}\right) = \frac{1}{10} - \frac{1}{200} + \frac{1}{3000} - \frac{300 - 15 + 1}{3000} = \frac{286}{1500} = \frac{143}{750}.
\]

Blunders (−3)

B1 Error in simplification other than slip.

Attempts (2 marks)

A1 \( \frac{11}{10} = 1 + \frac{1}{10} \).
A2 Correct value for \( x \).
8 (b)(iii) Write down the general term of the series \( f(x) \) and hence show that the series converges for \(-1 < x < 1\).

**General term/converges** 5 marks Att 2

\[
\text{(b) (iii)} \quad \text{General term} = u_n = \frac{(-1)^{n+1} x^n}{n}. \quad \therefore \quad u_{n+1} = \frac{(-1)^{n+2} x^{n+1}}{n+1}
\]

\[
\therefore \quad \lim_{n \to \infty} \frac{u_{n+1}}{u_n} = \lim_{n \to \infty} \frac{(-1)^{n+2} x^{n+1}}{n+1} \times \frac{n}{(-1)^{n+1} x^n} = \lim_{n \to \infty} \frac{(-1) x^n}{n+1} = \lim_{n \to \infty} \frac{x}{1 + \frac{1}{n}} = |x|.
\]

Series converges when \(|x| < 1 \Rightarrow -1 < x < 1\).

**Blunders (−3)**

B1 Incorrect power in general term.
B2 \((-1)\) omitted from general term.
B3 Error in \(u_{n+1}\).
B4 Error in evaluating limit other than slip.
B5 Evaluates limit as \(|x|\) and stops.

**Slips (−1)**

S1 Arithmetic error.

**Attempts (2 marks)**

A1 Power of \(x\) correct.
A2 Denominator correct.
A3 \(u_{n+1}\) correct, given that \(u_n\) is not worthless.
A4 Correct substitution into ratio test and fails to finish.
A cone has radius \( r \) cm, vertical height \( h \) cm and slant height \( 10\sqrt{3} \) cm.

Find the value of \( h \) for which the volume is a maximum.

Volume in terms of \( h \) or \( r \):

\[
V = \frac{1}{3} \pi r^2 h = \frac{1}{3} \pi h (300 - h^2)
\]

Correct differentiation:

\[
\frac{dV}{dh} = \frac{1}{3} \pi (300 - 3h^2) = 0 \quad \text{for maximum volume.}
\]

\[
\therefore 300 - 3h^2 = 0 \quad \Rightarrow \quad h = 10, \quad \text{(since } h > 0). 
\]

\[
\frac{d^2V}{dx^2} = -2\pi h < 0 \quad \text{for } h = 10.
\]

\[
\therefore h = 10 \text{ cm gives maximum volume.}
\]

* \( \frac{d^2V}{dh^2} < 0 \), for \( h = 10 \text{ cm not required.} \)

Blunders (-3)

B1 Incorrect application of Pythagoras.
B2 Error in differentiation.
B3 Error in solving for \( h \) or \( r \), other than slip.

Slips (-1)

S1 Arithmetic error.
S2 Correct value for \( r \), but value of \( h \) not given.

Attempts (2, 2, 2 marks)

A1 \( h^2 + r^2 = 300 \).
A2 Some part of differentiation correct.
A3 \( \frac{dV}{dh} = 0 \), given that candidate’s work is not worthless.
**QUESTION 9**

<table>
<thead>
<tr>
<th>Part (a)</th>
<th>10 marks</th>
<th>Att 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Part (b)</td>
<td>20 (10, 5, 5) marks</td>
<td>Att (3, 2, 2)</td>
</tr>
<tr>
<td>Part (c)</td>
<td>20 (10, 5, 5) marks</td>
<td>Att (3, 2, 2)</td>
</tr>
</tbody>
</table>

**Part (a) 10 marks Att 3**

9 (a) \( z \) is a random variable with standard normal distribution. Find \( P \left( 1 < z < 2 \right) \).

\[
9 \text{ (a)} \quad P \left( 1 < z < 2 \right) = 0.9772 - 0.8413 = 0.1359.
\]

**Blunders (−3)**
B1 \( P(z \leq 1) \) or \( P(z < 2) \) incorrect.

**Slips (−1)**
S1 Arithmetic error.

**Attempts (3 marks)**
A1 \( P(z \leq 1) \) or \( P(z < 2) \) correct.

<table>
<thead>
<tr>
<th>Part (b)</th>
<th>20 (10, 5, 5) marks</th>
<th>Att (3, 2, 2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Part (b) (i)</td>
<td>10 marks</td>
<td>Att 3</td>
</tr>
</tbody>
</table>

9 (b) (i) During a match John takes a number of penalty shots. The shots are independent of each other and his probability of scoring with each shot is \( \frac{4}{5} \).

(i) Find the probability that John misses each of his first four penalty shots.

**Probability 10 marks Att 3**

9 (b) (i) Probability \( = \left( \frac{1}{5} \right)^4 = \frac{1}{625} \) or \( 4 \binom{4}{0} \left( \frac{4}{5} \right)^0 \left( \frac{1}{5} \right)^4 = \frac{1}{625} \).

**Blunders (−3)**
B1 Error in binomial.
B2 Incorrect \( q \).

**Slips (−1)**
S1 Arithmetic error.

**Attempts (3 marks)**
A1 Correct \( q \).
Part (b) (ii)  5 marks  Att 2

9 (b) (ii) Find the probability that John scores exactly three of his first four penalty shots.

Probability  5 marks  Att 2

9 (b) (ii) Probability \( = 4 \binom{3}{5} \left( \frac{4}{5} \right)^3 \left( \frac{1}{5} \right) = \frac{256}{625} \).

Blunders (–3)
B1 Error in binomial.
B2 Incorrect \( q \).

Slips (–1)
S1 Arithmetic error.

Attempts (2 marks)
A1 \( \left( \frac{4}{5} \right)^3 \left( \frac{1}{5} \right) \).

Part (b) (iii)  5 marks  Att 2

9 (b) (iii) If John takes ten penalty shots during the match, find the probability that he scores at least eight of them.

Probability  5 marks  Att 2

9 (b) (iii) \[ P \text{ (scores at least eight)} = P \text{ (scores eight)} + P \text{ (scores nine)} + P \text{ (scores ten)}. \]
\[ = \binom{10}{8} \left( \frac{4}{5} \right)^8 \left( \frac{1}{5} \right)^2 + \binom{10}{9} \left( \frac{4}{5} \right)^9 \left( \frac{1}{5} \right)^1 + \binom{10}{10} \left( \frac{4}{5} \right)^{10} \left( \frac{1}{5} \right)^0 \]
\[ = \frac{2949120 + 2621440 + 1048576}{9765625} = \frac{6619136}{9765625} \approx 0.678. \]

Blunders (–3)
B1 Error in binomial.
B2 Omits one essential probability.

Slips (–1)
S1 Arithmetic error.

Attempts (2 marks)
A1 Finds one correct probability.
A2 Probability = \( P \) (scoring eight) + \( P \) (scoring nine) + \( P \) (scoring ten).
A survey was carried out to find the weekly rental costs of holiday apartments in a certain country. A random sample of 400 apartments was taken. The mean of the sample was €320 and the standard deviation was €50.

Form a 95% confidence interval for the mean weekly rental costs of holiday apartments in that country.

\[ \bar{x} = 320, \quad \sigma = 50, \quad n = 400. \]

\[ \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{50}{20} = 2.5. \]

The 95% confidence interval is

\[ [\bar{x} - 1.96(\sigma_{\bar{x}}), \bar{x} + 1.96(\sigma_{\bar{x}})] \]

\[ = [320 - 1.96(2.5), 320 + 1.96(2.5)] = [€315.10, €324.90] \]
## QUESTION 10

<table>
<thead>
<tr>
<th>Part (a)</th>
<th>15 (10, 5) marks</th>
<th>Att (3, 2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Part (b)</td>
<td>20 (10, 10) marks</td>
<td>Att (3, 3)</td>
</tr>
<tr>
<td>Part (c)</td>
<td>15 (5, 5, 5) marks</td>
<td>Att (2, 2, 2)</td>
</tr>
</tbody>
</table>

### Part (a) 15 (10, 5) marks Att (3, 2)

**10 (a)**

Show that \( \{0, 2, 4\} \) forms a group under addition modulo 6. You may assume associativity.

<table>
<thead>
<tr>
<th>Show closure</th>
<th>10 marks</th>
<th>Att 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Identity and inverses</td>
<td>5 marks</td>
<td>Att 2</td>
</tr>
</tbody>
</table>

**10 (a) (i)**

<table>
<thead>
<tr>
<th>+ (_{\text{mod } 6})</th>
<th>0</th>
<th>2</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>0</td>
<td>2</td>
</tr>
</tbody>
</table>

Closed: No new element.

Identity = 0.

Inverses: \(0^{-1} = 0\), \(2^{-1} = 4\), \(4^{-1} = 2\).

\(\therefore\) Group.

**Blunders (−3)**

- B1 Identity not given.
- B2 Inverses not stated.

**Slips (−1)**

- S1 Arithmetic error.
- S2 Each inverse not given.

**Attempts (3, 2 marks)**

- A1 Incomplete Cayley table or error in Cayley table.
- A2 Identity given.
- A3 One inverse given.
Part (b) (i) 20 (10, 10) marks  
10 (b) (i) 10 marks  
10 (b) (i) 3 marks

Part (b) (i)

10 (b) (i)

\( R_{90} \) and \( S_M \) are elements of \( D_4 \), the dihedral group of a square.

(i) List the other elements of the group.

---

List elements 10 marks  
10 (b) (i) Att 3

10 (b) (i)

\( R_{0}, R_{180}, R_{270}, S_N, S_L, S_K \).

---

Blunders (−3)
B1 Each incorrect element.
B2 Each missing element.

Slips (−1)
S1 Arithmetic error.

Attempts (3 marks)
A1 One correct element.

---

Part (b) (ii) 10 marks  
10 (b) (ii) Att 3

10 (b) (ii)

Find \( C(S_M) \), the centralizer of \( S_M \).

---

Find centralizer 10 marks  
10 (b) (ii) Att 3

10 (b) (ii)

\[ C(S_M) = R_{90}, S_M, S_N, R_{180}. \]

---

Blunders (−3)
B1 Each incorrect element.
B2 Each missing element.

Slips (−1)
S1 Arithmetic error.

Attempts (3 marks)
A1 One correct element.
Part (c)  15 (5, 5, 5) marks
Part (c) (i)  5 marks

A regular tetrahedron has twelve rotational symmetries. These form a group under composition. The symmetries can be represented as permutations of the vertices \(a, b, c\) and \(d\). 

\[
X = \left\{ \begin{pmatrix} a & b & c & d \\ a & b & c & d \\ a & b & c & d \end{pmatrix} \right\} \left\{ \begin{pmatrix} a & b & c & d \\ b & a & d & c \end{pmatrix} \right\}, \quad \circ \text{ is a subgroup of this tetrahedral group.}
\]

(i) Write down one other subgroup of order 2.

Subgroup of order two  5 marks

\[
10 \text{ (c) (i)} \left\{ \begin{pmatrix} a & b & c & d \\ a & b & c & d \\ a & b & c & d \end{pmatrix} \right\} \left\{ \begin{pmatrix} a & b & c & d \\ b & a & d & c \end{pmatrix} \right\}, \quad \text{or} \quad \left\{ \begin{pmatrix} a & b & c & d \\ a & b & c & d \\ a & b & c & d \end{pmatrix} \right\} \left\{ \begin{pmatrix} a & b & c & d \\ c & d & a & b \end{pmatrix} \right\}.
\]

* If subgroup is not of order 2 then 0 marks.

Blunders (−3)
B1 Incorrect element.

Slips (−1)
S1 Arithmetic error.

Attempts ( 2 marks)
A1 One correct element.

Part (c) (ii)  5 marks

Write down a subgroup of order 3.

Subgroup of order three  5 marks

\[
10 \text{ (c) (ii)} \left\{ \begin{pmatrix} a & b & c & d \\ a & b & c & d \\ a & b & c & d \end{pmatrix} \right\} \left\{ \begin{pmatrix} a & b & c & d \\ a & c & d & b \end{pmatrix} \right\} \left\{ \begin{pmatrix} a & b & c & d \\ a & d & b & c \end{pmatrix} \right\}.
\]

or

\[
10 \text{ (c) (ii)} \left\{ \begin{pmatrix} a & b & c & d \\ a & b & c & d \\ a & b & c & d \end{pmatrix} \right\} \left\{ \begin{pmatrix} a & b & c & d \\ c & b & d & a \end{pmatrix} \right\} \left\{ \begin{pmatrix} a & b & c & d \\ d & b & a & c \end{pmatrix} \right\}.
\]

or

\[
10 \text{ (c) (ii)} \left\{ \begin{pmatrix} a & b & c & d \\ a & b & c & d \\ a & b & c & d \end{pmatrix} \right\} \left\{ \begin{pmatrix} a & b & c & d \\ b & d & c & a \end{pmatrix} \right\} \left\{ \begin{pmatrix} a & b & c & d \\ d & a & c & b \end{pmatrix} \right\}.
\]

or

\[
10 \text{ (c) (ii)} \left\{ \begin{pmatrix} a & b & c & d \\ a & b & c & d \\ a & b & c & d \end{pmatrix} \right\} \left\{ \begin{pmatrix} a & b & c & d \\ b & c & a & d \end{pmatrix} \right\} \left\{ \begin{pmatrix} a & b & c & d \\ c & a & b & d \end{pmatrix} \right\}.
\]

* If subgroup is not of order 3 then 0 marks.
Blunders (−3)
B1 Each incorrect element.

Slips (−1)
S1 Arithmetic error.

Attempts (2 marks)
A1 One correct element.

<table>
<thead>
<tr>
<th>Part (c) (iii)</th>
<th>5 marks</th>
<th>Att 2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>10 (c) (iii)</strong> Write down the only subgroup of order four.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Subgroup of order four</th>
<th>5 marks</th>
<th>Att 2</th>
</tr>
</thead>
</table>
| **10 (c) (iii)** \(
\begin{align*}
(a & b & c & d) \\
(a & b & c & d)
\end{align*}
\) \( \times \)
\(
\begin{align*}
(a & b & c & d) \\
(a & b & c & d)
\end{align*}
\) \( \times \)
\(
\begin{align*}
(a & b & c & d) \\
(a & b & c & d)
\end{align*}
\) |         |       |

* If subgroup is not of order 4 then 0 marks.

Blunders (−3)
B1 Each incorrect element.

Slips (−1)
S1 Arithmetic error.

Attempts (2 marks)
A1 One correct element.
11(a) Find the equation of an ellipse with centre (0, 0), eccentricity \( \frac{5}{6} \) and one focus at (10, 0).

\[
\text{Focus } (10, 0) = (ae, 0) \implies ae = 10. \quad \therefore \frac{5}{6}a = 10 \implies a = 12.
\]

\[
b^2 = a^2 \left(1 - e^2 \right) \quad \Rightarrow b^2 = 144 \left(1 - \frac{25}{36}\right) = 144 \left(\frac{11}{36}\right) \quad \Rightarrow \quad b^2 = 44.
\]

\[
\therefore \quad \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \Rightarrow \quad \frac{x^2}{144} + \frac{y^2}{44} = 1.
\]

**Blunders (−3)**

B1 Values for \( a \) and \( b \) found but final equation not given.

**Slips (−1)**

S1 Arithmetic error.

**Attempts (2, 2 marks)**

A1 \( ae = 10 \).

A2 \( b^2 = a^2 \left(1 - e^2 \right) \).

A3 Correct value for \( b^2 \) and stops.
Part (b) 20 (5, 5, 5, 5) marks  
Att (2, 2, 2, 2)

<table>
<thead>
<tr>
<th>Question</th>
<th>Marks</th>
<th>Att</th>
</tr>
</thead>
<tbody>
<tr>
<td>11 (b)</td>
<td>5 marks</td>
<td>2</td>
</tr>
<tr>
<td>Prove that $</td>
<td>\angle abc</td>
<td>=</td>
</tr>
<tr>
<td>Finish</td>
<td>5 marks</td>
<td>2</td>
</tr>
</tbody>
</table>

**Cos$\angle abc$**

$$\cos \angle abc = \frac{|ab|^2 + |bc|^2 - |ac|^2}{2|ab||bc|}.$$  

**Cos$\angle a'b'c'$**

$$\cos \angle a'b'c' = \frac{|a'b'|^2 + |b'c'|^2 - |a'c'|^2}{2|a'b'||b'c'|}.$$  

But $|a'c'| = k|ac|$, $|a'b'| = k|ab|$ and $|b'c'| = k|bc|$ as $f$ is a similarity transformation.

$$\therefore \cos \angle a'b'c' = \frac{k^2|ab|^2 + k^2|bc|^2 - k^2|ac|^2}{2k^2|ab||bc|} = \frac{|ab|^2 + |bc|^2 - |ac|^2}{2|ab||bc|} = \cos \angle abc.$$  

$$\cos \angle abc = \cos \angle a'b'c' \Rightarrow |\angle abc| = |\angle a'b'c'|, \text{ as } 0^\circ \leq |\angle abc| \leq 180^\circ.$$

**Blunders (-3)**

B1 Error in cosine formula.

B2 Error in definition of similarity transformation.

B3 Fails to square $k$.

B4 Reason why $|\angle abc| = |\angle a'b'c'|$ not given.

**Slips (-1)**

S1 Arithmetic error.

**Attempts (2, 2, 2, 2 marks)**

A1 Use of cosine rule.

A2 Cos$\angle a'b'c'$ expressed in terms of sides of triangle $abc$. 

---

Page 77
Part (c) 20 (5, 5, 5, 5) marks Att (2, 2, 2, 2)

Part (c) (i) 5 marks Att 2

11 (c) (i)  $g$ is the transformation $(x, y) \rightarrow (x', y')$ where $x' = ax$ and $y' = by$ and $a > b > 0$.

(i) $C$ is the circle $x^2 + y^2 = 1$. Show that $g(C)$ is an ellipse.

Show that $g(C)$ is an ellipse 5 marks Att 2

11 (c) (i) $C: x^2 + y^2 = 1$. $x' = ax$ and $y' = by \Rightarrow x = \frac{x'}{a}$ and $y = \frac{y'}{b}$.

$\therefore g(C) = \frac{x'^2}{a^2} + \frac{y'^2}{b^2} = 1$. $\therefore g(C)$ is an ellipse.

Blunders (−3)
B1 Error in substitution.

Slips (−1)
S1 Arithmetic error.

Attempts (2 marks)
A1 $x$ in terms of $x'$ or $y$ in terms of $y'$.
Part (c) (ii)  

$L$ and $K$ are tangents at the end points of a diameter of the ellipse $g(C)$. 
Prove that $L$ and $K$ are parallel.

### $g^{-1}$ mapping of $g(C), D, L$ and $K$  

**5 marks**  

### Showing $g^{-1}(L)$ or $g^{-1}(K) \perp g^{-1}(D)$  

**5 marks**  

### Prove $L$ and $K$ are parallel  

**5 marks**

---

By $g^{-1}, L, K$ and $D$ map onto $g^{-1}(L), g^{-1}(K)$ and $g^{-1}(D)$ respectively.  
But $g^{-1}(L)$ is perpendicular to $g^{-1}(D)$ and $g^{-1}(K)$ is perpendicular to $g^{-1}(D)$, 
as tangent to circle is perpendicular to diameter at point of contact.  
\[ \therefore g^{-1}(L) \text{ is parallel to } g^{-1}(K). \]
\[ \therefore L \text{ is parallel to } K, \text{ as parallelism is invariant.} \]

---

**Blunders (−3)**

B1 Error in mapping or mapping circle to ellipse.

B2 Reason why $g^{-1}(L)$ or $g^{-1}(K) \perp g^{-1}(D)$ not given.

B3 Reason why $L$ is parallel to $K$ not given.

**Slips (−1)**

S1 Arithmetic error.

**Attempts (2, 2, 2 marks)**

A1 One correct mapping.

A2 States $g^{-1}(L)$ or $g^{-1}(K) \perp g^{-1}(D)$ without reason given.

A3 $g^{-1}(L)$ parallel to $g^{-1}(K)$. 

---

Page 79
**BONUS MARKS FOR ANSWERING THROUGH IRISH**

Bonus marks are applied separately to each paper as follows:

If the mark achieved is less than 226, the bonus is 5% of the mark obtained, rounding *down*. (e.g. 198 marks × 5% = 9.9 ⇒ bonus = 9 marks.)

If the mark awarded is 226 or above, the following table applies:

<table>
<thead>
<tr>
<th>Marks obtained</th>
<th>Bonus</th>
</tr>
</thead>
<tbody>
<tr>
<td>226 – 231</td>
<td>11</td>
</tr>
<tr>
<td>232 – 238</td>
<td>10</td>
</tr>
<tr>
<td>239 – 245</td>
<td>9</td>
</tr>
<tr>
<td>246 – 251</td>
<td>8</td>
</tr>
<tr>
<td>252 – 258</td>
<td>7</td>
</tr>
<tr>
<td>259 – 265</td>
<td>6</td>
</tr>
<tr>
<td>266 – 271</td>
<td>5</td>
</tr>
<tr>
<td>272 – 278</td>
<td>4</td>
</tr>
<tr>
<td>279 – 285</td>
<td>3</td>
</tr>
<tr>
<td>286 – 291</td>
<td>2</td>
</tr>
<tr>
<td>292 – 298</td>
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</tr>
<tr>
<td>299 – 300</td>
<td>0</td>
</tr>
</tbody>
</table>