Instructions

There are two sections in this examination paper.

Section A  Concepts and Skills  150 marks  6 questions
Section B  Contexts and Applications  150 marks  2 questions

Answer all eight questions, as follows:

In Section A, answer

Questions 1 to 5 and

either Question 6A or Question 6B.

In Section B, answer Question 7 and Question 8.

Write your answers in the spaces provided in this booklet. You may lose marks if you do not do so. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the Formulae and Tables booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

Marks will be lost if all necessary work is not clearly shown.

Answers should include the appropriate units of measurement, where relevant.

Answers should be given in simplest form, where relevant.

Write the make and model of your calculator(s) here:
Question 1  (25 marks)

Katie tossed a coin 200 times and threw 109 heads. Joe tossed the same coin 400 times and threw 238 heads. Lucy tossed the same coin 500 times and threw 291 heads. Katie, Joe and Lucy now think the coin may be biased.

(a) Give a reason why they think that the coin may be biased.

(b) Lucy uses all the above data and calculates that the best estimate of the probability of throwing a head with this coin is 0.58. Show how Lucy might have calculated this probability.

(c) Joe agrees with Lucy’s estimate of 0.58 as the probability of throwing a head with this coin. He claims that the probability of throwing 3 successive heads with this coin is less than the probability of throwing 2 successive tails. Calculate the probability of each event and state whether Joe’s claim is true or not.
Question 2

An unbiased circular spinner has a movable pointer and five equal sectors, two coloured green and three coloured red.

(a) (i) Find the probability that the pointer stops on green for one spin of the spinner.

(ii) List all the possible outcomes of 3 successive spins of the spinner.
(b) A game consists of spinning the spinner 3 times. Each time the spinner stops on green the player wins €1; otherwise the player wins nothing. For example, if the outcome of one game is “green, red, green” the player wins €2.

Complete the following table:

<table>
<thead>
<tr>
<th>Player wins</th>
<th>€0</th>
<th>€1</th>
<th>€2</th>
<th>€3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Required outcomes</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(c) Is one spin of the spinner above an example of a Bernoulli trial?

Answer: ___________________

Explain what a Bernoulli trial is.
Question 3  

(25 marks)

(a) \( l \) is the line \( 3x + 2y + 18 = 0 \). Find the slope of \( l \).

(b) The line \( k \) is perpendicular to \( l \) and cuts the \( x \)-axis at the point \( (7, 0) \). Find the equation of \( k \).

(c) Find the co-ordinates of the point of intersection of the lines \( l \) and \( k \).
Question 4

(25 marks)

The point $A$ has co-ordinates (8, 6) and $O$ is the origin.
The diagram shows two circles $c_1$ and $c_2$.
$c_1$ has centre (0, 0) and radius $|OA|$.  
$c_2$ has a diameter of $[OA]$.

(a) Find the equation of $c_1$.

(b) Find the equation of $c_2$.

(c) The circle $c_2$ cuts the $x$-axis at the point $P$. Find the co-ordinates of $P$. 
Question 5  
(a) Draw a sketch of the net of the surface of the cylinder and write its dimensions on the sketch.
(b) Calculate the volume of the cylinder. Give your answer in terms of \( \pi \).

(c) A sphere has the same volume as the cylinder. Find the surface area of the sphere. Give your answer in terms of \( \pi \).
Question 6  
(25 marks)

Answer either 6A or 6B.

Question 6A

(a) Construct the triangle $ABC$ such that $|AB| = 8$ cm, $|BC| = |AC| = 5$ cm. The point $A$ is given to you.

(b) On the same diagram, construct the image of the triangle $ABC$ under the axial symmetry in $AB$.

(c) Justify the statement “$AC'BC$ is a parallelogram” where $C'$ is the image of $C$ under the axial symmetry in $AB$. 
OR

Question 6B

In the acute-angled triangle $ABC$

$AP \perp BC$, $BQ \perp AC$ and $CR \perp AB$.

Prove that

$| \angle ABQ | + | \angle BCR | + | \angle CAP | = 90^\circ$. 
Answer both Question 7 and Question 8.

**Question 7**

(75 marks)

The table below shows the rates of births, marriages and deaths in Ireland from 1990 to 2010. The rates are per 10,000 of the estimated population.

<table>
<thead>
<tr>
<th>Year</th>
<th>Births</th>
<th>Marriages</th>
<th>Deaths</th>
</tr>
</thead>
<tbody>
<tr>
<td>1990</td>
<td>151</td>
<td>51</td>
<td>90</td>
</tr>
<tr>
<td>1991</td>
<td>150</td>
<td>49</td>
<td>89</td>
</tr>
<tr>
<td>1992</td>
<td>144</td>
<td>47</td>
<td>87</td>
</tr>
<tr>
<td>1993</td>
<td>138</td>
<td>47</td>
<td>90</td>
</tr>
<tr>
<td>1994</td>
<td>135</td>
<td>46</td>
<td>86</td>
</tr>
<tr>
<td>1995</td>
<td>135</td>
<td>43</td>
<td>90</td>
</tr>
<tr>
<td>1996</td>
<td>140</td>
<td>45</td>
<td>87</td>
</tr>
<tr>
<td>1997</td>
<td>144</td>
<td>43</td>
<td>86</td>
</tr>
<tr>
<td>1998</td>
<td>146</td>
<td>45</td>
<td>85</td>
</tr>
<tr>
<td>1999</td>
<td>144</td>
<td>50</td>
<td>87</td>
</tr>
<tr>
<td>2000</td>
<td>145</td>
<td>51</td>
<td>83</td>
</tr>
<tr>
<td>2001</td>
<td>150</td>
<td>50</td>
<td>79</td>
</tr>
<tr>
<td>2002</td>
<td>155</td>
<td>52</td>
<td>76</td>
</tr>
<tr>
<td>2003</td>
<td>155</td>
<td>51</td>
<td>73</td>
</tr>
<tr>
<td>2004</td>
<td>153</td>
<td>52</td>
<td>71</td>
</tr>
<tr>
<td>2005</td>
<td>148</td>
<td>52</td>
<td>68</td>
</tr>
<tr>
<td>2006</td>
<td>154</td>
<td>52</td>
<td>67</td>
</tr>
<tr>
<td>2007</td>
<td>163</td>
<td>52</td>
<td>64</td>
</tr>
<tr>
<td>2008</td>
<td>168</td>
<td>50</td>
<td>63</td>
</tr>
<tr>
<td>2009</td>
<td>167</td>
<td>48</td>
<td>63</td>
</tr>
<tr>
<td>2010</td>
<td>165</td>
<td>46</td>
<td>61</td>
</tr>
</tbody>
</table>

(a) Complete the back to back stem and leaf plot below to show the marriage rate and death rate in Ireland during the period covered in the table above.

<table>
<thead>
<tr>
<th>Marriage rate</th>
<th>Death rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
</tr>
</tbody>
</table>

Key:

(b) State one difference that can be observed between the distributions of the marriage rate and the death rate in your plot.

(c) Find the median and interquartile range of the yearly marriage rates in Ireland from 1990 to 2010.

<table>
<thead>
<tr>
<th>Median</th>
<th>Interquartile range</th>
</tr>
</thead>
</table>

(d) (i) Find the mean of the death rate in Ireland from 1990 to 2010. Give your answer correct to one decimal place.
(ii) The standard deviation of the death rates in the table over is 10·3. List all of the death rates that are within 1 standard deviation of the mean.

(e) In 2010, the number of children born in Ireland was 75 174. Use this number to estimate the total population of Ireland in 2010.

(f) Use your answer to (e) to estimate the number of people who died in Ireland in 2010.

(g) “More children were born in Ireland in 1990 than in 2000.” Give a reason, based on the data, why this statement is not necessarily true.
(h) Find the ratio, Birth rate : Death rate, for the two years 1990 and 2010. Based on your answers for the two years, what would you predict about the population of Ireland in future years. Give a reason for your answer.

<table>
<thead>
<tr>
<th>1990 Ratio</th>
<th>2010 Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prediction</td>
<td></td>
</tr>
<tr>
<td>Reason</td>
<td></td>
</tr>
</tbody>
</table>

(i) The birth rate and death rate over the 21 years are plotted against each other in the scatter plot below. The correlation coefficient between the two sets of data is $-0.85$. Describe the relationship between the two sets of data and suggest a reason why this might be the case.

![Scatter plot of birth rate vs. death rate with correlation coefficient -0.85](image-url)
Question 8

A search is begun for a buoy that has become detached from its mooring at sea. The area to be searched is a circle of radius 30 km from the last known position, \( K \), of the buoy. The search area is divided into six equal sectors as indicated by the letters \( A, B, C, D, E \) and \( F \).

(a) Fishing boats search the triangular area \( KAB \).

(i) Find \( \angle BKA \).

Answer: ____________________

(ii) Find the area of the triangle \( KAB \).
(iii) Write the area of the triangle $KAB$ as a percentage of the area of the sector $KAB$.

(iv) Use the cosine rule to find the length of $[AB]$.

(v) What does your answer to (iv) above show about the triangle $KAB$?
(b) A helicopter took part in the search.

(i) The helicopter flew from the point $F$ around the perimeter of the search area. What distance did the helicopter fly, correct to the nearest km?

(ii) The helicopter then flew in a straight line from $F$ to $D$ and from $D$ on to $C$, also in a straight line. Draw the path of the helicopter on the diagram.

(iii) A theorem on your course can be used to find $|\angle FDC|$. Write down $|\angle FDC|$ and state the theorem.

$|\angle FDC| = \underline{\text{______________}}$

Statement of theorem:

(iv) The helicopter flew at a speed of 80 km/h. How long did it take to fly from $F$ to $D$ and on to $C$?
(c) A lifeboat taking part in the search sailed, in a straight line, from the point $K$ until it reached a point $X$, the midpoint of $[ED]$.

(i) Calculate $|KX|$.

(ii) The buoy was located at the point where the path $KX$, of the lifeboat, crossed the path $FD$ of the helicopter. How far was the buoy from $X$?