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BONUS MARKS FOR ANSWERING THROUGH IRISH .............................................. 62
MARKING SCHEME

LEAVING CERTIFICATE EXAMINATION 2006

MATHEMATICS – HIGHER LEVEL – PAPER 1

GENERAL GUIDELINES FOR EXAMINERS – PAPER 1

1. Penalties of three types are applied to candidates’ work as follows:
   • Blunders - mathematical errors/omissions (-3)
   • Slips - numerical errors (-1)
   • Misreadings (provided task is not oversimplified) (-1).

   Frequently occurring errors to which these penalties must be applied are listed in the scheme. They are labelled: B1, B2, B3,…, S1, S2,…, M1, M2,…etc. These lists are not exhaustive.

2. When awarding attempt marks, e.g. Att(3), note that
   • any correct, relevant step in a part of a question merits at least the attempt mark for that part
   • if deductions result in a mark which is lower than the attempt mark, then the attempt mark must be awarded
   • a mark between zero and the attempt mark is never awarded.

3. Worthless work is awarded zero marks. Some examples of such work are listed in the scheme and they are labelled as W1, W2,…etc.

4. The phrase “hit or miss” means that partial marks are not awarded – the candidate receives all of the relevant marks or none.

5. The phrase “and stops” means that no more work is shown by the candidate.

6. Special notes relating to the marking of a particular part of a question are indicated by an asterisk. These notes immediately follow the box containing the relevant solution.

7. The sample solutions for each question are not intended to be exhaustive lists – there may be other correct solutions. Any examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his/her advising examiner.

8. Unless otherwise indicated in the scheme, accept the best of two or more attempts – even when attempts have been cancelled.

9. The same error in the same section of a question is penalised once only.

10. Particular cases, verifications and answers derived from diagrams (unless requested) qualify for attempt marks at most.

11. A serious blunder, omission or misreading results in the attempt mark at most.

12. Do not penalise the use of a comma for a decimal point, e.g. €5.50 may be written as €5,50.
Part (a) 10 marks

1. (a) Find the real number $a$ such that for all $x \neq 9$,

$$\frac{x - 9}{\sqrt{x - 3}} = \sqrt{x + a}.$$
Part 1(b) 20 (5, 5, 5, 5) marks  Att (2, 2, 2, 2)

1 (b) \( f(x) = 3x^3 + mx^2 - 17x + n \), where \( m \) and \( n \) are constants.

Given that \( x - 3 \) and \( x + 2 \) are factors of \( f(x) \), find the value of \( m \) and the value of \( n \).

\[ f(3) = 5 \text{ marks} \]
\[ f(-2) = 5 \text{ marks} \]
\[ \text{Equations} \quad 5 \text{ marks} \]
\[ \text{Solving} \quad 5 \text{ marks} \]

**1 (b)**

\( f(3) = 3(3)^3 + m(3)^2 - 17(3) + n = 0 \)
\[ 9m + n = -30 \quad \text{.......... (i)} \]

\( f(-2) = 3(-2)^3 + m(-2)^2 - 17(-2) + n = 0 \)
\[ 4m + n = -10 \quad \text{.......... (ii)} \]

\( (x - 3) \) a factor \( \Rightarrow f(3) = 0 \)

\( (x + 2) \) factor \( \Rightarrow f(-2) = 0 \)

\( (i) : \quad 9m + n = -30 \)
\( (ii) : \quad 4m + n = -10 \)

\[ 5m = -20 \]
\[ m = -4 \]

or

\[ \frac{3x + (m + 3)}{x^2 - x - 6} \]

\[ \frac{3x^3 - 3x^2 - 18x}{(m + 3)x^2 + x + n} \]

\[ \frac{(m + 3)x^2 - (m + 3)x - 6(m + 3)}{(m + 4)x + (6m + n + 18) = (0)x + (0)} \]

\( (i) : \quad m + 4 = 0 \)
\( m = -4 \)

\( (ii) : \quad 6m + n + 18 = 0 \)
\( -24 + n + 18 = 0 \)
\( n = 6 \)

or

\( (x + 2)(x - 3) = x^2 - x - 6 \)

Other factor must be linear = \( (ax + b) \)

\[ (x^2 - x - 6)(ax + b) = 3x^3 + mx^2 - 17x + n \]
\[ ax^3 - ax^2 - 6ax + bx^2 - bx - 6b = 3x^3 + mx^2 - 17x + n \]
\[ ax^3 + (a + b)x^2 + (-6a - b)x + (-6b) = 3x^3 + mx^2 + (-17)x + n \]

Equating Coefficients

\( (i) : \quad a = 3 \)
\( (ii) : \quad -a + b = m \)
\( (iii) : \quad -6a - b = -17 \)
\( (iv) : \quad -6b = n \)

(ctd...)
(iii) \[-6a - b = -17 \]
\[-18 - b = -17 \]
\[-1 = b \]

(ii) \[m = -a + b \]
\[= -3 - 1 \]
\[= -4 \]

(iv) \[n = -6b = -6(-1) = 6 \]

**Blunders (-3)**

B1 Deduction root from factor

B2 Indices

B3 2\textsuperscript{nd} value not found (having found 1\textsuperscript{st})

B4 In equating coefficients (not like to like)

**Slips (-1)**

S1 Not changing sign when subtracting in division

**Part 1(c) 20 (5, 10, 5) marks Att (2, 3, 2)**

1 (c) \[x^2 - t \text{ is a factor of } x^3 - px^2 - qx + r. \]

(i) Show that \(pq = r.\)

(ii) Express the roots of \(x^3 - px^2 - qx + r = 0\) in terms of \(p\) and \(q.\)

**Division 5 marks Att 2**

(i) Show 10 marks Att 3

(ii) Express 5 marks Att 2

1 (c) (i)

\[\frac{x - p}{x^2 - t}\frac{x^3 - px^2 - qx + r}{x^3 - tx} = -pt + (t - q)x + r = 0 \]

\(= pt + px^2 + (t - q)x + r = 0) \]

\(= pt + (t - q)x + (r - pt) = 0 \)

\(= pt + (r - pq) = 0 \)

Equating Coefficients:

(i) \(t - q = 0 \) \(\Rightarrow t = q \)

(ii) \(r - pt = 0 \) \(\Rightarrow r = pt \)

\(= r = pq \)

1 (c) (ii)

\(f(x) = (x^2 - t)(x - p) = 0 \)

\(\Rightarrow x^2 - t = 0 \) \(\text{or} \) \(x - p = 0 \)

\(x^2 = t \) \(x = p \)

\(x = \pm \sqrt{t} \)

\(x = \pm \sqrt{q} \)

Roots: \(\left\{ \pm \sqrt{q}, p \right\} \)
1 (c) (i) \[ f(x) = \left(x^2 - t\right) \left(x - \frac{r}{t}\right) \]
\[ = x^3 - \frac{r}{t}x^2 - tx + r \]
\[ = x^3 - px^2 - qx + r \]

Equating coefficients:
(i) \[ \frac{r}{t} = p \]
(ii) \[ t = q \]
\[ r = pt \]
\[ r = pq \]

1 (c) (ii) as above

*Remainder ≠ 0 in division

Blunders (-3)
B1 Indices
B2 In equating coefficients (not like to like)
B3 Root from factor
B4 Root omitted
B5 Roots not in \( p \) and \( q \)
B6 Show not in required form

Slips (-1)
S1 Not changing sign when subtracting in division.

Attempts
A1 Any attempt at division
A2 Other factor not linear
2. (a) Solve the simultaneous equations

\[ y = 2x - 5 \]
\[ x^2 + xy = 2. \]

(ii) \( x^2 + xy - 2 = 0 \)
\[ x^2 + x(2x - 5) - 2 = 0 \]
\[ 3x^2 - 5x - 2 = 0 \]
\[ (3x + 1)(x - 2) = 0 \]
\[ 3x + 1 = 0 \quad \text{or} \quad x - 2 = 0 \]
\[ x = -\frac{1}{3} \quad \text{or} \quad x = 2 \]

(iii) \( x = -\frac{1}{3} \) : \[ y = 2\left(-\frac{1}{3}\right) - 5 = -\frac{17}{3} \Rightarrow \left(-\frac{1}{3}, -\frac{17}{3}\right) \]
\[ x = 2 \] : \[ y = 2x - 5 = 2(2) - 5 = -1 \Rightarrow (2, -1) \]
2 (b) (i) Find the range of values of \( t \in \mathbb{R} \) for which the quadratic equation 
\[ (2t - 1)x^2 + 5tx + 2t = 0 \] 
has real roots.

(ii) Explain why the roots are real when \( t \) is an integer.

---

**Correct substitution** in \( b^2 - 4ac \geq 0 \) 5 marks Att 2

**Inequality** 5 marks Att 2

**Finish** 10 marks Att 3

\[
(2t - 1)x^2 + 5tx + 2t = 0 \\
\text{Real Roots:} \quad b^2 - 4ac \geq 0 \\
(5t)^2 - 4(2t - 1)(2t) \geq 0 \\
25t^2 - 16t^2 + 8t \geq 0 \\
9t^2 + 8t \geq 0 \\
\]

**Graph** 
\[ 9t^2 + 8t = 0 \]
\[ t(9t + 8) = 0 \]
\[ t = 0 \quad \text{or} \quad t = -\frac{8}{9} \]

\[ \therefore \quad 9t^2 + 8t \geq 0 \quad \text{when} \quad \{t \leq -\frac{8}{9}\} \cup \{t \geq 0\} \]

(ii) Imaginary roots only when \(-\frac{8}{9} < t < 0\)
No integer included here.
\[ \Rightarrow \text{real roots for all integers.} \]

---

**Blunders (-3)**
B1 Inequality sign
B2 Indices
B3 Factors once only
B4 Deduction root from factor
B5 Range not stated (written down) or no range
B6 Incorrect range
B7 Shade graph only
B8 Incorrect deduction or no deduction in (ii)

**Misreading (-1)**
M1 Uses '>' for '≥'
Part (c) 15 (5, 5, 5) marks Att (2, 2, 2)

2 (c) \( f(x) = 1 - b^{2x} \) and \( g(x) = b^{1+2x} \), where \( b \) is a positive real number.

Find, in terms of \( b \), the value of \( x \) for which \( f(x) = g(x) \).

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<td>( b^{2x} ) isolated</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Value</td>
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</table>

2 (c) \( f(x) = 1 - b^{2x} \)
\( g(x) = b^{1+2x} \)

\( f(x) = g(x) \)
\( 1 - b^{2x} = b^{1+2x} \)
\( 1 - b^{2x} = bb^{2x} \)
\( 1 = bb^{2x} + b^{2x} \)
\( 1 = b^{2x}(b + 1) \)
\( b^{2x} = \frac{1}{b + 1} \)
\( \log_b(b^{2x}) = \log_b\left(\frac{1}{b+1}\right) \)
\( 2x \log_b b = -\log_b (b + 1) \)
\( 2x = -\log_b (b + 1) \)
\( x = -\frac{1}{2} \log_b (b + 1) \)
\( x = -\log_b \sqrt{b + 1} \)

* Accept logs to any base

Blunders (-3)
B1  Indices
B2  Factors
B3  Logs
QUESTION 3

Part (a) 5 marks Att 2
Part (b) 25 (10, 5, 5, 5) marks Att (3, 2, 2, 2)
Part (c) 20 (5, 5, 5, 5) marks Att (2, 2, 2, 2)

Part (a) 5 marks Att 2
3. (a) Given that \( z = 2 + i \), where \( i^2 = -1 \), find the real number \( d \) such that \( z + \frac{d}{z} \) is real.

Part (a) 5 marks Att 2
3 (a) 

\[
\frac{d}{z} = d \left[ \frac{1}{2i\pi} \cdot \frac{2+i}{2+i} \right] = \frac{d(2+i)}{5} \\
(2 + i) + \frac{d}{5}(2 - i) = a + (0)i \\
(2 + \frac{2d}{5}) + (1 - \frac{d}{5})i = (a) + (0)i \\
\]

Equating Coefficients:

\[
1 - \frac{d}{5} = 0 \\
1 = \frac{d}{5} \\
d = 5
\]

Blunders (-3)

B1 \( i \)
B2 Not real to real etc
B3 \( (2 + i)(2 - i) \neq 5 \)
B4 Conjugate

Attempts

A1 \( f(z) = 0 \)

Part (b)(i) 15(10, 5) marks Att (3, 2)

3 (b) (i) Use matrix methods to solve the simultaneous equations

\[
\begin{align*}
4x - 2y &= 5 \\
8x + 3y &= -4
\end{align*}
\]

Part (b)(i) Matrix form 10 marks Att 3
Solution 5 marks Att 2

\[
\begin{pmatrix} 4 & -2 \\ 8 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ -4 \end{pmatrix} \\
A = \begin{pmatrix} 4 & -2 \\ 8 & 3 \end{pmatrix} \Rightarrow A^{-1} = \frac{1}{28} \begin{pmatrix} 3 & 2 \\ -8 & 4 \end{pmatrix} \\
\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{28} \begin{pmatrix} 3 & 2 \\ -8 & 4 \end{pmatrix} \begin{pmatrix} 5 \\ -4 \end{pmatrix} \\
= \frac{1}{28} \begin{pmatrix} 7 \\ -56 \end{pmatrix} = \begin{pmatrix} \frac{1}{4} \\ -2 \end{pmatrix}
\]
**Blunders (-3)**

B1 Incorrect matrix $A$

B2 Incorrect $\frac{1}{\det}$ or no $\frac{1}{\det}$

B3 $A^{-1}.A \neq I$

B4 Incorrect matrix

---

### Part (b) (ii) 10 (5, 5)marks Att (2, 2)

#### 3 (b) (ii) Find the two values of $k$ which satisfy the matrix equation

$\begin{pmatrix} 1 & k \\ -2 & 1 \end{pmatrix} \begin{pmatrix} 3 & 4 \\ -1 & k \end{pmatrix} = 11.$

---

#### Part (b)(ii) Complete Multiplication 5 marks Att 2

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<td>$3 + 4k - 2k + k^2 - 11 = 0$</td>
<td>$3 - 2k + 4k + k^2 = 11$</td>
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<tr>
<td>$k^2 + 2k - 8 = 0$</td>
<td>$k^2 + 2k - 8 = 0$</td>
</tr>
<tr>
<td>$(k + 4)(k - 2) = 0$</td>
<td>$(k + 4)(k - 2) = 0$</td>
</tr>
<tr>
<td>$k = -4$ or $k = 2$</td>
<td>$k = -4$ or $k = 2$</td>
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**Blunders (-3)**

B1 Indices

B2 Factors once only

B3 Deduction root from factor or no deduction.

B4 Incorrect matrix

Note: Cannot get 2nd 5 marks if equation is linear.
3 (c)  

(i) Express $-8 - 8\sqrt{3}i$ in the form $r(\cos \theta + isin \theta)$.

(ii) Hence find $\left(-8 - 8\sqrt{3}i\right)^3$.

(iii) Find the four complex numbers $z$ such that $z^4 = -8 - 8\sqrt{3}i$.

Give your answers in the form $a + bi$, with $a$ and $b$ fully evaluated.

(i) 5 marks  

(ii) 5 marks  

(iii) 5 marks  

Complex numbers 5 marks  

3(c)(i)  

$$z = -8 - 8\sqrt{3}i$$

$$r = \sqrt{(-8)^2 + (-8\sqrt{3})^2}$$

$$= \sqrt{64 + 192}$$

$$= \sqrt{256}$$

$$= 16$$

$$z = r[\cos \theta + isin \theta]$$

$$= 16[\cos \frac{4\pi}{3} + isin \frac{4\pi}{3}]$$

$$= 2^4(\cos \frac{4\pi}{3} + isin \frac{4\pi}{3})$$

3(c)(ii)  

$$z = 2^4(\cos \frac{4\pi}{3} + isin \frac{4\pi}{3})$$

$$z^3 = [2^4(\cos \frac{4\pi}{3} + isin \frac{4\pi}{3})]^3$$

$$= 2^{12}(\cos 4\pi + isin 4\pi)$$

$$= 2^{12}(1 + i(0))$$

$$= 2^{12} = 4096$$

3(c)(iii)  

$$z^4 = -8 - 8\sqrt{3}i = 2^4[\cos(2n\pi + \frac{4\pi}{3}) + isin(2n\pi + \frac{4\pi}{3})]$$

$$\Rightarrow z = \left(2^4[\cos(2n\pi + \frac{4\pi}{3}) + isin(2n\pi + \frac{4\pi}{3})]\right)^{\frac{1}{4}}$$

$$= 2\left[\cos\left(\frac{4\pi}{3}\right) + isin\left(\frac{4\pi}{3}\right)\right]^{\frac{1}{4}}$$

$$n = 0: z_0 = 2\left[\cos\left(\frac{4\pi}{3}\right) + isin\left(\frac{4\pi}{3}\right)\right]^{\frac{1}{4}} = 1 + i\sqrt{3}$$

$$n = 1: z_1 = 2\left[\cos\left(\frac{4\pi}{3}\right) + isin\left(\frac{4\pi}{3}\right)\right]^{\frac{1}{4}} = -\sqrt{3} + i$$

$$n = 2: z_2 = 2\left[\cos\left(\frac{4\pi}{3}\right) + isin\left(\frac{4\pi}{3}\right)\right]^{\frac{1}{4}} = -1 - i\sqrt{3}$$

$$n = 3: z_3 = 2\left[\cos\left(\frac{4\pi}{3}\right) + isin\left(\frac{4\pi}{3}\right)\right]^{\frac{1}{4}} = \sqrt{3} - i$$
Blunders (-3)
B1 Argument
B2 Modulus
B3 Trig definition
B4 Indices
B5 $i$
B6 Statement De Moivre once only
B7 Application De Moivre
B8 Root omitted
B9 No general solution
B10 $a$ and $b$ not fully evaluated
B11 Improper use of polar formula

Slips (-1)
S1 Trig value
QUESTION 4

Part (a) 10 (5, 5) marks  Att (2, 2)
Part (b) 20 (5, 5, 5, 5) marks  Att (2, 2, 2, 2)
Part (c) 20 (5, 5, 5, 5) marks  Att (2, 2, 2, 2)

Part (a) 10 (5, 5) marks  Att (2, 2)

4 (a) \(-2 + 2 + 6 + \ldots + (4n - 6)\) are the first \(n\) terms of an arithmetic series. 
\(S_n\), the sum of these \(n\) terms, is 160. Find the value of \(n\).

Correct substitution in formula  5 marks  Att 2
Value of \(n\)  5 marks  Att 2

\[4 (a) \quad -2 + 2 + 6 + \ldots + (4n - 6)\]
\[a = -2 \quad ; \quad d = 4 \quad ; \quad S_n = 160\]
\[S_n = \frac{n}{2} [2a + (n - 1)d] = 160\]
\[\frac{n}{2} [-4 + (n - 1)4] = 160\]
\[\frac{n}{2} [-4 + 4n - 4] = 160\]
\[n(2n - 4) = 160\]
\[2n^2 - 4n - 160 = 0\]
\[n^2 - 2n - 80 = 0\]
\[(n - 10)(n + 8) = 0\]
\[n = 10 \quad \text{or} \quad n = -8\]
\[n = 10\]

Blunders (-3)
B1 Formula \(AP\) once only
B2 Incorrect ‘\(a\)’ in formula
B3 Incorrect ‘\(d\)’ in formula
B4 Indices
B5 Factors once only
B6 Incorrect deduction root from factor
B7 Roots formula

Slips
S1 Excess value or incorrect value indicated.

Worthless
W1 treats as \(G.P.\)
Part (b)  The sum to infinity of a geometric series is \( \frac{9}{2} \).

The second term of the series is \(-2\).

Find the value of \( r \), the common ratio of the series.

\[
S_\infty = \frac{a}{1-r} \quad \text{and} \quad 9(1-r) = 2a \quad \text{............}(i)
\]

\(a, ar, ar^2\)

\(ar = -2\)

\(a = \frac{-2}{r} \quad \text{............}(ii)\)

\((i): 9(1-r) = 2(a)\)

\(9(1-r) = 2\left(-\frac{2}{r}\right)\)

\(9 - 9r = -\frac{4}{r}\)

\(9r - 9r^2 = 4\)

\(0 = 9r^2 - 9r - 4\)

\(0 = (3r + 1)(3r - 4)\)

\(\Rightarrow r = \frac{-1}{3} \quad \text{or} \quad r = \frac{4}{3}\)

Since sum to infinity \(\Rightarrow r = \frac{-1}{3}\)

\[\text{Blunders (-3)}\]

B1 Formula sum to infinity
B2 Definition of term of a G.P.
B3 Indices
B4 Factors once only
B5 Incorrect deduction root from factor
B6 Incorrect ‘a’

\[\text{Slips}\]
S1 Excess value or incorrect value indicated

\[\text{Worthless}\]
W1 Uses AP formula
W2 Trial and error
4 (c) The sequence \( u_1, u_2, u_3, \ldots \), defined by \( u_1 = 3 \) and \( u_{n+1} = 2u_n + 3 \), is as follows:

\[ 3, 9, 21, 45, 93, \ldots \]

(i) Find \( u_6 \), and verify that it is equal to the sum of the first six terms of a geometric series with first term 3 and common ratio 2.

(ii) Given that, for all \( k \), \( u_k \) is the sum of the first \( k \) terms of a geometric series with first term 3 and common ratio 2, find \( \sum_{k=1}^{n} u_k \).

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<td>Verify</td>
<td>5 marks</td>
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Part (c)(ii) Showing Pattern

| Sum        | 5 marks |

Q4(c)(ii) \((3), (3+6), (3+6+12), (3+6+12+24), \ldots \)

\((3), (9), (21), (45)\)

\[ u_k = 3 \left( 2^k - 1 \right) = 3 \left( 2^k \right) - 3 \]

\[ u_n = 3 \cdot 2^n - 3 \]

\[ u_{n-1} = 3 \cdot 2^{n-1} - 3 \]

\[ u_{n-2} = 3 \cdot 2^{n-2} - 3 \]

\[ \ldots \ldots \ldots \]

\[ u_3 = 3 \cdot 2^3 - 3 \]

\[ u_2 = 3 \cdot 2^2 - 3 \]

\[ u_1 = 3 \cdot 2^1 - 3 \]

\[ \sum = 3 \left( 2^1 + 2^2 + \ldots + 2^n \right) - 3n \]

\[ = 3 \left[ \frac{2^{n+1} - 1}{2-1} \right] - 3n = 6(2^n - 1) - 3n \]

Blunders (-3)

B1 Error in \( U_6 \)

B2 Formula sum of G.P.

B3 Incorrect ‘a’

B4 Incorrect ‘R’

B5 Error in \( u_k \)

B6 Indices
QUESTION 5

Part (a) 10 marks Att 3

5 (a) Find the value of the middle term of the binomial expansion of
\[ \left( \frac{x - y}{y - x} \right)^8. \]

\[
\begin{align*}
5 (a) & \quad u_5 = \frac{8}{4} \left( \frac{x}{y} \right)^4 \left( -\frac{y}{x} \right)^4 = \frac{8.7.6.5}{1.2.3.4} (1) = 70 \\
\text{or} \\
& \quad \left( \frac{x - y}{y - x} \right)^8 = \left( \frac{x}{y} \right)^8 + 8 \left( \frac{x}{y} \right)^7 \left( -\frac{y}{x} \right) + 28 \left( \frac{x}{y} \right)^6 \left( -\frac{y}{x} \right)^2 + \cdots \\
& \quad u_5 = \frac{8}{4} \left( \frac{x}{y} \right)^4 \left( -\frac{y}{x} \right)^4 = 70
\end{align*}
\]

* Answer must be in simplest form

Blunders (-3)

B1 General term
B2 Errors in binominal expansion once only
B3 Indices
B4 error value \( \binom{n}{r} \) or no value \( \binom{n}{r} \)
B5 \( x \) and \( y \) in answer
B6 \( x^* \neq 1 \)
B7 Incorrect term

Part (b) 20 (5, 10, 5) marks Att (2, 3, 2)

5 (b) (i) Express \( \frac{2}{(r+1)(r+3)} \) in the form \( \frac{A}{r+1} + \frac{B}{r+3} \).

(ii) Hence find \( \sum_{r=1}^{n} \frac{2}{(r+1)(r+3)} \).

(iii) Hence evaluate \( \sum_{r=1}^{\infty} \frac{2}{(r+1)(r+3)} \).
5 (b) (i)  
\[ \frac{a}{r+1} + \frac{b}{r+3} = \frac{2}{(r+1)(r+3)} \]
\[ a(r+3) + b(r+1) = 2 \]
\[ ar + 3a + br + b = (0)r + (2) \]
\[ (a+b)r + (3a+b) = (0)r + 2 \]
Equating Coefficients:
(i) \[ a + b = 0 \Rightarrow a = -b \]
(ii) \[ 3a + b = 2 \]
\[ 3a - a = 2 \]
\[ 2a = 2 \Rightarrow a = 1 \] and \[ b = -1 \]
\[ \frac{2}{(r+1)(r+3)} = \frac{1}{r+1} - \frac{1}{r+3} \]

Q5(b)(ii)  
\[ \sum_{r=1}^{n} \frac{2}{(r+1)(r+3)} \]
\[ u_n = \frac{2}{(n+1)(n+3)} = \frac{1}{n+1} - \frac{1}{n+3} \]
\[ u_{n-1} = \frac{2}{n(n+2)} = \frac{1}{n} - \frac{1}{n+2} \]
\[ u_{n-2} = \frac{2}{(n-1)(n+1)} = \frac{1}{n-1} - \frac{1}{n+1} \]
\[ \vdots \]
\[ u_3 = \frac{2}{4.6} = \frac{1}{4} - \frac{1}{6} \]
\[ u_2 = \frac{2}{3.5} = \frac{1}{3} - \frac{1}{5} \]
\[ u_1 = \frac{2}{2.4} = \frac{1}{2} - \frac{1}{4} \]
\[ S_n = \frac{1}{2} + \frac{1}{3} - \frac{1}{n+2} - \frac{1}{n+3} = \frac{5}{6} - \frac{1}{n+2} - \frac{1}{n+3} \]

Q5(b)(iii)  
\[ n \rightarrow \infty \quad s_\infty = \frac{5}{6} \]

Blunders (-3)
B1 Indices
B2 Cancellation must be shown or implied
B3 In equating coefficients not like to like
B4 Term or terms omitted
B5 \( S_r \)
B6 limits

Note: Must show 3 terms at start and 2 terms at finish or vice versa otherwise attempt.
**Part (c) 20 (5, 5, 5) marks  Att (2, 2, 2)**

<table>
<thead>
<tr>
<th>5 (c) (i)</th>
<th>Given two real numbers $a$ and $b$, where $a&gt;1$ and $b&gt;1$, prove that $\frac{1}{\log_b a} + \frac{1}{\log_a b} \geq 2$.</th>
</tr>
</thead>
<tbody>
<tr>
<td>(ii)</td>
<td>Under what condition is $\frac{1}{\log_b a} + \frac{1}{\log_a b} = 2$.</td>
</tr>
</tbody>
</table>

### Part 5(c) (i) Change of Base 5 marks  Att 2

**Inequality** 5 marks  Att 2

**Prove** 5 marks  Att 2

<table>
<thead>
<tr>
<th>5 (c) (i)</th>
<th>$\frac{1}{\log_b a} + \frac{1}{\log_a b} \geq 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\Leftrightarrow \log_a b + \frac{1}{\log_a b} \geq 2$ (since $\log_b a = \frac{1}{\log_a b}$)</td>
</tr>
<tr>
<td></td>
<td>$\Leftrightarrow (\log_a b)^2 + 1 \geq 2 \log_a b$ (since $a &gt; 1, b &gt; 1 \Rightarrow \log_a b &gt; 0$)</td>
</tr>
<tr>
<td></td>
<td>$\Leftrightarrow (\log_a b)^2 + 1 - 2 \log_a b \geq 0$</td>
</tr>
<tr>
<td></td>
<td>$\Leftrightarrow (\log_a b)^2 - 2 (\log_a b) + 1 \geq 0$</td>
</tr>
<tr>
<td></td>
<td>$\Leftrightarrow (\log_a b - 1)^2 \geq 0$</td>
</tr>
<tr>
<td></td>
<td>True, so $\frac{1}{\log_b a} + \frac{1}{\log_a b} \geq 2$.</td>
</tr>
</tbody>
</table>

**or**

**Q5 (c) (i)** To prove: $\log_a b + \frac{1}{\log_a b} \geq 2$

Let $\log_a b = x$. Then $x > 0$, since $a > 1, b > 1 \Rightarrow \log_a b > 0$.

To prove: $x + \frac{1}{x} \geq 2$

|         | $\Leftrightarrow x^2 + 1 \geq 2x$ |
|         | $\Leftrightarrow x^2 - 2x + 1 \geq 0$ |
|         | $\Leftrightarrow (x-1)^2 \geq 0$ |
|         | True, so $x + \frac{1}{x} \geq 2$. That is, $\log_a b + \frac{1}{\log_a b} \geq 2$. |

**Q5(c)(ii)** Equality holds in above solution when $(\log_a b - 1)^2 = 0$ [or $(x-1)^2 = 0$ in 2nd version]

|         | $\Leftrightarrow \log_a b = 1$ |
|         | $\Leftrightarrow a = b$ |

**Blunders** (-3)

- B1 Log laws
- B2 Change of base
- B3 Inequality sign
- B4 Incorrect deduction or no deduction
- B5 Indices
- B6 Factors once only
- B7 $(x^2 - 2x + 1) \neq (x-1)^2$

Note: Inequality must be quadratic

Page 21
**QUESTION 6**

<table>
<thead>
<tr>
<th>Part (a)</th>
<th>15 marks</th>
<th>Att 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Part (b)</td>
<td>20 (10, 5, 5) marks</td>
<td>Att (3, 2, 2)</td>
</tr>
<tr>
<td>Part (c)</td>
<td>15 (5, 5, 5) marks</td>
<td>Att (2, 2, 2)</td>
</tr>
</tbody>
</table>

**Part (a) 15 marks**

6 (a) Differentiate \( \sqrt{x}(x + 2) \) with respect to \( x \).

\[
\begin{align*}
6 \text{ (a)} \quad & y = \sqrt{x}(x + 2) \\
& = x^{\frac{1}{2}}(x + 2) \\
& = x^{\frac{3}{2}} + 2x^{\frac{1}{2}} \\
& \frac{dy}{dx} = \frac{3}{2}(x^{\frac{1}{2}}) + \frac{1}{x^{\frac{1}{2}}} \\
& = \frac{3}{2}x^{\frac{1}{2}} + \frac{1}{x^{\frac{1}{2}}} \\
& \text{or} \\
6 \text{(a)} \quad & y = x^{\frac{3}{2}}(x + 2) \\
& \frac{dy}{dx} = \frac{3}{2}(x^{\frac{1}{2}}) + (x + 2)\left(\frac{1}{2}x^{-\frac{1}{2}}\right) \\
& = x^{\frac{1}{2}} + \frac{x + 2}{2x^{\frac{1}{2}}} \\
& = \sqrt{x} + \frac{x + 2}{2\sqrt{x}}
\end{align*}
\]

**Blunders (-3)**

B1 Indices

B2 Differentiation

**Attempts**

A1 Error in differentiation formula
The equation of a curve is \( y = 3x^4 - 2x^3 - 9x^2 + 8 \).

(i) Show that the curve has a local maximum at the point (0, 8).

(ii) Find the coordinates of the two local minimum points on the curve.

(iii) Draw a sketch of the curve.

\[ dy \over dx = 12x^3 - 6x^2 - 18x \]
\[ \frac{d^2y}{dx^2} = 36x^2 - 12x - 18 \]

Local Max/Min: \( \frac{dy}{dx} = 0 \Rightarrow 12x^3 - 6x^2 - 18x = 0 \)

\( 6x(2x^2 - x - 3) = 0 \)

\( x = 0 \) or \( 2x^2 - x - 3 = 0 \)

\( (2x - 3)(x + 1) = 0 \)

\( x = \frac{3}{2} \) or \( x = -1 \)

Test \( x = 0 \) in \( \frac{d^2y}{dx^2} = 36(0) - 12(0) - 18 = -18 < 0 \) ⇒ local max at \( x = 0 \).

When \( x = 0 \): \( y = 3(0)^4 - 2(0)^3 - 9(0)^2 + 8 = 8 \) \( \therefore \) Local max at (0,8)

\[ \frac{d^2y}{dx^2} = 36 \left( x^2 \right) - 12 \left( x \right) - 18 \]
\[ \text{At } x = \frac{3}{2}: \]
\[ y = 3 \left( \frac{3}{2} \right)^4 - 2 \left( \frac{3}{2} \right)^3 - 9 \left( \frac{3}{2} \right)^2 + 8 \]
\[ = \frac{243}{16} - \frac{108}{16} - \frac{324}{16} + \frac{128}{16} \]
\[ = -\frac{61}{16} = -3 \frac{13}{16} \approx -3.8 \] ⇒ \( y \approx -3.8 \)

And at \( x = \frac{3}{2} \):

\[ \text{At } x = -1: \]
\[ y = 3(-1)^4 - 2(-1)^3 - 9(-1)^2 + 8 \]
\[ = 3 + 2 - 9 + 8 \]
\[ = 4 \] ⇒ \( y = 4 \)

And at \( x = -1 \):

\[ \frac{d^2y}{dx^2} = 36(-1)^2 - 12(-1) - 18 = 36 + 12 - 18 > 0 \] ⇒ local min at (–1, 4).
**Blunders (-3)**

B1 Differentiation
B2 Indices
B3 Deduction from 2\textsuperscript{nd} derivative or no deduction
B4 Not 3 values from \( f'(x) = 0 \)
B5 Not testing in \( f''(x) \) for max
B6 Incorrect \( y \) values or no \( y \) values in (ii)
B7 Factors once only
B8 Incorrect root from factor
B9 Not getting \( f''(x) \)

**Attempts**

A1 Error in differentiation formula

**Worthless**

W1 Integration
Part (c) 15 (5, 5, 5) marks Att (2, 2, 2)

6 (c) Prove by induction that \( \frac{d}{dx}(x^n) = nx^{n-1}, n \geq 1, n \in \mathbb{N} \).

6 (c) P(I) 5 marks Att 2
P(k) 5 marks Att 2
P(k +1) 5 marks Att 2

\[
P(n) : \frac{d}{dx}(x^n) = nx^{n-1}
\]

\( P(1) \): show that \( \frac{d}{dx}(x) = 1 \):

\[f(x) = x \Rightarrow f(x + h) - f(x) = \frac{h}{h} = 1.
\]

\[\therefore \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} = 1 \quad \therefore P(1) \text{ is true.}
\]

Assume \( P(k) \) true: \( \frac{d}{dx}(x^k) = kx^{k-1} \)

Deduce \( P(k +1) \):

\[\frac{d}{dx}(x^{k+1}) = \frac{d}{dx}(x^k \cdot x)\]
\[= x^k(1) + x \frac{d}{dx}(x^k) \quad \text{(by product rule)}\]
\[= x^k(1) + x(kx^{k-1}) \quad \text{(by } P(k) \text{ assumption)}\]
\[= x^k + kx^k\]
\[= x^k(k +1)\]

\[\therefore \text{ True for } P(k +1)
\]

As \( P(1) \) is true, and \( P(k) \Rightarrow P(k +1) \) for all \( k, \ P(n) \) is true for all \( n \geq 1 \).

**Blunders (-3)**
B1 Failure to prove case \( n = 1 \) or uses rule to prove true for \( n = 1 \)
B2 Definition of \( f'(x) \)
B3 Error in \( f(x + h) \) or \( (x + \Delta x) \)
B4 Indices
B5 Limit or no limit shown or implied
B6 Differentiation
B7 \( n = 0 \)

**Attempts**
A1 Error in differentiation formula
7 (a) Taking \( x_1 = 2 \) as the first approximation to the real root of the equation
\[ x^3 + x - 9 = 0, \]
use the Newton-Raphson method to find \( x_2 \), the second approximation.

\[
x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}
\]

\[
x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}
\]

\[
x_1 = 2 : \quad f(x) = x^3 + x - 9 \quad \Rightarrow f(2) = (2)^3 + 2 - 9 = 1
\]
\[
f''(x) = 3x^2 + 1 \quad \Rightarrow f''(2) = 3(2)^2 + 1 = 13
\]
\[
x_2 = 2 - \frac{1}{13} = \frac{25}{13}
\]

Blunder (-3)
B1 Newton-Raphson formula once only
B2 Differentiation
B3 Indices
B4 \( x_1 \neq 2 \)

Worthless
W1 Incorrect answer and no work
7 (b) The parametric equations of a curve are:

\[ x = 3\cos\theta - \cos^3\theta \]
\[ y = 3\sin\theta - \sin^3\theta, \text{ where } 0 < \theta < \frac{\pi}{2}. \]

(i) Find \( \frac{dy}{d\theta} \) and \( \frac{dx}{d\theta} \).

(ii) Hence show that \( \frac{dy}{dx} = \frac{-1}{\tan^3\theta}. \)

\[
\begin{align*}
\frac{dx}{d\theta} &= -3\sin\theta - 3(\cos\theta)^3(-\sin\theta) \\
&= -3\sin\theta + 3\sin\theta\cos^2\theta \\
&= -3\sin\theta(1 - \cos^2\theta) \\
&= -3\sin^3\theta \\

\frac{dy}{d\theta} &= 3\cos\theta - 3(\sin\theta)^3.\cos\theta \\
&= 3\cos\theta(1 - \sin^2\theta) \\
&= 3\cos^3\theta \\
\frac{dy}{dx} &= \left( \frac{dy}{d\theta} \right) \left( \frac{dx}{d\theta} \right)^{-1} = \frac{3\cos^3\theta}{-3\sin^3\theta} = \frac{1}{\tan^3\theta}
\end{align*}
\]

**Blunders (-3)**

B1 Indices
B2 Differentiation
B3 Trig laws
B4 Not in required form

**Attempts**

A1 Error in differentiation formula
7 (c) Given \( y = \ln\left(\frac{3+x}{\sqrt{9-x^2}}\right) \), find \( \frac{dy}{dx} \) and express it in the form \( \frac{a}{b-x^n} \).

\[
\frac{dy}{dx} = \frac{1}{3+x} - \frac{1}{2}\left[\frac{1}{9-x^2}(-2x)\right]
\]

\[
= \frac{1}{3+x} + \frac{x}{9-x^2}
\]

\[
= \frac{1}{3+x} + \frac{x}{(3-x)(3+x)}
\]

\[
= \frac{(3-x)+x}{(3-x)(3+x)} = \frac{3}{9-x^2}
\]

or

Q7(c) \( y = \ln\frac{3+x}{\sqrt{(3-x)(3+x)}} \)

\[
= \ln\left(\frac{3+x}{(3-x)^{\frac{1}{2}}}\right)
\]

\[
= \frac{1}{2}\ln(3+x) - \frac{1}{2}\ln(3-x)
\]

\[
\frac{dy}{dx} = \frac{1}{2}\left[\frac{1}{3+x} - \frac{1}{3-x}(-1)\right]
\]

\[
= \frac{1}{2}\left[\frac{1}{3+x} + \frac{1}{3-x}\right]
\]

\[
= \frac{1}{2}\left[\frac{(3-x)+(3+x)}{9-x^2}\right] = \frac{1}{2}\left(\frac{6}{9-x^2}\right) = \frac{3}{9-x^2}
\]

or
Q7(c)  

\[ y = \ln \left( \frac{3 + x}{\sqrt[3]{9 - x^2}} \right) = \ln \left( \frac{3 + x}{(9 - x^2)^{\frac{1}{3}}} \right) \]

\[ \frac{dy}{dx} = \frac{1}{\left( \frac{3 + x}{9 - x^2} \right)^{\frac{1}{3}}} \left( (9 - x^2)^{\frac{1}{3}} - (3 + x) \frac{1}{3} (9 - x^2)^{-\frac{2}{3}} (-2x) \right) \]

\[ = (9 - x^2)^{\frac{1}{3}} \left( \frac{9 - x^2 + x(3 + x)}{(3 + x)(9 - x^2)^{\frac{2}{3}}} \right) \]

\[ = \frac{9 - x^2 + 3x + x^2}{(3 + x)(9 - x^2)^{\frac{2}{3}}} \]

\[ = \frac{9 + 3x}{(3 + x)(9 - x^2)^{\frac{2}{3}}} \]

\[ = \frac{3(3 + x)}{(3 + x)(9 - x^2)^{\frac{2}{3}}} \]

\[ = \frac{3}{9 - x^2} \]

* \( \frac{dy}{dx} \) and simplifying can be in any order

\textbf{Blunders (-3)}

B1 Differentiation  
B2 Log laws  
B3 Indices  
B4 Not simplified to required form  
B5 Factors once only.

\textbf{Attempts}

A1 Error in differentiation formula
QUESTION 8

Part (a) 10 (5, 5) marks  Att (2, 2)
Part (b) 20 (5, 5, 5, 5) marks  Att (2, 2, 2, 2)
Part (c) 20 (10, 10) marks  Att (3, 3)

Part (a) 10 (5, 5) marks  Att (2, 2)

8. (a) Find
   \( (i) \int \sqrt{x} \, dx \)
   \( (ii) \int e^{-2x} \, dx \).

\[ \text{Q8 (a)(i)} \int \sqrt{x} \, dx = \int x^{\frac{1}{2}} \, dx = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c = \frac{2}{3}x^{\frac{3}{2}} + c \]

\[ \text{Q8 (a)(ii)} \int e^{-2x} \, dx = -\frac{1}{2}e^{-2x} + c \]

* If \( c \) shown once, then no penalty

Blunders (-3)
B1 Integration
B2 Indices
B3 No ‘c’ (penalize 1st integration)

Attempts
A1 Only ‘c’ correct

Worthless
W1 Differentiation instead of integration

Part (b) 20 (5, 5, 5, 5) marks  Att (2, 2, 2, 2)

8 (b) Evaluate
   \( (i) \int_{1}^{2} x(1 + x^2)^3 \, dx \)
   \( (ii) \int_{0}^{\frac{\pi}{2}} \sin 5\theta \cos 3\theta \, d\theta \).

Part (b)(i) Integration 5 marks  Att 2
Value 5 marks  Att 2
Part (b)(ii) Integration 5 marks  Att 2
Value 5 marks  Att 2

8 (b) (i) \[ \int_{1}^{2} x(1 + x^2)^3 \, dx \]
\[ = \int_{1}^{2} (1 + x^2)^3 \cdot x \, dx \]
\[ = \int_{1}^{2} (1 + x^2)^3 \cdot u \, \frac{du}{x} \]
\[ = \frac{1}{2} \int_{1}^{2} u^4 \, du \]
\[ = \frac{1}{2} \left[ \frac{u^5}{5} \right]_{1}^{2} \]
\[ = \frac{1}{2} \left[ (5)^5 - (2)^5 \right] \]
\[ = \frac{609}{8} \]

or
Q8(b)(i) \[ \int_{1}^{2} x(1 + x^2)^3 \, dx \]
\[ = \int_{1}^{2} x(3x^2 + 3x^4 + x^6) \, dx \]
\[ = \int_{1}^{2} (x^3 + 3x^5 + x^7) \, dx \]
\[ = \left[ \frac{x^2}{2} + \frac{3x^4}{4} + \frac{x^6}{8} \right]_{1}^{2} \]
\[ = (2^2 + 3(2)^4 + (2)^6) - (1^2 + 3(1)^4 + (1)^6) \]
\[ = 78 - \frac{17}{8} = 76 \frac{1}{8} = \frac{609}{8} \]

\[ (1 + x^2)^3 = (1)^3 + \left( \frac{3}{1} \right)(1)(x^2) + \left( \frac{3}{2} \right)(1)(x^2)^2 + (x^2)^3 \]
\[ = 1 + 3x^2 + 3x^4 + x^6 \]

\[ \text{Blunders (-3)} \]
B1 Integration
B2 Indices
B3 Differentiation
B4 Limits
B5 Incorrect order in applying limits
B6 Not calculating substituted limits
B7 Not changing limits
B8 Trig formula

\[ \text{Slips} \]
S1 Trig value

\[ \text{Worthless} \]
W1 Differentiation instead of integration except where other work merits attempt

Note Incorrect substitution and unable to finish yields attempt at most.
8 (c) The diagram shows the graphs of the curves \( y = f(x) \) and \( y = g(x) \), where \( f(x) = 12 - 3x^2 \) and \( g(x) = 9x^2 \).

(i) Calculate the area of the region enclosed by the curve \( y = f(x) \) the \( x \)-axis.

(ii) Show that the region enclosed by the curves \( y = f(x) \) and \( y = g(x) \) has half that area.

\[ f(x) = 0 \Rightarrow 12 - 3x^2 = 0 \]
\[ 4 = x^2 \quad \Rightarrow \quad x = \pm 2 \]

\[ A = \int_{-2}^{2} y \, dx = 2 \int_{0}^{2} (12 - 3x^2) \, dx \]
\[ = 2 \left[ (24 - 8) - 0 \right] \]
\[ = 32 \]

\[ f(x) = g(x) \]
\[ 12 - 3x^2 = 9x^2 \]
\[ 12 = 12x^2 \]
\[ 1 = x^2 \quad \Rightarrow \quad x = \pm 1 \]

Enclosed Area
\[ = \int_{-1}^{1} f(x) \, dx - \int_{-1}^{1} g(x) \, dx \]
\[ = 2 \left[ \int_{0}^{1} (12 - 3x^2) \, dx - \int_{0}^{1} 9x^2 \, dx \right] \]
\[ = 2 \left[ (2x - x^3) \bigg|_{0}^{1} - (3x^3) \bigg|_{0}^{1} \right] \]
\[ = 2 \left[ (2 - 1) - 3 \right] \]
\[ = 2 \left[ (12 - 4) - (0) \right] \]
\[ = 16 \]

or
Q8(c)(ii) Enclosed Area = 32 − \left[ A_1 + A_2 + A_3 + A_4 \right] \\
= 32 - 2(A_3 + A_4) \\
A_3 = \int_{0}^{1} 9x^2 \, dx = 3x^3 \bigg|_{0}^{1} = 3 - 0 = 3 \\
A_4 = \int_{1}^{2} (12 - 3x^2) \, dx = 12x - x^3 \bigg|_{1}^{2} \\
= (24 - 8) - (12 - 1) = 5 \\
\therefore 2(A_3 + A_4) = 2(3 + 5) = 16 \\
\therefore \text{Enclosed Area} = 32 - 16 = 16

\textbf{Blunders (-3)}
B1 Indices \\
B2 Integration \\
B3 Calculating roots of \( f(x)=0 \) \\
B4 Calculating \( f \cap g \) \\
B5 Error in area formula \\
B6 Incorrect order in applying limits \\
B7 Not calculating substituted limits \\
B8 Error with \( f(x) \) or \( g(x) \) \\
B9 Uses \( \pi \int y \, dx \) for area formula

\textbf{Attempts}
A1 Uses volume formula \\
A2 Uses \( y^2 \) in formula

\textbf{Worthless}
W1 Differentiation instead of integration except where other work merits attempt \\
W2 Wrong area formula and no work
MARKING SCHEME
LEAVING CERTIFICATE EXAMINATION 2006
MATHEMATICS – HIGHER LEVEL – PAPER 2

GENERAL GUIDELINES FOR EXAMINERS – PAPER 2

1. Penalties of three types are applied to candidates’ work as follows:
   - **Blunders** - mathematical errors/omissions (-3)
   - **Slips** - numerical errors (-1)
   - **Misreadings** (provided task is not oversimplified) (-1).

   Frequently occurring errors to which these penalties must be applied are listed in the scheme. They are labelled: B1, B2, B3,…, S1, S2,…, M1, M2,…etc. These lists are not exhaustive.

2. When awarding attempt marks, e.g. Att(3), note that
   - any correct, relevant step in a part of a question merits at least the attempt mark for that part
   - if deductions result in a mark which is lower than the attempt mark, then the attempt mark must be awarded
   - a mark between zero and the attempt mark is never awarded.

3. Worthless work is awarded zero marks. Some examples of such work are listed in the scheme and they are labelled as W1, W2,…etc.

4. The phrase “hit or miss” means that partial marks are not awarded – the candidate receives all of the relevant marks or none.

5. The phrase “and stops” means that no more work is shown by the candidate.

6. Special notes relating to the marking of a particular part of a question are indicated by an asterisk. These notes immediately follow the box containing the relevant solution.

7. The sample solutions for each question are not intended to be exhaustive lists – there may be other correct solutions. Any examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his/her advising examiner.

8. Unless otherwise indicated in the scheme, accept the best of two or more attempts – even when attempts have been cancelled.

9. The same error in the same section of a question is penalised once only.

10. Particular cases, verifications and answers derived from diagrams (unless requested) qualify for attempt marks at most.

11. A serious blunder, omission or misreading results in the attempt mark at most.

12. Do not penalise the use of a comma for a decimal point, e.g. €5.50 may be written as €5,50.
QUESTION 1

Part (a) 15 marks
Part (b) 20 (10, 10) marks
Part (c) 15 (10,5) marks

1 (a) $a (-1, -3)$ and $b (3, 1)$ are the end-points of a diameter of a circle.
Write down the equation of the circle.

1 (a) Mid-point of $[ab] = Centre$ of circle $c = (1, -1)$
Radius $= |ac| = \sqrt{4 + 4} = \sqrt{8}$.
$\therefore$ Equation of circle: $(x - 1)^2 + (y + 1)^2 = 8.$

Blunders
B1 Error in mid-point formula (apply once).
B2 Error in distance formula (apply once)
B3 Incorrect substitution for each formula.

Slips
S1 Arithmetic errors

Attempts
A1 Radius or midpoint only
A2 General form of equation written down and some correct substitution

(b) Circle $C$ has centre $(5, -1)$.
The line $L: 3x - 4y + 11 = 0$ is a tangent to $C$.

(i) Show that the radius of $C$ is 6.

(ii) The line $x + py + 1 = 0$ is also a tangent to $C$.
Find two possible values of $p$.

1 (b) (i)
Radius = Distance from centre $(5, -1)$ to line $3x - 4y + 11 = 0$.
Radius $= \frac{|15 + 4 + 11|}{\sqrt{9 + 16}} = 6.$

Blunders
B1 Error in distance formula

Slips
S1 Arithmetic errors

Attempts
A1 Picks point on $3x - 4y + 11 = 0$ and finds distance using $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
Part (b) (ii) 10 marks Att 3

1 (b) (ii) Perpendicular distance from centre (5, -1) to \( x + py + 1 = 0 \) equals 6.

\[ \left| \frac{5 - p + 1}{\sqrt{1 + p^2}} \right| = 6 \Rightarrow (6 - p)^2 = 36 + 36p^2. \]

\[ \Rightarrow 35p^2 + 12p = 0 \Rightarrow p(35p + 12) = 0 \Rightarrow p = 0 \text{ or } p = -\frac{12}{35} \]

Blunders
B1 Error in perpendicular distance formula
B2 Error in squaring or fails to square
B3 Error in solving quadratic

Slips
S1 Arithmetic

Attempts
A1 Attempts to substitute and solve for p

Part (c) 15 (10,5) marks Att (3,2)

1 (c) \( S \) is the circle \( x^2 + y^2 + 4x + 4y - 17 = 0 \) and \( K \) is the line \( 4x + 3y = 12 \).

(i) Show that the line \( K \) does not intersect \( S \).

(ii) Find the co-ordinates of the point on \( S \) that is closest to \( K \).

Part (c) (i) 10 marks Att 3

1 (c) (i)

Centre = \((-2, -2)\), radius = \( \sqrt{4^2 + 4^2 - 17} = \sqrt{4 + 4 + 17} = 5 \).

Distance from line to centre = \( \frac{-8 - 6 - 12}{5} = \frac{26}{5} > 5 \). \( \therefore \) \( K \) does not intersect \( S \).

Blunders
B1 Error centre or radius formula (each formula)
B2 Error in perpendicular distance formula (mod omitted)
B3 Error in squaring
B4 No conclusion

Slips
S1 Arithmetic
Part (c) (ii)  \hspace{1cm} 5 \text{ marks}  \hspace{1cm} \text{Att 2}

Required point is on line containing centre and perpendicular to \(4x + 3y = 12\).

\[
y + 2 = \frac{3}{4}(x + 2) \Rightarrow 3x - 4y = 2.
\]

Required point is \(3x - 4y = 2 \cap S\).

\[
x = \frac{2 + 4y}{3} \Rightarrow \left(\frac{2 + 4y}{3}\right)^2 + y^2 + 4\left(\frac{2 + 4y}{3}\right) + 4y - 17 = 0.
\]

\[
\therefore \frac{4 + 16y + 16y^2}{9} + y^2 + \frac{8 + 16y}{3} + 4y - 17 = 0.
\]

\[
4 + 16y + 16y^2 + 9y^2 + 24 + 48y + 36y - 153 = 0 \Rightarrow 25y^2 + 100y - 125 = 0.
\]

\[
\therefore y^2 + 4y - 5 = 0 \Rightarrow (y - 1)(y + 5) = 0 \Rightarrow y = 1 \text{ or } y = -5.
\]

\[
\therefore (2, 1) \text{ or } (-6, -5). \text{ But (2,1) is closest point.} \therefore \text{Solution} = (2, 1).
\]

**Blunders**

B1 Error in equation of line  
B2 Error in substitution  
B3 Error in squaring  
B4 Error in forming quadratic  
B5 Error in solving quadratic  
B5 No conclusion

**Slips**  
S1 Arithmetic

**Attempts**

A1 Writes down the centre or the radius
QUESTION 2

Part (a) 10 marks Att 3

2 (a) \( \vec{x} = -3 \hat{i} + \hat{j} \). Express \( \left( \begin{array}{c} x \\ x \end{array} \right) \) in terms of \( \hat{i} \) and \( \hat{j} \).

Part (a) 10 marks Att 3

2(a)

\[ \vec{x} = -3 \hat{i} + \hat{j} \Rightarrow \vec{x} = \left( -3 \hat{i} + \hat{j} \right) = -\hat{i} - 3 \hat{j} \Rightarrow \left( \begin{array}{c} x \\ x \end{array} \right) \] = 3 \hat{i} - \hat{j}.

Blunders

B1 Incorrect sign

B2 Incorrect scalar

Attempts

A1 Correct formula

Part (b) 30 (5, 5, 10, 10) marks Att (2, 2, 3, 3)

(b) \( \vec{p} = -5 \hat{i} + 2 \hat{j} \), \( \vec{q} = \hat{i} - 6 \hat{j} \) and \( \vec{r} = -\hat{i} + 5 \hat{j} \).

(i) Express \( \vec{pq} \) and \( \vec{pr} \) in terms of \( \hat{i} \) and \( \hat{j} \).

(ii) Given that \( 10 \vec{s} = \vec{pr} + \vec{pq} \), express \( \vec{s} \) in terms of \( \hat{i} \) and \( \hat{j} \).

(iii) Find the measure of the angle between \( \vec{s} \) and \( \vec{pr} \).

(i) Express \( \vec{pq} \) 5 marks Att 2

Express \( \vec{pr} \) 5 marks Att 2

2 (b) (i)

\[ \vec{pq} = \vec{q} - \vec{p} = \hat{i} - 6 \hat{j} + 5 \hat{j} - 2 \hat{j} = 6 \hat{i} - 8 \hat{j} \]  
\[ \vec{pr} = \vec{r} - \vec{p} = -\hat{i} + 5 \hat{j} + 5 \hat{j} - 2 \hat{j} = 4 \hat{i} + 3 \hat{j} \]

Blunders

B1 \( \vec{pq} \neq \vec{q} - \vec{p} \) or \( \vec{pr} \neq \vec{r} - \vec{p} \)

B2 Error in signs

Slips

S1 Arithmetic
Part (b) (ii) 10 marks

\[ | \vec{pr} | = \sqrt{16 + 9} = 5 \quad \text{and} \quad | \vec{pq} | = \sqrt{36 + 64} = 10. \]

\[ \therefore \ 10 \vec{s} = 5 \left( 6 \vec{i} - 8 \vec{j} \right) + 10 \left( 4 \vec{i} + 3 \vec{j} \right) \Rightarrow 2 \vec{s} = 6 \vec{i} - 8 \vec{j} + 8 \vec{i} + 6 \vec{j} = 14 \vec{i} - 2 \vec{j}. \]

\[ \therefore \vec{s} = 7 \vec{i} - \vec{j}. \]

**Blunders**

B1 Error in \( | \vec{pq} | \) or in \( | \vec{pr} | \)

**Slips**

S1 Arithmetic

---

Part (b) (iii) 10 marks

\[ \vec{s} \cdot \vec{pr} \Rightarrow \left( 7 \vec{i} - \vec{j} \right) \left( 4 \vec{i} + 3 \vec{j} \right) = \left| 7 \vec{i} - \vec{j} \right| \left| 4 \vec{i} + 3 \vec{j} \right| \cos \theta. \]

\[ \therefore \ \cos \theta = \frac{28 - 3}{\sqrt{50} \sqrt{25}} = \frac{25}{25 \sqrt{2}} = \frac{1}{\sqrt{2}}. \quad \therefore \ \theta = \frac{\pi}{4}. \]

**Blunders**

B1 Error in form of scalar product each time

B2 Error in calculating \( \cos^{-1} \frac{1}{\sqrt{2}} \)

**Slips**

S1 Arithmetic
(c) The origin $o$ is the circumcentre of the triangle $abc$.

If $\vec{h} = \vec{a} + \vec{b} + \vec{c}$, show that $\vec{ah} \perp \vec{bc}$.

\[ \vec{ah} \cdot \vec{bc} = (\vec{h} - \vec{a}) \cdot (\vec{c} - \vec{b}) = (\vec{b} + \vec{c}) \cdot (\vec{c} - \vec{b}) = |\vec{c}|^2 - |\vec{b}|^2. \]

But since $o$ is the circumcentre, $|\vec{b}| = |\vec{c}|$. \therefore $\vec{ah} \cdot \vec{bc} = 0 \Rightarrow \vec{ah} \perp \vec{bc}$.

**Blunders**

B1 $\vec{ah} \neq \vec{h} - \vec{a}$
B2 $\vec{bc} \neq \vec{c} - \vec{b}$
B3 Error in vector multiplication

**Slips**

S1 Arithmetic errors

**Attempts**

A1 States condition for perpendicular vectors correctly
A2 $\vec{ah} = \vec{h} - \vec{a}$
QUESTION 3

Part (a) 15 marks  Att 5

Part (b) 10 marks  Att 3

Part (c) 25 (10, 15) marks  Att 3,5

Part (a) 15 marks  Att 5

3 (a) Show that the line containing the points (3, −6) and (−7, 12) is perpendicular to the line $5x − 9y + 6 = 0$.

3 (a) Slope of line containing points (3, −6) and (−7, 12) is $m_1 = \frac{12 + 6}{-7 - 3} = \frac{18}{-10} = -\frac{9}{5}$.

The line $5x − 9y + 6 = 0$ has slope $m_2 = \frac{5}{9}$.

But $m_1 m_2 = -1$, ∴ lines perpendicular.

Blunders
B1 Error in finding either slope
B2 Product of slopes not shown = -1
B3 No conclusion

Slips
S1 Arithmetic

Part (b) 10 marks  Att 3

3 (b) The line $K$ has positive slope and passes through the point $p (2, −9)$. $K$ intersects the x-axis at $q$ and the y-axis at $r$ and $|pq| : |pr| = 3 : 1$.

Find the co-ordinates of $q$ and the co-ordinates of $r$.

3 (b) Equation of $K : y + 9 = m(x − 2)$. $q$ is $\left(\frac{9}{m} + 2, 0\right)$ and $r$ is $(0, −2m − 9)$.

$\frac{9}{m} + 2 + 0 \Rightarrow \frac{9}{m} + 2 = 8 \Rightarrow \frac{9}{m} = 6 \Rightarrow m = \frac{3}{2}$.

∴ $q$ is $(8, 0)$ and $r$ is $(0, −12)$.

Blunders
B1 Error in finding equation of line
B2 Error in finding q or r each time
B3 Error in ratio formula
B4 Error in simplification if not a slip

Slips
S1 Arithmetic errors

Attempts
Correct formula for K for ratio for distance with some correct substitution
3 (c) (i) Prove that the measure of one of the angles between two lines with slopes $m_1$ and $m_2$ is given by $\tan \theta = \frac{m_1 - m_2}{1 + m_1 m_2}$.

(ii) $L$ is the line $y = 4x$ and $K$ is the line $x = 4y$.

$f$ is the transformation $(x, y) \rightarrow (x', y')$, where $x' = 2x - y$ and $y' = x + 3y$.

Find the measure of the acute angle between $f(L)$ and $f(K)$, correct to the nearest degree.

**Blunders**

B1 Error in expressing $q$ in terms of $q_1$ and $q_2$

B2 Error in $\tan (\theta_1 - \theta_2)$

**Part (c) (i) 10 marks**

3 (c) (i)

Slope $L_1 = m_1 = \tan \theta_1$.
Slope $L_2 = m_2 = \tan \theta_2$.

$\theta_1 = \theta + \theta_2 \Rightarrow \theta = \theta_1 - \theta_2$.

$\therefore \tan \theta = \tan (\theta_1 - \theta_2) = \frac{\tan \theta_1 - \tan \theta_2}{1 + \tan \theta_1 \tan \theta_2}$

$\therefore \tan \theta = \frac{m_1 - m_2}{1 + m_1 m_2}$.

**Part (c) (ii) 15 marks**

3 (c) (ii)

$3x' = 6x - 3y$

$y' = x + 3y$

$7x = 3x' + y' \Rightarrow x = \frac{1}{7}(3x' + y')$. But $y' = x + 3y \Rightarrow y' = \frac{1}{7}(3x' + y') + 3y$.

$\therefore y = \frac{1}{7}(-x' + 2y')$.

$f(L): \frac{1}{7}(-x' + 2y') = \frac{4}{7}(3x' + y') \Rightarrow f(L): 2y' = -13x' \Rightarrow \text{slope } f(L) = -\frac{13}{2}$.

$f(K): \frac{1}{7}(3x' + y') = \frac{4}{7}(-x' + 2y') \Rightarrow f(K): y' = x' \Rightarrow \text{slope } f(K) = 1$.

$\tan \theta = \frac{-\frac{13}{2}}{1 - \frac{13}{2}} = \frac{15}{11} \Rightarrow \theta = 54^\circ$.

**Blunders**

B1 Error in setting up/solving simultaneous equations each time

B2 Error in calculating slope for $F(L)$ and $F(K)$ each time for different blunder

B3 Error in applying formula

B4 Error in calculating $\tan^{-1}\frac{15}{11}$

**Slips**

S1 Arithmetic errors
QUESTION 4

<table>
<thead>
<tr>
<th>Part (a)</th>
<th>15 marks</th>
<th>Att 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Part (b)</td>
<td>20 (10, 10) marks</td>
<td>Att (3,3)</td>
</tr>
<tr>
<td>Part (c)</td>
<td>15 (10,5) marks</td>
<td>Att (3,2)</td>
</tr>
</tbody>
</table>

**Part (a) 15 marks Att 5**

4 (a) Write down the values of $A$ for which $\cos A = \frac{1}{2}$, where $0^\circ \leq A \leq 360^\circ$.

4 (a) $\cos A = \frac{1}{2} \Rightarrow A = 60^\circ, 300^\circ$.

**Blunders**
B1 Each incorrect or omitted value

**Part (b) 20 (10, 10) marks Att (3,3)**

4 (b) (i) Express $\sin(3x + 60^\circ) - \sin x$ as a product of sine and cosine.

(ii) Find all the solutions of the equation $\sin(3x + 60^\circ) - \sin x = 0$, where $0^\circ \leq x \leq 360^\circ$.

**Part (b) (i) 10 marks Att 3**

4 (b) (i) $\sin(3x + 60^\circ) - \sin x = 2\cos(2x + 30^\circ)\sin(x + 30^\circ)$

**Blunders**
B1 Error in simplifying the expression

**Part (b) (ii) 10 marks Att 3**

4 (b) (ii) $\sin(3x + 60^\circ) - \sin x = 0 \Rightarrow 2\cos(2x + 30^\circ)\sin(x + 30^\circ) = 0$

$\therefore \cos(2x + 30^\circ) = 0$ or $\sin(x + 30^\circ) = 0$

$2x + 30^\circ = 90^\circ, 270^\circ, 450^\circ, 630^\circ$ or $x + 30^\circ = 180^\circ, 360^\circ.$

$\therefore x = 30^\circ, 120^\circ, 210^\circ, 300^\circ$ or $x = 150^\circ, 330^\circ$.

$\therefore$ Solution $= \{30^\circ, 120^\circ, 150^\circ, 210^\circ, 300^\circ, 330^\circ\}$.

**Blunders**
B1 Error in solving
B2 Each incorrect or missing solution

Page 43
The diagram shows a sector (solid line) circumscribed by a circle (dashed line).

(i) Find the radius of the circle in terms of $k$.

(ii) Show that the circle encloses an area which is double that of the sector.

### Part (c) (i) 10 marks

\[
\cos 30^\circ = \frac{k}{r} \quad \Rightarrow \quad \frac{r \sqrt{3}}{2} = \frac{k}{2} \\
\Rightarrow \quad r = \frac{k}{\sqrt{3}}.
\]

**Blunders**

B1 Incorrect use of cosine rule or sine rule or area of triangle.
B2 Error in cos from right-angled triangle

### Part (c) (ii) 5 marks

\[
\text{Area of circle} = \pi r^2 = \pi \left(\frac{k}{\sqrt{3}}\right)^2 = \frac{\sqrt{3}}{3} k^2.
\]

\[
\text{Area of sector} = \frac{1}{2} k^2 \theta = \frac{k^2}{2} \cdot 60^\circ = \frac{\pi}{6} k^2.
\]

\[
\therefore \quad \text{Area of circle} = 2 \times \text{area of sector}.
\]

**Blunders**

B1 Error in area of sector
B2 Error in area of circle
B3 No conclusion

**Slips**

S1 Arithmetic
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QUESTION 5

Part (a)  25 (10, 5, 5, 5) marks  Att (3, 2, 2, 2)
Part (b)  25 (15, 10) marks  Att (5, 3)

Part (a)  25 (10, 5, 5, 5) marks  Att (3, 2, 2, 2)

5 (a) (i)  Copy and complete the table below for \( f : x \to \tan^{-1}x \),
giving the values for \( f(x) \) in terms of \( \pi \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>(-\sqrt{3})</th>
<th>(-1)</th>
<th>(-\frac{1}{\sqrt{3}})</th>
<th>(0)</th>
<th>(\frac{1}{\sqrt{3}})</th>
<th>(1)</th>
<th>(\sqrt{3})</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>(-\frac{\pi}{3})</td>
<td>(-\frac{\pi}{4})</td>
<td>(-\frac{\pi}{6})</td>
<td>(0)</td>
<td>(\frac{\pi}{6})</td>
<td>(\frac{\pi}{4})</td>
<td>(\frac{\pi}{3})</td>
</tr>
</tbody>
</table>

(ii) Draw the graph of \( y = f(x) \) in the domain \(-2 \leq x \leq 2\),
scaling the \( y \)-axis in terms of \( \pi \).

(iii) Draw the two horizontal asymptotes of the graph.

(iv) For some values of \( k \in \mathbb{R} \), but not all values, \( \tan^{-1}(\tan k) = k \).

State the range of values of \( k \) for which \( \tan^{-1}(\tan k) = k \).

Show, by means of an example, what happens outside the range.

5 (a) (i)  10 marks  Att 3

Mark as follows:

<table>
<thead>
<tr>
<th>No. of values correct</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mark</td>
<td>Att</td>
<td>Att</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
</tr>
</tbody>
</table>

Part (a) (ii)  5 marks  Att 2

Part (a) (iii)  5 marks  Att 2

5 (a) (ii) & (iii)
Blunders
B1 Asymptotes not horizontal
B2 Asymptotes do not contain $-\frac{\pi}{2}$ and $\frac{\pi}{2}$

Slips
S1 Each incorrectly plotted point

### Part (a) (iv) 5 marks  

<table>
<thead>
<tr>
<th>5 (a) (iv)</th>
<th>Range of values: $-\frac{\pi}{2} &lt; k &lt; \frac{\pi}{2}$.</th>
</tr>
</thead>
<tbody>
<tr>
<td>e.g. if $k = \frac{5\pi}{4}$, then $\tan^{-1}\left(\tan\frac{5\pi}{4}\right) = \tan^{-1}1 = \frac{\pi}{4} \neq \frac{5\pi}{4}$.</td>
<td></td>
</tr>
</tbody>
</table>

Blunders
B1 Incorrect endpoint of range each time
B2 No example given

### Part (b) 25 (15, 10) marks  

5 (b) The great pyramid at Giza in Egypt has a square base and four triangular faces. The base of the pyramid is of side 230 metres and the pyramid is 146 metres high. The top of the pyramid is directly above the centre of the base.

(i) Calculate the length of one of the slanted edges, correct to the nearest metre.

(ii) Calculate, correct to two significant figures, the total area of the four triangular faces of the pyramid (assuming they are smooth flat surfaces).
5 (b) (i) Diagonal of square base \( d = \sqrt{230^2 + 230^2} = \sqrt{105800}. \)

Let length of slant edge \( s \) and height of pyramid \( h. \)

\[
s^2 = h^2 + \left(\frac{1}{2}d\right)^2 \implies s = \sqrt{21316 + 26450} = \sqrt{47766} = 218.5 \approx 219 \text{ metres.}
\]

Blunders

B1 Incorrect application of Pythagoras each time or incorrect trig ratio

Slips

S1 Error in calculations

5 (b) (ii)

\[
h = \sqrt{219^2 - 115^2} = \sqrt{47961 - 13225} = \sqrt{34736} = 186.37.
\]

Total surface area \( = 4 \times \frac{1}{2}(230)(186.37) = 85730.2 = 86000 \text{ m}^2.\)

Blunders

B1 Error in Pythagoras or in area of triangle formula

Slips

Arithmetic errors or failure to round off.
QUESTION 6

Part (a)  10 (5, 5) marks  Att (- , 2)
Part (b)  25 (5, 5, 5, 5, 5) marks  Att (2, 2, 2, 2, 2)
Part (c)  15 (5, 5, 5) marks  Att (2, 2, 2)

6 (a)  (i)  How many different teams of three people can be chosen from a panel of six boys and five girls?
       (ii)  If the team is chosen at random, find the probability that it consists of girls only?

Part (a) (i)  5 marks  Hit/Miss
6 (a) (i)  Answer = \( \binom{11}{3} = 165 \).

Part (a) (ii)  5 marks  Att 2
6 (a) (ii)  3 girls to choose from 5 ⇒ Solution = \( \binom{5}{3} = 10 \).

\[
P(\text{all girls}) = \frac{10}{165} = \frac{2}{33}
\]

Blunders
B1  Incorrect total possible
B2  Incorrect total favourable

Slips
S1  Arithmetic errors

Part (b)  25 (5, 5, 5, 5, 5) marks  Att (2, 2, 2, 2, 2)

6 (b)  (i)  Solve the difference equation \( 6u_{n+2} - 7u_{n+1} + u_n = 0 \), where \( n \geq 0 \),
given that \( u_0 = 8 \) and \( u_1 = 3 \).
(ii)  Verify that the solution to part (i) also satisfies the difference equation
\( 6u_{n+1} - u_n - 10 = 0 \).

(b) (i)  Char. Eqn.  5 marks  Att 2
Roots  5 marks  Att 2
Sim. Eqns.  5 marks  Att 2
Finish  5 marks  Att 2

6 (b) (i)  \[
6u_{n+2} - 7u_{n+1} + u_n = 0.
\]
\[
\therefore 6x^2 - 7x + 1 = 0 \Rightarrow (x - 1)(6x - 1) = 0 \Rightarrow x = 1 \text{ or } x = \frac{1}{6}.
\]
\[
\therefore u_n = l(1)^n + m\left(\frac{1}{6}\right)^n = l + m\left(\frac{1}{6}\right)^n.
\]
\[
u_0 = 8 \Rightarrow l + m = 8 \text{ and } u_1 = 3 \Rightarrow l + \frac{1}{6}m = 3.
\]
\[
\frac{5}{6}m = 5 \Rightarrow m = 6 \text{ and } l = 2. \therefore u_n = 2 + 6\left(\frac{1}{6}\right)^n = 2 + \left(\frac{1}{6}\right)^{n-1}.
\]
Blunders
B1 Error in characteristic equation
B2 Error in factors or quadratic formula
B3 Incorrect use of initial conditions

Slips
S1 Arithmetic errors

Attempts
A1 Char equation
A2 Eqn in \( l \) and \( m \)

\[
\text{Part (b) (ii) 5 marks}\quad \text{Att } 2
\]

\[
u_n = 2 + \left( \frac{1}{6} \right)^{n-1} \quad \text{and} \quad 6u_{n+1} - u_n - 10 = 0.
\]

\[
\therefore 6 \left[ 2 + \left( \frac{1}{6} \right)^n \right] - 10 = 12 + \left( \frac{1}{6} \right)^{n-1} - 2 - \left( \frac{1}{6} \right)^{n-1} - 10 = 0. \quad \therefore \text{solution.}
\]

Blunders
B1 Error in \( U_{n+1} \) or \( U_n \)
B2 Error in indices
6 (c) There are thirty days in June. Seven students have their birthdays in June. The birthdays are independent of each other and all dates are equally likely.

(i) What is the probability that all seven students have the same birthday?
(ii) What is the probability that all seven students have different birthdays?
(iii) Show that the probability that at least two have the same birthday is greater than 0.5.

<table>
<thead>
<tr>
<th>Part (c) (i)</th>
<th>5 marks</th>
<th>Att 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 (c) (i)</td>
<td>P=total favourable [=\frac{30 \times 1 \times 1 \times 1 \times 1 \times 1}{30 \times 30 \times 30 \times 30 \times 30} = \frac{1}{(30)^6}]</td>
<td></td>
</tr>
</tbody>
</table>

**Blunders**

B1 Incorrect total possible  
B2 Incorrect total favourable  
B3 No fraction

<table>
<thead>
<tr>
<th>Part (c) (ii)</th>
<th>5 marks</th>
<th>Att 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 (c) (ii)</td>
<td>P=total favourable [=\frac{30 \times 29 \times 28 \times 27 \times 26 \times 25 \times 24}{30 \times 30 \times 30 \times 30 \times 30 \times 30} = \frac{2639}{5625}]</td>
<td></td>
</tr>
</tbody>
</table>

**Blunders**

B1 Incorrect total possible  
B2 Incorrect total favourable  
B3 No fraction

<table>
<thead>
<tr>
<th>Part (c) (iii)</th>
<th>5 marks</th>
<th>Att 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 (c) (iii)</td>
<td>Probability [= 1 - P(\text{all seven have different birthdays})] [= 1 - \frac{29 \times 28 \times 27 \times 26 \times 25 \times 24}{(30)^6}] [= 1 - 0.4691 = 0.5309 &gt; 0.5.]</td>
<td></td>
</tr>
</tbody>
</table>

**Blunders**

B1 Error in correct total possible  
B2 Error in correct total favourable  
B3 No conclusion
QUESTION 7

<table>
<thead>
<tr>
<th>Part (a)</th>
<th>10 (5, 5) marks</th>
<th>Att (2, 2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Part (b)</td>
<td>25 (5, 10, 5, 5) marks</td>
<td>Att (-, 3, 2, 2)</td>
</tr>
<tr>
<td>Part (c)</td>
<td>15 marks</td>
<td>Att 5</td>
</tr>
</tbody>
</table>

**Part (a) 10 (5, 5) marks Att (2, 2)**

7 (a) The password for a mobile phone consists of five digits.

(i) How many passwords are possible?

(ii) How many of these passwords start with a 2 and finish with an odd digit?

**Part (a) (i) 5 marks Att 2**

7 (a) (i) Number of possible passwords = \(10^5 = 100,000\)

Slips
S1 Gives \(9^5 = 59049\)

**Part (a) (ii) 5 marks Att 2**

7 (a) (ii) Number of passwords = \(1 \times 10^3 \times 5 = 5,000\).

Blunders
B1 Adds \(1 + 10^3 + 5\)

**Part (b) 25 (5, 10, 5, 5) marks Att (-, 3, 2, 2)**

7 (b) For a lottery, 35 cards numbered 1 to 35 are placed in a drum. Five cards will be chosen at random from the drum as a winning combination.

(i) How many different combinations are possible?

(ii) How many of all the possible combinations will match exactly four numbers with the winning combination?

(iii) How many of all the possible combinations will match exactly three numbers with the winning combination?

(iv) Show that the probability of matching at least three numbers with the winning combination is approximately 0.014.

**Part (b) (i) 5 marks Hit/Miss**

7 (b) (i) Number of different possible combinations = \(\binom{35}{5} = 324,632\).

**Part (b) (ii) 10 marks Att 3**

7 (b) (ii) Match four = \(\binom{5}{4} \times \binom{30}{1} = 150\).

Blunders
B1 Addition for multiplication
B2 31 for 30
**Part (b) (iii)** 5 marks

Match three \( \binom{5}{3} \times 30 \binom{2}{2} = 4350 \).

**Blunders**

B1 Addition for multiplication
B2 32 for 30

---

**Part (b) (iv)** 5 marks

P(of matching at least three numbers)

\[
P(\text{matching at least three numbers}) = P(\text{matching three}) + P(\text{matching four}) + P(\text{matching five})
\]

\[
= \frac{4350 + 150 + 1}{324632} = \frac{4501}{324632} = 0.01386493 = 0.014.
\]

**Blunders**

B1 Error in total favourable
B2 Error in total possible
B3 No fraction
B4 Incorrect or no conclusion

---

**Part (c)** 15 marks

The mean of the integers from \(-n\) to \(n\), inclusive, is 0.

Show that the standard deviation is \( \sqrt{\frac{n(n+1)}{3}} \).

**Part (c)** 15 marks

\[
\sigma^2 = \frac{(-n)^2 + (n-1)^2 + \ldots + (n-2)^2 + (n-1)^2 + 0^2 + 1^2 + \ldots + (n-2)^2 + (n-1)^2 + n^2}{2n+1}
\]

\[
\therefore \sigma^2 = \frac{2[n^2 + (n+1)^2 + \ldots + n^2]}{2n+1} = \frac{2n+1}{2n+1} \times \frac{n(n+1)(2n+1)}{6} = \frac{n(n+1)}{3}.
\]

\[
\therefore \sigma = \sqrt{\frac{n(n+1)}{3}}.
\]

**Blunders**

B1 Not squared
B2 No square root
B3 Mean not found or incorrect denominator
B5 Error in sum to \(n\) terms
**QUESTION 8**

Part (a) 15 marks Att 5

8 (a) Derive the Maclaurin series for \( f(x) = e^x \) up to and including the term containing \( x^3 \).

\[
8 (a) \quad f(x) = f(0) + \frac{f'(0)x}{1!} + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} + \ldots
\]

\[
f(x) = e^x \quad \Rightarrow \quad f(0) = e^0 = 1.
\]

\[
f'(x) = e^x \quad \Rightarrow \quad f'(0) = 1.
\]

\[
f''(x) = e^x \quad \Rightarrow \quad f''(0) = 1.
\]

\[
f'''(x) = e^x \quad \Rightarrow \quad f'''(0) = 1.
\]

\[
\therefore \quad f(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots
\]

**Blunders**

B1 Error in differentiation each time if error not consistent

B2 Error in evaluating \( f^{(n)}(0) \)

B3 Each term missing

---

Part (b) 15 (10, 5) marks Att (3, 2)

8 (b) A line passes through the point (4, 2) and has slope \( m \), where \( m < 0 \). The line intersects the axes at the points \( a \) and \( b \).

(i) Find the co-ordinates of \( a \) and \( b \), in terms of \( m \).

(ii) Hence, find the value of \( m \) for which the area of triangle \( aob \) is a minimum.

---

8 (b) (i) Equation of line \( y - 2 = m(x - 4) \) \( \Rightarrow \) \( mx - y = 4m - 2 \).

\[
\therefore \quad a \left( 0, -4m + 2 \right) \text{ and } b \left( 4 - \frac{2}{m}, 0 \right).
\]

---

**Blunders**

B1 Error in finding the equation of the line

B2 \( a \) and \( b \) coordinates must have 0 in correct position

B3 Not in coordinate form
Part (b) (ii)  

\[ a (0, -4m + 2), \ b \left( 4 - \frac{2}{m}, 0 \right) \].

Area of triangle = \( A = \frac{1}{2} \left( 4 - \frac{2}{m} \right) \left( -4m + 2 \right) \).

\[ A = \frac{1}{2} \left( -16m + 8 + \frac{4}{m} \right) = \frac{1}{2} \left( -16m + 16 - 4m^{-1} \right) = -8m + 8 - 2m^{-1} \].

\[ \frac{dA}{dm} = -8 + 2m^{-2} = 0, \text{ for minimum.} \]

\[ \therefore -4 + \frac{1}{m^2} = 0 \implies 4m^2 = 1 \implies m = -\frac{1}{2} \text{ as } m < 0. \]

\[ \therefore A = -8(-0.5) + 8 - \frac{2}{-0.5} = 4 + 8 + 4 = 16, \text{ minimum area.} \]

Blunders
B1 Error in formula for area of a triangle
B2 Error in derivative
B3 Error in solving equation

Slips
S1 Arithmetic errors

Part (c)  

Use the ratio test to test each of the following series for convergence.

In each case, specify clearly the range of values of \( x \) for which the series converges, the range of values for which it diverges, and the value(s) of \( x \) for which the test is inconclusive.

(i) \( \sum_{n=1}^{\infty} n^3 x^n \)  
(ii) \( \sum_{n=1}^{\infty} \frac{(n+1)!}{(2n)!} x^n \).

Part (c) (i)  

\[ u_n = n^3 x^n \implies u_{n+1} = (n+1)^3 x^{n+1}. \]

\[ \lim_{n \to \infty} \left| \frac{u_{n+1}}{u_n} \right| = \lim_{n \to \infty} \left| \frac{(n+1)^3 x^{n+1}}{n^3 x^n} \right| = \lim_{n \to \infty} 3x \left( 1 + \frac{1}{n} \right) = |3x|. \]

Converges for \(|3x| < 1 \implies -\frac{1}{3} < x < \frac{1}{3}\).

Diverges for \(|3x| > 1 \implies x > \frac{1}{3} \text{ or } x < -\frac{1}{3}\).

Inconclusive for \(|3x| = 1 \text{ i.e. } x = \pm \frac{1}{3}\).
Blunders
B1 Error in $U_{n+1}$
B2 Error in evaluating $\frac{U_{n+1}}{U_n}$
B3 Error in evaluating limit
B4 Range incorrectly or not applied

Misreading (-1) for each case omitted

Part (c) (ii)  10 marks  Att 3

$8 \text{ (c) (ii)} \sum_{n=1}^{\infty} \frac{(n+1)!n^!}{(2n)!} x^n.$

Converges for $\left|\frac{x}{4}\right| < 1 \Rightarrow -4 < x < 4.$

Diverges for $\left|\frac{x}{4}\right| > 1 \Rightarrow x > 4 \text{ or } x < -4.$

Inconclusive for $\left|\frac{x}{4}\right| = 1 \text{ i.e. } x = \pm 4.$

Blunders
B1 Error in $U_{n+1}$
B2 Error in evaluating $\frac{U_{n+1}}{U_n}$
B3 Error in evaluating limit
B4 Range incorrectly or not applied

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### QUESTION 9

<table>
<thead>
<tr>
<th>Part (a)</th>
<th>10 marks</th>
<th>Att 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Part (b)</td>
<td>20 (5, 5, 5, 5) marks</td>
<td>Att (2, 2, 2, 2)</td>
</tr>
<tr>
<td>Part (c)</td>
<td>20 (5, 5, 5, 5) marks</td>
<td>Att (2, 2, 2, 2)</td>
</tr>
</tbody>
</table>

#### Part (a) 10 marks

9 (a)  
\[ z \text{ is a random variable with standard normal distribution.} \]
Find the value of \( z_1 \) for which \( P(z > z_1) = 0.0808 \).

\[
P(z > z_1) = 0.0808 \Rightarrow 1 - P(z < z_1) = 0.0808.
\]
\[
\therefore P(z < z_1) = 0.9192 \Rightarrow z_1 = 1.4.
\]

**Blunders**
B1 Incorrect reading of tables or incorrect area each time

#### Part (b) 20 (5, 5, 5, 5) marks

9 (b) A bag contains the following cardboard shapes:
- 10 red squares, 15 green squares, 8 red triangles and 12 green triangles.
One of the shapes is drawn at random from the bag.

- \( E \) is the event that a square is drawn.
- \( F \) is the event that a green shape is drawn.

(i) Find \( P(E \cap F) \).
(ii) Find \( P(E \cup F) \).
(iii) State whether \( E \) and \( F \) are independent events, giving a reason for your answer.
(iv) State whether \( E \) and \( F \) are mutually exclusive events, giving a reason for your answer.

**Blunders**
B1 Incorrect total possible
B2 Incorrect total favourable
B3 No fraction

#### Part (b) (i) 5 marks

9 (b) (i) \[
P(E \cap F) = \frac{15}{45}.
\]

**Blunders**
B1 Incorrect total possible
B2 Incorrect total favourable
B3 No fraction

#### Part (b) (ii) 5 marks

9 (b) (ii) \[
P(E \cup F) = P(E) + P(F) - P(E \cap F) = \frac{25}{45} + \frac{27}{45} - \frac{15}{45} = \frac{37}{45}.
\]

**Blunders**
B1 Incorrect total possible
B2 Incorrect total favourable
B3 No fraction
Part (b) (iii)  5 marks  Att 2

9 (b) (iii) \[ P(E)P(F) = \frac{25}{45} \cdot \frac{27}{45} = \frac{15}{45} = P(E \cap F). \therefore \text{ Independent events.} \]

**Blunders**

B1 reason not given

Part (b) (iv)  5 marks  Att 2

9 (b) (iv) \[ P(E \cap F) \neq 0 \text{ hence not mutually exclusive} \]

Part (c)  20 (5,5,5,5) marks  Att (2,2,2,2)

9 (c) The marks awarded in an examination are normally distributed with a mean mark of 60 and a standard deviation of 10.
A sample of 50 students has a mean mark of 63.
Test, at the 5% level of significance, the hypothesis that this is a random sample from the population.

9 (c)
\[
\bar{x} = 60, \sigma = 10 \implies \sigma_x = \frac{\sigma}{\sqrt{n}} = \frac{10}{\sqrt{50}} = 1.4142.
\]
\[
\frac{x - \bar{x}}{\sigma_x} = \frac{63 - 60}{1.4142} = 2.1213 > 1.96. \therefore \text{Not a random sample.}
\]

**Blunders**

B1 Error in standard error of mean
B2 Error in finding Z value
B3 Error in confidence interval

**Slips**

S1 Arithmetic errors
QUESTION 10

Part (a) 30 (10, 5, 10, 5) marks  
Part (b) 20 (10, 10) marks

Part (a) 30 (10, 5, 10, 5) marks  

10 (a)  

G is the set of rotations that map a regular hexagon onto itself. 

\((G, \circ)\) is a group, where \(\circ\) denotes composition. 

The anti-clockwise rotation through 60° is written as \(R_{60^\circ}\).

(i) List the elements of \(G\).

(ii) State which elements of the group, if any, are generators.

(iii) List all the proper subgroups of \((G, \circ)\).

(iv) Find \(Z(G)\), the centre of \((G, \circ)\). Justify your answer.

Part (a) (i) 10 marks  

10 (a) (i) 

\[ G = \{ R_0^\circ, R_{60^\circ}, R_{120^\circ}, R_{180^\circ}, R_{240^\circ}, R_{300^\circ} \} \]

Blunders

B1 Each one omitted or incorrect

Part (a) (ii) 5 marks  

10 (a) (ii) 

Generators are \(R_{60^\circ}\) and \(R_{300^\circ}\).

Blunders

B1 Each one omitted

Part (a) (iii) 10 marks  

10 (a) (iii) 

Proper subgroups of \((G, \circ)\) are \(\{ R_0^\circ, R_{180^\circ} \}\) and \(\{ R_0^\circ, R_{120^\circ}, R_{240^\circ} \}\).

Blunders

B1 Each one omitted or incorrect

Part (a) (iv) 5 marks  

10 (a) (iv) 

Each element of \(G\) commutes with each of the elements of \(G\). 

\[ \therefore Z(G) = G. \]

Blunders

B1 \(Z(G)\) not found

B3 Error in justification
Part (b)  

20 (10, 10) marks 

Att (3, 3)

10 (b) (i) Show that the group \( \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \right\} \) under matrix multiplication is isomorphic to the group \( \{0, 1\} \) under addition modulo 2.

(ii) Prove that any infinite cyclic group is isomorphic to \( (\mathbb{Z}, +) \).

---

Part (b) (i)  

10 marks 

Att 3

Let \( I = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \) and \( A = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \). Let \( G = \{ I, A \} \) under matrix multiplication.

\[
A^2 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} . \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I
\]

Thus \( I \) is of order 1 and \( A \) is of order 2.

Let \( H = \{0, 1\} \) under addition modulo 2.

\( 1+1 = 2 = 0 \mod(2) \).

Thus 0 is of order 1 and 1 is of order 2.

As \( G \) and \( H \) both have one element of order 1 and one element of order 2, they are isomorphic. The isomorphism \( G \rightarrow H \) is

\[
I \rightarrow 0 \\
A \rightarrow 1.
\]

---

Blunders

B1 Any property of isomorphism not included

Attempts

A1 Elements of \( H \) found

Part (b) (ii)  

10 marks 

Att 3

Let \( G = \langle g \rangle \) be any infinite cyclic group generated by \( g \) under \( * \).

Define \( \phi : (G, \ast) \rightarrow (\mathbb{Z}, +) : g^k \rightarrow k \).

\[
\phi(g^a \ast g^b) = \phi(g^{a+b}) = a + b \\
= \phi(g^a) + \phi(g^b).
\]

\( \therefore \phi \) is an isomorphism \( \Rightarrow (G, \ast) \) and \( (\mathbb{Z}, +) \) are isomorphic.

---

Blunders

B1 Error with indices

B2 No conclusion
QUESTION 11

Part (a)  10 (5, 5) marks Att (2, 2)
Part (b)  20 (10, 10) marks Att (3, 3)
Part (c)  20 marks Att 6

Part (a)  10 (5, 5) marks Att (2, 2)
11 (a) (i) Find the image of \(a(-1, 2)\) and \(b(0, 4)\) under the transformation
\[
\begin{pmatrix} x' \\ y' \\ \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \\ \end{pmatrix} \begin{pmatrix} x \\ y \\ \end{pmatrix} + \begin{pmatrix} 2 \\ -4 \\ \end{pmatrix}.
\]

(ii) Show that \(ab\) is parallel to \(a'b'\).

Blunders
B1 Error in matrix multiplication
B2 Error in addition

Part (a) (i)  5 marks Att 2
11 (a) (i) 
\[
\begin{pmatrix} 2 & 0 \\ 0 & 2 \\ \end{pmatrix} \begin{pmatrix} -1 \\ 2 \\ \end{pmatrix} + \begin{pmatrix} 2 \\ -4 \\ \end{pmatrix} = \begin{pmatrix} -2 \\ 4 \\ \end{pmatrix} \quad \quad \begin{pmatrix} 2 \\ -4 \\ \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \end{pmatrix}
\]

B1 Error in each distance formula
B2 Error in addition

Part (a) (ii)  5 marks Att 2
11 (a) (ii) Slope \(ab = \frac{4-2}{0+1} = 2\) and slope \(a'b' = \frac{4-0}{2-0} = 2\). \(\therefore\) \(ab\) is parallel to \(a'b'\).

Blunders
B1 Error in slope formula
B2 No conclusion

Part (b)  20 (10, 10) marks Att (3, 3)
11 (b) \(p(x, y)\) is a point such that the distance from \(p\) to the point \((2, 0)\) is half the distance from \(p\) to the line \(x = 8\).

(i) Find the equation of the locus of \(p\).

(ii) Show that this locus is an ellipse centred at the origin, by expressing
its equation in the form \(\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1\).

Part (b) (i)  10 marks Att 3
11 (b) (i) 
\[
p(x, y), \quad s(2, 0). \quad \therefore |ps| = \frac{1}{2} \sqrt{(x-2)^2 + (y-0)^2}.
\]
Distance from \(p(x, y)\) to line \(x = 8\) is \(\left| \frac{8-x}{1} \right|\).
\(\therefore\) Locus of \(p\): \(3x^2 + 4y^2 = 48\).

Blunders
B1 Error in each distance formula
Part (b) (ii)  

11 (b) (ii) 

\[ 3x^2 + 4y^2 = 48 \]
\[ \Rightarrow \frac{3x^2}{48} + \frac{4y^2}{48} = 1 \]
\[ \Rightarrow \frac{x^2}{16} + \frac{y^2}{12} = 1 \]

Blunders

B1 Error in format of ellipse equation
B2 Error in squaring

---

Part (c)  

11 (c)  

Prove that the areas of all parallelograms circumscribed about a given ellipse at the endpoints of conjugate diameters are equal.

\[ [w'z'] \text{and} [u'v'] \text{are conjugate diameters of ellipse } E. \]
Tangents at their end-points form the parallelogram \( p'q'r's' \).

Under an affine transformation \( f^{-1} \), the ellipse maps to the circle \( x^2 + y^2 = 1 \)
and \( p'q'r's' \) is mapped to \( pQRS \).

\([uv]\text{and} [wz] \text{are conjugate diameters of the circle and} uv \perp wz \).
The square \( pQRS \) has fixed area 4 sq units.

Area \( pQRS = 2 \) area \( pqr \) \( \Rightarrow \) area \( p'q'r's' = 2 \) area \( p'q'r' \) as ratio is an invariant map.

Area \( p'q'r's' = 2 |\det f| \text{area } \Delta pqr \)
\[ = |\det f| \text{area } pQRS. \]

But \( \det f \) is constant and area \( pQRS \) is also constant \( \Rightarrow \) area \( p'q'r's' \) is constant.

\( \therefore \) Areas of all parallelograms at end points of conjugate diameters are equal.

Blunders

B1 Error in mapping
B2 No statement regarding constant area of square
B3 No statement of ratio being invariant and \( \det f \) being constant
B4 No conclusion
BONUS MARKS FOR ANSWERING THROUGH IRISH

Bonus marks are applied separately to each paper as follows:

If the mark achieved is less than 226, the bonus is 5% of the mark obtained, rounding down. (e.g. 198 marks × 5% = 9.9 ⇒ bonus = 9 marks.)

If the mark awarded is 226 or above, the following table applies:

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<thead>
<tr>
<th>Marks obtained</th>
<th>Bonus</th>
</tr>
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<tbody>
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<td>11</td>
</tr>
<tr>
<td>232 – 238</td>
<td>10</td>
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<tr>
<td>239 – 245</td>
<td>9</td>
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<tr>
<td>246 – 251</td>
<td>8</td>
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<td>252 – 258</td>
<td>7</td>
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<td>6</td>
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