Coimisiún naScrúduithe Stáit
State Examinations Commission

Leaving Certificate 2012

Marking Scheme

Mathematics
(Project Maths – Phase 1)

Ordinary Level
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1. Penalties of three types are applied to candidates’ work as follows:
   - Blunders - mathematical errors/omissions (-3)
   - Slips - numerical errors (-1)
   - Misreadings (provided task is not oversimplified) (-1).

   Frequently occurring errors to which these penalties must be applied are listed in the scheme. They are labelled: B1, B2, B3,…, S1, S2,…, M1, M2,…etc. These lists are not exhaustive.

2. When awarding attempt marks, e.g. Att (3), note that
   - any correct, relevant step in a part of a question merits at least the attempt mark for that part
   - if deductions result in a mark which is lower than the attempt mark, then the attempt mark must be awarded
   - a mark between zero and the attempt mark is never awarded.

3. Worthless work is awarded zero marks. Some examples of such work are listed in the scheme and they are labelled as W1, W2,…etc.

4. The phrase “hit or miss” means that partial marks are not awarded – the candidate receives all of the relevant marks or none.

5. The phrase “and stops” means that no more work is shown by the candidate.

6. Special notes relating to the marking of a particular part of a question are indicated by an asterisk. These notes immediately follow the box containing the relevant solution.

7. The sample solutions for each question are not intended to be exhaustive lists – there may be other correct solutions. Any examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his/her advising examiner.

8. Unless otherwise indicated in the scheme, accept the best of two or more attempts – even when attempts have been cancelled.

9. The same error in the same section of a question is penalised once only.

10. Particular cases, verifications and answers derived from diagrams (unless requested) qualify for attempt marks at most.

11. A serious blunder, omission or misreading results in the attempt mark at most.

12. Do not penalise the use of a comma for a decimal point, e.g. €5.50 may be written as €5,50.
APPLYING THE GUIDELINES

Examples (not exhaustive) of the different types of error:

**Blunders** (i.e. mathematical errors) (-3)
- Algebraic errors: \( 8x + 9x = 17x^2 \) or \( 5p \times 4p = 20p \) or \((-3)^2 = 6\)
- Sign error: \(-3(-4) = -12\)
- Decimal errors
- Fraction error (incorrect fraction, inversion etc.)
- Cross-multiplication error
- Operation chosen is incorrect. (e.g., multiplication instead of division)
- Transposition error: e.g. \(-2x - k + 3 \Rightarrow -2x = 3 + k\) or \(-3x = 6 \Rightarrow x = 2\) or \(4x = 12 \Rightarrow x = 8\).
- Distribution error e.g. \(3(2x + 4)\) has \(6x + 4\) or \(\frac{x}{6}(3 - x) = 5 \Rightarrow 6 - x = 5\)
- Omission, if not oversimplified.
- Index error.
- Factorisation: error in one or both factors of a quadratic: \(2x^2 - 2x - 3 = (2x - 1)(x + 3)\)
- Root errors from candidate’s factors: error in one or both roots:
- Error(s) in transcribing formulae from tables (assuming it generates mathematical acceptable answer(s)). Serious errors or over simplifications will merit Attempt marks at most (check relevant section of scheme)
- Central sign error in \(uv\) or \(u/v\) formulae
- Omission of \(\div v^2\) or division not done in \(u/v\) formula
- Vice-versa substitution in \(uv\) or \(u/v\) formulae
- Quadratic formula and its application apply a maximum of two blunders

**Slips** (-1)
- Numerical slips: \(4 + 7 = 10\) or \(3 \times 6 = 24\), but \(5 + 3 = 15\) is a blunder.
- An omitted round-off or incorrect round off to a required degree of accuracy, or early rounding off which effects final answer are penalised as a slip each time.
- However an early round-off which has the effect of simplifying the work is at least a blunder
- Omission of units of measurement or giving the incorrect units of measurement in an answer is treated as a slip, once per section of each question. Only applies where a candidate would otherwise have achieved full marks

**Misreadings** (-1)
- Writing 2436 for 2346 will not alter the nature of the question so M(-1)
  However, writing 5000 for 5026 will simplify the work and is penalised as at least a blunder.

*Note:* Correct relevant formula *isolated* and stops: if formula is *not* in Tables, award attempt mark.
QUESTION 1

Part (a) 15 marks  Att 5

When Katie had travelled 140 km, she had completed $\frac{4}{9}$ of her journey.

Find the length of her journey.

(a) 15 marks  Att 5

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
<th>Length of Journey</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>$\frac{4}{9}$ of journey = 140 km $\Rightarrow$ length of journey $= \frac{140 \times 9}{4} = 315$ km</td>
<td>315 km (15)</td>
</tr>
<tr>
<td>or</td>
<td></td>
<td></td>
</tr>
<tr>
<td>II</td>
<td>$\frac{140}{4} = 35$ $\Rightarrow$ length of journey $= 35 \times 9 = 315$ km</td>
<td>315 km (15)</td>
</tr>
<tr>
<td>or</td>
<td></td>
<td></td>
</tr>
<tr>
<td>III</td>
<td>$\frac{4}{9} = 0.4444$ $\Rightarrow$ length of journey $= \frac{140}{0.4444} = 315.0315$ km</td>
<td>315.0315 km (15)</td>
</tr>
</tbody>
</table>

* Accept correct answers without work

Blunders (-3)
B1 Mathematical error e.g. decimal point (apply if method III does not give answer 315 to nearest whole number)

Slips (-1)
S1 Units omitted

Attempts (5 marks)
A1 Any relevant work

Worthless (0)
W1 Incorrect answer without work
W2 $140 \div 9$ and stops
W3 Divides 140 in the ratio 4:9 and stops
Robert’s electricity bill gave the following data:

<table>
<thead>
<tr>
<th>Unit type</th>
<th>Present reading</th>
<th>Previous reading</th>
<th>Unit price</th>
</tr>
</thead>
<tbody>
<tr>
<td>Day rate</td>
<td>35 087</td>
<td>34 537</td>
<td>€0.1506</td>
</tr>
<tr>
<td>Night rate</td>
<td>17 213</td>
<td>16 853</td>
<td>€0.0745</td>
</tr>
</tbody>
</table>

(i) Calculate the total cost of the units used. Robert also pays a standing charge of €24.89 and a levy of €5.46. VAT at the rate of 13.5% is charged on all amounts.

(ii) Calculate the total amount of Robert’s electricity bill.

(b) (i) 5 marks Att 2

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Day:</td>
<td>35087</td>
<td>34537</td>
</tr>
<tr>
<td>Night:</td>
<td>17213</td>
<td>16853</td>
</tr>
</tbody>
</table>

Cost: 550 × 0.1506 = €82.83

Cost: 360 × 0.0745 = €26.82

Total cost: €82.83 + €26.82 = €109.65.

Blunders (-3)
B1 Mathematical error

Slips (-1)
S1 Numerical slips

Attempts (2 marks)
A1 Any relevant work e.g. mentions/states cost = units × rate

(b) (ii) 10 (5, 5) marks Att 2

<p>| | | |</p>
<table>
<thead>
<tr>
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</tbody>
</table>

Total:
€109.65 + €24.89 + €5.46 = €140 [5]

Amount with VAT

I Vat = 140 × 0.135 = €18.90 ⇒ Total amount = €140 + €18.90 = €158.90 [4]

or

II Total amount = 140 × 1.135 = €158.90 [2] [5]

* Accept candidates work from (i)

Blunders(-3)
B1 Mathematical error

Slips (-1)
S1 Numerical slips

Misreading (-1)
M1 Omits standing charge or levy

Attempts (2 marks)
A1 Any relevant work
A retailer bought 40 toys at €24.75 each. He sold 10 of the toys at €33.88 each and sold the remaining 30 toys at a reduced price. His total sales amounted to €1270.

(i) Write his total profit on the transaction as a percentage of his cost. Give your answer correct to one decimal place.

(ii) Find the reduced selling price of each of the remaining 30 toys.

* Answers to parts of question must be clearly identified/labelled otherwise assume order in paper

(c) (i) 10 (5, 5) marks

<table>
<thead>
<tr>
<th>Cost:</th>
<th>40 × €24.75 = €990 [5]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Profit:</td>
<td>€1270 – €990 = €280</td>
</tr>
<tr>
<td>Percentage profit:</td>
<td>( \frac{280}{990} \times 100 = 28.28 = 28.3% ) [2] [4] [5]</td>
</tr>
</tbody>
</table>

Blunders (-3)
B1 Mathematical error e.g. finding percentage

Slips (-1)
S1 Fails to round off or rounds-off incorrectly

Attempts (2 marks)
A1 Any relevant work (two attempts possible) e.g. uses single unit costs.

Note: \( \frac{1270}{990} = 1.2828 \) and continues can merit full marks

(c) (ii) 10 marks

<table>
<thead>
<tr>
<th>Selling price, 10 toys:</th>
<th>10 × €33.88 = €338.80 [4]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Selling price, 30 toys:</td>
<td>€1270 – €338.80 = €931.20 [7]</td>
</tr>
<tr>
<td>Price per toy:</td>
<td>€931.20 ÷ 30 = €31.04 [10]</td>
</tr>
</tbody>
</table>

Blunders (-3)
B1 Mathematical error

Slips (-1)
S1 Numerical slips

Attempts (3 marks)
A1 Any relevant work
Solve for $x$ and $y$

\begin{align*}
  x - y &= 4 \\
  2x + y &= 5
\end{align*}

\[3x = 9 \Rightarrow x = 3 \quad [12]\]

\[x - y = 4 \Rightarrow 3 - y = 4 \Rightarrow -y = 1 \Rightarrow y = -1 \quad [15]\]
Let \( f(x) = x^3 + 2x^2 - x - 2 \).

(i) Show, by division, that \( x - 1 \) is a factor of \( f(x) \).

(ii) Hence, or otherwise, find the other factors of \( f(x) \).

\[
\begin{array}{c|cc}
& x^2 & + 3x & + 2 \\
\hline
x & x^3 & + 3x^2 & + 2x \\
-1 & -x^2 & -3x & -2 \\
\end{array}
\]

Any method of division.

\( f(x) = x^3 + 2x^2 - x - 2 = (x-1)(x^2 + 3x + 2) = (x-1)(x+1)(x+2) \)

Other factors: \((x+1)\) and \((x+2)\)

\( \star \) No back/retrospective marking
\( \star \) No penalty if continues to find roots - extra work
\( \star \) Accept candidates quadratic from (i) if not oversimplified, see A1
\( \star \) Candidates may find quadratic by comparing coefficients
\( \star \) If incorrect answer at b(i) is “unfactorisable” award attempt marks at most in b(ii) unless uses relevant formula to find roots and then factors

Blunders (-3)
B1 Mathematical error e.g. finds, using formula, roots of quadratic at (i) and stops

Attempts (2 marks)
A1 If quotient at (i) is linear award attempt marks at most in (ii)
Part (c) 15 (5, 10) marks Att (2, 3)

Let \( g(x) = \frac{1}{x^2} - \frac{1}{2x} \) and \( h(x) = 1 - \frac{2}{x} \), where \( x \neq 0 \) and \( x \in \mathbb{R} \).

(i) Show that \( h(x) = -2x \left[ g(x) \right] \).

(ii) Find the values of \( x \) for which \( g(x) = h(x) \).

(c) (i) 5 marks Att 2

\[
-2x[g(x)] = -2x \left( \frac{1}{x^2} - \frac{1}{2x} \right) = \frac{-2x}{x^2} + \frac{2x}{2x} = \frac{-2}{x} + \frac{1}{1} = 1 - \frac{2}{x} = h(x)
\]

\[ \text{[2]} \quad \text{[2]} \quad \text{[5]} \]

*Blunders (-3)*

B1 Mathematical error e.g. index error

*Attempts (2 marks)*

A1 Any relevant work e.g. correct substitution

(c) (ii) 10 marks Att 3

\[
\frac{1}{x^2} - \frac{1}{2x} = 1 - \frac{2}{x}
\]

\[ \Rightarrow 2 - x = 2x^2 - 4x \]

\[ \Rightarrow 2x^2 - 3x - 2 = 0 \]

\[ \Rightarrow (2x + 1)(x - 2) = 0 \]

\[ \Rightarrow x = -\frac{1}{2} \quad \text{or} \quad x = 2 \]

* If equation becomes linear candidates will merit at most attempt marks

Award marks as follows:

10 marks: Fully correct answer
3 marks: Work of some merit
0 marks: Otherwise
Given that \((t - 1)x = 2 - 5t\), find the value of \(x\) when \(t = 7\).

\[
(t - 1)x = 2 - 5t \quad \Rightarrow \quad (7 - 1)x = 2 - 5(7)
\]

\[
6x = 2 - 35 = -33 \quad \Rightarrow \quad x = -5 \frac{1}{2} \quad \text{or} \quad -5.5 \quad \text{or equivalent e.g.} \quad \frac{-33}{6}
\]

or

\[
x = \frac{2 - 5t}{t - 1} = \frac{2 - 5(7)}{7 - 1} = -5 \frac{1}{2} \quad \text{or} \quad x = -5.5 \quad \text{or equivalent e.g.} \quad \frac{-33}{6}
\]
(b) (i) 15 (10, 5) marks

Step 1  Isolation of one variable: 10 marks

Step 2  Finding second variable:  5 marks

\[
\begin{align*}
\text{I} & : \quad x - y + 5 = 0 \implies x = y - 5 \\
\text{II} & : \quad y = x + 5 \\
& \quad x^2 + y^2 = 17 \implies (y - 5)^2 + y^2 = 17 \\
& \quad (y - 5)^2 + y^2 = 17 \implies (x + 5)^2 + x^2 = 17 \\
\quad \implies y^2 - 10y + 25 + y^2 - 17 = 0 \implies x^2 + 10x + 25 + x^2 - 17 = 0 \\
\quad \implies 2y^2 - 10y + 8 = 0 \implies y^2 - 5y + 4 = 0 \implies 2x^2 + 10x + 8 = 0 \implies x^2 + 5x + 4 = 0 \\
\quad \implies (y - 1)(y - 4) = 0 \implies y = 1 \text{ or } y = 4 \implies (x + 4)(x + 1) = 0 \implies x = -4 \text{ or } x = -1 \\
\end{align*}
\]

\[
\begin{align*}
y = 1 & \implies x - 1 + 5 = 0 \implies x = -4 \\
y = 4 & \implies x - 4 + 5 = 0 \implies x = -1 \\
x = -4 & \implies -4 - y + 5 = 0 \implies y = 1 \\
x = -1 & \implies -1 - y + 5 = 0 \implies y = 4 \\
\end{align*}
\]

* Candidates may use other acceptable algebraic approaches

Blunders (-3)

B1  Mathematical error

Attempts (3 or 2 marks)

A1  Any relevant work

A2  Correct answers by trial and error or without relevant work, verified in:

- One equation – one Att 2
- Both equations – Att 3 + Att 2

If tests only one solution award Att 2 only once

A3  Some effort at a graphical solution or some effort at trial and error – one attempt only

A4  Fully correct graphical solutions merits Att 3 and Att 2 at most
(b) (ii) 5 marks

\[
\begin{align*}
(-4, 1) & \Rightarrow x - 2y = -4 - 2(1) = -6 \quad [2] \\
(-1, 4) & \Rightarrow x - 2y = -1 - 2(4) = -9 \quad [4] \\
(-1, 4) & \text{gives the lesser value of } -9 \quad [5]
\end{align*}
\]

* Accept candidates answer from (i) if not oversimplified. Note if only one answer found at (i) can earn at most att 2 for this section

**Blunders (-3)**

B1 Mathematical error e.g. only tests one point

**Slip (-1)**

S1 Incorrect or no conclusion

**Attempt (2 marks)**

A1 Any relevant work e.g. correct substitution for \(x\) and/or \(y\) and stops

**Worthless (0)**

W1 Invents values for \(x\) and \(y\)
### Part (c)

**15 (5, 5, 5) marks**  
**Att (2, 2, 2)**

(i) Simplify \( \left( \sqrt{x} - \frac{2}{\sqrt{x}} \right) \left( \sqrt{x} + \frac{2}{\sqrt{x}} \right) \), where \( x > 0 \) and \( x \in \mathbb{R} \).

(ii) Hence, solve \( \left( \sqrt{x} - \frac{2}{\sqrt{x}} \right) \left( \sqrt{x} + \frac{2}{\sqrt{x}} \right) = 3 \), where \( x > 0 \).

(iii) Verify your solution.

---

### (c) (i) 5 marks  
**Att 2**

\[
\left( \sqrt{x} - \frac{2}{\sqrt{x}} \right) \left( \sqrt{x} + \frac{2}{\sqrt{x}} \right) = \left( \sqrt{x} \right)^2 - \left( \frac{2}{\sqrt{x}} \right)^2 = x - \frac{4}{x} \quad \text{or} \quad \frac{x^2 - 4}{x} \quad \text{accept either}
\]

or

\[
\left( \sqrt{x} - \frac{2}{\sqrt{x}} \right) \left( \sqrt{x} + \frac{2}{\sqrt{x}} \right) = \left( \sqrt{x} \right)^2 + 2\sqrt{x} - 2\sqrt{x} - \left( \frac{2}{\sqrt{x}} \right)^2 = x - \frac{4}{x} \quad \text{or} \quad \frac{x^2 - 4}{x} \quad \text{accept either}
\]

or

\[
\left( \sqrt{x} - \frac{2}{\sqrt{x}} \right) \left( \sqrt{x} + \frac{2}{\sqrt{x}} \right) = \left( x - 2 \right) \left( x + 2 \right) = \frac{x^2 - 4}{x} \quad \text{or} \quad x - \frac{4}{x} \quad \text{accept either}
\]

* Candidates may offer other correct methods

**Blunders (-3)**

B1  Mathematical error e.g. sign

**Attempts (2 marks)**

A1  Any relevant work e.g. starts multiplication

A2  Correct answer without work

---

### (c) (ii) 5 marks  
**Att 2**

\[
\left( \sqrt{x} - \frac{2}{\sqrt{x}} \right) \left( \sqrt{x} + \frac{2}{\sqrt{x}} \right) = 3
\]

\[
\Rightarrow x - \frac{4}{x} = 3 \quad \text{or} \quad \frac{x^2 - 4}{x} = 3 \quad [2]
\]

\[
\Rightarrow x^2 - 4 = 3x \quad \Rightarrow x^2 - 3x - 4 = 0
\]

\[
\Rightarrow (x - 4)(x + 1) = 0 \quad \Rightarrow x = 4 \quad \text{or} \quad x = -1 \quad [4]
\]

\[
x > 0 \quad \Rightarrow \quad x = 4 \quad [5]
\]

* Accept candidates answer from (i) – if linear award attempt marks at most

**Blunders (-3)**

B1  Mathematical error

**Slips (-1)**

S1  Does not isolate correct value

**Attempts (2 marks)**

A1  Any relevant work
\[
x = 4 \Rightarrow \left( \sqrt{4} - \frac{2}{\sqrt{4}} \right) \left( \sqrt{4} + \frac{2}{\sqrt{4}} \right) = \left( \frac{2}{2} - \frac{2}{2} \right) \left( \frac{2}{2} + \frac{2}{2} \right) = (1)(3) = 3
\]

<table>
<thead>
<tr>
<th>5 marks</th>
<th>Att 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>[x = 4 \Rightarrow \left( \sqrt{4} - \frac{2}{\sqrt{4}} \right) \left( \sqrt{4} + \frac{2}{\sqrt{4}} \right) = \left( \frac{2}{2} - \frac{2}{2} \right) \left( \frac{2}{2} + \frac{2}{2} \right) = (1)(3) = 3]</td>
<td></td>
</tr>
</tbody>
</table>

* Accept candidates answer from (ii)
* Accept verification of \(-1\) in \(\sqrt{x} - \frac{2}{\sqrt{x}} \left( \sqrt{x} + \frac{2}{\sqrt{x}} \right)\) - uses \(\sqrt{-1} = i\)
* Must verify in \(\sqrt{x} - \frac{2}{\sqrt{x}} \left( \sqrt{x} + \frac{2}{\sqrt{x}} \right)\)

**Blunders (-3)**
B1  Mathematical error e.g. square root error

**Slips (-1)**
S1  Incorrect or no conclusion, if one is required

**Attempts (2 marks)**
A1  Any relevant work e.g. substitutes answer from (ii) into expression and stops or verifies in simplified version.
**QUESTION 4**

<table>
<thead>
<tr>
<th>Part (a)</th>
<th>15 (5, 10) marks</th>
<th>Att (2, 3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Part (b)</td>
<td>15 (5, 5, 5) marks</td>
<td>Att (2, 2, 2)</td>
</tr>
<tr>
<td>Part (c)</td>
<td>20 marks</td>
<td>Att (3, 3)</td>
</tr>
</tbody>
</table>

**Part (a) 15 (5, 10) marks Att (2, 3)**

Given that \(6 - 4i + 3u = 5i\), where \(i^2 = -1\),

(i) find \(u\),

(ii) plot \(u\) on an Argand diagram.

| (a) (i) | 5 marks | Att 2 |
| (a) (ii) | 10 marks | Att 3 |

(i) \[6 - 4i + 3u = 5i\] 
\[\Rightarrow 3u = -6 + 9i \Rightarrow u = -2 + 3i\] 

(ii) ![Argand Diagram](image)

* No penalty for interchange of real and imaginary axes if consistent
* Accept candidates answer from (i)

**Blunders (-3)**

B1 Mathematical error e.g. mixes up axes but check * above.

**Attempts (2 or 3 marks)**

A1 Any relevant work e.g. construction of axes and stops – once only
Part (b) 15 (5, 5, 5) marks Att (2, 2, 2)

Let \( z = 1 + i \).

(i) Find \( |z| \).

(ii) Show that \( z^2 + \overline{z}^2 = 0 \), where \( \overline{z} \) is the complex conjugate of \( z \).

(iii) Verify that \( \frac{1 + 5i}{3 + 2i} = z \).

(b) (i) 5 marks Att 2

\[
|1 + i| = \sqrt{1^2 + 1^2} = \sqrt{2}
\]

* Formula not in tables

Blunders (-3)
B1 Mathematical error e.g. includes \( i^2 \) or \( i^2 \neq -1 \)

Attempt (2 marks)
A1 Any relevant work e.g. correct formula

(b) (ii) 5 marks Att 2

\[
z^2 + \overline{z}^2 = (1 + i)^2 + (1 - i)^2 = 1 + 2i + i^2 + 1 - 2i + i^2 = 1 + 2i - 1 + 2i - 1 = 2 - 2 = 0
\]

* Accept “conjugate of square = square of conjugate” i.e. \( \overline{z^2} = (\overline{z})^2 \)

Blunders (-3)
B1 Mathematical error

Attempt (2 marks)
A1 Any relevant work e.g. correct substitution or identification of \( \overline{z} \), specific or general

(b) (iii) 5 marks Att 2

\[
\frac{1 + 5i}{3 + 2i} = \frac{1 + 5i}{3 + 2i} \cdot \frac{3 - 2i}{3 - 2i} = \frac{3 - 2i + 15i - 10i^2}{9 + 4} = \frac{13 + 13i}{13} = 1 + i = z.
\]

or

\[
\frac{1 + 5i}{3 + 2i} = 1 + i
\]

\[\Rightarrow 1 + 5i = (3 + 2i)(1 + i) \quad [2]\]

\[= 3(1 + i) + 2i(1 + i) = 3 + 3i + 2i + 2i^2 = 3 + 5i - 2 = 1 + 5i \quad [5]\]

Blunders(-3)
B1 Mathematical error e.g. \( i^2 \neq -1 \)

Slips (-1)
S1 Incorrect answer and no conclusion or wrong conclusion

Attempts (2 marks)
A1 Any relevant work e.g. correct substitution for \( z \)
Let $w = 3 + 4i$.
Find the real numbers $k$ and $t$ such that

$$w^2 - (k + t)w + t = 0.$$ 

### Step 1: Substitution 10 marks

Step 2: Finish 10 marks

\[w^2 - (k + t)w + t = 0\]

\[\Rightarrow (3 + 4i)^2 - (k + t)(3 - 4i) + t = 0\]

\[\Rightarrow 9 + 24i - 16 - 3k - 4ki - 3t - 4ti + t = 0\]

Real parts: \[9 - 16 - 3k - 3t + t = 0 \Rightarrow -3k - 2t - 7 = 0\]

Imaginary parts: \[24 - 4k - 4t = 0\]

\[6k + 4t = -14\]
\[-4k - 4t = -24\]

\[2k = -38 \Rightarrow k = -19\]

\[-3k - 2t - 7 = 0 \Rightarrow -3(-19) - 2t - 7 = 0 \Rightarrow -2t = -50 \Rightarrow t = 25\]

Step 1: Substitution

Award marks as follows:

- 10 marks: Fully correct substitution
- 7 marks: Partial correct substitution
- 3 marks: Attempt at substitution
- 0 marks: Otherwise

Step 2: Finish

Award marks as follows:

- 10 marks: Fully correct answer
- 3 marks: Work of some merit
- 0 marks: Otherwise
QUESTION 5

Part (a) 15 marks Att 5
Part (b) 20 (10, 10) marks Att (3, 3)
Part (c) 15 (5, 5, 5) marks Att (2, 2, 2)

* Do not penalise notation

Part (a) 15 marks Att 5

The $n^{th}$ term of a sequence is $T_n = \frac{2n-1}{n+1}$.

Find the sum of the second and third terms of the sequence.

(a) 15 marks Att 5

\[
T_n = \frac{2n-1}{n+1}
\]

\[
T_2 = \frac{4-1}{2+1} = 1 \quad [9]
\]

\[
T_3 = \frac{6-1}{3+1} = \frac{5}{4} \quad [12]
\]

\[
T_2 + T_3 = 1 + \frac{5}{4} = 2\frac{1}{4} \text{ or } 2.25 \quad [15]
\]

* Accept correct answers without work
* $T_2$ and $T_3$ found merits 12 marks without work

Blunders (-3)
B1 Mathematical error e.g. decimal or fractions

Attempts (5 marks)
A1 Any relevant work e.g. finds $T_1$ and stops

[18]
The first term of an arithmetic series is 2 and the eighth term is 30.

(i) Find $T_3$, the third term of the series.

(ii) Find $S_{10}$, the sum of the first ten terms of the series.

* Answers to parts of question must be clearly identified otherwise order in paper

<table>
<thead>
<tr>
<th>(i)</th>
<th>10 marks</th>
<th>Att 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a = 2, \quad T_8 = a + 7d = 30 \quad \Rightarrow \quad 2 + 7d = 30 \quad \Rightarrow \quad 7d = 28 \quad \Rightarrow \quad d = 4$</td>
<td>[3]</td>
<td>[4]</td>
</tr>
<tr>
<td>$T_3 = a + 2d = 2 + 2(4) = 10$</td>
<td>[10]</td>
<td></td>
</tr>
<tr>
<td>or $T_1 = 2, \quad T_2 = 2 + 4 = 6, \quad T_3 = 6 + 4 = 10$</td>
<td>[3]</td>
<td>[7]</td>
</tr>
</tbody>
</table>

* Accept correct answer without work
* Accept difference, $d$, as 4 without work for 7 marks

**Blunders (-3)**
B1 Mathematical error e.g. incorrect term found

**Attempts (3 marks)**
A1 Any relevant work e.g. writes $a = 2$ or $T_1 = 2$ or similar

**Worthless (0)**
W1 Treats as a geometric series but check for A1

<table>
<thead>
<tr>
<th>(ii)</th>
<th>10 marks</th>
<th>Att 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_n = \frac{n}{2}(2a + (n-1)d)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Rightarrow S_{10} = \frac{10}{2}(2(2) + (10-1)4) = 5(4 + 36) = 200$</td>
<td>[4]</td>
<td>[7]</td>
</tr>
<tr>
<td>or $2 + 6 + 10 + 14 + 18 + 22 + 26 + 30 + 34 + 38 = 200$</td>
<td></td>
<td>[7]</td>
</tr>
</tbody>
</table>

* Accept correct answer without work
* No back/retrospective marking

**Blunders (-3)**
B1 Mathematical error e.g. each missing or extra term in method II

**Attempts (3 marks)**
A1 Writes $a = 2$ in this part

**Worthless (0)**
W1 Treats as a geometric series but check for A1
The \(n^{\text{th}}\) term of a series is \(T_n = \frac{2}{3^n+1}\).

(i) Write, in terms of \(n\), an expression for \(T_{n-1}\), the \((n-1)^{\text{st}}\) term.

(ii) Prove that the series is geometric.

(iii) Show that \(S_9 = \frac{1}{3} - \frac{1}{3^9}\), where \(S_9\) is the sum of the first nine terms of the series.

* Answers to parts of question must be clearly identified otherwise order in paper

**Part (c)**

<table>
<thead>
<tr>
<th></th>
<th>15 (5, 5, 5) marks</th>
<th>Att (2, 2, 2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(c) (i)</td>
<td>5 marks</td>
<td></td>
</tr>
<tr>
<td>[ T_n = \frac{2}{3^{n+1}} \Rightarrow T_{n-1} = \frac{2}{3^{n-1}+1} = \frac{2}{3^n} ]</td>
<td>2 [ 5 ]</td>
<td></td>
</tr>
<tr>
<td>* Accept correct answer without work</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Blunders (-3)**
B1 Mathematical error

**Attempts (3 marks)**
A1 Any relevant work e.g. finds \(T_1\) and stops

| (c) (ii) | 5 marks | |
| \[ \frac{T_n}{T_{n-1}} = \frac{2}{3^{n+1}} \div \frac{2}{3^n} = \frac{2}{3^{n+1}} \times \frac{3^n}{2} = \frac{1}{3^n} \], a constant. Thus, the series is geometric. | 2 \[ 5 \] |
| * Accept other correct versions of proof |

**Blunders (-3)**
B1 Mathematical error
B2 Proves for specific terms only

**Attempts (2 marks)**
A1 Writes out the correct value of at least one term e.g. \(T_1\) (at this part)

**Worthless (0)**
W1 Treats as an arithmetic series but see A1
\[
\begin{align*}
\text{a} &= T_1 = \frac{2}{3^{1+1}} = \frac{2}{9} \quad \text{and/or} \quad r = \frac{1}{3} \\
S_9 &= \frac{\frac{2}{9} \left(1 - \frac{1}{3^9}\right)}{1 - \frac{1}{3}} = \frac{1}{3} \left(1 - \frac{1}{3^9}\right) = \frac{1}{3} - \frac{1}{3^{10}} \quad [5]
\end{align*}
\]

* No back/retrospective marking
* Accept proof by correct lists method

**Blunders (-3)**
B1  Mathematical error e.g. indices error

**Attempts (2 marks)**
A1  Any relevant work
Let \( h(x) = ax + b \), where \( x \in \mathbb{R} \).
Given that \( h(0) = 3 \) and \( h(2) = -5 \), find the value of \( a \) and the value of \( b \).

\[
\begin{align*}
\text{(a)} & \quad 10 \text{ marks} & \text{Att 3} \\
& \quad \text{Given that } h(0) = 3 \text{ and } h(2) = -5, \text{ find the value of } a \text{ and the value of } b. \\
\end{align*}
\]

\[
\begin{align*}
& h(x) = ax + b \\
& h(0) = a(0) + b = 3 \quad \Rightarrow \quad b = 3 \quad [7] \\
& h(2) = a(2) + 3 = -5 \quad \Rightarrow \quad 2a = -8 \quad \Rightarrow \quad a = -4 \quad [10] \\
\end{align*}
\]

* Accept correct answers for \( a \) and \( b \) without work.

**Blunders (-3)**

B1 Mathematical error e.g. confuses \( x \) and \( h(x) \), has \( h(3) = 0 \)

B2 Only finds one variable i.e. \( a \) or \( b \) – accept without work

**Misreading (-1)**

M1 Has \( h(2) = 3 \) and/or \( h(0) = -5 \)

**Attempts (3 marks)**

A1 Any relevant work
Part (b) 20 (10, 5, 5) marks

The diagram shows part of the graph of a function $f$.

Use the graph to estimate

(i) the values of $x$ for which $f(x) = 0$,
(ii) the values of $x$ for which $f'(x) = 0$, where $f'(x)$ is the derivative of $f(x)$,
(iii) the range of values of $x$ for which $f'(x) < 0$.

* Answers to parts of question must be clearly identified/labelled otherwise order in paper
* No back/retrospective marking

(b) (i) 10 marks
(b) (ii) 5 marks
(b) (iii) 5 marks

<table>
<thead>
<tr>
<th>(i)</th>
<th>(ii)</th>
<th>(iii)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x = -1.75, \quad x = 0, \quad x = 2.5$</td>
<td>$x = -1, \quad x = 1.5$</td>
<td>$-1 &lt; x &lt; 1.5$</td>
</tr>
</tbody>
</table>

* Answer on enclosed paper or copies graph: only, award full marks if answers clearly identified otherwise attempt marks at most.
* Accept candidates answer in part (ii) for part (iii) assuming part (ii) merited at least attempt marks

Blunders (-3)
B1 Each incorrect or missing value in (i) and (ii) apply a tolerance ±0.2

B2 Mathematical error e.g. in forming inequality in (iii)

Slips (-1)
S1 Includes equal sign in inequality once only

Attempts (2 or 3 marks)
A1 Some relevant work e.g. in (ii) mentions maximum and/or minimum or in (iii) mentions decreasing
A2 Some valid attempt at forming $f(x)$ to use in (ii)

Worthless (0)
W1 Copies diagram from paper and stops
Let \( g(x) = x(3x^2 - 9) \), where \( x \in \mathbb{R} \).

(i) Find \( g'(x) \), the derivative of \( g(x) \).

(ii) Find the co-ordinates of the local maximum point and of the local minimum point of the curve \( y = g(x) \).

(iii) Draw the graph of the function \( g'(x) \), the derivative of \( g(x) \), in the domain \(-2 \leq x \leq 2\).

\[ (c) \text{ (i)} \quad 5 \text{ marks} \quad \text{Att} 2 \]

<p>| | | | | |</p>
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<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>I</td>
<td>( g(x) = x(3x^2 - 9) = 3x^3 - 9x ) ( \Rightarrow ) ( g'(x) = 9x^2 - 9 )</td>
<td>[2]</td>
<td>[5]</td>
<td></td>
</tr>
<tr>
<td>or</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>II</td>
<td>( g(x) = x(3x^2 - 9) )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Let ( u(x) = x ) ( \Rightarrow ) ( u'(x) = 1 )</td>
<td>[2]</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>( v(x) = 3x^2 - 9 ) ( \Rightarrow ) ( v'(x) = 6x )</td>
<td>[2]</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>( g'(x) = (3x^2 - 9)(1) + (x)(6x) = 3x^2 - 9 + 6x^2 = 9x^2 - 9 )</td>
<td>[5]</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Blunders (-3)**

B1 Mathematical error e.g. multiplication method I

**Attempts (2 marks)**

A1 Some relevant work e.g. identifies \( u \) and /or \( v \) and stops

\[ (c) \text{ (ii)} \quad 10 \text{ marks} \quad \text{Att} 3 \]

<p>| | | | | |</p>
<table>
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</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( g'(x) = 0 ) ( \Rightarrow ) ( 9x^2 - 9 = 0 )</td>
<td>[3]</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \Rightarrow ) ( 9(x + 1)(x - 1) = 0 ) ( \Rightarrow ) ( x = -1 ) or ( x = 1 )</td>
<td>[4]</td>
<td>[7]</td>
<td></td>
</tr>
<tr>
<td>or</td>
<td>( x^2 - 1 = 0 ) ( \Rightarrow ) ( (x + 1)(x - 1) = 0 ) ( \Rightarrow ) ( x = -1 ) or ( x = 1 )</td>
<td>[4]</td>
<td>[7]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( g(-1) = -1(3 - 9) = 6 ) ( \Rightarrow ) ( (-1, 6) )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>( g(1) = 1(3 - 9) = -6 ) ( \Rightarrow ) ( (1, -6) )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>[9]</td>
<td>[10]</td>
</tr>
</tbody>
</table>

* Not required to identify local maximum and/or local minimum
* Only one point found [4] marks e.g. only uses \( x = 1 \)

**Blunders (-3)**

B1 Mathematical error e.g. fails to find \( g(x) \) value

**Attempts (3 marks)**

A1 Mentions local maximum/minimum occurs when \( g'(x) = 0 \) or mentions “turning points”

A2 Draws a graph of \( g(x) \) in this part – must have a domain of \(-2 \leq x \leq 2\)

A3 Reads from a graph only (no calculus used)
\[ g'(x) = 9x^2 - 9 \]
\[ g'(-2) = 9(-2)^2 - 9 = 27 \]
\[ g'(-1) = 9(-1)^2 - 9 = 0 \]
\[ g'(0) = 9(0) - 9 = -9 \]
\[ g'(1) = 9(1)^2 - 9 = 0 \]
\[ g'(2) = 9(2)^2 - 9 = 27 \]

* No penalty if domain extended

**Blunders (-3)**
B1 Mathematical error e.g. incomplete domain or plots incorrect function correctly e.g. \( g(x) \)

**Slips (-1)**
S1 Each missing or incorrectly plotted point to a maximum of 3 slips

**Attempts (2 marks)**
A1 Any relevant work e.g. sets up grid on graph paper
QUESTION 7

Part (a) 15 marks Att 5
Part (b) 20 (10, 10) marks Att (3, 3)
Part (c) 15 (5, 5, 5) marks Att (2, 2, 2)

Differentiate \( y = 6x - x^2 - 5x^4 \) with respect to \( x \).

(a) 15 marks Att 5

\[
y = 6x - x^2 - 5x^4 \Rightarrow \frac{dy}{dx} = 6 - 2x - 20x^3
\]

* Correct answer without work or notation: full marks.
* If done from first principles, ignore errors in procedure – just mark the answer.
* Only one non zero term correct, award 9 marks.

Blunders (-3)
B1 Mathematical error e.g. differentiation error once per term – includes sign

Attempts (5 marks)
A1 Any relevant step e.g. mentions \( \frac{dy}{dx} \) or \( f'(x) \)
A2 A correct coefficient or a correct index of \( x \) in one of the term(s)/part(s)
Part (b) 20 (10, 10) marks  Att (3, 3)

(i) Differentiate \( y = (3x^2 + 2)(x^3 - x) \) with respect to \( x \).

(ii) Given that \( y = (x^3 - 2x^2 + 4)^5 \), find the value of \( \frac{dy}{dx} \) when \( x = -1 \).

(b) (i) 10 marks  Att 3

\[
\begin{align*}
I & \quad y = (3x^2 + 2)(x^3 - x) \\
& \quad \text{Let } u = 3x^2 + 2 \quad \Rightarrow \quad \frac{du}{dx} = 6x \\
& \quad \text{Let } v = x^3 - x \quad \Rightarrow \quad \frac{dv}{dx} = 3x^2 - 1.
\end{align*}
\]

\[
\frac{dy}{dx} = \left(3x^2 + 2\right)(3x^2 - 1) + \left(x^3 - x\right)(6x) \quad \text{[10]}
\]

\[
= 6x^4 - 6x^2 + 9x^4 + 6x^2 - 3x^2 - 2 = 15x^4 - 3x^2 - 2
\]

or

\[
\begin{align*}
I & \quad y = (3x^2 + 2)(x^3 - x) = 3x^5 - 3x^3 + 2x^3 - 2x \quad \text{[4]}
\end{align*}
\]

\[
\Rightarrow y = 3x^5 - x^3 - 2x
\]

\[
\frac{dy}{dx} = 15x^4 - 3x^2 - 2 \quad \text{or} \quad \frac{dy}{dx} = 15x^4 - 3x^2 - 2 = 15x^4 - 9x^2 + 6x^2 - 2 \quad \text{[10]}
\]

* Check list of blunders re use of formulae

Blunders (-3)

B1 Differentiation errors, once per part
B2 Multiplication, method II, once only unless oversimplified. (Must contain two terms)

Attempts (3 marks)

A1 Any relevant work e.g. \( u \) and/or \( v \) correctly identified and stops
(b) (ii) 10 marks

I

\[ y = (x^2 - 2x^2 + 4)^5 \]

\[ \frac{dy}{dx} = 5(x^2 - 2x^2 + 4)^4(3x^2 - 4x) \quad [9] \]

\[ x = -1: \quad \frac{dy}{dx} = 5((-1)^3 - 2(-1)^2 + 4)^4(3(-1)^2 - 4(-1)) = 5(-1 - 2 + 4)^4(7) = 35 \quad [10] \]

or

\[ y = (x^3 - 2x^2 + 4)^5 \]

II

Let \( u = x^3 - 2x^2 + 4 \Rightarrow \frac{du}{dx} = 3x^2 - 4x \)

\[ y = u^5 \Rightarrow \frac{dy}{du} = 5u^4 \quad [4] \]

\[ \frac{dy}{dx} = 5u^4(3x^2 - 4x) = 5(x^3 - 2x^2 + 4)^4(3x^2 - 4x) \quad [9] \]

\[ x = -1: \quad \frac{dy}{dx} = 5((-1)^3 - 2(-1)^2 + 4)^4(3(-1)^2 - 4(-1)) = 5(-1 - 2 + 4)^4(7) = 35 \quad [10] \]

* Accept \( \frac{dy}{dx} = 5((-1)^3 - 2(-1)^2 + 4)^4(3(-1)^2 - 4(-1)) = 5(-1 - 2 + 4)^4(7) = 35 \) for full marks

**Blunder (-3)**
B1 Differentiation errors once per part

**Attempts (3 marks)**
A1 Any relevant work e.g. relevant differentiation

**Worthless (0)**
W1 Substitutes \( x = -1 \) into \( y \) and stops or evaluates \( y \)
A ball is thrown vertically down from the top of a high building. The distance, $s$ metres, the ball falls is given by 

$$s = 3t + 5t^2$$

where $t$ is the time in seconds from the instant the ball is thrown.

(i) Find the speed of the ball after 3 seconds.

(ii) Find the time $t$ when the ball is falling at a speed of $23\text{ ms}^{-1}$.

(iii) The ball hits the ground at a speed of $38\text{ ms}^{-1}$.

How high is the building?

* Units: Penalise as per guidelines
* No retrospective marking
* If parts of (c) are unlabelled, and the context doesn’t identify which part is which, assume the questions were answered in sequence from (c)(i) to (c)(iii)

\[
\frac{ds}{dt} = 3 + 10t = 3 + 30 = 33\text{ m s}^{-1}\text{ at } t = 3
\]

[2] [5]

Blunders (-3)
B1 Mathematical error e.g. differentiation error

Slip (-1)
S1 No units or incorrect units only applies if answer numerically correct e.g. 33 (no units)

Attempts (2 marks)
A1 Any correct relevant work e.g. $\frac{ds}{dt}$ or $\frac{dy}{dx}$ mentioned

Worthless (0)
W1 No differentiation e.g. $t = 3$ substituted into original equation
(c) (ii) 5 marks

\[ \frac{ds}{dt} = 3 + 10t = 23 \Rightarrow 10t = 20 \Rightarrow t = 2 \text{ seconds} \]

[2] [5]

* Must use \( \frac{ds}{dt} \) explicitly at this part

* Accept candidates \( \frac{ds}{dt} \) from (i) if involved differentiation

**Blunders (-3)**

B1 Mathematical error e.g. solving equation

**Slips (-1)**

S1 No units or incorrect units

**Attempts (2 marks)**

A1 Any relevant work

**Worthless (0)**

W1 No differentiation e.g. solves \( s = 23 \)

W2 \( \frac{d^2 s}{dt^2} = 10 \) only

---

(c) (iii) (5) marks

\[ \frac{ds}{dt} = 3 + 10t = 38 \Rightarrow 10t = 35 \Rightarrow t = 3.5 \]

\[ s = 3t + 5t^2 = 3(3.5) + 5(3.5)^2 = 10.5 + 61.25 = 71.75 \text{ m} \]

* Must use \( \frac{ds}{dt} \) explicitly at this part

* Accept candidates \( \frac{ds}{dt} \) from (i) if involved differentiation

**Blunders (-3)**

B1 Mathematical error

**Slips (-1)**

S1 No units or incorrect units

**Attempts (2 marks)**

A1 Any relevant work e.g. uses \( s \) as height.
**QUESTION 8**

<table>
<thead>
<tr>
<th>Part (a)</th>
<th>15 marks</th>
<th>Att 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Part (b)</td>
<td>20 (10, 5, 5) marks</td>
<td>Att (3, 2, 2)</td>
</tr>
<tr>
<td>Part (c)</td>
<td>15 (5, 5, 5) marks</td>
<td>Att (2, 2, 2)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Part (a)</th>
<th>15 marks</th>
<th>Att 5</th>
</tr>
</thead>
</table>

Let \( g(x) = k(1 - x) \), where \( x \in \mathbb{R} \).

Given that \( g(-5) = 20 \), find the value of \( k \).

(a) 15 marks  

\[
g(x) = k(1 - x) \\
g(-5) = k(1 + 5) = 20 \\
\Rightarrow 6k = 20 \\
\Rightarrow k = \frac{20}{6} = \frac{10}{3} \quad \text{or} \quad 3 \frac{1}{3} \quad \text{or equivalent}
\]

\begin{tabular}{lll}
\end{tabular}

* Accept correct answer without work.

**Blunder (-3)**

B1 Mathematical error e.g. solves \( g(20) = -5 \) or substitutes for \( k \) then solves correctly for \( x \)

**Attempts (5 marks)**

A1 Some relevant work e.g. multiplies out RHS and stops
Part (b) 20 (10, 5, 5) marks Att (3, 2, 2)

Let \( f(x) = \frac{5 + x^2}{2 - x} \), where \( x \in \mathbb{R} \) and \( x \neq 2 \).

(i) Find \( f(5) \).

(ii) Find \( f'(x) \), the derivative of \( f(x) \).

(iii) Show that \( f'(x) = 0 \) at \( x = -1 \).

(b) (i) 10 marks Att 3

\[
f(x) = \frac{5 + x^2}{2 - x} \Rightarrow f(5) = \frac{5 + 5^2}{2 - 5} = \frac{5 + 25}{2 - 5} = \frac{30}{-3} = -10
\]

<table>
<thead>
<tr>
<th>Blunders (-3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1 Mathematical error e.g. solves ( f(x) = 5 )</td>
</tr>
</tbody>
</table>

Attempts (3 marks)
A1 Some relevant work e.g. work at substitution

(b) (ii) 5 marks Att 2

\[
f(x) = \frac{5 + x^2}{2 - x} \quad \text{or} \quad f(x) = (5 + x^2)(2 - x)^{-1}
\]

Let \( u(x) = 5 + x^2 \Rightarrow u'(x) = 2x \)
Let \( v(x) = 2 - x \Rightarrow v'(x) = -1 \)

\[
f'(x) = \frac{(2 - x)(2x) - (5 + x^2)(-1)}{(2 - x)^2} = \frac{4x - 2x^2 + 5 + x^2}{(2 - x)^2} = \frac{-x^2 + 4x + 5}{(2 - x)^2}
\]

* See blunder list at start on use of calculus formulae

<table>
<thead>
<tr>
<th>Blunders (-3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1 Mathematical error</td>
</tr>
</tbody>
</table>

Attempts (2 marks)
A1 Some relevant work e.g. identifies \( u \) and/or \( v \) and stops
(b) (iii)  

\[ f'(x) = \frac{-x^2 + 4x + 5}{(2-x)^2} \]

\[ x = -1: \quad f'(-1) = \frac{-(-1)^2 + 4(-1) + 5}{(2-(-1))^2} = \frac{-1 - 4 + 5}{9} = 0 \]

* Accept candidates \( f''(x) \) from (ii), penalise error in simplification at this stage

**Blunders (-3)**

B1 Mathematical errors e.g. simplification of \( f'(x) \) in (ii) or incorrect substitution in \( f''(x) \)

**Slip (-1)**

S1 Incorrect or no conclusion if not proven

**Attempts (2 marks)**

A1 Some relevant work e.g. uses \( f(x) \) instead of \( f'(x) \) and substitutes \(-1\)

**Worthless (0)**

W1 No effort at substitution
Part (e) 15 (5, 5, 5) marks  Att (2, 2, 2)

Let \( h(x) = 5 + 3x - x^2 \), where \( x \in \mathbb{R} \).

(i) Find the co-ordinates of the point \( P \) at which the curve \( y = h(x) \) cuts the y-axis.

(ii) Find the equation of the tangent to the curve \( y = h(x) \) at \( P \).

(iii) The tangent to the curve \( y = h(x) \) at \( x = t \) is perpendicular to the tangent at \( P \).

Find the value of \( t \).

(c) (i) 5 marks  Att 2

Cuts y-axis when \( x = 0 \)  [2]

\[ h(x) = 5 + 3x - x^2 \Rightarrow h(0) = 5 + 3(0) - (0)^2 = 5 \Rightarrow P(0, 5) \]


* Accept (0, 5) without work

Blunders (-3)

B1 Mathematical error e.g. uses incorrect axis

Attempts (2 marks)

A1 Some relevant work e.g. states \( y = 5 \) without work

(c) (ii) 5 marks  Att 2

\[ h(x) = 5 + 3x - x^2 \Rightarrow h'(x) = 3 - 2x \Rightarrow h'(0) = 3 - 2(0) = 3 \]

Equation of tangent: \( y - 5 = 3(x - 0) \) or \( 3x - y + 5 = 0 \)

* Accept candidates value for \( P \) from (i) if not oversimplified - No retrospective marking

* No differentiation merits attempt marks at most

Blunders (-3)

B1 Mathematical error e.g. solves \( h'(x) = 0 \) and continues i.e. incorrect slope

Attempts (2 marks)

A1 Some relevant work e.g. connects slope of tangent with differentiation or equation of line formula with some correct substitution

(c) (iii) 5 marks  Att 2

\[ h(x) = 5 + 3x - x^2 \Rightarrow h'(x) = 3 - 2x \]

Slope of tangent at \( P = 3 \) \( \Rightarrow \) slope of perpendicular tangent = \( -\frac{1}{3} \)  [2]

\[ h'(t) = 3 - 2t = -\frac{1}{3} \Rightarrow -2t = -\frac{10}{3} \Rightarrow t = \frac{5}{3} \] or equivalent  [5]

* Accept candidate's value of slope from (ii) if not oversimplified

Blunders (-3)

B1 Mathematical error

Attempts (2 marks)

A1 Some relevant work e.g. mentions \( m_1 \times m_2 = -1 \)
Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate Examination, 2012

Mathematics
(Project Maths – Phase 1)

Paper 2

Ordinary Level

Monday 11 June     Morning 9:30 – 12:00

300 marks

Model Solutions – Paper 2

Note: the model solutions for each question are not intended to be exhaustive – there may be other correct solutions. Any examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his / her advising examiner.
Instructions

There are three sections in this examination paper:

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Marks</th>
<th>Questions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Section A</td>
<td>Concepts and Skills</td>
<td>125</td>
<td>5</td>
</tr>
<tr>
<td>Section B</td>
<td>Contexts and Applications</td>
<td>125</td>
<td>2</td>
</tr>
<tr>
<td>Section C</td>
<td>Area and Volume (old syllabus)</td>
<td>50</td>
<td>1</td>
</tr>
</tbody>
</table>

Answer all eight questions, as follows:

In Section A, answer:

Questions 1 to 4 and

either Question 5A or Question 5B.

In Section B, answer Questions 6 and 7.

In Section C, answer Question 8.

Write your answers in the spaces provided in this booklet. You will lose marks if you do not do so. There is space for extra work on the back cover of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the Formulae and Tables booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

Marks will be lost if all necessary work is not clearly shown.

Answers should include the appropriate units of measurement, where relevant.

Answers should be given in simplest form, where relevant.

Write the make and model of your calculator(s) here:
Question 1 (25 marks)

Peter and Niamh go to a large school. One morning, they arrive early. While they are waiting, they decide to guess whether each of the next three students to come in the door will be a boy or a girl.

(a) Write out the sample space showing all the possible outcomes. For example, BGG is one outcome, representing Boy, Girl, Girl.

BBB, BBG, BGB, GBB, BGG, GBG, GGB, GGG

(b) Peter says these outcomes are equally likely. Niamh says they are not. What do you need to know about the students in the school to decide which of them is correct?

The number of boys and the number of girls in the school.

(c) If all the outcomes are equally likely, what is the probability that the three students will be two girls followed by a boy?

\[ P(\text{GGB}) = \frac{1}{8} \text{ or } 0.125 \text{ or } 12.5\% \]

(d) Niamh guesses that there will be at least one girl among the next three students. Peter guesses that the next three students will be either three boys or two boys and a girl. Who is more likely to be correct, assuming all outcomes are equally likely? Justify your answer.

\[ P(\text{at least one girl}) = \frac{7}{8} \text{ or } 0.875 \text{ or } 87.5\% \]
\[ P(\text{three boys or two boys and a girl}) = \frac{4}{8} \text{ or } \frac{1}{2} \text{ or } 0.5 \text{ or } 50\% \]

Niamh is more likely to be correct because of the greater probability.
Question 2  

(a) In the Venn diagram below, the universal set is a normal deck of 52 playing cards. The two sets shown represent clubs and picture cards (kings, queens and jacks).

Show on the diagram the number of elements in each region.

(b) (i) A card is drawn from a pack of 52 cards. Find the probability that the card drawn is the king of clubs.

\[ P(\text{king of clubs}) = \frac{1}{52} \]

(ii) A card is drawn from a pack of 52 cards. Find the probability that the card drawn is a club or a picture card.

\[ P(\text{club or picture card}) = \frac{26}{52} = \frac{11}{26} \]

(iii) Two cards are drawn from a pack of 52 cards. Find the probability that neither of them is a club or a picture card. Give your answer correct to two decimal places.

\[ P(\text{not club or picture card}) = \frac{10}{52} \times \frac{26}{51} \approx 0.33 \]
Question 3  

$A(6, -1), B(12, -3), C(8, 5)$ and $D(2, 7)$ are four points.

(a) Plot the four points on the diagram below.

(b) Describe two different ways of showing, using co-ordinate geometry techniques, that the points form a parallelogram $ABCD$.

Any TWO of:

- Show that opposite sides are parallel by showing the slopes of opposite lines are equal.
- Show that the diagonals bisect each other by showing the midpoint of $[AC]$ equals the midpoint of $[DB]$.
- Show that the opposite sides are equal in length using the length formula.
- Show that $\overline{AB}$ maps $D$ onto $C$ or similar.
(c) Use one of the ways you have described to show that $ABCD$ is a parallelogram.

Slope $AB = \frac{3+1}{12-6} = -\frac{2}{6}$, Slope $DC = \frac{2+4}{2-8} = -\frac{2}{6} \implies AB \parallel DC$

Slope $BC = \frac{5+1}{8-12} = -2$, Slope $AD = \frac{7+1}{2-6} = -2 \implies BC \parallel AD$

Hence, $ABCD$ a parallelogram

or

Midpoint $[AC] = (\frac{6+4}{2}, -\frac{1+5}{2}) = (7,2)$, Midpoint $[BD] = (\frac{12+2}{2}, -\frac{3+7}{2}) = (7,2)$,

$\implies$ Diagonals bisect. Hence, $ABCD$ a parallelogram

or

Length $[AB] = \sqrt{(12 - 6)^2 + (-3 + 1)^2} = \sqrt{40}$

Length $[DC] = \sqrt{(2 - 8)^2 + (7 - 5)^2} = \sqrt{40}$

Length $[AD] = \sqrt{(6 - 2)^2 + (-1 - 7)^2} = \sqrt{80}$

Length $[BC] = \sqrt{(12 - 8)^2 + (-3 - 5)^2} = \sqrt{80}$

$\implies$ Opposite sides are equal. Hence, $ABCD$ a parallelogram

or

$A(6, -1) \rightarrow B(12, -3) \text{ maps } D(2, 7) \rightarrow (2 + 6, 7 - 2) = C(8, 5) \text{ or similar}$

$\implies \overrightarrow{AB} = \overrightarrow{DC}$ . Hence, $ABCD$ a parallelogram

---

**Question 4**  
(25 marks)

The diagram shows two circles $c_1$ and $c_2$ of equal radius.  
$c_1$ has centre $(0, 0)$ and it cuts the $x$-axis at $(5, 0)$.

(a) Find the equation of $c_1$.

$$x^2 + y^2 = 5^2 = 25$$

(b) Show that the point $P(-3, 4)$ is on $c_1$.

$$x^2 + y^2 = (-3)^2 + 4^2 = 9 + 16 = 25 = r^2$$
(c) The two circles touch at $P(-3, 4)$.  
$P$ is on the line joining the two centres.  
Find the equation of $c_2$.

\[
(0, 0) \rightarrow (-3, 4) \text{ maps } (-3, 4) \rightarrow (-6, 8)
\]

\[
c_2 : (x + 6)^2 + (y - 8)^2 = 25
\]

(d) Find the equation of the common tangent at $P$.

Slope of line of centres: \[
\frac{8 - 0}{-6 - 0} = -\frac{4}{3}
\]

Perpendicular slope, the slope of the tangent: \[
\frac{3}{4}
\]

Equation of tangent: \[
y - 4 = \frac{3}{4}(x + 3) \implies 3x - 4y + 25 = 0
\]
Question 5A

(a) (i) Write down a geometrical result that can be used to construct a tangent to a circle at a point.

The tangent is perpendicular to the radius at the point of contact

(ii) On the diagram shown, construct the tangent to the circle at $A$.

(b) Construct the circumcentre and circumcircle of the triangle below, using only a straight edge and compass. Show all construction marks clearly.
OR

Question 5B

$ABCD$ is a parallelogram.

The points $A$, $B$ and $C$ lie on the circle which cuts $[AD]$ at $P$.
The line $CP$ meets the line $BA$ at $Q$.

Prove that $|CD| = |CP|$.

\[
\begin{align*}
|\angle ABC| &= |\angle CDP|, \text{ opposite angles in a parallelogram} \\
|\angle ABC| + |\angle CPA| &= 180^\circ, \text{ opposite angles in cyclic quadrilateral} \\
|\angle DPC| + |\angle CPA| &= 180^\circ \\
\text{Hence, } |\angle ABC| &= |\angle DPC| \\
\text{Hence, } |\angle CDP| &= |\angle DPC| \Rightarrow \text{isosceles triangle or } |CD| = |CP|
\end{align*}
\]
Answer Question 6 and Question 7.

Question 6  (75 marks)

The following table gives data on new private cars sold in Ireland in each quarter of each year from 2006 to 2011.

New private cars sales

<table>
<thead>
<tr>
<th>Year</th>
<th>January to March</th>
<th>April to June</th>
<th>July to Sept.</th>
<th>October to Dec.</th>
<th>Annual Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Petrol</td>
</tr>
<tr>
<td>2006</td>
<td>75 769</td>
<td>54 572</td>
<td>32 873</td>
<td>10 059</td>
<td>173 273</td>
</tr>
<tr>
<td>2007</td>
<td>81 750</td>
<td>57 124</td>
<td>32 418</td>
<td>9 462</td>
<td>180 754</td>
</tr>
<tr>
<td>2008</td>
<td>77 441</td>
<td>37 128</td>
<td>27 361</td>
<td>4 540</td>
<td>146 470</td>
</tr>
<tr>
<td>2009</td>
<td>27 140</td>
<td>15 225</td>
<td>9 049</td>
<td>3 018</td>
<td>54 432</td>
</tr>
<tr>
<td>2010</td>
<td>34 555</td>
<td>26 806</td>
<td>17 011</td>
<td>6 535</td>
<td>84 907</td>
</tr>
<tr>
<td>2011</td>
<td>39 484</td>
<td>29 770</td>
<td>13 467</td>
<td>4 211</td>
<td>86 932</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Engine type of cars sold</th>
<th>Petrol</th>
<th>Diesel</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
<td>Annual Total</td>
<td>128 634</td>
<td>44 010</td>
<td>629</td>
</tr>
<tr>
<td>2006</td>
<td>128 346</td>
<td>50 560</td>
<td>1 848</td>
</tr>
<tr>
<td>2007</td>
<td>92 298</td>
<td>50 283</td>
<td>3 889</td>
</tr>
<tr>
<td>2008</td>
<td>22 802</td>
<td>30 645</td>
<td>985</td>
</tr>
<tr>
<td>2009</td>
<td>27 124</td>
<td>53 998</td>
<td>3 785</td>
</tr>
<tr>
<td>2010</td>
<td>23 246</td>
<td>61 730</td>
<td>1 956</td>
</tr>
</tbody>
</table>


(a)  (i) Show the annual total sales of cars over the six years, using a suitable chart.

(ii) Find the mean number of cars sold per year over the six years.

\[
\text{Mean } = \frac{1}{6} \left( 173273 + 180754 + 146470 + 54432 + 84907 + 86932 \right) = \frac{1}{6} \left( 726768 \right) = 121128
\]
(iii) Calculate the percentage increase in annual car sales between 2009 and 2011.

\[
\text{Increase} = 86\,932 - 54\,432 = 32\,500 \Rightarrow \%\text{ increase} = \frac{32500}{54432} \times 100 = 59.71\% 
\]

(iv) Aoife says that this increase shows car sales are currently going well. Paul says that car sales are currently going badly. He says that sales have fallen by 52% since 2007 and that they are well below average. Complete the sentences below to give a criticism of each argument.

Aoife’s argument does not recognise that this increase is from a very low base, and that sales had been much better before 2009.

Paul’s argument does not recognise that, although sales are much lower now than in 2007, they have recovered a lot since their lowest point in 2009.

(v) Give a more balanced description of the pattern of car sales over the six years.

Sales fell dramatically from 2007 to 2009; they recovered a lot since then, but are still much lower than they were at the start.

(b) (i) Describe how the sales of the cars are distributed over the four quarters of each year.

Highest quarterly sales are in the first quarter and decrease significantly in each subsequent quarter.

(ii) Suggest a reason for this pattern of sales.

People like to buy new cars early in the year so that they have a new year number plate.

(iii) The sales for the first quarter of 2012 are 36,081. Find, with justification, an estimate for the total annual sales for 2012.

First quarter sales in 2012 are about one-third of the range between those quarters in 2010 and 2011. Assuming annual sales will retain that proportion would give an estimate of 85,500 for annual sales.
(c) (i) Two pie charts are being used to show the change from 2006 to 2011 in the popularity of petrol and diesel cars. Complete the second pie chart.

\[
\text{Diesel} = \frac{61730}{86932} \times 360 = 256^\circ \\
\text{Petrol} = \frac{23246}{86932} \times 360 = 96^\circ \\
\text{Other} = \frac{1956}{86932} \times 360 = 8^\circ 
\]

(ii) Which of the following statements best describes the change over time in the popularity of diesel cars as a percentage of the total?

A. Diesel cars have suddenly become very popular in the last year or two.
B. Diesel cars have increased very steadily in popularity over the last six years.
C. Diesel cars have become very popular since car sales started to improve.
D. Diesel cars got more popular each year, with an especially big increase in 2009.
E. Diesel cars became popular as car sales fell but have been getting less popular as they rise again.

Write the letter corresponding to the correct answer in the box.  

D
A survey of some of the most popular models of private cars sold in 2011 examined the CO₂ emissions in g/km from diesel engines and petrol engines. The data are as follows:

<table>
<thead>
<tr>
<th>Diesel engines</th>
<th>Petrol engines</th>
</tr>
</thead>
</table>

(i) Construct a back-to-back stem-and-leaf plot of the above data.

(ii) Does the information suggest that diesel engines produce lower CO₂ emissions than petrol engines? In your answer you should refer to the stem-and-leaf plot and to an appropriate measure of central tendency.

Yes. The diesel engines grouped at the top of the plot have a smaller median /mean value.

(iii) Does the information suggest that there is a greater variation in the CO₂ emissions of diesel engines than petrol engines? In your answer you should refer to the stem-and-leaf plot and an appropriate measure of variability.

No. The emissions for the petrol engines are more spread out than for the diesel ones. The range [/interquartile range /standard deviation] for the petrol engines is greater than that for the diesel engines.
Question 7

The planned supports for the roof of a building form scalene triangles of different sizes.

(a) Explain what is meant by a **scalene triangle**.

A triangle in which the three sides have different lengths

The triangle $EFG$ is the image of the triangle $CDE$ under an enlargement and the triangle $CDE$ is the image of the triangle $ABC$ under the same enlargement.

(b) Find the length of $[FG]$.

Scale factor $= \frac{7.2}{8} = 1.25$

$|DE| = 1.25 \cdot |BC| = 1.25 \cdot 8 = 10 \quad \Rightarrow \quad |FG| = 1.25 \cdot |DE| = 1.25 \cdot 10 = 12.5 \text{ m}$

(c) Find the length of $[BD]$, correct to three decimal places.

$|BD|^2 = 8^2 + 9^2 - (2)(8)(9) \cos 60^\circ = 64 + 81 - 72 = 73$

$\Rightarrow \quad |BD| = \sqrt{73} = 8.544 \text{ m}$
(d) The centre of the enlargement is $O$. Find the distance from $O$ to the point $B$. 

\[
\frac{|OD|}{|OB|} = \frac{x + 8.544}{x} = 1.25
\]

$\Rightarrow x + 8.544 = 1.25x$

$\Rightarrow 0.25x = 8.544$

$\Rightarrow x = 34.176 \text{ m}$
A condition of the planning is that the height of the point $G$ above the horizontal line $BF$ cannot exceed 11.6 m.

Does the plan meet this condition? Justify your answer by calculation.

\[
| \angle GFH | = \alpha = | \angle CBD |
\]

In triangle $CBD$:

\[
\sin \alpha = \frac{\sin 60^\circ}{9} = \frac{8.544}{8.544} \Rightarrow \sin \alpha = \frac{9\sin 60^\circ}{8.544}
\]

In triangle $GFH$:

\[
\sin \alpha = \frac{h}{12.5} = \frac{9\sin 60^\circ}{8.544}
\]

\[
\Rightarrow h = \frac{12.5 \times 9\sin 60^\circ}{8.544} = 11.4 < 11.6
\]

Yes, the plan meets the condition.
Section C  Area and Volume (old syllabus)  50 marks

Answer Question 8 from this section.

**Question 8**

(a) The diagram shows a circle inscribed in a square. The area of the square is 16 cm².

(i) Find the radius length of the circle.

\[ l^2 = 16 \Rightarrow l = 4 \Rightarrow \text{radius} = 2 \text{ cm} \]

(ii) Find the area of the shaded region, in cm², correct to one decimal place.

Shaded area: \[ 16 - \pi(2)^2 = 16 - 12.566 = 3.433 = 3.4 \text{ cm}^2 \]
(b) In order to estimate the area of the irregular shape shown below, a horizontal line was drawn across the widest part of the shape and five offsets (perpendicular lines) were drawn at equal intervals along this line.

(i) Find the lengths of the horizontal line and the offsets, taking each grid unit as 5 mm, and record the lengths on the diagram.

(ii) Use Simpson’s rule to estimate the area of the shape.

\[
A = \frac{20}{3} \left(0 + 0 + 2(45 + 40) + 4(35 + 60 + 40)\right)
\]
\[
= \frac{20}{3} (0 + 170 + 540)
\]
\[
= \frac{20}{3} (710)
\]
\[
= 4733\frac{1}{3} \text{ mm}^2
\]
(c) A solid wax candle is in the shape of a cylinder with a cone on top, as shown in the diagram.

The diameter of the base of the cylinder is 3 cm and the height of the cylinder is 8 cm.

The volume of the wax in the candle is $21\pi \text{ cm}^3$.

(i) Find the height of the candle.

\[
\text{Volume of cylinder} = \pi (1.5)^2 \times 8 = 18\pi \\
\text{Volume of cone} = 21\pi - 18\pi = 3\pi \\
\frac{1}{3} \pi r^2 h = 3\pi \Rightarrow \frac{1}{3} \pi (1.5)^2 h = 3\pi \Rightarrow h = \frac{9}{2.25} = 4 \text{ cm}
\]

Height of candle: $8 + 4 = 12 \text{ cm}$

(ii) Nine of these candles fit into a rectangular box. The base of the box is a square. Find the volume of the smallest rectangular box that the candles will fit into.

Square base $\Rightarrow$ 3 candles wide $\times$ 3 candles deep.
Dimensions of base $= 3(3) \times 3(3)$.
Area of base of box: $9 \times 9 = 81 \text{ cm}^2$

Height of box: 12 cm
Volume: $81 \times 12 = 972 \text{ cm}^3$
Marking Scheme – Paper 2, Section A, Section B and Section C

Structure of the marking scheme

Candidate responses are marked according to different scales, depending on the types of response anticipated. Scales labelled A divide candidate responses into two categories (correct and incorrect). Scales labelled B divide responses into three categories (correct, partially correct, and incorrect), and so on. The scales and the marks that they generate are summarised in this table:

<table>
<thead>
<tr>
<th>Scale label</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>No of categories</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>5 mark scale</td>
<td>0, 5</td>
<td>0, 2, 5</td>
<td>0, 2, 4, 5</td>
<td></td>
</tr>
<tr>
<td>10 mark scale</td>
<td>0, 5, 10</td>
<td>0, 4, 7, 10</td>
<td>0, 2, 5, 8, 10</td>
<td></td>
</tr>
<tr>
<td>15 mark scale</td>
<td>0, 5, 10, 15</td>
<td>0, 4, 7, 11, 15</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20 mark scale</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>25 mark scale</td>
<td></td>
<td></td>
<td>0, 8, 17, 25</td>
<td></td>
</tr>
</tbody>
</table>

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the scheme, where necessary.

Marking scales – level descriptors

A-scales (two categories)
- incorrect response (no credit)
- correct response (full credit)

B-scales (three categories)
- response of no substantial merit (no credit)
- partially correct response (partial credit)
- correct response (full credit)

C-scales (four categories)
- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

D-scales (five categories)
- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- response about half-right (middle partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

In certain cases, typically involving incorrect rounding or omission of units, a mark that is one mark below the full-credit mark may also be awarded. Such cases are flagged with an asterisk. Thus, for example, scale 10C* indicates that 9 marks may be awarded.
Summary of mark allocations and scales to be applied

### Section A

<table>
<thead>
<tr>
<th>Question 1</th>
<th>(a)</th>
<th>10C</th>
</tr>
</thead>
<tbody>
<tr>
<td>(b)</td>
<td>5B</td>
<td></td>
</tr>
<tr>
<td>(c)</td>
<td>5B</td>
<td></td>
</tr>
<tr>
<td>(d)</td>
<td>5C</td>
<td></td>
</tr>
</tbody>
</table>

<table>
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<tr>
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<td>(d)</td>
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| Question 5A | (a) (i) | 5B   |
|             | (a) (ii)| 5B   |
|             | (b)     | 15D  |

| Question 5B | 25C |

### Section B

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### Section C

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Detailed marking notes

Section A

Question 1

(a) Scale 10C (0, 4, 7, 10)
   Low partial credit:
   ▪ Any work of merit e.g. one correct outcome other than BGG.

   High partial credit:
   ▪ An almost correct response such as one/two outcomes missing or extra outcomes.

(b) Scale 5B (0, 2, 5)
   Partial credit:
   ▪ Any work of merit e.g. the number of boys or the number of girls.

(c) Scale 5B (0, 2, 5)
   Partial credit:
   ▪ Any work of merit e.g. correct numerator or denominator in fraction format
     e.g. \( P(2 \text{ girls and a boy}) = \frac{3}{8} \)

   Full credit:
   ▪ Correct answer without work shown.

(d) Scale 5C (0, 2, 4, 5)
   Low partial credit:
   ▪ Any work of merit e.g. outcomes for Niamh or Peter given or a correct answer with no
     work shown.

   High partial credit:
   ▪ An almost correct response such as the outcomes for both Niamh and Peter given.
   ▪ Answer with one element missing.
Question 2

(a) Scale 5C (0, 2, 4, 5)

*Low partial credit:*

- Any work of merit such as one correct element.

*High partial credit:*

- An almost correct response such as two or three correct elements.

(b)(i) Scale 10B (0, 5, 10)

*Partial credit:*

- Any work of merit such as correct or consistent numerator or denominator in fraction format.

*Full credit:*

- Correct answer without work shown.

(b)(ii) Scale 5C (0, 2, 4, 5)

*Low partial credit:*

- Any work of merit such as a correct or consistent partial probability e.g. $\frac{10}{52}$.

*High partial credit:*

- Almost correct or consistent response e.g. $\frac{13}{52} + \frac{9}{52}$ and stops.

*Full credit:*

- Correct answer without work shown.

(b)(iii) Scale 5C* (0, 2, 4, 5)

*Low partial credit:*

- Any work of merit such as a correct or consistent numerator or denominator in fraction format.

*High partial credit:*

- Almost correct or consistent response e.g. $\frac{30}{52} \times \frac{20}{51}$ and stops.

*Full credit:*

- Correct answer without work shown.
Question 3

(a) Scale 15C (0, 5, 10, 15)

*Low partial credit:*
- Any work of merit such as showing knowledge of plotting a point.

*High partial credit:*
- An almost correct response such as two or three points plotted correctly or a consistent error in plotting all four points.

(b) Scale 5B (0, 2, 5)

*Partial credit:*
- Any work of merit e.g. incomplete statement(s) with some merit.

(c) Scale 5C (0, 2, 4, 5)

*Low partial credit:*
- Any work of merit such as identification of a relevant formula.

*High partial credit:*
- Almost correct response such as one pair of lines shown as parallel or a correct response using a method not described in (b).
Question 4

(a) Scale 10C (0, 4, 7, 10)

Low partial credit:
- Any work of merit such as identification of a relevant formula.

High partial credit:
- An almost correct response e.g. mishandles $r^2$.

(b) Scale 5C (0, 2, 4, 5)

Low partial credit:
- Any work of merit such as a correct substitution.

High partial credit:
- Almost correct response such as required conclusion not given.

(c) Scale 5C (0, 2, 4, 5)

Low partial credit:
- Any work of merit e.g. use of translation indicated.

High partial credit:
- Almost correct response such as correct centre and radius indicated.
- Correct answer without work shown.

(d) Scale 5C (0, 2, 4, 5)

Low partial credit:
- Any work of merit such as identification of a relevant formula.

High partial credit:
- An almost correct response such as slope of tangent found correctly but fails to finish.
- Correct answer without work shown.
Question 5A

(a)(i) Scale 5B (0, 2, 5)
Partial credit:
- Any work of merit such as an incomplete statement with radius or diameter or point of contact mentioned.

(a)(ii) Scale 5B (0, 2, 5)
Partial credit:
- Any work of merit e.g. any incorrect line through the point of contact.

(b) Scale 15D (0, 4, 7, 11, 15)
Low partial credit:
- Any work of merit such as an arc drawn from any vertex of the triangle.
- Midpoint of a triangle side indicated or finds the incentre or centroid.

Middle partial credit:
- Further work of merit such as one perpendicular bisector of the triangle side constructed correctly.

High partial credit:
- An almost correct response where the circumcentre is established correctly but no circle drawn or both mediators and circle drawn correctly but no evidence of construction.

Question 5B

Scale 25C (0, 8, 17, 25).
Low partial credit:
- Any one correct step.

High partial credit:
- One or two steps missing or incorrect.
Section B

Question 6

(a)(i) Scale 10C  (0, 4, 7, 10)
Low partial credit:
- Any work of merit such as a rectangle for one year represented correctly or a correct scaled axis shown.

High partial credit:
- An almost correct response such as one or two incorrect rectangles or scales omitted.

(a)(ii) Scale 10C  (0, 4, 7, 10).
Low partial credit:
- Any work of merit e.g. at least two correct elements identified.

High partial credit:
An almost correct response such as formula for mean with fully correct substitution.

Full credit:
- 121 128 without work.

(a)(iii) Scale 5C  (0, 2, 4, 5)
Low partial credit:
- Any work of merit e.g. a correct relevant figure identified 86 932 or 54 432.

High partial credit:
- An almost correct response such as one step missing.

Full credit:
- Correct answer without work shown.

(a)(iv) Scale 5B  (0, 2, 5)
Partial credit:
- Any work of merit which includes any mention of the year 2009, a decline in sales or an increase in sales.

(a)(v) Scale 5B  (0, 2, 5)
Partial credit:
- Any work of merit which includes any mention of an increase of sales in year 2007, a decline in sales in years 2008/2009 or an increase in sales in years 2010/2011.

(b)(i) Scale 5B  (0, 2, 5)
Partial credit:
- Any work of merit such as a correct relevant observation that is incomplete.

(b)(ii) Scale 5A  (0, 5)
No credit:
- No reason given.
(b)(iii) Scale 5B (0, 2, 5)
Partial credit:
- Any work of merit such as an estimate within the range of values 72 000 to 90 000 without justification or any work with 36 081.

(c)(i) Scale 5C (0, 2, 4, 5)
Low partial credit:
- Any work of merit such as finding any or all of the correct percentages or fractions.

High partial credit:
- An almost correct response such as all angles indicated but drawn incorrectly or not drawn.
- Correct answer without work shown

(c)(ii) Scale 5B (0, 2, 5)
Partial credit:
- Selects B or C.

(d)(i) Scale 5C (0, 2, 4, 5)
Low partial credit:
- Any work of merit e.g. a correct element of stem-leaf plot indicated.

High partial credit:
- An almost complete response such as an incomplete plot with one/two elements missing or incorrect or no key indicated.

(d)(ii) Scale 5C (0, 2, 4, 5)
Low partial credit:
- Any work of merit such as an answer with no reference to stem-leaf or central tendency.

High partial credit:
- An almost complete response such as reference to median/mean but no conclusion.

(d)(iii) Scale 5C (0, 2, 4, 5).
Low partial credit:
- Any work of merit such as an answer with no reference to stem-leaf or measure of variability.

High partial credit:
- An almost complete response such as reference to range or measurement of variability but no conclusion.
Question 7

(a) Scale 5B* (0, 2, 5).

*Partial credit:*

- Any work of merit such as any triangle drawn with no measurement of sides indicated or defines an isosceles triangle.

*Full credit:*

- A triangle drawn with three different lengths indicated.

(b) Scale 15D* (0, 4, 7, 11, 15).

*Low partial credit:*

- Any work of merit e.g. an indication that \([FG]\) is an enlargement of \([BC]\) or \([ED]\) or work towards scale factor.

*Middle partial credit:*

- Further work of merit such as scale factor found.

*High partial credit:*

- An almost complete response such as scale factor applied to \([BC]\) to get \([ED]\) correctly or fails to finish e.g. leaves \([FG]\) as \(1 \cdot 25^2 \times 8\) or \(1 \cdot 25 \times 10\).

*Full credit:*

- Correct answer without work shown.

(c) Scale 15D* (0, 4, 7, 11, 15)

*Low partial credit:*

- Any work of merit such as identifying the cosine rule as a method of solution.

*Middle partial credit:*

- Further work of merit such as significant correct substitution into the cosine formula.

*High partial credit:*

- An almost complete response such as cosine rule worked to \(\sqrt{73}\).
- Correct answer without work shown.

(d) Scale 5B* (0, 2, 5)

*Partial credit:*

- Any work of merit such as an effort at finding or explaining the centre of enlargement or an effort at the use of the scale factor to find the centre of enlargement.
- Correct answer without work shown.

(e) Scale 10D (0, 2, 5, 8, 10)

*Low partial credit:*

- Any work of merit e.g. vertical height indicated (from point \(G\) or in triangles \(CBD, EDF\)) or correct relevant formula identified or \(|FG|=12 \cdot 5\).
- Correct answer without work shown.

*Middle partial credit:*

- Further work of merit such as \(\alpha\) or sine \(\alpha\) found.

*High partial credit:*

- An almost complete response whereby \(h\) is found but no conclusion is given.
Section C

Question 8

(a)(i) Scale 5B* (0, 2, 5)

*Partial credit:*
- Any work of merit e.g. leaves diameter = 4 or works with \( l = \frac{16}{4} \).

*Full credit:*
- Correct answer without work shown.

(a)(ii) Scale 5B* (0, 2, 5)

*Partial credit:*
- Any work of merit such as identification of \( r \) in this part.

*Full credit:*
- Correct answer without work shown.

(b)(i) Scale 10C* (0, 4, 7, 10)

*Low partial credit:*
- Any work of merit such as one or two correct measurements shown.
- If (b) (i) not done but at least one correct measurement is shown in (b) (ii).

*High partial credit:*
- An almost correct response such as one or two incorrect measurements.
- If (b) (i) not done but all correct measurements are shown in (b) (ii).

(b)(ii) Scale 15D* (0, 4, 7, 11, 15)

*Low partial credit:*
- Any work of merit such as a correct substitution of \( h \), \( F + L \) or TOFE.
- Correct answer without work shown.

*Middle partial credit:*
- Further work of merit such as two correct substitutions of \( h \), \( F + L \) or TOFE.
- One correct substitution followed by some correct calculation.

*High partial credit:*
- An almost correct response such as fully correct substitution followed by an error in calculation.
- Two correct substitutions followed by fully correct calculation.
(c)(i) Scale 10D* (0, 2, 5, 8, 10)

Low partial credit:
▪ Any work of merit such as \( r \) identified as \( 1.5 \).

Middle partial credit:
▪ Further work of merit e.g. correct substitution for \( r \) and \( h \) or volume of cylinder or cone correct.

High partial credit:
▪ An almost correct response such as \( h \) isolated.

(c)(ii) Scale 5B* (0, 2, 5)

Partial credit:
▪ Any work of merit e.g. finds base or height of the box.
MARCANNA BREISE AS UCHT FREGAIRT TRÍ GHAELGE

(Bonus marks for answering through Irish)

Ba chóirmarcanna de réiranghnáthráta a bhronadh ariarrthóiríngnóthaimníosmóná 75% d’iomlánnamarcanna don pháipéar. Ba chóirfreisin marc bónais sin a shlánú síos.

Déantarancinneadhagus an riomhaireacht faoin marc bónais i gcásgach páipéir ar leithligh.

Is é 5% an gnáthráta agus is é 300 iomlánnamarcanna don pháipéar. Mar sin, bainúsáid as an gnáthráta 5% i gcásiarrthóirí a ghnóthaíonn 225 marc nónioslú, e.g. 198 marc \(\times 5\% = 9.9\) \(\Rightarrow\) bónas = 9 marc.

Mághnóthaíonn an t-iarrthóirníosmóná 225 marc, riomhtar an bónas de réirnafoirmle \([300 – \text{bunmharc}] \times 15\%\), agus marc bónais sin a shlánú síos. In ionadan riomhaireacht sin a dhéanamh, is féidir úsáid a bhaint as an táblathíos.

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