LEAVING CERTIFICATE EXAMINATION, 2012

MATHEMATICS — ORDINARY LEVEL

PAPER 1 (300 marks)

FRIDAY, 8 JUNE – AFTERNOON, 2:00 to 4:30

Attempt SIX QUESTIONS (50 marks each).

WARNING: Marks will be lost if all necessary work is not clearly shown.

Answers should include the appropriate units of measurement, where relevant.
1. (a) When Katie had travelled 140 km, she had completed \( \frac{4}{9} \) of her journey. Find the length of her journey.

(b) Robert’s electricity bill gave the following data:

<table>
<thead>
<tr>
<th>Unit type</th>
<th>Present reading</th>
<th>Previous reading</th>
<th>Unit price</th>
</tr>
</thead>
<tbody>
<tr>
<td>Day rate</td>
<td>35 087</td>
<td>34 537</td>
<td>€0.1506</td>
</tr>
<tr>
<td>Night rate</td>
<td>17 213</td>
<td>16 853</td>
<td>€0.0745</td>
</tr>
</tbody>
</table>

(i) Calculate the total cost of the units used.

Robert also pays a standing charge of €24.89 and a levy of €5.46. VAT at the rate of 13.5% is charged on all amounts.

(ii) Calculate the total amount of Robert’s electricity bill.

(c) A retailer bought 40 toys at €24.75 each. He sold 10 of the toys at €33.88 each and sold the remaining 30 toys at a reduced price. His total sales amounted to €1270.

(i) Write his total profit on the transaction as a percentage of his cost. Give your answer correct to one decimal place.

(ii) Find the reduced selling price of each of the remaining 30 toys.

2. (a) Solve for \( x \) and \( y \)

\[
\begin{align*}
  x - y &= 4 \\
  2x + y &= 5.
\end{align*}
\]

(b) Let \( f(x) = x^3 + 2x^2 - x - 2 \).

(i) Show, by division, that \( x - 1 \) is a factor of \( f(x) \).

(ii) Hence, or otherwise, find the other factors of \( f(x) \).

(c) Let \( g(x) = \frac{1}{x^2} - \frac{1}{2x} \) and \( h(x) = 1 - \frac{2}{x} \), where \( x \neq 0 \) and \( x \in \mathbb{R} \).

(i) Show that \( h(x) = -2x[g(x)] \).

(ii) Find the values of \( x \) for which \( g(x) = h(x) \).
3. (a) Given that \((t - 1)x = 2 - 5t\), find the value of \(x\) when \(t = 7\).

(b) (i) Solve for \(x\) and \(y\)

\[
\begin{align*}
x - y + 5 &= 0 \\
x^2 + y^2 &= 17.
\end{align*}
\]

(ii) Which solution gives the lesser value of \(x - 2y\)? Write down this value.

(c) (i) Simplify \(\left(\sqrt{x} - \frac{2}{\sqrt{x}}\right)\left(\sqrt{x} + \frac{2}{\sqrt{x}}\right)\), where \(x > 0\) and \(x \in \mathbb{R}\).

(ii) Hence, solve \(\left(\sqrt{x} - \frac{2}{\sqrt{x}}\right)\left(\sqrt{x} + \frac{2}{\sqrt{x}}\right) = 3\), where \(x > 0\).

(iii) Verify your solution.

4. (a) Given that \(6 - 4i + 3u = 5i\), where \(i^2 = -1\),

(i) find \(u\),

(ii) plot \(u\) on an Argand diagram.

(b) Let \(z = 1 + i\).

(i) Find \(|z|\).

(ii) Show that \(z^2 + \overline{z}^2 = 0\), where \(\overline{z}\) is the complex conjugate of \(z\).

(iii) Verify that \(\frac{1 + 5i}{3 + 2i} = z\).

(c) Let \(w = 3 + 4i\).

Find the real numbers \(k\) and \(t\) such that

\[w^2 - (k + t)w + t = 0\].
5. (a) The $n^{\text{th}}$ term of a sequence is $T_n = \frac{2n - 1}{n + 1}$.

Find the sum of the second and third terms of the sequence.

(b) The first term of an arithmetic series is 2 and the eighth term is 30.

(i) Find $T_3$, the third term of the series.

(ii) Find $S_{10}$, the sum of the first ten terms of the series.

(c) The $n^{\text{th}}$ term of a series is $T_n = \frac{2}{3^n} + 1$.

(i) Write, in terms of $n$, an expression for $T_{n-1}$, the $(n-1)^{\text{st}}$ term.

(ii) Prove that the series is geometric.

(iii) Show that $S_9 = \frac{1}{3} - \frac{1}{3^{10}}$, where $S_9$ is the sum of the first nine terms of the series.

6. (a) Let $h(x) = ax + b$, where $x \in \mathbb{R}$.

Given that $h(0) = 3$ and $h(2) = -5$, find the value of $a$ and the value of $b$.

(b) The diagram shows part of the graph of a function $f$.

Use the graph to estimate

(i) the values of $x$ for which $f(x) = 0$,

(ii) the values of $x$ for which $f''(x) = 0$, where $f''(x)$ is the derivative of $f(x)$,

(iii) the range of values of $x$ for which $f''(x) < 0$.

(c) Let $g(x) = x(3x^2 - 9)$, where $x \in \mathbb{R}$.

(i) Find $g'(x)$, the derivative of $g(x)$.

(ii) Find the co-ordinates of the local maximum point and of the local minimum point of the curve $y = g(x)$.

(iii) Draw the graph of the function $g'(x)$, the derivative of $g(x)$, in the domain $-2 \leq x \leq 2$. 
7. (a) Differentiate $y = 6x - x^2 - 5x^4$ with respect to $x$.

(b) (i) Differentiate $y = (3x^2 + 2x^3 - x)$ with respect to $x$.

(ii) Given that $y = (x^3 - 2x^2 + 4)^5$, find the value of $\frac{dy}{dx}$ when $x = -1$.

(c) A ball is thrown vertically down from the top of a high building. The distance, $s$ metres, the ball falls is given by 
$$s = 3t + 5t^2$$
where $t$ is the time in seconds from the instant the ball is thrown.

(i) Find the speed of the ball after 3 seconds.

(ii) Find the time $t$ when the ball is falling at a speed of $23$ ms$^{-1}$.

(iii) The ball hits the ground at a speed of $38$ ms$^{-1}$. How high is the building?

8. (a) Let $g(x) = k(1 - x)$, where $x \in \mathbb{R}$. 
Given that $g(-5) = 20$, find the value of $k$.

(b) Let $f(x) = \frac{5 + x^2}{2 - x}$, where $x \in \mathbb{R}$ and $x \neq 2$.

(i) Find $f(5)$.

(ii) Find $f'(x)$, the derivative of $f(x)$.

(iii) Show that $f'(x) = 0$ at $x = -1$.

(c) Let $h(x) = 5 + 3x - x^2$, where $x \in \mathbb{R}$.

(i) Find the co-ordinates of the point $P$ at which the curve $y = h(x)$ cuts the $y$-axis.

(ii) Find the equation of the tangent to the curve $y = h(x)$ at $P$.

(iii) The tangent to the curve $y = h(x)$ at $x = t$ is perpendicular to the tangent at $P$. Find the value of $t$. 

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