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MARKING SCHEME

LEAVING CERTIFICATE EXAMINATION 2006

MATHEMATICS – HIGHER LEVEL – PAPER 1

GENERAL GUIDELINES FOR EXAMINERS – PAPER 1

1. Penalties of three types are applied to candidates' work as follows:

- Blunders - mathematical errors/omissions (-3)
- Slips - numerical errors (-1)
- Misreadings (provided task is not oversimplified) (-1).

Frequently occurring errors to which these penalties must be applied are listed in the scheme. They are labelled: B1, B2, B3,..., S1, S2,..., M1, M2,...etc. These lists are not exhaustive.

2. When awarding attempt marks, e.g. Att(3), note that

- any *correct, relevant* step in a part of a question merits at least the attempt mark for that part
- if deductions result in a mark which is lower than the attempt mark, then the attempt mark must be awarded
- a mark between zero and the attempt mark is never awarded.

3. Worthless work is awarded zero marks. Some examples of such work are listed in the scheme and they are labelled as W1, W2,...etc.

4. The phrase “hit or miss” means that partial marks are not awarded – the candidate receives all of the relevant marks or none.

5. The phrase “and stops” means that no more work is shown by the candidate.

6. Special notes relating to the marking of a particular part of a question are indicated by an asterisk. These notes immediately follow the box containing the relevant solution.

7. The sample solutions for each question are not intended to be exhaustive lists – there may be other correct solutions. Any examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his/her advising examiner.

8. Unless otherwise indicated in the scheme, accept the best of two or more attempts – even when attempts have been cancelled.

9. The *same* error in the *same* section of a question is penalised *once* only.

10. Particular cases, verifications and answers derived from diagrams (unless requested) qualify for attempt marks at most.

11. A serious blunder, omission or misreading results in the attempt mark at most.

12. Do not penalise the use of a comma for a decimal point, e.g. €5.50 may be written as €5,50.

QUESTION 1

| | | |
|-----------------|------------------------------|-------------------------|
| Part (a) | 10 marks | Att 3 |
| Part (b) | 20 (5, 5, 5, 5) marks | Att (2, 2, 2, 2) |
| Part (c) | 20 (5, 10, 5) marks | Att (2, 3, 2) |

Part (a) **10 marks** **Att 3**

1. (a) Find the real number a such that for all $x \neq 9$,

$$\frac{x-9}{\sqrt{x}-3} = \sqrt{x} + a.$$

Part 1(a) **10 marks** **Att 3**

$$\mathbf{1(a)} \quad \frac{x-9}{\sqrt{x}-3} \cdot \frac{\sqrt{x}+3}{\sqrt{x}+3} = \frac{(x-9)(\sqrt{x}+3)}{(x-9)} = \sqrt{x}+3 \quad \Rightarrow a=3$$

or

$$\mathbf{1(a)} \quad \frac{x-9}{\sqrt{x}-3} = \frac{(\sqrt{x})^2 - (3)^2}{(\sqrt{x}-3)} = \frac{(\sqrt{x}-3)(\sqrt{x}+3)}{(\sqrt{x}-3)}$$

$$= \sqrt{x}+3$$

$$\Rightarrow a=3$$

or

$$\mathbf{1(a)} \quad \frac{x-9}{\sqrt{x}-3} = \sqrt{x} + a$$

True for all $x \neq 9$, so let $x = 0$ (or any other chosen value):

$$\therefore \frac{0-9}{0-3} = 0 + a$$

$$\Rightarrow a=3$$

or

$$\mathbf{1(a)} \quad \frac{x-9}{\sqrt{x}-3} = \sqrt{x} + a$$

$$(x-9) = (\sqrt{x}-3)(\sqrt{x}+a)$$

$$(x-9) = x - 3\sqrt{x} + a\sqrt{x} - 3a$$

$$x + (0)\sqrt{x} + (-9) = x + (a-3)\sqrt{x} + (-3a)$$

Equating Coefficients:

$$\mathbf{(i)} \quad \begin{aligned} 0 &= a - 3 \\ a &= 3 \end{aligned}$$

$$\mathbf{(ii)} \quad \begin{aligned} -9 &= -3a \\ a &= 3 \end{aligned}$$

Blunders (-3)

B1 Indices

B2 $(\sqrt{x}-3)(\sqrt{x}+3) \neq x-9$

B3 In equating coefficients (not like to like)

B4 Squares both sides initially

Attempts

A1 No conjugate

A2 If oversimplified (no surd in answer) or x in answer

1 (b) $f(x) = 3x^3 + mx^2 - 17x + n$, where m and n are constants.
 Given that $x - 3$ and $x + 2$ are factors of $f(x)$, find the value of m and the value of n .

f(3) **5 marks** **Att 2**
f(-2) **5 marks** **Att 2**
Equations **5 marks** **Att 2**
Solving **5 marks** **Att 2**

1 (b) $f(x) = 3x^3 + mx^2 - 17x + n$
 $(x - 3)$ a factor $\Rightarrow f(3) = 0$
 $f(3) = 3(3)^3 + m(3)^2 - 17(3) + n = 0$
 $9m + n = -30 \dots \dots \dots (i)$
 $(x + 2)$ factor $\Rightarrow f(-2) = 0$
 $f(-2) = 3(-2)^3 + m(-2)^2 - 17(-2) + n = 0$
 $4m + n = -10 \dots \dots \dots (ii)$

| | |
|---|---|
| <p>(i) : $9m + n = -30$ (ii) : $4m + n = -10$ <hr style="width: 100%;"/> $5m = -20$ $m = -4$</p> | <p>(ii) : $4m + n = -10$ $-16 + n = -10$ $n = 6$</p> |
|---|---|

or

1 (b)
$$x^2 - x - 6 \overline{) 3x^3 + mx^2 - 17x + n}$$

$$\begin{array}{r} 3x^3 - 3x^2 - 18x \\ \hline (m+3)x^2 + x + n \\ (m+3)x^2 - (m+3)x - 6(m+3) \\ \hline (m+4)x + (6m+n+18) = (0)x + (0) \end{array}$$

| | |
|--|---|
| <p>(i) : $m + 4 = 0$ $m = -4$</p> | <p>(ii) $6m + n + 18 = 0$ $-24 + n + 18 = 0$ $n = 6$</p> |
|--|---|

or

1 (b) $(x + 2)(x - 3) = x^2 - x - 6$
 Other factor must be linear = $(ax + b)$

$$(x^2 - x - 6)(ax + b) = 3x^3 + mx^2 - 17x + n$$

$$ax^3 - ax^2 - 6ax + bx^2 - bx - 6b = 3x^3 + mx^2 - 17x + n$$

$$ax^3 + (-a + b)x^2 + (-6a - b)x + (-6b) = 3x^3 + mx^2 + (-17)x + n$$

Equating Coefficients

| | | | |
|-------------|-------------------|-----------------------|----------------|
| (i) $a = 3$ | (ii) $-a + b = m$ | (iii) $-6a - b = -17$ | (iv) $-6b = n$ |
|-------------|-------------------|-----------------------|----------------|

(ctd...)

$$\begin{aligned} \text{(iii)} : -6a - b &= -17 \\ -18 - b &= -17 \\ -1 &= b \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad m &= -a + b \\ &= -3 - 1 & m &= -4 \\ &= -4 & n &= 6 \end{aligned}$$

$$\text{(iv)} \quad n = -6b = -6(-1) = 6$$

Blunders (-3)

- B1 Deduction root from factor
- B2 Indices
- B3 2nd value not found (having found 1st)
- B4 In equating coefficients (not like to like)

Slips (-1)

- S1 Not changing sign when subtracting in division

Part 1(c)

20 (5, 10, 5) marks

Att (2, 3, 2)

- 1 (c)** $x^2 - t$ is a factor of $x^3 - px^2 - qx + r$.
- (i)** Show that $pq = r$.
- (ii)** Express the roots of $x^3 - px^2 - qx + r = 0$ in terms of p and q .

Division

5 marks

Att 2

(i) Show

10 marks

Att 3

(ii) Express

5 marks

Att 2

1 (c) (i)

$$\begin{array}{r} x^2 - t \overline{) x^3 - px^2 - qx + r} \\ \underline{x^3 - tx} \\ - px^2 + (t - q)x + r \\ \underline{- px^2 + pt} \\ (t - q)x + (r - pt) = (0)x + (0) \end{array}$$

Equating Coefficients:

$$\begin{aligned} \text{(i)} : t - q &= 0 & \text{(ii)} \quad r - pt &= 0 \\ t &= q & r &= pt \\ & & r &= pq \end{aligned}$$

1 (c) (ii)

$$\begin{aligned} f(x) &= (x^2 - t)(x - p) = 0 \\ \Rightarrow x^2 - t &= 0 & \text{or} & \quad x - p = 0 \\ x^2 &= t & & \quad x = p \\ x &= \pm\sqrt{t} \\ x &= \pm\sqrt{q} \\ \text{Roots: } & \{ \pm\sqrt{q}, p \} \end{aligned}$$

or

$$\begin{aligned} \mathbf{1 (c) (i)} \quad f(x) &= (x^2 - t)\left(x - \frac{r}{t}\right) \\ &= x^3 - \frac{r}{t}x^2 - tx + r \\ &= x^3 - px^2 - qx + r \end{aligned}$$

Equating coefficients:

$$\begin{array}{ll} \text{(i)} & \frac{r}{t} = p \\ & r = pt \\ & r = pq \end{array} \qquad \begin{array}{ll} \text{(ii)} & t = q \end{array}$$

1 (c) (ii) *as above*

*Remainder $\neq 0$ in division

Blunders (-3)

- B1 Indices
- B2 In equating coefficients (not like to like)
- B3 Root from factor
- B4 Root omitted
- B5 Roots not in p and q
- B6 Show not in required form

Slips (-1)

- S1 Not changing sign when subtracting in division.

Attempts

- A1 Any attempt at division
- A2 Other factor not linear

QUESTION 2

| | | |
|-----------------|----------------------------|----------------------|
| Part (a) | 15 (5, 5, 5)marks | Att (2, 2, 2) |
| Part (b) | 20 (5, 5, 10) marks | Att (2, 2, 3) |
| Part (c) | 15 (5, 5, 5) marks | Att (2, 2, 2) |

Part (a) **15 (5, 5, 5)marks** **Att (2, 2, 2)**

2. (a) Solve the simultaneous equations

$$y = 2x - 5$$

$$x^2 + xy = 2.$$

| | | |
|--------------------------------------|----------------|--------------|
| Part (a) (i) Substitution | 5 marks | Att 2 |
| (ii) 1st Variable | 5 marks | Att 2 |
| (iii) 2nd Variable | 5 marks | Att 2 |

2 (a)

(i) : $y = 2x - 5$

(ii) : $x^2 + xy = 2$

(ii) : $x^2 + xy - 2 = 0$

$$x^2 + x(2x - 5) - 2 = 0$$

$$3x^2 - 5x - 2 = 0$$

$$(3x + 1)(x - 2) = 0$$

$3x + 1 = 0$ or $x - 2 = 0$

$$x = -\frac{1}{3}$$

$$x = 2$$

(i) : $y = 2x - 5$

$$x = -\frac{1}{3} : y = 2\left(-\frac{1}{3}\right) - 5 = -\frac{17}{3} \Rightarrow \left(-\frac{1}{3}, -\frac{17}{3}\right)$$

$$x = 2 : y = 2x - 5 = 2(2) - 5 = -1 \Rightarrow (2, -1)$$

Blunders (-3)

- B1 Indices
- B2 Factors once only
- B3 Deduction root from factor
- B4 Root formula once only

Attempts

- A1 Not quadratic

Worthless

- W1 Trial and error only

2 (b) (i) Find the range of values of $t \in \mathbf{R}$ for which the quadratic equation $(2t-1)x^2 + 5tx + 2t = 0$ has real roots.

(ii) Explain why the roots are real when t is an integer.

Correct substitution in $b^2 - 4ac \geq 0$

5 marks

Att 2

Inequality

5 marks

Att 2

Finish

10 marks

Att 3

2 (b) (i) $(2t-1)x^2 + 5tx + 2t = 0$

Real Roots: $b^2 - 4ac \geq 0$

$$(5t)^2 - 4(2t-1)(2t) \geq 0$$

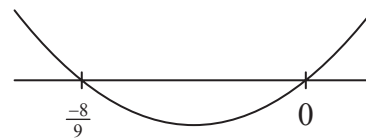
$$25t^2 - 16t^2 + 8t \geq 0$$

$$9t^2 + 8t \geq 0$$

Graph $9t^2 + 8t = 0$

$$t(9t+8) = 0$$

$$t = 0 \quad \text{or} \quad t = -\frac{8}{9}$$



$$\therefore 9t^2 + 8t \geq 0 \quad \text{when} \quad \left\{t \leq -\frac{8}{9}\right\} \cup \{t \geq 0\}$$

(ii) Imaginary roots only when $-\frac{8}{9} < t < 0$

No integer included here.

\Rightarrow real roots for all integers.

Blunders (-3)

B1 Inequality sign

B2 Indices

B3 Factors once only

B4 Deduction root from factor

B5 Range not stated (written down) or no range

B6 Incorrect range

B7 Shade graph only

B8 Incorrect deduction or no deduction in (ii)

Misreading (-1)

M1 Uses ' $>$ ' for ' \geq '

Part (c)

15 (5, 5, 5) marks

Att (2, 2, 2)

2 (c) $f(x) = 1 - b^{2x}$ and $g(x) = b^{1+2x}$, where b is a positive real number.
Find, in terms of b , the value of x for which $f(x) = g(x)$.

Equation

5 marks

Att 2

b^{2x} isolated

5 marks

Att 2

Value

5 marks

Att 2

2 (c)

$$f(x) = 1 - b^{2x}$$

$$g(x) = b^{1+2x}$$

$$f(x) = g(x)$$

$$1 - b^{2x} = b^{1+2x}$$

$$1 - b^{2x} = b \cdot b^{2x}$$

$$1 = b \cdot b^{2x} + b^{2x}$$

$$1 = b^{2x}(b + 1)$$

$$b^{2x} = \frac{1}{b + 1}$$

$$\log_b(b^{2x}) = \log_b\left(\frac{1}{b+1}\right)$$

$$2x \log_b b = -\log_b(b + 1)$$

$$2x = -\log_b(b + 1)$$

$$x = -\frac{1}{2} \log_b(b + 1)$$

$$x = -\log_b \sqrt{b + 1}$$

* Accept logs to any base

Blunders (-3)

B1 Indices

B2 Factors

B3 Logs

QUESTION 3

| | | |
|-----------------|-------------------------------|-------------------------|
| Part (a) | 5 marks | Att 2 |
| Part (b) | 25 (10, 5, 5, 5) marks | Att (3, 2, 2, 2) |
| Part (c) | 20 (5, 5, 5, 5) marks | Att (2, 2, 2, 2) |

Part (a) **5 marks** **Att 2**

3. (a) Given that $z = 2 + i$, where $i^2 = -1$, find the real number d such that $z + \frac{d}{z}$ is real.

Part (a) **5 marks** **Att 2**

3 (a) $(2 + i) + \left(\frac{d}{2+i}\right)$

$$\frac{d}{2+i} = d \left[\frac{1}{2+i} \cdot \frac{2-i}{2-i} \right] = \frac{d(2-i)}{5}$$

$$(2 + i) + \frac{d}{5}(2 - i) = a + (0)i$$

$$\left(2 + \frac{2d}{5}\right) + \left(1 - \frac{d}{5}\right)i = (a) + (0)i$$

Equating Coefficients :

$$1 - \frac{d}{5} = 0$$

$$1 = \frac{d}{5}$$

$$d = 5$$

Blunders (-3)

- B1 i
- B2 Not real to real etc
- B3 $(2 + i)(2 - i) \neq 5$
- B4 Conjugate

Attempts

A1 $f(z) = 0$

Part (b)(i) **15(10, 5) marks** **Att (3, 2)**

3 (b) (i) Use matrix methods to solve the simultaneous equations

$$4x - 2y = 5$$

$$8x + 3y = -4$$

Part (b)(i) Matrix form **10 marks** **Att 3**
Solution **5 marks** **Att 2**

3 (b) (i) Solve : $4x - 2y = 5$
 $8x + 3y = -4$

$$\begin{pmatrix} 4 & -2 \\ 8 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ -4 \end{pmatrix}$$

$$A = \begin{pmatrix} 4 & -2 \\ 8 & 3 \end{pmatrix} \Rightarrow A^{-1} = \frac{1}{28} \begin{pmatrix} 3 & 2 \\ -8 & 4 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{28} \begin{pmatrix} 3 & 2 \\ -8 & 4 \end{pmatrix} \begin{pmatrix} 5 \\ -4 \end{pmatrix}$$

$$= \frac{1}{28} \begin{pmatrix} 7 \\ -56 \end{pmatrix} = \begin{pmatrix} \frac{1}{4} \\ -2 \end{pmatrix}$$

Blunders (-3)

B1 Incorrect matrix A

B2 Incorrect $\frac{1}{\det}$ or no $\frac{1}{\det}$

B3 $A^{-1}.A \neq I$

B4 Incorrect matrix

Part (b) (ii)

10 (5, 5)marks

Att (2, 2)

3 (b) (ii) Find the two values of k which satisfy the matrix equation

$$(1 \quad k) \begin{pmatrix} 3 & 4 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ k \end{pmatrix} = 11.$$

**Part (b)(ii) Complete Multiplication
Values**

**5 marks
5 marks**

**Att 2
Att 2**

| | | | |
|-------------------|---|----|---|
| 3 (b) (ii) | $(1 \quad k) \begin{pmatrix} 3 & 4 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ k \end{pmatrix} = 11$ | or | $(1 \quad k) \begin{pmatrix} 3 & 4 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ k \end{pmatrix} = 11$ |
| | $(1 \quad k) \begin{pmatrix} 3+4k \\ -2+k \end{pmatrix} = 11$ | | $(3-2k \quad 4+k) \begin{pmatrix} 1 \\ k \end{pmatrix} = 11$ |
| | $3+4k-2k+k^2-11=0$ | | $3-2k+4k+k^2=11$ |
| | $k^2+2k-8=0$ | | $k^2+2k-8=0$ |
| | $(k+4)(k-2)=0$ | | $(k+4)(k-2)=0$ |
| | $k=-4$ or $k=2$ | | $k=-4$ or $k=2$ |

Blunders (-3)

B1 Indices

B2 Factors once only

B3 Deduction root from factor or no deduction.

B4 Incorrect matrix

Note: Cannot get 2nd 5 marks if equation is linear.

3 (c) (i) Express $-8 - 8\sqrt{3}i$ in the form $r(\cos\theta + i\sin\theta)$.

(ii) Hence find $(-8 - 8\sqrt{3}i)^3$.

(iii) Find the four complex numbers z such that

$$z^4 = -8 - 8\sqrt{3}i.$$

Give your answers in the form $a + bi$, with a and b fully evaluated

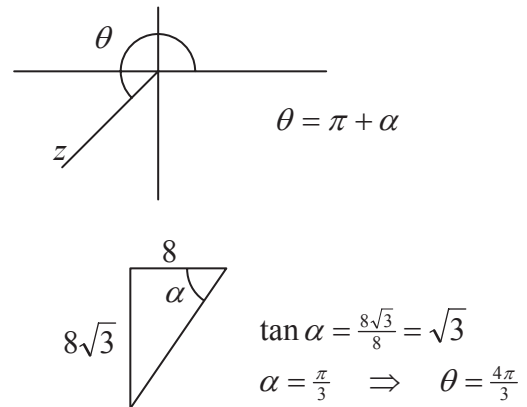
| | | |
|-----------------|---------|-------|
| (i) | 5 marks | Att 2 |
| (ii) | 5 marks | Att 2 |
| (iii) z | 5 marks | Att 2 |
| Complex numbers | 5 marks | Att 2 |

3(c)(i)

$$z = -8 - 8\sqrt{3}i$$

$$\begin{aligned} r &= \sqrt{(-8)^2 + (-8\sqrt{3})^2} \\ &= \sqrt{64 + 192} \\ &= \sqrt{256} \\ &= 16 \end{aligned}$$

$$\begin{aligned} z &= r[\cos\theta + i\sin\theta] \\ &= 16\left[\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right] \\ &= 2^4\left(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right) \end{aligned}$$



3(c)(ii) $z = 2^4\left(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right)$

$$\begin{aligned} z^3 &= \left[2^4\left(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right)\right]^3 \\ &= 2^{12}\left(\cos 4\pi + i\sin 4\pi\right) \\ &= 2^{12}(1 + i(0)) \\ &= 2^{12} = 4096 \end{aligned}$$

3(c)(iii) $z^4 = -8 - 8\sqrt{3}i = 2^4\left[\cos\left(2n\pi + \frac{4\pi}{3}\right) + i\sin\left(2n\pi + \frac{4\pi}{3}\right)\right]$

$$\Rightarrow z = \left\langle 2^4\left[\cos\left(2n\pi + \frac{4\pi}{3}\right) + i\sin\left(2n\pi + \frac{4\pi}{3}\right)\right] \right\rangle^{\frac{1}{4}}$$

$$z = 2\left[\cos\left(\frac{n\pi}{2} + \frac{\pi}{3}\right) + i\sin\left(\frac{n\pi}{2} + \frac{\pi}{3}\right)\right]$$

$$n = 0: z_0 = 2\left[\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right] = 2\left[\frac{1}{2} + i\frac{\sqrt{3}}{2}\right] = 1 + i\sqrt{3}$$

$$\begin{aligned} n = 1: z_1 &= 2\left[\cos\left(\frac{\pi}{2} + \frac{\pi}{3}\right) + i\sin\left(\frac{\pi}{2} + \frac{\pi}{3}\right)\right] \\ &= 2\left[\cos\left(\frac{5\pi}{6}\right) + i\sin\left(\frac{5\pi}{6}\right)\right] = 2\left[-\frac{\sqrt{3}}{2} + i\left(\frac{1}{2}\right)\right] = -\sqrt{3} + i \end{aligned}$$

$$\begin{aligned} n = 2: z_2 &= 2\left[\cos\left(\pi + \frac{\pi}{3}\right) + i\sin\left(\pi + \frac{\pi}{3}\right)\right] \\ &= 2\left[\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right] = 2\left[-\frac{1}{2} - i\left(\frac{\sqrt{3}}{2}\right)\right] = -1 - i\sqrt{3} \end{aligned}$$

$$\begin{aligned} n = 3: z_3 &= 2\left[\cos\left(\frac{3\pi}{2} + \frac{\pi}{3}\right) + i\sin\left(\frac{3\pi}{2} + \frac{\pi}{3}\right)\right] \\ &= 2\left[\cos\frac{11\pi}{6} + i\sin\frac{11\pi}{6}\right] = 2\left[\frac{\sqrt{3}}{2} + i\left(-\frac{1}{2}\right)\right] = \sqrt{3} - i \end{aligned}$$

Blunders (-3)

- B1 Argument
- B2 Modulus
- B3 Trig definition
- B4 Indices
- B5 i
- B6 Statement De Moivre once only
- B7 Application De Moivre
- B8 Root omitted
- B9 No general solution
- B10 a and b not fully evaluated
- B11 Improper use of polar formula

Slips (-1)

- S1 Trig value

QUESTION 4

| | | |
|-----------------|------------------------------|-------------------------|
| Part (a) | 10 (5, 5) marks | Att (2, 2) |
| Part (b) | 20 (5, 5, 5, 5) marks | Att (2, 2, 2, 2) |
| Part (c) | 20 (5, 5, 5, 5) marks | Att (2, 2, 2, 2) |

Part (a) **10 (5, 5) marks** **Att (2, 2)**

4 (a) $-2 + 2 + 6 + \dots + (4n - 6)$ are the first n terms of an arithmetic series.
 S_n , the sum of these n terms, is 160. Find the value of n .

Correct substitution in formula **5 marks** **Att 2**
Value n **5 marks** **Att 2**

4 (a) $-2 + 2 + 6 + \dots + (4n - 6)$
 $a = -2$; $d = 4$; $S_n = 160$
 $S_n = \frac{n}{2}[2a + (n - 1)d] = 160$
 $\frac{n}{2}[-4 + (n - 1)4] = 160$
 $\frac{n}{2}[-4 + 4n - 4] = 160$
 $n(2n - 4) = 160$
 $2n^2 - 4n - 160 = 0$
 $n^2 - 2n - 80 = 0$
 $(n - 10)(n + 8) = 0$
 $n = 10$ or $n = -8$
 $n = 10$

Blunders (-3)

- B1 Formula *AP* once only
- B2 Incorrect 'a' in formula
- B3 Incorrect 'd' in formula
- B4 Indices
- B5 Factors once only
- B6 Incorrect deduction root from factor
- B7 Roots formula

Slips

- S1 Excess value or incorrect value indicated.

Worthless

- W1 treats as *G.P.*

- 4 (b) The sum to infinity of a geometric series is $\frac{9}{2}$.
The second term of the series is -2 .
Find the value of r , the common ratio of the series.

| | | |
|-----------------------------------|---------|-------|
| Part (b) 1 st equation | 5 marks | Att 2 |
| 2 nd equation | 5 marks | Att 2 |
| Quadratic simplified | 5 marks | Att 2 |
| Value r | 5 marks | Att 2 |

4 (b)
$$S_{\infty} = \frac{9}{2} = \frac{a}{1-r}$$

$$9(1-r) = 2a \dots\dots\dots (i)$$

a, ar, ar^2

$$ar = -2$$

$$a = -\frac{2}{r} \dots\dots\dots (ii)$$

(i): $9(1-r) = 2(a)$

$$9(1-r) = 2\left(-\frac{2}{r}\right)$$

$$9 - 9r = -\frac{4}{r}$$

$$9r - 9r^2 = -4$$

$$0 = 9r^2 - 9r - 4$$

$$0 = (3r + 1)(3r - 4)$$

$$\Rightarrow r = -\frac{1}{3} \quad \text{or} \quad r = \frac{4}{3}$$

Since sum to infinity $\Rightarrow r = -\frac{1}{3}$

Blunders (-3)

- B1 Formula sum to infinity
- B2 Definition of term of a *G.P.*
- B3 Indices
- B4 Factors once only
- B5 Incorrect deduction root from factor
- B6 Incorrect ' a '

Slips

- S1 Excess value or incorrect value indicated

Worthless

- W1 Uses *AP* formula
- W2 Trial and error

4 (c) The sequence u_1, u_2, u_3, \dots , defined by $u_1 = 3$ and $u_{n+1} = 2u_n + 3$, is as follows:

3, 9, 21, 45, 93, ...

(i) Find u_6 , and verify that it is equal to the sum of the first six terms of a geometric series with first term 3 and common ratio 2.

(ii) Given that, for all k , u_k is the sum of the first k terms of a geometric series with first term 3 and common ratio 2, find $\sum_{k=1}^n u_k$.

| | | |
|------------------------------|---------|-------|
| Part (c)(i) U_6 | 5 marks | Att 2 |
| Verify | 5 marks | Att 2 |
| Part (c)(ii) Showing Pattern | 5 marks | Att 2 |
| Sum | 5 marks | Att 2 |

4 (c) (i) $u_1 = 3$; $u_{n+1} = 2u_n + 3$
 $u_6 = 2(u_5) + 3 = 2(93) + 3 = 189$

G.P.: $a = 3$; $r = 2$
 $s_6 = \frac{3[2^6 - 1]}{2 - 1} = 3[64 - 1] = 189$

or

$s_6 = 3 + 6 + 12 + 24 + 48 + 96 = 189$

Q4(c)(ii) (3), (3+6), (3+6+12), (3+6+12+24), ...
 (3), (9), (21), (45)

$u_k = 3(2^k - 1) = 3(2^k) - 3$

$u_n = 3 \cdot 2^n - 3$

$u_{n-1} = 3 \cdot 2^{n-1} - 3$

$u_{n-2} = 3 \cdot 2^{n-2} - 3$

.....

$u_3 = 3 \cdot 2^3 - 3$

$u_2 = 3 \cdot 2^2 - 3$

$u_1 = 3 \cdot 2^1 - 3$

$\Sigma = 3(2^1 + 2^2 + \dots + 2^n) - 3n$
 $= 3 \left[\frac{2(2^n - 1)}{2 - 1} \right] - 3n = 6(2^n - 1) - 3n$

Blunders (-3)

B1 Error in U_6

B2 Formula sum of G.P.

B3 Incorrect 'a'

B4 Incorrect 'R'

B5 Error in U_k

B6 Indices

QUESTION 5

| | | |
|-----------------|------------------------------|-------------------------|
| Part (a) | 10 marks | Att 3 |
| Part (b) | 20 (5, 10, 5) marks | Att (2, 3, 2) |
| Part (c) | 20 (5, 5, 5, 5) marks | Att (2, 2, 2, 2) |

Part (a) **10 marks** **Att 3**

5 (a) Find the value of the middle term of the binomial expansion of

$$\left(\frac{x}{y} - \frac{y}{x}\right)^8.$$

Part (a) **10 marks** **Att 3**

$$\mathbf{5 (a)} \quad u_5 = \binom{8}{4} \left[\left(\frac{x}{y}\right)^4 \left(-\frac{y}{x}\right)^4 \right] = \frac{8 \cdot 7 \cdot 6 \cdot 5}{1 \cdot 2 \cdot 3 \cdot 4} (1) = 70$$

or

5(a)

$$\left(\frac{x}{y} - \frac{y}{x}\right)^8 = \binom{8}{0} \left(\frac{x}{y}\right)^8 + \binom{8}{1} \left(\frac{x}{y}\right)^7 \left(-\frac{y}{x}\right) + \binom{8}{2} \left(\frac{x}{y}\right)^6 \left(-\frac{y}{x}\right)^2 + \binom{8}{3} \left(\frac{x}{y}\right)^5 \left(-\frac{y}{x}\right)^3 + \binom{8}{4} \left(\frac{x}{y}\right)^4 \left(-\frac{y}{x}\right)^4 + \dots$$

$$u_5 = \binom{8}{4} \left(\frac{x}{y}\right)^4 \left(-\frac{y}{x}\right)^4 = 70$$

* Answer must be in simplest form

Blunders (-3)

- B1 General term
- B2 Errors in binominal expansion once only
- B3 Indices
- B4 error value $\binom{n}{r}$ or no value $\binom{n}{r}$
- B5 x and y in answer
- B6 $x^\circ \neq 1$
- B7 Incorrect term

Part (b) **20 (5, 10, 5) marks** **Att (2, 3, 2)**

- 5 (b)**
- (i) Express $\frac{2}{(r+1)(r+3)}$ in the form $\frac{A}{r+1} + \frac{B}{r+3}$.
- (ii) Hence find $\sum_{r=1}^n \frac{2}{(r+1)(r+3)}$.
- (iii) Hence evaluate $\sum_{r=1}^{\infty} \frac{2}{(r+1)(r+3)}$.

Part (b) (i)
(ii)
(iii)

5 marks
10 marks
5 marks

Att 2
Att 3
Att 2

5 (b) (i)

$$\frac{a}{r+1} + \frac{b}{r+3} = \frac{2}{(r+1)(r+3)}$$

$$a(r+3) + b(r+1) = 2$$

$$ar + 3a + br + b = (0)r + (2)$$

$$(a+b)r + (3a+b) = (0)r + 2$$

Equating Coefficients:

(i) $a + b = 0 \Rightarrow a = -b$

(ii) $3a + b = 2$

$$3a - a = 2$$

$$2a = 2 \Rightarrow a = 1 \quad \text{and} \quad b = -1$$

$$\frac{2}{(r+1)(r+3)} = \frac{1}{r+1} - \frac{1}{r+3}$$

Q5(b)(ii)

$$\sum_{r=1}^n \frac{2}{(r+1)(r+3)}$$

$$u_n = \frac{2}{(n+1)(n+3)} = \frac{\cancel{1}}{n+1} - \frac{1}{n+3}$$

$$u_{n-1} = \frac{2}{n(n+2)} = \frac{\cancel{1}}{n} - \frac{1}{n+2}$$

$$u_{n-2} = \frac{2}{(n-1)(n+1)} = \frac{\cancel{1}}{n-1} - \frac{\cancel{1}}{n+1}$$

⋮

$$u_3 = \frac{2}{4 \cdot 6} = \frac{\cancel{1}}{4} - \frac{\cancel{1}}{6}$$

$$u_2 = \frac{2}{3 \cdot 5} = \frac{1}{3} - \frac{\cancel{1}}{5}$$

$$u_1 = \frac{2}{2 \cdot 4} = \frac{1}{2} - \frac{\cancel{1}}{4}$$

$$s_n = \frac{1}{2} + \frac{1}{3} - \frac{1}{n+2} - \frac{1}{n+3} = \frac{5}{6} - \frac{1}{n+2} - \frac{1}{n+3}$$

Q5(b)(iii)

$$n \rightarrow \infty \quad s_\infty = \frac{5}{6}$$

Blunders (-3)

- B1 Indices
- B2 Cancellation must be shown or implied
- B3 In equating coefficients not like to like
- B4 Term or terms omitted
- B5 S_r
- B6 limits

Note: Must show 3 terms at start and 2 terms at finish or vice versa otherwise attempt.

5 (c) (i) Given two real numbers a and b , where $a > 1$ and $b > 1$, prove that

$$\frac{1}{\log_b a} + \frac{1}{\log_a b} \geq 2.$$

(ii) Under what condition is $\frac{1}{\log_b a} + \frac{1}{\log_a b} = 2$.

Part 5(c) (i) Change of Base

5 marks

Att 2

Inequality

5 marks

Att 2

Prove

5 marks

Att 2

(ii)

5 marks

Att 2

5 (c) (i)

$$\frac{1}{\log_b a} + \frac{1}{\log_a b} \geq 2$$

$$\Leftrightarrow \log_a b + \frac{1}{\log_a b} \geq 2 \quad (\text{since } \log_a b = \frac{1}{\log_b a})$$

$$\Leftrightarrow (\log_a b)^2 + 1 \geq 2 \log_a b \quad (\text{since } a > 1, b > 1 \Rightarrow \log_a b > 0)$$

$$\Leftrightarrow (\log_a b)^2 + 1 - 2 \log_a b \geq 0$$

$$\Leftrightarrow (\log_a b)^2 - 2(\log_a b) + 1 \geq 0$$

$$\Leftrightarrow (\log_a b - 1)^2 \geq 0$$

$$\text{True, so } \frac{1}{\log_b a} + \frac{1}{\log_a b} \geq 2.$$

or

Q5 (c) (i)

To prove: $\log_a b + \frac{1}{\log_a b} \geq 2$

Let $\log_a b = x$. Then $x > 0$, since $a > 1, b > 1 \Rightarrow \log_a b > 0$.

To prove: $x + \frac{1}{x} \geq 2$

$$\Leftrightarrow x^2 + 1 \geq 2x$$

$$\Leftrightarrow x^2 - 2x + 1 \geq 0$$

$$\Leftrightarrow (x-1)^2 \geq 0$$

True, so $x + \frac{1}{x} \geq 2$. That is, $\log_a b + \frac{1}{\log_a b} \geq 2$

Q5(c)(ii) Equality holds in above solution when $(\log_a b - 1)^2 = 0$ [or $(x-1)^2 = 0$ in 2nd version]

$$\Leftrightarrow \log_a b = 1$$

$$\Leftrightarrow a = b$$

Blunders (-3)

B1 Log laws

B2 Change of base

B3 Inequality sign

B4 Incorrect deduction or no deduction

B5 Indices

B6 Factors once only

B7 $(x^2 - 2x + 1) \neq (x-1)^2$

Note : Inequality must be quadratic

QUESTION 6

| | | |
|-----------------|----------------------------|----------------------|
| Part (a) | 15 marks | Att 5 |
| Part (b) | 20 (10, 5, 5) marks | Att (3, 2, 2) |
| Part (c) | 15 (5, 5, 5) marks | Att (2, 2, 2) |

Part (a) **15 marks** **Att 5**

6 (a) Differentiate $\sqrt{x}(x+2)$ with respect to x .

Part (a) **15 marks** **Att 5**

$$\begin{aligned} \mathbf{6 (a)} \quad y &= \sqrt{x} \cdot (x+2) \\ &= x^{\frac{1}{2}}(x+2) \\ &= x^{\frac{3}{2}} + 2x^{\frac{1}{2}} \\ \frac{dy}{dx} &= \frac{3}{2}\left(x^{\frac{1}{2}}\right) + 2\left(\frac{1}{2} \cdot x^{-\frac{1}{2}}\right) \\ &= \frac{3}{2}\left(x^{\frac{1}{2}}\right) + \frac{1}{x^{\frac{1}{2}}} \end{aligned}$$

or

$$\begin{aligned} \mathbf{6(a)} \quad y &= x^{\frac{1}{2}} \cdot (x+2) \\ \frac{dy}{dx} &= x^{\frac{1}{2}}(1) + (x+2)\left(\frac{1}{2} \cdot x^{-\frac{1}{2}}\right) \\ &= x^{\frac{1}{2}} + \frac{x+2}{2x^{\frac{1}{2}}} \\ &= \sqrt{x} + \frac{x+2}{2\sqrt{x}} \end{aligned}$$

Blunders (-3)

B1 Indices

B2 Differentiation

Attempts

A1 Error in differentiation formula

- 6 (b)** The equation of a curve is $y = 3x^4 - 2x^3 - 9x^2 + 8$.
- (i) Show that the curve has a local maximum at the point (0, 8).
- (ii) Find the coordinates of the two local minimum points on the curve.
- (iii) Draw a sketch of the curve.

Part (b) (i)**10 marks****Att 3****(ii)****5 marks****Att 2****(iii)****5 marks****Att 2****6 (b) (i)**

$$y = 3x^4 - 2x^3 - 9x^2 + 8.$$

$$\frac{dy}{dx} = 12x^3 - 6x^2 - 18x$$

$$\frac{d^2y}{dx^2} = 36x^2 - 12x - 18$$

Local Max/Min: $\frac{dy}{dx} = 0 \Rightarrow 12x^3 - 6x^2 - 18x = 0$

$$6x(2x^2 - x - 3) = 0$$

$$x = 0 \quad \text{or} \quad 2x^2 - x - 3 = 0$$

$$(2x - 3)(x + 1) = 0$$

$$x = \frac{3}{2} \quad \text{or} \quad x = -1$$

Test $x = 0$ in $\frac{d^2y}{dx^2} = 36(0) - 12(0) - 18 = -18 < 0 \Rightarrow$ local max at $x = 0$.

When $x = 0$: $y = 3(0) - 2(0) - 9(0) + 8 = 8 \quad \therefore$ Local max at (0,8)

Q6(b)(ii)Other two turning points at $x = \frac{3}{2}$ and $x = -1$

At $x = \frac{3}{2}$: $y = 3\left(\frac{3}{2}\right)^4 - 2\left(\frac{3}{2}\right)^3 - 9\left(\frac{3}{2}\right)^2 + 8$

$$= \frac{243}{16} - \frac{108}{16} - \frac{324}{16} + \frac{128}{16}$$

$$= -\frac{61}{16} = -3\frac{13}{16} \approx -3.8 \quad \Rightarrow y \approx -3.8$$

And at $x = \frac{3}{2}$: $\frac{d^2y}{dx^2} = 36\left(\frac{9}{4}\right) - 12\left(\frac{3}{2}\right) - 18 = 81 - 18 - 18 > 0 \Rightarrow$ local min at $\left(\frac{3}{2}, -3.8\right)$

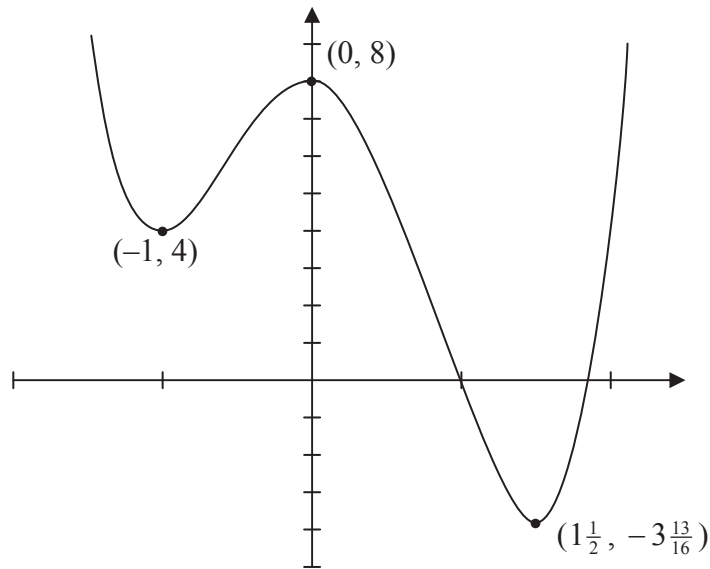
At $x = -1$: $y = 3(-1)^4 - 2(-1)^3 - 9(-1)^2 + 8$

$$= 3 + 2 - 9 + 8$$

$$= 4 \quad \Rightarrow y = 4$$

And at $x = -1$: $\frac{d^2y}{dx^2} = 36(-1)^2 - 12(-1) - 18 = 36 + 12 - 18 > 0 \Rightarrow$ local min at $(-1, 4)$.

Q6(b)(iii)



Blunders (-3)

- B1 Differentiation
- B2 Indices
- B3 Deduction from 2nd derivative or no deduction
- B4 Not 3 values from $f'(x) = 0$
- B5 Not testing in $f''(x)$ for max
- B6 Incorrect y values or no y values in (ii)
- B7 Factors once only
- B8 Incorrect root from factor
- B9 Not getting $f''(x)$

Attempts

- A1 Error in differentiation formula

Worthless

- W1 Integration

6 (c) Prove by induction that $\frac{d}{dx}(x^n) = nx^{n-1}$, $n \geq 1$, $n \in \mathbf{N}$.

6(c) P(1)

5 marks

Att 2

P(k)

5 marks

Att 2

P(k + 1)

5 marks

Att 2

6 (c) $P(n): \frac{d}{dx}(x^n) = nx^{n-1}$

P(1): show that $\frac{d}{dx}(x) = 1$:

$$f(x) = x \Rightarrow \frac{f(x+h) - f(x)}{h} = \frac{(x+h) - x}{h} = \frac{h}{h} = 1.$$

$$\therefore \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = 1 \quad \therefore P(1) \text{ is true.}$$

Assume $P(k)$ true: $\frac{d}{dx}(x^k) = kx^{k-1}$

Deduce $P(k+1)$: $\frac{d}{dx}(x^{k+1}) = \frac{d}{dx}(x^k \cdot x)$

$$= x^k(1) + x \frac{d}{dx}(x^k) \quad (\text{by product rule})$$

$$= x^k(1) + x(kx^{k-1}) \quad (\text{by } P(k) \text{ assumption})$$

$$= x^k + kx^k$$

$$= x^k(k+1)$$

\therefore True for $p(k+1)$

As $P(1)$ is true, and $P(k) \Rightarrow P(k+1)$ for all k , $P(n)$ is true for all $n \geq 1$.

Blunders (-3)

B1 Failure to prove case $n = 1$ or uses rule to prove true for $n = 1$

B2 Definition of $f'(x)$

B3 Error in $f(x+k)$ or $(x + \Delta x)$

B4 Indices

B5 Limit or no limit shown or implied

B6 Differentiation

B7 $n = 0$

Attempts

A1 Error in differentiation formula

QUESTION 7

| | | |
|-----------------|------------------------------|-------------------------|
| Part (a) | 15 (5, 5, 5) marks | Att (2, 2, 2) |
| Part (b) | 20 (5, 5, 5, 5) marks | Att (2, 2, 2, 2) |
| Part (c) | 15 (5, 5, 5) marks | Att (2, 2, 2) |

Part (a) **15 (5, 5, 5) marks** **Att (2, 2, 2)**

7 (a) Taking $x_1 = 2$ as the first approximation to the real root of the equation

$$x^3 + x - 9 = 0,$$
 use the Newton-Raphson method to find x_2 , the second approximation.

| | | |
|-------------------------|----------------|--------------|
| Part (a) Formula | 5 marks | Att 2 |
| Differentiation | 5 marks | Att 2 |
| Finish | 5 marks | Att 2 |

7 (a)

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$x_1 = 2 :$ $f(x) = x^3 + x - 9 \Rightarrow f(2) = (2)^3 + 2 - 9 = 1$
 $f'(x) = 3x^2 + 1 \Rightarrow f'(2) = 3(2)^2 + 1 = 13$
 $x_2 = 2 - \frac{1}{13} = \frac{25}{13}$

Blunder (-3)

- B1 Newton-Raphson formula once only
- B2 Differentiation
- B3 Indices
- B4 $x_1 \neq 2$

Worthless

- W1 Incorrect answer and no work

7 (b) The parametric equations of a curve are:

$$x = 3\cos\theta - \cos^3\theta$$

$$y = 3\sin\theta - \sin^3\theta, \text{ where } 0 < \theta < \frac{\pi}{2}.$$

(i) Find $\frac{dy}{d\theta}$ and $\frac{dx}{d\theta}$.

(ii) Hence show that $\frac{dy}{dx} = \frac{-1}{\tan^3\theta}$.

$$\frac{dx}{d\theta}$$

5 marks

Att 2

$$\frac{dy}{d\theta}$$

5 marks

Att 2

$$\frac{dy}{dx}$$

5 marks

Att 2

Show

5 marks

Att 2

7 (b)

$$x = 3\cos\theta - (\cos\theta)^3$$

$$\frac{dx}{d\theta} = -3\sin\theta - 3(\cos\theta)^2 \cdot (-\sin\theta)$$

$$= -3\sin\theta + 3\sin\theta\cos^2\theta$$

$$= -3\sin\theta(1 - \cos^2\theta)$$

$$= -3\sin^3\theta$$

$$y = 3\sin\theta - (\sin\theta)^3$$

$$\frac{dy}{d\theta} = 3\cos\theta - 3(\sin\theta)^2 \cdot \cos\theta$$

$$= 3\cos\theta(1 - \sin^2\theta)$$

$$= 3\cos^3\theta$$

$$\frac{dy}{dx} = \frac{\left(\frac{dy}{d\theta}\right)}{\left(\frac{dx}{d\theta}\right)} = \frac{3\cos^3\theta}{-3\sin^3\theta} = -\frac{1}{\left(\frac{\sin\theta}{\cos\theta}\right)^3} = -\frac{1}{\tan^3\theta}$$

Blunders (-3)

B1 Indices

B2 Differentiation

B3 Trig laws

B4 Not in required form

Attempts

A1 Error in differentiation formula

7 (c) Given $y = \ln\left(\frac{3+x}{\sqrt{9-x^2}}\right)$, find $\frac{dy}{dx}$ and express it in the form $\frac{a}{b-x^n}$.

$$\frac{dy}{dx}$$

5 marks

Att 2

Simplifying
Express

5 marks

Att 2

5 marks

Att 2

$$\begin{aligned}
 7 \text{ (c)} \quad y &= \ln\left(\frac{3+x}{\sqrt{9-x^2}}\right) \\
 &= \ln(3+x) - \ln\sqrt{9-x^2} \\
 &= \ln(3+x) - \frac{1}{2}\ln(9-x^2) \\
 \frac{dy}{dx} &= \frac{1}{3+x} - \frac{1}{2}\left[\frac{1}{9-x^2}(-2x)\right] \\
 &= \frac{1}{3+x} + \frac{x}{9-x^2} \\
 &= \frac{1}{3+x} + \frac{x}{(3-x)(3+x)} \\
 &= \frac{(3-x)+x}{(3-x)(3+x)} = \frac{3}{9-x^2}
 \end{aligned}$$

or

$$\begin{aligned}
 \text{Q7(c)} \quad y &= \ln\frac{3+x}{\sqrt{(3-x)(3+x)}} \\
 &= \ln\frac{(3+x)^{\frac{1}{2}}}{(3-x)^{\frac{1}{2}}} \\
 &= \frac{1}{2}\ln(3+x) - \frac{1}{2}\ln(3-x) \\
 \frac{dy}{dx} &= \frac{1}{2}\left[\frac{1}{3+x} - \frac{1}{3-x}(-1)\right] \\
 &= \frac{1}{2}\left[\frac{1}{3+x} + \frac{1}{3-x}\right] \\
 &= \frac{1}{2}\left[\frac{(3-x)+(3+x)}{9-x^2}\right] = \frac{1}{2}\left(\frac{6}{9-x^2}\right) = \frac{3}{9-x^2}
 \end{aligned}$$

or

Q7(c)

$$y = \ln\left(\frac{3+x}{\sqrt{9-x^2}}\right) = \ln\left(\frac{3+x}{(9-x^2)^{\frac{1}{2}}}\right)$$

$$\frac{dy}{dx} = \frac{1}{\left[\frac{3+x}{(9-x^2)^{\frac{1}{2}}}\right]} \cdot \frac{(9-x^2)^{\frac{1}{2}}(1) - (3+x)\frac{1}{2}(9-x^2)^{-\frac{1}{2}}(-2x)}{(9-x^2)^1}$$

$$= \frac{(9-x^2)^{\frac{1}{2}}}{(3+x)} \cdot \frac{(9-x^2)^{\frac{1}{2}} + \frac{x(3+x)}{(9-x^2)^{\frac{1}{2}}}}{(9-x^2)}$$

$$= \frac{(9-x^2) + x(3+x)}{(3+x)(9-x^2)}$$

$$= \frac{9-x^2+3x+x^2}{(3+x)(9-x^2)}$$

$$= \frac{9+3x}{(3+x)(9-x^2)}$$

$$= \frac{3(3+x)}{(3+x)(9-x^2)}$$

$$= \frac{3}{9-x^2}$$

* $\frac{dy}{dx}$ and simplifying can be in any order

Blunders (-3)

- B1 Differentiation
- B2 Log laws
- B3 Indices
- B4 Not simplified to required form
- B5 Factors once only.

Attempts

- A1 Error in differentiation formula

QUESTION 8

| | | |
|-----------------|------------------------------|-------------------------|
| Part (a) | 10 (5, 5) marks | Att (2, 2) |
| Part (b) | 20 (5, 5, 5, 5) marks | Att (2, 2, 2, 2) |
| Part (c) | 20 (10, 10) marks | Att (3, 3) |

| | | |
|-----------------|------------------------|-------------------|
| Part (a) | 10 (5, 5) marks | Att (2, 2) |
|-----------------|------------------------|-------------------|

| | |
|---|-------------------------|
| 8. (a) Find (i) $\int \sqrt{x} dx$ | (ii) $\int e^{-2x} dx.$ |
|---|-------------------------|

| | | |
|-----------------|------------------------|-------------------|
| Part (a) | 10 (5, 5) marks | Att (2, 2) |
|-----------------|------------------------|-------------------|

Q8 (a)(i) $\int \sqrt{x} dx = \int x^{\frac{1}{2}} dx = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c = \frac{2}{3} x^{\frac{3}{2}} + c$

Q8 (a)(ii) $\int e^{-2x} dx = -\frac{1}{2} e^{-2x} + c$

* If c shown once, then no penalty

Blunders (-3)

B1 Integration

B2 Indices

B3 No 'c' (penalize 1st integration)

Attempts

A1 Only 'c' correct

Worthless

W1 Differentiation instead of integration

| | | |
|-----------------|------------------------------|-------------------------|
| Part (b) | 20 (5, 5, 5, 5) marks | Att (2, 2, 2, 2) |
|-----------------|------------------------------|-------------------------|

| | |
|--|--|
| 8 (b) Evaluate (i) $\int_1^2 x(1+x^2)^3 dx$ | (ii) $\int_0^{\frac{\pi}{4}} \sin 5\theta \cos 3\theta d\theta.$ |
|--|--|

| | | |
|--------------------------------|----------------|--------------|
| Part (b)(i) Integration | 5 marks | Att 2 |
|--------------------------------|----------------|--------------|

| | | |
|--------------|----------------|--------------|
| Value | 5 marks | Att 2 |
|--------------|----------------|--------------|

| | | |
|---------------------------------|----------------|--------------|
| Part (b)(ii) Integration | 5 marks | Att 2 |
|---------------------------------|----------------|--------------|

| | | |
|--------------|----------------|--------------|
| Value | 5 marks | Att 2 |
|--------------|----------------|--------------|

8 (b) (i) $\int_1^2 x(1+x^2)^3 dx$

$$\int (1+x^2)^3 \cdot x dx = \int u^3 \frac{du}{2} = \frac{1}{2} \left[\frac{u^4}{4} \right]$$

$$= \frac{1}{8} \left[(1+x^2)^4 \right]_1^2$$

$$= \frac{1}{8} \left[(5)^4 - (2)^4 \right] = \frac{609}{8}$$

Let $u = 1 + x^2$

$$\frac{du}{dx} = 2x$$

$$\frac{du}{2} = x dx$$

or

Q8(b)(i)
$$\int_1^2 x(1+x^2)^3 dx$$

$$= \int x(1+3x^2+3x^4+x^6) dx$$

$$= \int (x+3x^3+3x^5+x^7) dx$$

$$= \left[\frac{x^2}{2} + \frac{3x^4}{4} + \frac{x^6}{2} + \frac{x^8}{8} \right]_1^2$$

$$= (2+12+32+32) - \left(\frac{1}{2} + \frac{3}{4} + \frac{1}{2} + \frac{1}{8} \right)$$

$$= 78 - \frac{17}{8} = 76\frac{1}{8} = \frac{609}{8}$$

$$(1+x^2)^3 = (1)^3 + \binom{3}{1}(1)^2(x^2) + \binom{3}{2}(1)(x^2)^2 + (x^2)^3$$

$$= 1 + 3x^2 + 3x^4 + x^6$$

Q8(b)(ii)
$$\int_0^{\frac{\pi}{4}} \sin 5\theta \cos 3\theta d\theta.$$

$$= \frac{1}{2} \int 2 \sin 5\theta \cos 3\theta d\theta$$

$$= \frac{1}{2} \int (\sin 8\theta + \sin 2\theta) d\theta$$

$$= \frac{1}{2} \left[-\frac{\cos 8\theta}{8} - \frac{\cos 2\theta}{2} \right]_0^{\frac{\pi}{4}}$$

$$= \frac{1}{2} \left[\left(-\frac{\cos 2\pi}{8} - \frac{\cos \frac{\pi}{2}}{2} \right) - \left(-\frac{\cos 0}{8} - \frac{\cos 0}{2} \right) \right]$$

$$= \frac{1}{2} \left[\left(-\frac{1}{8} - 0 \right) - \left(-\frac{1}{8} - \frac{1}{2} \right) \right]$$

$$= \frac{1}{2} \left[-\frac{1}{8} + \frac{1}{8} + \frac{1}{2} \right]$$

$$= \frac{1}{4}$$

Blunders (-3)

- B1 Integration
- B2 Indices
- B3 Differentiation
- B4 Limits
- B5 Incorrect order in applying limits
- B6 Not calculating substituted limits
- B7 Not changing limits
- B8 Trig formula

Slips

- S1 Trig value

Worthless

- W1 Differentiation instead of integration except where other work merits attempt

Note Incorrect substitution and unable to finish yields attempt at most.

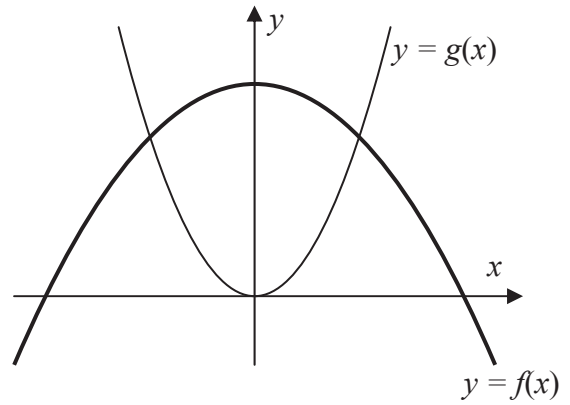
Part (c)

20 (10, 10) marks

Att (3, 3)

8 (c) The diagram shows the graphs of the curves $y = f(x)$ and $y = g(x)$, where $f(x) = 12 - 3x^2$ and $g(x) = 9x^2$.

- (i) Calculate the area of the region enclosed by the curve $y = f(x)$ the x -axis.
- (ii) Show that the region enclosed by the curves $y = f(x)$ and $y = g(x)$ has half that area.



Part (c) (i)
(ii)

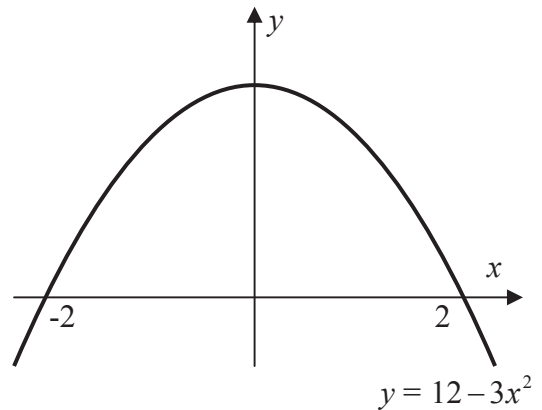
10 marks
10 marks

Att 3
Att 3

8 (c) (i)

$$f(x) = 0 \Rightarrow 12 - 3x^2 = 0$$
$$4 = x^2 \Rightarrow x = \pm 2$$

$$A = \int_{-2}^2 y \cdot dx = 2 \int_0^2 (12 - 3x^2) dx$$
$$= 2 [12x - x^3]_0^2$$
$$= 2 [(24 - 8) - 0]$$
$$= 32$$

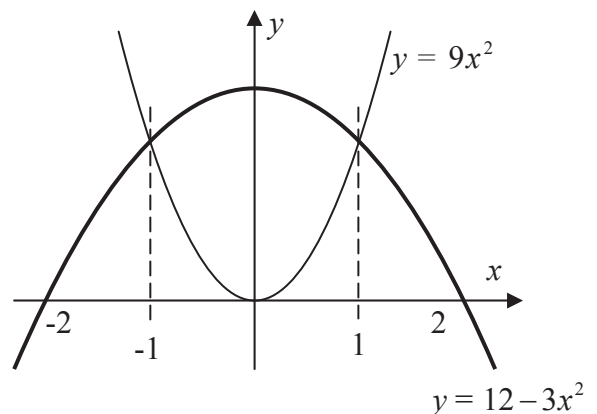


Q8(c)(ii)

$$f(x) = g(x)$$
$$12 - 3x^2 = 9x^2$$
$$12 = 12x^2$$
$$1 = x^2 \Rightarrow x = \pm 1$$

Enclosed Area

$$= \int_{-1}^1 f(x) dx - \int_{-1}^1 g(x) dx$$
$$= 2 \left[\int_0^1 (12 - 3x^2) dx - \int_0^1 9x^2 dx \right]$$
$$= 2 [12x - x^3 - 3x^3]_0^1$$
$$= 2 [12x - 4x^3]_0^1$$
$$= 2 [(12 - 4) - (0)]$$
$$= 16$$



or

Q8(c)(ii) Enclosed Area = $32 - [A_1 + A_2 + A_3 + A_4]$
 $= 32 - 2(A_3 + A_4)$

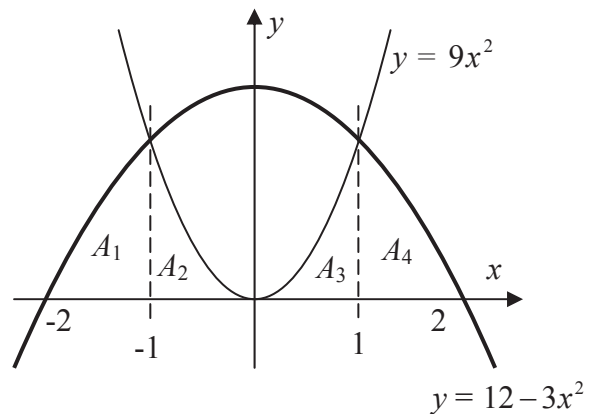
$$A_3 = \int_0^1 9x^2 dx = 3x^3 \Big|_0^1 = 3 - 0 = 3$$

$$A_4 = \int_1^2 (12 - 3x^2) dx = 12x - x^3 \Big|_1^2$$

$$= (24 - 8) - (12 - 1) = 5$$

$$\therefore 2(A_3 + A_4) = 2(3 + 5) = 16$$

$$\therefore \text{Enclosed Area} = 32 - 16 = 16$$



Blunders (-3)

- B1 Indices
- B2 Integration
- B3 Calculating roots of $f(x)=0$
- B4 Calculating $f \cap g$
- B5 Error in area formula
- B6 Incorrect order in applying limits
- B7 Not calculating substituted limits
- B8 Error with $f(x)$ or $g(x)$
- B9 Uses $\pi \int y dx$ for area formula

Attempts

- A1 Uses volume formula
- A2 Uses y^2 in formula

Worthless

- W1 Differentiation instead of integration except where other work merits attempt
- W2 Wrong area formula and no work

MARKING SCHEME

LEAVING CERTIFICATE EXAMINATION 2006

MATHEMATICS – HIGHER LEVEL – PAPER 2

GENERAL GUIDELINES FOR EXAMINERS – PAPER 2

1. Penalties of three types are applied to candidates' work as follows:

- Blunders - mathematical errors/omissions (-3)
- Slips - numerical errors (-1)
- Misreadings (provided task is not oversimplified) (-1).

Frequently occurring errors to which these penalties must be applied are listed in the scheme. They are labelled: B1, B2, B3,..., S1, S2,..., M1, M2,...etc. These lists are not exhaustive.

2. When awarding attempt marks, e.g. Att(3), note that

- any *correct, relevant* step in a part of a question merits at least the attempt mark for that part
- if deductions result in a mark which is lower than the attempt mark, then the attempt mark must be awarded
- a mark between zero and the attempt mark is never awarded.

3. Worthless work is awarded zero marks. Some examples of such work are listed in the scheme and they are labelled as W1, W2,...etc.

4. The phrase “hit or miss” means that partial marks are not awarded – the candidate receives all of the relevant marks or none.

5. The phrase “and stops” means that no more work is shown by the candidate.

6. Special notes relating to the marking of a particular part of a question are indicated by an asterisk. These notes immediately follow the box containing the relevant solution.

7. The sample solutions for each question are not intended to be exhaustive lists – there may be other correct solutions. Any examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his/her advising examiner.

8. Unless otherwise indicated in the scheme, accept the best of two or more attempts – even when attempts have been cancelled.

9. The *same* error in the *same* section of a question is penalised *once* only.

10. Particular cases, verifications and answers derived from diagrams (unless requested) qualify for attempt marks at most.

11. A serious blunder, omission or misreading results in the attempt mark at most.

12. Do not penalise the use of a comma for a decimal point, e.g. €5.50 may be written as €5,50.

QUESTION 1

| | | |
|-----------------|--------------------------|-------------------|
| Part (a) | 15 marks | Att 5 |
| Part (b) | 20 (10, 10) marks | Att (3, 3) |
| Part (c) | 15 (10,5) marks | Att (3, 2) |

Part (a) **15 marks** **Att 5**

1 (a) $a(-1, -3)$ and $b(3, 1)$ are the end-points of a diameter of a circle.
Write down the equation of the circle.

1 (a) Mid-point of $[ab] = \text{Centre of circle } c = (1, -1)$
 Radius = $|ac| = \sqrt{4+4} = \sqrt{8}$.
 \therefore Equation of circle : $(x-1)^2 + (y+1)^2 = 8$.

Blunders

- B1 Error in mid-point formula (apply once).
- B2 Error in distance formula (apply once)
- B3 Incorrect substitution for each formula.

Slips

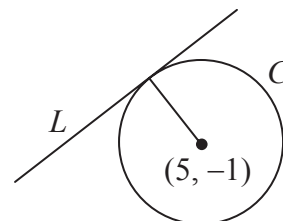
- S1 Arithmetic errors

Attempts

- A1 Radius or midpoint only
- A2 General form of equation written down and some correct substitution

Part (b) **20 (10, 10) marks** **Att (3, 3)**

(b) Circle C has centre $(5, -1)$.
The line $L: 3x - 4y + 11 = 0$ is a tangent to C .



- (i)** Show that the radius of C is 6.
- (ii)** The line $x + py + 1 = 0$ is also a tangent to C .
Find two possible values of p .

Part (b) (i) **10 marks** **Att 3**

1 (b) (i)
 Radius = Distance from centre $(5, -1)$ to line $3x - 4y + 11 = 0$.
 Radius = $\frac{|15 + 4 + 11|}{\sqrt{9+16}} = 6$.

Blunders

- B1 Error in distance formula

Slips

- S1 Arithmetic errors

Attempts

- A1 Picks point on $3x - 4y + 11 = 0$ and finds distance using $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Part (b) (ii)**10 marks****Att 3**

1 (b) (ii) Perpendicular distance from centre $(5, -1)$ to $x + py + 1 = 0$ equals 6.

$$\therefore \left| \frac{5 - p + 1}{\sqrt{1 + p^2}} \right| = 6 \Rightarrow (6 - p)^2 = 36 + 36p^2.$$

$$\therefore 35p^2 + 12p = 0 \Rightarrow p(35p + 12) = 0 \Rightarrow p = 0 \text{ or } p = -\frac{12}{35}$$

Blunders

- B1 Error in perpendicular distance formula
- B2 Error in squaring or fails to square
- B3 Error in solving quadratic

Slips

- S1 Arithmetic

Attempts

- A1 Attempts to substitute and solve for p

Part (c)**15 (10,5) marks****Att (3,2)**

1 (c) S is the circle $x^2 + y^2 + 4x + 4y - 17 = 0$ and K is the line $4x + 3y = 12$.

- (i) Show that the line K does not intersect S .
- (ii) Find the co-ordinates of the point on S that is closest to K .

Part (c) (i)**10 marks****Att 3****1 (c) (i)**

$$\text{Centre} = (-2, -2), \text{ radius} = \sqrt{g^2 + f^2 - c} = \sqrt{4 + 4 + 17} = 5.$$

$$\text{Distance from line to centre} = \left| \frac{-8 - 6 - 12}{5} \right| = \frac{26}{5} > 5. \therefore K \text{ does not intersect } S.$$

Blunders

- B1 Error centre or radius formula (each formula)
- B2 Error in perpendicular distance formula (mod omitted)
- B3 Error in squaring
- B4 No conclusion

Slips

- S1 Arithmetic

1 (c) (ii) Required point is on line containing centre and perpendicular to $4x + 3y = 12$.

$$y + 2 = \frac{3}{4}(x + 2) \Rightarrow 3x - 4y = 2. \text{ Required point is } 3x - 4y = 2 \cap S.$$

$$x = \frac{2 + 4y}{3} \Rightarrow \left(\frac{2 + 4y}{3}\right)^2 + y^2 + 4\left(\frac{2 + 4y}{3}\right) + 4y - 17 = 0.$$

$$\therefore \frac{4 + 16y + 16y^2}{9} + y^2 + \frac{8 + 16y}{3} + 4y - 17 = 0.$$

$$4 + 16y + 16y^2 + 9y^2 + 24 + 48y + 36y - 153 = 0 \Rightarrow 25y^2 + 100y - 125 = 0.$$

$$\therefore y^2 + 4y - 5 = 0 \Rightarrow (y - 1)(y + 5) = 0 \Rightarrow y = 1 \text{ or } y = -5.$$

$$\therefore (2, 1) \text{ or } (-6, -5). \text{ But } (2, 1) \text{ is closest point. } \therefore \text{Solution} = (2, 1).$$

Blunders

- B1 Error in equation of line
- B2 Error in substitution
- B3 Error in squaring
- B4 Error in forming quadratic
- B5 Error in solving quadratic
- B5 No conclusion

Slips

- S1 Arithmetic

Attempts

- A1 Writes down the centre or the radius

QUESTION 2

| | | |
|-----------------|-------------------------------|----------------------|
| Part (a) | 10 marks | Att 3 |
| Part (b) | 30 ([5,5],10,10) marks | Att (2,2,3,3) |
| Part (c) | 10(5,5)marks | Att (2,2) |

| | | |
|-----------------|-----------------|--------------|
| Part (a) | 10 marks | Att 3 |
|-----------------|-----------------|--------------|

2 (a) $\vec{x} = -3\vec{i} + \vec{j}$. Express $\left(\vec{x}^\perp\right)^\perp$ in terms of \vec{i} and \vec{j} .

| | | |
|-----------------|-----------------|--------------|
| Part (a) | 10 marks | Att 3 |
|-----------------|-----------------|--------------|

2(a)

$$\vec{x} = -3\vec{i} + \vec{j} \Rightarrow \vec{x}^\perp = \left(-3\vec{i} + \vec{j}\right)^\perp = -\vec{i} - 3\vec{j} \Rightarrow \left(\vec{x}^\perp\right)^\perp = 3\vec{i} - \vec{j}.$$

Blunders

- B1 Incorrect sign
- B2 Incorrect scalar

Attempts

- A1 Correct formula

| | | |
|-----------------|--------------------------------|-------------------------|
| Part (b) | 30 (5, 5, 10, 10) marks | Att (2, 2, 3, 3) |
|-----------------|--------------------------------|-------------------------|

(b) $\vec{p} = -5\vec{i} + 2\vec{j}$, $\vec{q} = \vec{i} - 6\vec{j}$ and $\vec{r} = -\vec{i} + 5\vec{j}$.

(i) Express \vec{pq} and \vec{pr} in terms of \vec{i} and \vec{j} .

(ii) Given that $10\vec{s} = \left|\vec{pr}\right|\vec{pq} + \left|\vec{pq}\right|\vec{pr}$, express \vec{s} in terms of \vec{i} and \vec{j} .

(iii) Find the measure of the angle between \vec{s} and \vec{pr} .

| | | |
|--|----------------|--------------|
| (i) Express \vec{pq} | 5 marks | Att 2 |
|--|----------------|--------------|

| | | |
|--------------------------------------|----------------|--------------|
| Express \vec{pr} | 5 marks | Att 2 |
|--------------------------------------|----------------|--------------|

2 (b) (i)

$$\vec{pq} = \vec{q} - \vec{p} = \vec{i} - 6\vec{j} + 5\vec{i} - 2\vec{j} = 6\vec{i} - 8\vec{j}.$$

$$\vec{pr} = \vec{r} - \vec{p} = -\vec{i} + 5\vec{j} + 5\vec{i} - 2\vec{j} = 4\vec{i} + 3\vec{j}.$$

Blunders

- B1 $\vec{pq} \neq \vec{q} - \vec{p}$ or $\vec{pr} \neq \vec{r} - \vec{p}$
- B2 Error in signs

Slips

- S1 Arithmetic

Part (b) (ii)**10 marks****Att 3****2 (b) (ii)**

$$|\vec{pr}| = |4\vec{i} + 3\vec{j}| = \sqrt{16+9} = 5 \quad \text{and} \quad |\vec{pq}| = |6\vec{i} - 8\vec{j}| = \sqrt{36+64} = 10.$$

$$\therefore 10\vec{s} = 5(6\vec{i} - 8\vec{j}) + 10(4\vec{i} + 3\vec{j}) \Rightarrow 2\vec{s} = 6\vec{i} - 8\vec{j} + 8\vec{i} + 6\vec{j} = 14\vec{i} - 2\vec{j}.$$

$$\therefore \vec{s} = 7\vec{i} - \vec{j}.$$

*Blunders*B1 Error in $|\vec{pq}|$ or in $|\vec{pr}|$ *Slips*

S1 Arithmetic

Part (b) (iii)**10 marks****Att 3****2 (b) (iii)**

$$\vec{s} \cdot \vec{pr} \Rightarrow (7\vec{i} - \vec{j}) \cdot (4\vec{i} + 3\vec{j}) = |7\vec{i} - \vec{j}| |4\vec{i} + 3\vec{j}| \cos\theta.$$

$$\therefore \cos\theta = \frac{28-3}{\sqrt{50}\sqrt{25}} = \frac{25}{25\sqrt{2}} = \frac{1}{\sqrt{2}}. \quad \therefore \theta = \frac{\pi}{4}.$$

Blunders

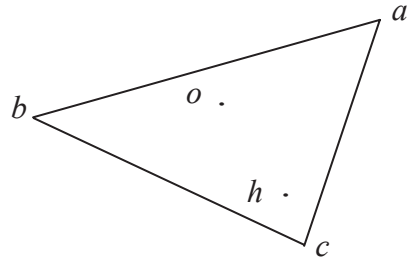
B1 Error in form of scalar product each time

B2 Error in calculating $\cos^{-1} \frac{1}{\sqrt{2}}$ *Slips*

S1 Arithmetic

(c) The origin o is the circumcentre of the triangle abc .

If $\vec{h} = \vec{a} + \vec{b} + \vec{c}$, show that $\vec{ah} \perp \vec{bc}$.



Simplify \vec{ah} and \vec{bc}

5 marks

Att 2

Finish

5 marks

Att 2

2 (c)

$$\vec{ah} \cdot \vec{bc} = (\vec{h} - \vec{a}) \cdot (\vec{c} - \vec{b}) = (\vec{b} + \vec{c}) \cdot (\vec{c} - \vec{b}) = |\vec{c}|^2 - |\vec{b}|^2.$$

But since o is the circumcentre, $|\vec{b}| = |\vec{c}|$. $\therefore \vec{ah} \cdot \vec{bc} = 0 \Rightarrow \vec{ah} \perp \vec{bc}$.

Blunders

B1 $\vec{ah} \neq \vec{h} - \vec{a}$

B2 $\vec{bc} \neq \vec{c} - \vec{b}$

B3 Error in vector multiplication

Slips

S1 Arithmetic errors

Attempts

A1 States condition for perpendicular vectors correctly

A2 $\vec{ah} = \vec{h} - \vec{a}$

QUESTION 3

| | | |
|-----------------|--------------------------|----------------|
| Part (a) | 15 marks | Att 5 |
| Part (b) | 10 marks | Att 3 |
| Part (c) | 25 (10, 15) marks | Att 3,5 |
| Part (a) | 15 marks | Att 5 |

3 (a) Show that the line containing the points (3, -6) and (-7, 12) is perpendicular to the line $5x - 9y + 6 = 0$.

| | | |
|-----------------|-----------------|--------------|
| Part (a) | 15 marks | Att 5 |
|-----------------|-----------------|--------------|

3 (a) Slope of line containing points (3, -6) and (-7, 12) is $m_1 = \frac{12+6}{-7-3} = \frac{18}{-10} = -\frac{9}{5}$.

The line $5x - 9y + 6 = 0$ has slope $m_2 = \frac{5}{9}$.

But $m_1 \cdot m_2 = -1$, \therefore lines perpendicular.

Blunders

- B1 Error in finding either slope
- B2 Product of slopes not shown = -1
- B3 No conclusion

Slips

- S1 Arithmetic

| | | |
|-----------------|-----------------|--------------|
| Part (b) | 10 marks | Att 3 |
|-----------------|-----------------|--------------|

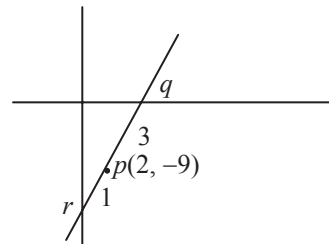
3 (b) The line K has positive slope and passes through the point p (2, -9).
 K intersects the x -axis at q and the y -axis at r and $|pq| : |pr| = 3 : 1$.
 Find the co-ordinates of q and the co-ordinates of r .

3 (b)

Equation of $K : y + 9 = m(x - 2)$. q is $\left(\frac{9}{m} + 2, 0\right)$ and r is $(0, -2m - 9)$.

$$\therefore \frac{\frac{9}{m} + 2 + 0}{4} = 2 \Rightarrow \frac{9}{m} + 2 = 8 \Rightarrow \frac{9}{m} = 6 \Rightarrow m = \frac{3}{2}$$

$\therefore q$ is (8, 0) and r is (0, -12).



Blunders

- B1 Error in finding equation of line
- B2 Error in finding q or r each time
- B3 Error in ratio formula
- B4 Error in simplification if not a slip

Slips

- S1 Arithmetic errors

Attempts

Correct formula for K for ratio for distance with some correct substitution

3 (c) (i) Prove that the measure of one of the angles between two lines

$$\text{with slopes } m_1 \text{ and } m_2 \text{ is given by } \tan \theta = \frac{m_1 - m_2}{1 + m_1 m_2}.$$

(ii) L is the line $y = 4x$ and K is the line $x = 4y$.

f is the transformation $(x, y) \rightarrow (x', y')$, where $x' = 2x - y$ and $y' = x + 3y$.

Find the measure of the acute angle between $f(L)$ and $f(K)$, correct to the nearest degree.

Part (c) (i)

10 marks

Att 3

3 (c) (i)

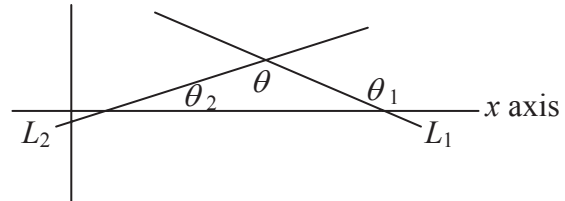
$$\text{Slope } L_1 = m_1 = \tan \theta_1.$$

$$\text{Slope } L_2 = m_2 = \tan \theta_2.$$

$$\theta_1 = \theta + \theta_2 \Rightarrow \theta = \theta_1 - \theta_2.$$

$$\therefore \tan \theta = \tan(\theta_1 - \theta_2) = \frac{\tan \theta_1 - \tan \theta_2}{1 + \tan \theta_1 \tan \theta_2}$$

$$\therefore \tan \theta = \frac{m_1 - m_2}{1 + m_1 m_2}.$$

*Blunders*B1 Error in expressing θ in terms of θ_1 and θ_2 B2 Error in $\tan(\theta_1 - \theta_2)$

Part (c) (ii)

15 marks

Att 5

$$3x' = 6x - 3y$$

$$y' = x + 3y$$

$$7x = 3x' + y' \Rightarrow x = \frac{1}{7}(3x' + y'). \text{ But } y' = x + 3y \Rightarrow y' = \frac{1}{7}(3x' + y') + 3y.$$

$$\therefore y = \frac{1}{7}(-x' + 2y').$$

$$f(L): \frac{1}{7}(-x' + 2y') = \frac{4}{7}(3x' + y'). \therefore f(L): 2y' = -13x' \Rightarrow \text{slope } f(L) = -\frac{13}{2}.$$

$$f(K): \frac{1}{7}(3x' + y') = \frac{4}{7}(-x' + 2y'). \therefore f(K): y' = x' \Rightarrow \text{slope } f(K) = 1.$$

$$\tan \theta = \frac{-\frac{13}{2} - 1}{1 - \frac{13}{2}} = \frac{15}{11} \Rightarrow \theta = 54^\circ.$$

Blunders

B1 Error in setting up/solving simultaneous equations each time

B2 Error in calculating slope for $F(L)$ and $F(K)$ each time for different blunder

B3 Error in applying formula

B4 Error in calculating $\tan^{-1} \frac{15}{11}$ *Slips*

S1 Arithmetic errors

QUESTION 4

| | | |
|-----------------|--------------------------|------------------|
| Part (a) | 15 marks | Att 5 |
| Part (b) | 20 (10, 10) marks | Att (3,3) |
| Part (c) | 15 (10,5) marks | Att (3,2) |

| | | |
|-----------------|-----------------|--------------|
| Part (a) | 15 marks | Att 5 |
|-----------------|-----------------|--------------|

4 (a) Write down the values of A for which $\cos A = \frac{1}{2}$, where $0^\circ \leq A \leq 360^\circ$.

4 (a) $\cos A = \frac{1}{2} \Rightarrow A = 60^\circ, 300^\circ$.

Blunders

B1 Each incorrect or omitted value

| | | |
|-----------------|--------------------------|------------------|
| Part (b) | 20 (10, 10) marks | Att (3,3) |
|-----------------|--------------------------|------------------|

4 (b) (i) Express $\sin(3x + 60^\circ) - \sin x$ as a product of sine and cosine.
(ii) Find all the solutions of the equation
 $\sin(3x + 60^\circ) - \sin x = 0$, where $0^\circ \leq x \leq 360^\circ$.

| | | |
|---------------------|-----------------|--------------|
| Part (b) (i) | 10 marks | Att 3 |
|---------------------|-----------------|--------------|

4 (b) (i) $\sin(3x + 60^\circ) - \sin x = 2\cos(2x + 30^\circ)\sin(x + 30^\circ)$

Blunders

B1 Error in simplifying the expression

| | | |
|----------------------|-----------------|--------------|
| Part (b) (ii) | 10 marks | Att 3 |
|----------------------|-----------------|--------------|

4 (b) (ii)

$$\sin(3x + 60^\circ) - \sin x = 0 \Rightarrow 2\cos(2x + 30^\circ)\sin(x + 30^\circ) = 0$$
$$\therefore \cos(2x + 30^\circ) = 0 \text{ or } \sin(x + 30^\circ) = 0$$
$$2x + 30^\circ = 90^\circ, 270^\circ, 450^\circ, 630^\circ \text{ or } x + 30^\circ = 180^\circ, 360^\circ$$
$$\therefore x = 30^\circ, 120^\circ, 210^\circ, 300^\circ \text{ or } x = 150^\circ, 330^\circ$$
$$\therefore \text{Solution} = \{30^\circ, 120^\circ, 150^\circ, 210^\circ, 300^\circ, 330^\circ\}$$

Blunders

B1 Error in solving

B2 Each incorrect or missing solution

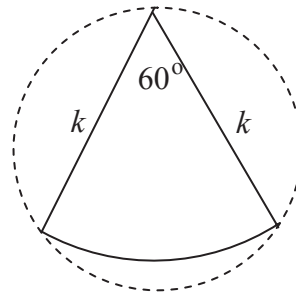
Part (c)

15 (10, 5) marks

Att (3, 2)

4 (c) The diagram shows a sector (solid line) circumscribed by a circle (dashed line).

- (i) Find the radius of the circle in terms of k .
- (ii) Show that the circle encloses an area which is double that of the sector.



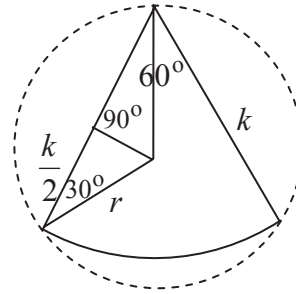
Part (c) (i)

10 marks

Att 3

4 (c) (i)

$$\cos 30^\circ = \frac{\frac{k}{2}}{r} \Rightarrow \frac{r\sqrt{3}}{2} = \frac{k}{2}$$
$$\Rightarrow r = \frac{k}{\sqrt{3}}$$



Blunders

B1 Incorrect use of cosine rule or sine rule or area of triangle.

B2 Error in cos from right-angled triangle

Part (c) (ii)

5 marks

Att 2

4 (c) (ii)

$$\text{Area of circle} = \pi r^2 = \frac{\pi k^2}{3}$$

$$\text{Area of sector} = \frac{1}{2} k^2 \theta = \frac{k^2}{2} \cdot \frac{\pi}{3} = \frac{\pi k^2}{6}$$

\therefore Area of circle = $2 \times$ area of sector.

Blunders

B1 Error in area of sector

B2 Error in area of circle

B3 No conclusion

Slips

S1 Arithmetic

QUESTION 5

Part (a)

25 (10, 5, 5, 5) marks

Att (3, 2, 2, 2)

Part (b)

25 (15, 10) marks

Att (5, 3)

Part (a)

25 (10, 5, 5, 5) marks

Att (3, 2, 2, 2)

5 (a) (i) Copy and complete the table below for $f : x \rightarrow \tan^{-1} x$, giving the values for $f(x)$ in terms of π .

| | | | | | | | |
|--------|-------------|------|-----------------------|-----|----------------------|-----------------|------------|
| x | $-\sqrt{3}$ | -1 | $-\frac{1}{\sqrt{3}}$ | 0 | $\frac{1}{\sqrt{3}}$ | 1 | $\sqrt{3}$ |
| $f(x)$ | | | | | | $\frac{\pi}{4}$ | |

(ii) Draw the graph of $y = f(x)$ in the domain $-2 \leq x \leq 2$, scaling the y -axis in terms of π .

(iii) Draw the two horizontal asymptotes of the graph.

(iv) For some values of $k \in \mathbf{R}$, but not all values, $\tan^{-1}(\tan k) = k$.

State the range of values of k for which $\tan^{-1}(\tan k) = k$.

Show, by means of an example, what happens outside the range.

Part (a) (i)

10 marks

Att 3

5 (a) (i)

| | | | | | | | |
|--------|------------------|------------------|-----------------------|-----|----------------------|-----------------|-----------------|
| x | $-\sqrt{3}$ | -1 | $-\frac{1}{\sqrt{3}}$ | 0 | $\frac{1}{\sqrt{3}}$ | 1 | $\sqrt{3}$ |
| $f(x)$ | $-\frac{\pi}{3}$ | $-\frac{\pi}{4}$ | $-\frac{\pi}{6}$ | 0 | $\frac{\pi}{6}$ | $\frac{\pi}{4}$ | $\frac{\pi}{3}$ |

Mark as follows:

| | | | | | | |
|-----------------------|-----|-----|---|---|---|----|
| No. of values correct | 1 | 2 | 3 | 4 | 5 | 6 |
| Mark | Att | Att | 7 | 8 | 9 | 10 |

Part (a) (ii)

5 marks

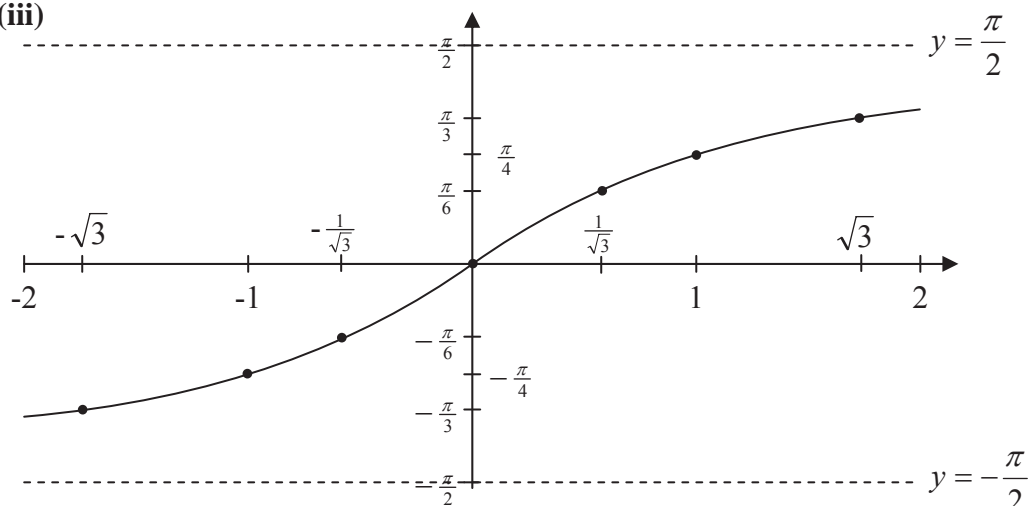
Att 2

Part (a) (iii)

5 marks

Att 2

5 (a) (ii) & (iii)



Blunders

B1 Asymptotes not horizontal

B2 Asymptotes do not contain $-\frac{\pi}{2}$ and $\frac{\pi}{2}$

Slips

S1 Each incorrectly plotted point

Part (a) (iv)

5 marks

Att 2

5 (a) (iv)

Range of values: $-\frac{\pi}{2} < k < \frac{\pi}{2}$.

e.g. if $k = \frac{5\pi}{4}$, then $\tan^{-1}\left(\tan \frac{5\pi}{4}\right) = \tan^{-1}1 = \frac{\pi}{4} \neq \frac{5\pi}{4}$.

Blunders

B1 Incorrect endpoint of range each time

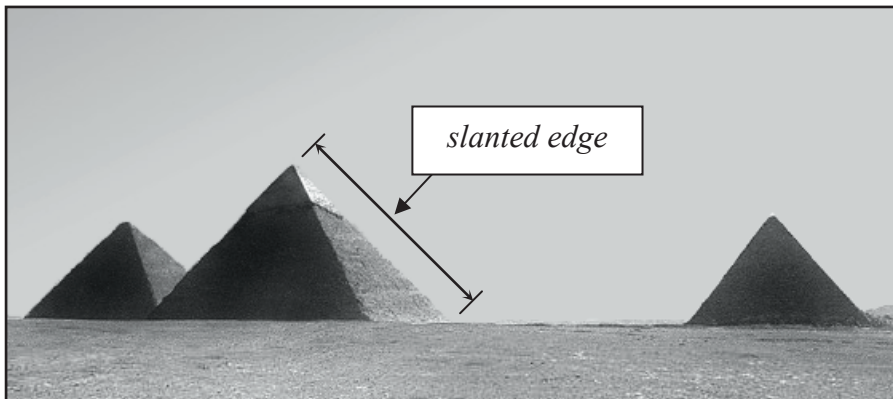
B2 No example given

Part (b)

25 (15, 10) marks

Att (5, 3)

5 (b) The great pyramid at Giza in Egypt has a square base and four triangular faces. The base of the pyramid is of side 230 metres and the pyramid is 146 metres high. The top of the pyramid is directly above the centre of the base.



- (i) Calculate the length of one of the slanted edges, correct to the nearest metre.
- (ii) Calculate, correct to two significant figures, the total area of the four triangular faces of the pyramid (assuming they are smooth flat surfaces).

Part (b) (i)**15 marks****Att 5**

5 (b) (i) Diagonal of square base = $d = \sqrt{230^2 + 230^2} = \sqrt{105800}$.

Let length of slant edge = s and height of pyramid = h .

$$s^2 = h^2 + \left(\frac{1}{2}d\right)^2 \Rightarrow s = \sqrt{21316 + 26450} = \sqrt{47766} = 218.5 = 219 \text{ metres.}$$

Blunders

B1 Incorrect application of Pythagoras each time or incorrect trig ratio

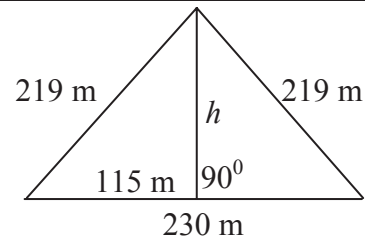
Slips

S1 Error in calculations

Part (b) (ii)**10 marks****Att 3****5 (b) (ii)**

$$h = \sqrt{219^2 - 115^2} = \sqrt{47961 - 13225}$$

$$h = \sqrt{34736} = 186.37.$$



$$\text{Total surface area} = 4 \times \frac{1}{2}(230) \cdot 186.37 = 85730.2 = 86000 \text{ m}^2.$$

Blunders

B1 Error in Pythagoras or in area of triangle formula

Slips

Arithmetic errors or failure to round off.

QUESTION 6

| | | |
|-----------------|---------------------------------|----------------------------|
| Part (a) | 10 (5, 5) marks | Att (-, 2) |
| Part (b) | 25 (5, 5, 5, 5, 5) marks | Att (2, 2, 2, 2, 2) |
| Part (c) | 15 (5, 5, 5) marks | Att (2, 2, 2) |

Part (a) **10 (5, 5) marks** **Att (-, 2)**

- 6 (a)** **(i)** How many different teams of three people can be chosen from a panel of six boys and five girls?
- (ii)** If the team is chosen at random, find the probability that it consists of girls only?

Part (a) (i) **5 marks** **Hit/Miss**

6 (a) (i) Answer = ${}^{11}C_3 = 165$.

Part (a) (ii) **5 marks** **Att 2**

6 (a) (ii) 3 girls to choose from 5 \Rightarrow Solution = ${}^5C_3 = 10$.

$$P(\text{all girls}) = \frac{10}{165} = \frac{2}{33}$$

Blunders

- B1 Incorrect total possible
 B2 Incorrect total favourable

Slips

- S1 Arithmetic errors

Part (b) **25 (5, 5, 5, 5, 5) marks** **Att (2, 2, 2, 2, 2)**

- 6 (b)** **(i)** Solve the difference equation $6u_{n+2} - 7u_{n+1} + u_n = 0$, where $n \geq 0$, given that $u_0 = 8$ and $u_1 = 3$.
- (ii)** Verify that the solution to part (i) also satisfies the difference equation $6u_{n+1} - u_n - 10 = 0$.

(b) (i) Char. Eqn. **5 marks** **Att 2**
Roots **5 marks** **Att 2**
Sim. Eqns. **5 marks** **Att 2**
Finish **5 marks** **Att 2**

6 (b) (i)

$$6u_{n+2} - 7u_{n+1} + u_n = 0.$$

$$\therefore 6x^2 - 7x + 1 = 0 \Rightarrow (x-1)(6x-1) = 0 \Rightarrow x = 1 \text{ or } x = \frac{1}{6}.$$

$$\therefore u_n = l(1)^n + m\left(\frac{1}{6}\right)^n = l + m\left(\frac{1}{6}\right)^n.$$

$$u_0 = 8 \Rightarrow l + m = 8 \text{ and } u_1 = 3 \Rightarrow l + \frac{1}{6}m = 3.$$

$$\frac{5}{6}m = 5 \Rightarrow m = 6 \text{ and } l = 2. \therefore u_n = 2 + 6\left(\frac{1}{6}\right)^n = 2 + \left(\frac{1}{6}\right)^{n-1}.$$

Blunders

- B1 Error in characteristic equation
- B2 Error in factors or quadratic formula
- B3 Incorrect use of initial conditions

Slips

- S1 Arithmetic errors

Attempts

- A1 Char equation
- A2 Eqn in l and m

Part (b) (ii)

5 marks

Att 2

6 (b) (ii)

$$u_n = 2 + \left(\frac{1}{6}\right)^{n-1} \quad \text{and} \quad 6u_{n+1} - u_n - 10 = 0.$$

$$\therefore 6 \left[2 + \left(\frac{1}{6}\right)^n \right] - \left[2 + \left(\frac{1}{6}\right)^{n-1} \right] - 10 = 12 + \left(\frac{1}{6}\right)^{n-1} - 2 - \left(\frac{1}{6}\right)^{n-1} - 10 = 0. \quad \therefore \text{solution.}$$

Blunders

- B1 Error in U_{n+1} or U_n
- B2 Error in indices

Part (c)**15 (5, 5, 5) marks****Att (2, 2, 2)**

- 6 (c)** There are thirty days in June. Seven students have their birthdays in June. The birthdays are independent of each other and all dates are equally likely.
- (i)** What is the probability that all seven students have the same birthday?
- (ii)** What is the probability that all seven students have different birthdays?
- (iii)** Show that the probability that at least two have the same birthday is greater than 0.5.

Part (c) (i)**5 marks****Att 2**

6 (c) (i)
$$P = \frac{\text{total favourable}}{\text{total possible}} = \frac{30 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1}{30 \times 30 \times 30 \times 30 \times 30 \times 30 \times 30} = \frac{1}{(30)^6}$$

Blunders

- B1 Incorrect total possible
 B2 Incorrect total favourable
 B3 No fraction

Part (c) (ii)**5 marks****Att 2**

6 (c) (ii)
$$P = \frac{\text{total favourable}}{\text{total possible}} = \frac{30 \times 29 \times 28 \times 27 \times 26 \times 25 \times 24}{30 \times 30 \times 30 \times 30 \times 30 \times 30 \times 30} = \frac{2639}{5625}$$

Blunders

- B1 Incorrect total possible
 B2 Incorrect total favourable
 B3 No fraction

Part (c) (iii)**5 marks****Att 2**

6 (c) (iii)

$$\begin{aligned} \text{Probability} &= 1 - P(\text{all seven have different birthdays}) \\ &= 1 - \frac{29 \times 28 \times 27 \times 26 \times 25 \times 24}{(30)^6} = 1 - 0.4691 = 0.5309 > 0.5. \end{aligned}$$

Blunders

- B1 Error in correct total possible
 B2 Error in correct total favourable
 B3 No conclusion

QUESTION 7

| | | |
|-----------------|------------------------------|-------------------------|
| Part (a) | 10 (5, 5) marks | Att (2, 2) |
| Part (b) | 25 (5,10, 5, 5) marks | Att (-, 3, 2, 2) |
| Part (c) | 15 marks | Att 5 |

| | | |
|-----------------|------------------------|-------------------|
| Part (a) | 10 (5, 5) marks | Att (2, 2) |
|-----------------|------------------------|-------------------|

- 7 (a)** The password for a mobile phone consists of five digits.
- (i)** How many passwords are possible?
 - (ii)** How many of these passwords start with a 2 and finish with an odd digit?

| | | |
|---------------------|----------------|--------------|
| Part (a) (i) | 5 marks | Att 2 |
|---------------------|----------------|--------------|

- 7 (a) (i)** Number of possible passwords = $10^5 = 100,000$

Slips

S1 Gives $9^5 = 59049$

| | | |
|----------------------|----------------|--------------|
| Part (a) (ii) | 5 marks | Att 2 |
|----------------------|----------------|--------------|

- 7 (a) (ii)** Number of passwords = $1 \times 10^3 \times 5 = 5,000$.

Blunders

B1 Adds $1 + 10^3 + 5$

| | | |
|-----------------|-------------------------------|-------------------------|
| Part (b) | 25 (5, 10, 5, 5) marks | Att (-, 3, 2, 2) |
|-----------------|-------------------------------|-------------------------|

- 7 (b)** For a lottery, 35 cards numbered 1 to 35 are placed in a drum. Five cards will be chosen at random from the drum as a winning combination.
- (i)** How many different combinations are possible?
 - (ii)** How many of all the possible combinations will match exactly four numbers with the winning combination?
 - (iii)** How many of all the possible combinations will match exactly three numbers with the winning combination?
 - (iv)** Show that the probability of matching at least three numbers with the winning combination is approximately 0.014.

| | | |
|---------------------|----------------|-----------------|
| Part (b) (i) | 5 marks | Hit/Miss |
|---------------------|----------------|-----------------|

- 7 (b) (i)** Number of different possible combinations = ${}^{35}C_5 = 324,632$.

| | | |
|----------------------|-----------------|--------------|
| Part (b) (ii) | 10 marks | Att 3 |
|----------------------|-----------------|--------------|

- 7 (b) (ii)** Match four = ${}^5C_4 \times {}^{30}C_1 = 150$.

Blunders

B1 Addition for multiplication

B2 31 for 30

Part (b) (iii)**5 marks****Att 2**

| | |
|--------------------|--|
| 7 (b) (iii) | Match three = ${}^5C_3 \times {}^{30}C_2 = 4350$. |
|--------------------|--|

Blunders

B1 Addition for multiplication

B2 32 for 30

Part (b) (iv)**5 marks****Att 2**

| | |
|-------------------|--|
| 7 (b) (iv) | P(of matching at least three numbers) = P(matching three) + P(matching four) + P(matching five) $= \frac{4350 + 150 + 1}{324632} = \frac{4501}{324632} = 0.01386493 = 0.014$. |
|-------------------|--|

Blunders

B1 Error in total favourable

B2 Error in total possible

B3 No fraction

B4 Incorrect or no conclusion

Part (c)**15 marks****Att 5**

| | |
|--------------|--|
| 7 (c) | The mean of the integers from $-n$ to n , inclusive, is 0. |
|--------------|--|

| | |
|--|---|
| | Show that the standard deviation is $\sqrt{\frac{n(n+1)}{3}}$. |
|--|---|

Part (c)**15 marks****Att 5****7 (c)**

$$\sigma^2 = \frac{(-n)^2 + (-n+1)^2 + \dots + (-2)^2 + (-1)^2 + (0)^2 + (1)^2 + (2)^2 + \dots + (n-2)^2 + (n-1)^2 + (n)^2}{2n+1}$$

$$\therefore \sigma^2 = \frac{2[1^2 + 2^2 + 3^2 + \dots + n^2]}{2n+1} = \frac{2}{2n+1} \times \frac{n}{6} (n+1)(2n+1) = \frac{n(n+1)}{3}$$

$$\therefore \sigma = \sqrt{\frac{n(n+1)}{3}}$$

Blunders

B1 Not squared

B2 No square root

B3 Mean not found or incorrect denominator

B5 Error in sum to n terms

QUESTION 8

| | | |
|-----------------|--------------------------|-------------------|
| Part (a) | 15 marks | Att 5 |
| Part (b) | 15 (10, 5) marks | Att (3,2) |
| Part (c) | 20 (10, 10) marks | Att (3, 3) |

Part (a) **15 marks** **Att 5**

8 (a) Derive the Maclaurin series for $f(x) = e^x$ up to and including the term containing x^3 .

8 (a)

$$f(x) = f(0) + \frac{f'(0)x}{1!} + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} + \dots$$

$$f(x) = e^x \Rightarrow f(0) = e^0 = 1.$$

$$f'(x) = e^x \Rightarrow f'(0) = 1.$$

$$f''(x) = e^x \Rightarrow f''(0) = 1.$$

$$f'''(x) = e^x \Rightarrow f'''(0) = 1.$$

$$\therefore f(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

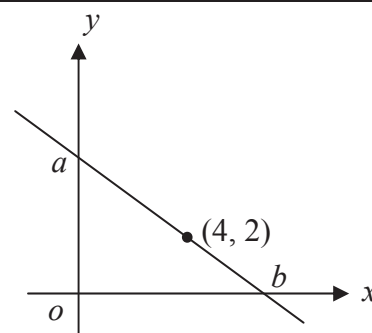
Blunders

- B1 Error in differentiation each time if error not consistent
- B2 Error in evaluating $f^{(n)}(0)$
- B3 Each term missing

Part (b) **15 (10, 5) marks** **Att (3,2)**

8 (b) A line passes through the point (4, 2) and has slope m , where $m < 0$. The line intersects the axes at the points a and b .

- (i) Find the co-ordinates of a and b , in terms of m .
- (ii) Hence, find the value of m for which the area of triangle aob is a minimum.



Part (b) (i) **10 marks** **Att 3**

8 (b) (i) Equation of line $y - 2 = m(x - 4) \Rightarrow mx - y = 4m - 2$.

$$\therefore a(0, -4m + 2) \text{ and } b\left(4 - \frac{2}{m}, 0\right).$$

Blunders

- B1 Error in finding the equation of the line
- B2 a and b coordinates must have 0 in correct position
- B3 Not in coordinate form

8 (b) (ii)

$$a(0, -4m + 2), b\left(4 - \frac{2}{m}, 0\right).$$

$$\text{Area of triangle} = A = \frac{1}{2}\left(4 - \frac{2}{m}\right)(-4m + 2).$$

$$A = \frac{1}{2}\left(-16m + 8 + 8 - \frac{4}{m}\right) = \frac{1}{2}(-16m + 16 - 4m^{-1}) = -8m + 8 - 2m^{-1}.$$

$$\frac{dA}{dm} = -8 + 2m^{-2} = 0, \text{ for minimum.}$$

$$\therefore -4 + \frac{1}{m^2} = 0 \Rightarrow 4m^2 = 1 \Rightarrow m = -\frac{1}{2} \text{ as } m < 0.$$

$$\therefore A = -8(-0.5) + 8 - \frac{2}{-0.5} = 4 + 8 + 4 = 16, \text{ minimum area.}$$

Blunders

- B1 Error in formula for area of a triangle
 B2 Error in derivative
 B3 Error in solving equation

Slips

- S1 Arithmetic errors

Part (c)

20 (10, 10) marks

Att (3, 3)

8 (c) Use the ratio test to test each of the following series for convergence.

In each case, specify clearly the range of values of x for which the series converges, the range of values for which it diverges, and the value(s) of x for which the test is inconclusive.

$$(i) \sum_{n=1}^{\infty} n3^n x^n \quad (ii) \sum_{n=1}^{\infty} \frac{(n+1)!n!}{(2n)!} x^n.$$

Part (c) (i)

10 marks

Att 3

$$u_n = n3^n x^n \Rightarrow u_{n+1} = (n+1)3^{n+1} x^{n+1}.$$

$$\text{Lim}_{n \rightarrow \infty} \left| \frac{u_{n+1}}{u_n} \right| = \text{Lim}_{n \rightarrow \infty} \left| \frac{(n+1)3^{n+1} x^{n+1}}{n3^n x^n} \right| = \text{Lim}_{n \rightarrow \infty} \left| 3x \left(1 + \frac{1}{n}\right) \right| = |3x|.$$

$$\text{Converges for } |3x| < 1 \Rightarrow -\frac{1}{3} < x < \frac{1}{3}.$$

$$\text{Diverges for } |3x| > 1 \Rightarrow x > \frac{1}{3} \text{ or } x < -\frac{1}{3}.$$

$$\text{Inconclusive for } |3x| = 1 \text{ i.e. } x = \pm \frac{1}{3}.$$

Blunders

B1 Error in U_{n+1}

B2 Error in evaluating $\frac{U_{n+1}}{U_n}$

B3 Error in evaluating limit

B4 Range incorrectly or not applied

Misreading (-1) for each case omitted

Part (c) (ii)

10 marks

Att 3

8 (c) (ii) $\sum_{n=1}^{\infty} \frac{(n+1)!n!}{(2n)!} x^n.$

$$u_n = \frac{(n+1)!n!}{(2n)!} x^n \Rightarrow u_{n+1} = \frac{(n+2)!(n+1)!}{(2n+2)!} x^{n+1}.$$

$$\lim_{n \rightarrow \infty} \left| \frac{u_{n+1}}{u_n} \right| = \lim_{n \rightarrow \infty} \left| \frac{(n+2)!(n+1)!x^{n+1}}{(2n+2)!} \times \frac{(2n)!}{(n+1)!n!x^n} \right|$$

$$= \lim_{n \rightarrow \infty} \left| \frac{(n+2)(n+1)x}{(2n+2)(2n+1)} \right| = \lim_{n \rightarrow \infty} \left| \frac{\left(1 + \frac{3}{n} + \frac{2}{n^2}\right)x}{4 + \frac{6}{n} + \frac{2}{n^2}} \right| = \left| \frac{x}{4} \right|.$$

Converges for $\left| \frac{x}{4} \right| < 1 \Rightarrow -4 < x < 4.$

Dirverges for $\left| \frac{x}{4} \right| > 1 \Rightarrow x > 4$ or $x < -4.$

Inconclusive for $\left| \frac{x}{4} \right| = 1$ i.e. $x = \pm 4.$

Blunders

B1 Error in U_{n+1}

B2 Error in evaluating $\frac{U_{n+1}}{U_n}$

B3 Error in evaluating limit

B4 Range incorrectly or not applied

QUESTION 9

| | | |
|-----------------|------------------------------|-------------------------|
| Part (a) | 10 marks | Att 3 |
| Part (b) | 20 (5, 5, 5, 5) marks | Att (2, 2, 2, 2) |
| Part (c) | 20 (5, 5, 5, 5) marks | Att (2, 2, 2, 2) |

| | | |
|-----------------|-----------------|--------------|
| Part (a) | 10 marks | Att 3 |
|-----------------|-----------------|--------------|

9 (a) z is a random variable with standard normal distribution.
Find the value of z_1 for which $P(z > z_1) = 0.0808$.

| | | |
|-----------------|-----------------|--------------|
| Part (a) | 10 marks | Att 3 |
|-----------------|-----------------|--------------|

9 (a)

$$P(z > z_1) = 0.0808 \Rightarrow 1 - P(z < z_1) = 0.0808.$$

$$\therefore P(z < z_1) = 0.9192 \Rightarrow z_1 = 1.4.$$

Blunders

B1 Incorrect reading of tables or incorrect area each time

| | | |
|-----------------|------------------------------|-------------------------|
| Part (b) | 20 (5, 5, 5, 5) marks | Att (2, 2, 2, 2) |
|-----------------|------------------------------|-------------------------|

9 (b) A bag contains the following cardboard shapes:
10 red squares, 15 green squares, 8 red triangles and 12 green triangles.
One of the shapes is drawn at random from the bag.
 E is the event that a square is drawn.
 F is the event that a green shape is drawn.

- (i) Find $P(E \cap F)$.
- (ii) Find $P(E \cup F)$.
- (iii) State whether E and F are independent events, giving a reason for your answer.
- (iv) State whether E and F are mutually exclusive events, giving a reason for your answer.

| | | |
|---------------------|----------------|--------------|
| Part (b) (i) | 5 marks | Att 2 |
|---------------------|----------------|--------------|

9 (b) (i) $P(E \cap F) = \frac{15}{45}$.

Blunders

B1 Incorrect total possible
B2 Incorrect total favourable
B3 No fraction

| | | |
|----------------------|----------------|--------------|
| Part (b) (ii) | 5 marks | Att 2 |
|----------------------|----------------|--------------|

9 (b) (ii) $P(E \cup F) = P(E) + P(F) - P(E \cap F) = \frac{25}{45} + \frac{27}{45} - \frac{15}{45} = \frac{37}{45}$.

Blunders

B1 Incorrect total possible
B2 Incorrect total favourable
B3 No fraction

Part (b) (iii)

5 marks

Att 2

$$9 \text{ (b) (iii)} \quad P(E) \cdot P(F) = \frac{25}{45} \cdot \frac{27}{45} = \frac{15}{45} = P(E \cap F). \quad \therefore \text{Independent events.}$$

Blunders

B1 reason not given

Part (b) (iv)

5 marks

Att 2

$$9 \text{ (b) (iv)} \quad P(E \cap F) \neq 0 \text{ hence not mutually exclusive}$$

Part (c)

20 (5,5, 5,5) marks

Att (2,2,2,2)

9 (c) The marks awarded in an examination are normally distributed with a mean mark of 60 and a standard deviation of 10.
A sample of 50 students has a mean mark of 63.
Test, at the 5% level of significance, the hypothesis that this is a random sample from the population.

Part (c)

20 (5,5, 5,5) marks

Att (2,2,2,2)

9 (c)

$$\bar{x} = 60, \sigma = 10 \Rightarrow \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{10}{\sqrt{50}} = 1.4142.$$

$$\frac{x - \bar{x}}{\sigma_{\bar{x}}} = \frac{63 - 60}{1.4142} = 2.1213 > 1.96. \quad \therefore \text{Not a random sample.}$$

Blunders

B1 Error in standard error of mean

B2 Error in finding Z value

B3 Error in confidence interval

Slips

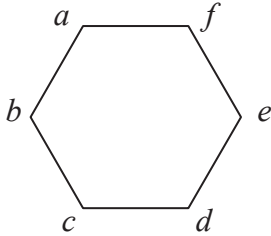
S1 Arithmetic errors

QUESTION 10

| | | |
|-----------------|--------------------------------|-------------------------|
| Part (a) | 30 (10, 5, 10, 5) marks | Att (3, 2, 3, 2) |
| Part (b) | 20 (10, 10) marks | Att (3, 3) |

Part (a) **30 (10, 5, 10,5) marks** **Att (3, 2, 3,2)**

10 (a) G is the set of rotations that map a regular hexagon onto itself.
 (G, \circ) is a group, where \circ denotes composition.
 The anti-clockwise rotation through 60° is written as R_{60° .



(i) List the elements of G .
 (ii) State which elements of the group, if any, are generators.
 (iii) List all the proper subgroups of (G, \circ) .
 (iv) Find $Z(G)$, the centre of (G, \circ) . Justify your answer.

Part (a) (i) **10 marks** **Att 3**

10 (a) (i) $G = \{R_{0^\circ}, R_{60^\circ}, R_{120^\circ}, R_{180^\circ}, R_{240^\circ}, R_{300^\circ}\}$

Blunders

B1 Each one omitted or incorrect

Part (a) (ii) **5 marks** **Att 2**

10 (a) (ii) Generators are R_{60° and R_{300° .

Blunders

B1 Each one omitted

Part (a) (iii) **10 marks** **Att 3**

10 (a) (iii) Proper subgroups of (G, \circ) are $\{R_{0^\circ}, R_{180^\circ}\}$, and $\{R_{0^\circ}, R_{120^\circ}, R_{240^\circ}\}$.

Blunders

B1 Each one omitted or incorrect

Part (a) (iv) **5 marks** **Att 2**

10 (a) (iv) Each element of G commutes with each of the elements of G .
 $\therefore Z(G) = G$.

Blunders

B1 $Z(G)$ not found

B3 Error in justification

Part (b)**20 (10, 10) marks****Att (3, 3)**

- 10 (b) (i)** Show that the group $\left\{ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \right\}$ under matrix multiplication is isomorphic to the group $\{0, 1\}$ under addition modulo 2.
- (ii)** Prove that any infinite cyclic group is isomorphic to $(\mathbf{Z}, +)$.

Part (b) (i)**10 marks****Att 3**

- 10 (b) (i)** Let $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ and $A = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$. Let $G = \{I, A\}$ under matrix multiplication.

$$A^2 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$$

Thus I is of order 1 and A is of order 2.

Let $H = \{0, 1\}$ under addition modulo 2.

$$1+1=2=0 \pmod{2}.$$

Thus 0 is of order 1 and 1 is of order 2.

As G and H both have one element of order 1 and one element of order 2, they are isomorphic. The isomorphism $G \rightarrow H$ is

$$I \rightarrow 0$$

$$A \rightarrow 1.$$

Blunders

B1 Any property of isomorphism not included

Attempts

A1 Elements of H found

Part (b) (ii)**10 marks****Att 3**

- 10 (b) (ii)** Let $G = \langle g \rangle$ be any infinite cyclic group generated by g under $*$.

Define $\phi : (G, *) \rightarrow (\mathbf{Z}, +) : g^k \rightarrow k$.

$$\begin{aligned} \phi(g^a * g^b) &= \phi(g^{a+b}) = a+b \\ &= \phi(g^a) + \phi(g^b). \end{aligned}$$

$\therefore \phi$ is an isomorphism $\Rightarrow (G, *)$ and $(\mathbf{Z}, +)$ are isomorphic.

Blunders

B1 Error with indices

B2 No conclusion

QUESTION 11

| | | |
|-----------------|--------------------------|-------------------|
| Part (a) | 10 (5, 5) marks | Att (2, 2) |
| Part (b) | 20 (10, 10) marks | Att (3, 3) |
| Part (c) | 20 marks | Att 6 |

Part (a) **10 (5, 5) marks** **Att (2, 2)**

11 (a) (i) Find the image of $a(-1, 2)$ and $b(0, 4)$ under the transformation

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 2 \\ -4 \end{pmatrix}.$$

(ii) Show that ab is parallel to $a'b'$.

Part (a) (i) **5 marks** **Att 2**

11 (a) (i)

$$\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} -1 \\ 2 \end{pmatrix} + \begin{pmatrix} 2 \\ -4 \end{pmatrix} = \begin{pmatrix} -2 \\ 4 \end{pmatrix} + \begin{pmatrix} 2 \\ -4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}.$$

$$\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} 0 \\ 4 \end{pmatrix} + \begin{pmatrix} 2 \\ -4 \end{pmatrix} = \begin{pmatrix} 0 \\ 8 \end{pmatrix} + \begin{pmatrix} 2 \\ -4 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}.$$

Blunders

B1 Error in matrix multiplication

B2 Error in addition

Part (a) (ii) **5 marks** **Att 2**

11 (a) (ii) Slope $ab = \frac{4-2}{0-1} = 2$ and slope $a'b' = \frac{4-0}{2-0} = 2$. $\therefore ab$ is parallel to $a'b'$.

Blunders

B1 Error in slope formula

B2 No conclusion

Part (b) **20 (10, 10) marks** **Att (3, 3)**

11 (b) $p(x, y)$ is a point such that the distance from p to the point $(2, 0)$ is half the distance from p to the line $x = 8$.

(i) Find the equation of the locus of p .

(ii) Show that this locus is an ellipse centred at the origin, by expressing

its equation in the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

Part (b) (i) **10 marks** **Att 3**

11 (b) (i)

$$p(x, y), s(2, 0). \quad \therefore |ps| = \frac{1}{2} \sqrt{(x-2)^2 + (y-0)^2}.$$

$$\text{Distance from } p(x, y) \text{ to line } x = 8 \text{ is } \left| \frac{8-x}{1} \right|.$$

$$\therefore \text{Locus of } p : 3x^2 + 4y^2 = 48.$$

Blunders

B1 Error in each distance formula

Part (b) (ii)

10 marks

Att 3

11 (b) (ii)

$$\begin{aligned}
3x^2 + 4y^2 &= 48 \\
\Rightarrow \frac{3x^2}{48} + \frac{4y^2}{48} &= 1 \\
\Rightarrow \frac{x^2}{16} + \frac{y^2}{12} &= 1
\end{aligned}$$

Blunders

- B1 Error in format of ellipse equation
- B2 Error in squaring

Part (c)

20 marks

Att 6

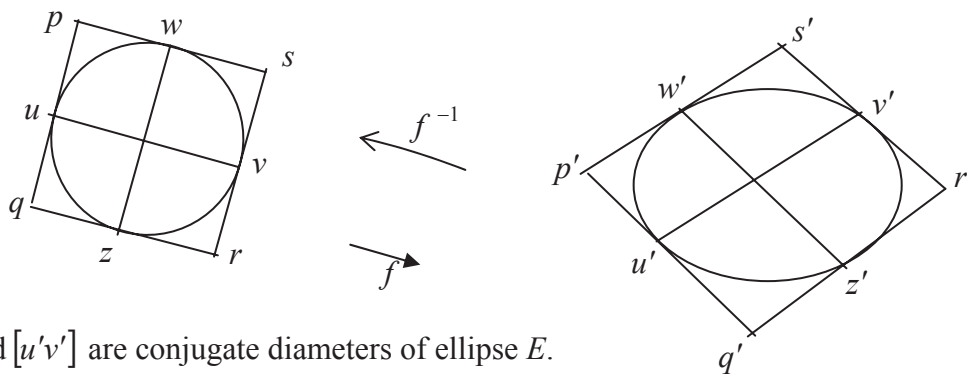
11 (c) Prove that the areas of all parallelograms circumscribed about a given ellipse at the endpoints of conjugate diameters are equal.

Part (c)

20 marks

Att 6

11 (c)



$[w'z']$ and $[u'v']$ are conjugate diameters of ellipse E .

Tangents at their end-points form the parallelogram $p'q'r's'$.

Under an affine transformation f^{-1} , the ellipse maps to the circle $x^2 + y^2 = 1$

and $p'q'r's'$ is mapped to $p q r s$.

$[uv]$ and $[wz]$ are conjugate diameters of the circle and $uv \perp wz$.

The square $p q r s$ has fixed area 4 sq units.

Area $p q r s = 2 \text{ area } p q r \Rightarrow \text{area } p'q'r's' = 2 \text{ area } p'q'r'$ as ratio is an invariant map.

Area $p'q'r's' = 2|\det. f| \text{ area } \Delta p q r$

$$= |\det. f| \text{ area } p q r s.$$

But $\det. f$ is constant and area $p q r s$ is also constant $\Rightarrow \text{area } p'q'r's'$ is constant.

\therefore Areas of all parallelograms at end points of conjugate diameters are equal.

Blunders

- B1 Error in mapping
- B2 No statement regarding constant area of square
- B3 No statement of ratio being invariant and $\det f$ being constant
- B4 No conclusion

BONUS MARKS FOR ANSWERING THROUGH IRISH

Bonus marks are applied separately to each paper as follows:

If the mark achieved is less than 226, the bonus is 5% of the mark obtained, rounding *down*.
(e.g. 198 marks \times 5% = 9.9 \Rightarrow bonus = 9 marks.)

If the mark awarded is 226 or above, the following table applies:

| Marks obtained | Bonus |
|----------------|-------|
| 226 – 231 | 11 |
| 232 – 238 | 10 |
| 239 – 245 | 9 |
| 246 – 251 | 8 |
| 252 – 258 | 7 |
| 259 – 265 | 6 |
| 266 – 271 | 5 |
| 272 – 278 | 4 |
| 279 – 285 | 3 |
| 286 – 291 | 2 |
| 292 – 298 | 1 |
| 299 – 300 | 0 |

