

Contents

Page

GENERAL GUIDELINES FOR EXAMINERS – PAPER 1.....	2
QUESTION 1.....	4
QUESTION 2.....	8
QUESTION 3.....	11
QUESTION 4.....	14
QUESTION 5.....	17
QUESTION 6.....	21
QUESTION 7.....	26
QUESTION 8.....	30
GENERAL GUIDELINES FOR EXAMINERS – PAPER 2.....	33
QUESTION 1.....	35
QUESTION 2.....	39
QUESTION 3.....	43
QUESTION 4.....	47
QUESTION 5.....	51
QUESTION 6.....	55
QUESTION 7.....	58
QUESTION 8.....	63
QUESTION 9.....	66
QUESTION 10.....	70
QUESTION 11.....	74
BONUS MARKS FOR ANSWERING THROUGH IRISH.....	79

MARKING SCHEME

LEAVING CERTIFICATE EXAMINATION 2006

MATHEMATICS – ORDINARY LEVEL – PAPER 1

GENERAL GUIDELINES FOR EXAMINERS – PAPER 1

1. Penalties of three types are applied to candidates' work as follows:
 - Blunders - mathematical errors/omissions (-3)
 - Slips - numerical errors (-1)
 - Misreadings (provided task is not oversimplified) (-1).

Frequently occurring errors to which these penalties must be applied are listed in the scheme. They are labelled: B1, B2, B3,..., S1, S2,..., M1, M2,...etc. These lists are not exhaustive.

2. When awarding attempt marks, e.g. Att(3), note that
 - any *correct, relevant* step in a part of a question merits at least the attempt mark for that part
 - if deductions result in a mark which is lower than the attempt mark, then the attempt mark must be awarded
 - a mark between zero and the attempt mark is never awarded.
3. Worthless work is awarded zero marks. Some examples of such work are listed in the scheme and they are labelled as W1, W2,...etc.
4. The phrase "hit or miss" means that partial marks are not awarded – the candidate receives all of the relevant marks or none.
5. The phrase "and stops" means that no more work is shown by the candidate.
6. Special notes relating to the marking of a particular part of a question are indicated by an asterisk. These notes immediately follow the box containing the relevant solution.
7. The sample solutions for each question are not intended to be exhaustive lists – there may be other correct solutions. Any examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his/her advising examiner.
8. Unless otherwise indicated in the scheme, accept the best of two or more attempts – even when attempts have been cancelled.
9. The *same* error in the *same* section of a question is penalised *once* only.
10. Particular cases, verifications and answers derived from diagrams (unless requested) qualify for attempt marks at most.
11. A serious blunder, omission or misreading results in the attempt mark at most.
12. Do not penalise the use of a comma for a decimal point, e.g. €5.50 may be written as €5,50.

APPLYING THE GUIDELINES

Examples of the different types of error:

Blunders (i.e. mathematical errors) (-3)

- Algebraic errors : $8x + 9x = 17x^2$ or $5p \times 4p = 20p$ or $(-3)^2 = 6$
- Sign error $-3(-4) = -12$
- Fraction error (incorrect fraction, inversion etc); apply once.
- Cross-multiplication error
- Operation chosen is incorrect. (e.g, multiplication instead of division)
- Transposition error : eg $-2x - k + 3 \Rightarrow -2x = 3 + k$ or $-3x = 6 \Rightarrow x = 2$ or $4x = 12 \Rightarrow x = 8$ each time.
- Distribution error (once per term, unless directed otherwise) eg $3(2x + 4) = 6x + 4$ or $\frac{1}{2}(3 - x) = 5 \Rightarrow 6 - x = 5$
- Expanding brackets incorrectly: apply once unless directed otherwise, eg $(2x - 3)(x + 4) = 8x^2 - 12$
- Omission, if not oversimplified.
- Index error, each time unless directed otherwise
- Factorisation: error in one or both factors of a quadratic: apply once $2x^2 - 2x - 3 = (2x - 1)(x + 3)$
- Root errors from candidate's factors: error in one or both roots: apply once.
- Error in formulae: eg $T_n = 2a + (n - 1)d$
- central sign error in uv or u/v formulae
- omission of $\div v^2$ or division not done in u/v formula (apply once)
- Vice-versa substitution in uv or u/v formulae (apply once)

Slips (-1)

- Numerical slips: $4 + 7 = 10$ or $3 \times 6 = 24$ but $5 + 3 = 15$ is a blunder.
- An omitted round-off or incorrect round off to a required degree of accuracy, or an early round off is penalised as a slip each time,
- However an early round-off which has the effect of simplifying the work is at least a blunder
- Omission of units of measurement or giving the incorrect units of measurement in an answer is treated as a slip, once per section (a), (b) and (c) of each question.

Misreadings (-1)

- Writing 2436 for 2346 will not alter the nature of the question so MR(-1)
However, writing 5000 for 5026 will simplify the work and is penalised as at least a blunder.

Note: Correct relevant formula *isolated* and stops: if formula is *not* in Tables, award attempt mark.

QUESTION 1

Part (a)	10 marks	Att 3
Part (b)	25 marks	Att 8
Part (c)	15 marks	Att 5

*No penalty for omitting € symbol, but if answer given in cent, then must include c, otherwise S-1.

Part (a)	10 marks	Att 3
-----------------	-----------------	--------------

(a) €320 is $\frac{4}{9}$ of a prize fund. Find the total prize fund.

(a)	10 marks	Att 3
------------	-----------------	--------------

$\frac{1}{9} = \frac{320}{4} \quad \dots\dots 3m$ $= 80 \quad \dots\dots 7m$ $\frac{2880}{4} \text{ or } 80 \times 9 \quad \dots\dots 7m$ $\left[\frac{9}{9} = \frac{320 \times 9}{4} \right] = \text{€}720 \quad \dots\dots 10m$	or	$\frac{1}{9} = \frac{320}{4} \quad \dots\dots 3m$ $\frac{5}{9} = \frac{320 \times 5}{4} \quad \dots\dots 7m$ $= 400 \quad \dots\dots 7m$ $320 + 400 = \text{€}720 \quad \dots\dots 10m$
--	----	---

*Correct answer without work: full marks.

*400 without work, award 7marks.

Blunders (-3)

B1 An incorrect numerator, eg 320×4 and continues.

B2 An incorrect denominator, eg $320/9$ and continues.

Note: $\frac{320 \times 4}{9} = 142.(222\dots)$ is $1 \times B$ (inversion); (if then not calculated: $2 \times B = 4m$).

B3 Gets 2880 ($=320 \times 9$) and stops (1 step missing).

B4 Incorrect or no simplification, or simplification not possible.

Slips (-1)

S1 A numerical slip.

Attempts (3 marks)

A1 Mentions 9 times and stops or 5 times and stops.

A2 Mentions $\frac{1}{4}$ or $\div 4$ and stops.

A3 Mentions $5/9$ or gets $5/9$ of 320 or 177.77 with work.

Worthless (0)

W1 Mentions $1/9$ and stops.

W2 Incorrect answer with no work (other than 400).

Aoife pays a fixed monthly charge of €15 for her mobile phone. This charge includes 100 free text messages and 50 minutes free call time each month. Further call time costs 28 cent per minute and additional text messages cost 11 cent each.

In one month Aoife sends 140 text messages and her call time is 2 hours.

- (i) Find the total cost of her fixed charge, text messages and call time.
- (ii) VAT is added to this cost at the rate of 21%.
Find the amount paid, including VAT.

(b)(i)

15 marks

Att 5

140-100 = 40 (first subtr.)5m	} interchangeable
120-50 = 70 (2 nd one)9m	
40×11c = €4.40 (first mult.)12m	} interchangeable
and 70×28c = €19.60 (2 nd mult.)14m	
Total = €4.40 + €19.60 + €15 = €39.0015m	

*Correct answer without work: Att5.

Blunders (-3)

B1 40×28 and/or 70×11 and continues.

B2 Incorrect use or no use of either the free calls or free texts (once only).

Note: 140×11 and 120×28 and continues correctly is B2 (12m).

B3 Using 1hr = 100mins and continues (4.40+42.00+15.00 ⇒ €61.40).

Slips (-1)

S1 Each numerical slip.

S2 Incorrect or no total.

Attempts (5 marks)

A1 Mentions 40 only or the 70 only and stops.

A2 Mentions 120 and no other relevant work.

A3 Mentions multiplying by 11 and/or by 28 and stops.

A4 Some relevant use of the 15 (addition only).

Worthless (0)

W1 Mentions 150 and stops.

(b)(ii)

10 marks

Att 3

$100\%+21\% = 121\%$3m	or	$21\% \text{ of } \text{€}39$3m
$\frac{39 \times 121}{100}$ 7m		$= \text{€}8.19$7m
$= \text{€}47.19$10m		$\text{€}39 + \text{€}8.19$9 m
			$= \text{€}47.19$10m

*Correct answer without work: full marks.

*Accept candidate's answer from part (b)(i).

*€8.19 with no work: 7m.

Case: *without work*

4719c ... 10m

4719 9m

€4719 7m

Blunders (-3)

B1 Uses 100/121 (getting 32.23) or similar.

B2 Subtracts the 21% (or 8.19) instead of adding (getting 30.81).

B3 Decimal error in calculations.

Attempts (3 marks)

A1 Mentions 121% or 21/100 or 121 only and stops.

A2 Some relevant use of the 39 (or candidate answer from (b)(i)). eg 39×21 only.

A3 Mentions 100 or 100% and stops.

Part (c)

15 (10, 5) marks

Att (3, 2)

(c) The standard rate of income tax is 20% and the higher rate is 42%.
 Colm has weekly tax credits of €50 and a standard-rate cut-off point of €240
 Until recently, Colm had a gross weekly income of €900.

(i) Calculate the tax Colm paid each week.

(ii) After getting a pay rise, Colm's weekly after-tax income increased by €20.30.
 Calculate the increase in Colm's gross weekly income.

(c)(i)

10 marks

Att 3

$900-240 = 660$3 m	} interchangeable
$240 \times 0.2 = \text{€}48$4 m	
$660 \times 0.42 = \text{€}277.20$7 m	
$48 + 277 = \text{€}325.20$		
$325.20 - 50 = \text{€}275.20$10 m	

* Correct answer without work: Att3.

Blunders (-3)

B1 No subtraction, ie $900 \times 42\%$ used with the $240 \times 20\%$ and continues.

B2 Decimal error in calculations.

B3 20% of an incorrect figure.

B4 Incorrect or no use of tax credit.

B5 Tax rates interchanged, ie $240 \times 42\%$ with $660 \times 20\%$ (once) and continues.
 (ie getting 100.80 and 132.00 respectively \Rightarrow Gross 232.80 \Rightarrow Nett €182.80).

Slips (-1)

S1 Each numerical slip.

Attempts (3 marks)

- A1 Mention of 660 and no other relevant work.
- A2 Some relevant tax calculation and stops.
- A3 Two tax costs appearing which are then added and stops.
- A4 Some relevant use of the tax credits ie by subtracting.

(c)(ii)

5 marks

Att 2

$$\begin{aligned} 100 - 42 &= 58 && \dots\dots 2m \\ 58\% &= 20.30 \\ \text{Increase} &= \frac{20.30 \times 100}{58} \\ &= \text{€}35 && \dots\dots 5m \end{aligned}$$

- * Correct answer without work: Att2.
- * Accept 935 for full marks (includes the increase)with work; otherwise Att2.

Blunders (-3)

- B1 $\frac{20.30 \times 100}{42} = 48.333$ calculated out.
- B2 Uses 20% \Rightarrow 80% getting $\frac{20.30 \times 100}{80} = 25.375$.
- B3 Decimal error in calculations.

Slips (-1)

- S1 Each numerical slip.
- S2 Decimal part of an incorrect answer omitted e.g. 48.33 written as 48.

Attempts (2 marks)

- A1 58% appearing.
- A2 Gets 42% of 20.30 (8.526) and stops.
- A3 Some relevant subtraction from 100 indicated.e.g. 100 – 20.
- A4 Gets 900 – answer (i) with or without adding 20.30.

Worthless (0)

- W1 Mentions 42/100 or 20/100 and stops.

QUESTION 2

Part (a)	15 marks	Att 5
Part (b)	20 marks	Att 7
Part (c)	15 marks	Att 5

Part (a) **15 marks** **Att 5**

(a) Simplify $3(2x + 4) - 5(x + 1)$.

(a) **15 marks** **Att 5**

$$\begin{aligned} 6x + 12 - 5x - 5 & 12\text{m} \\ = x + 7 & 15\text{m} \end{aligned}$$

* Ignore extra work after $x + 7$.

Blunders (-3)

- B1 Distribution error to max $2 \times B$.
- B2 Algebraic errors.
- B3 $3(2x + 4) = 5(x + 1)$ and continues.

Attempts (5 marks)

- A1 Any correct multiplication or addition

Part (b) **20 (5, 15) marks** **Att (2, 5)**

Let $f(x) = 2x^3 + ax^2 + bx + 14$.

(i) Express $f(2)$ in terms of a and b .

(ii) If $f(2) = 0$ and $f(-1) = 0$, find the value of a and the value of b .

(b)(i) **5 marks** **Att 2**

$$\begin{aligned} f(2) &= 2(2)^3 + a(2)^2 + b(2) + 14 \quad [5\text{m}] \\ &\text{or } 16 + 4a + 2b + 14 \quad \text{or } 4a + 2b + 30 \quad [5\text{m}] \end{aligned}$$

* Errors, if any, in simplifying $f(2)$ are applied in (ii).

* Accept correct long division (Remainder = $4a + 2b + 30$) or synthetic division. Otherwise apply slips and blunders.

Blunders (-3)

- B1 Substitutes $x = -2$

Attempts (2 marks)

- A1 Replaces a and/or b by ± 2 .
- A2 Attempt at division by $x \pm 2$.
- A3 Any correct substitution.

b(ii)**15 marks****att 5**

$$f(-1) : 2(-1)^3 + a(-1)^2 + b(-1) + 14 \dots 5m$$

$$2(-1)^3 + a(-1)^2 + b(-1) + 14 = 0 \dots 6m$$

$$f(2) : 2(2)^3 + a(2)^2 + b(2) + 14 = 0 \dots 9m$$

} interchangeable

$$\left. \begin{array}{l} a - b = -12 \\ 2a + b = -15 \end{array} \right\} \dots 12m$$

$$3a = -27 \Rightarrow a = -9; b = 3 \dots 15m$$

*Blunders (-3)*B1 Mathematical error in simplifying $f(2) = 0$.B2 Error in substitution $f(-1)$.B3 Error in simplifying $f(-1) = 0$.

B4 Mathematical error in solving simultaneous equations (once).

Slips (-1)

S1 Numerical errors.

*Attempts (5 marks)*A1 Any correct simplification of $f(2)$ or $f(-1)$ and stops.A2 Any correct substitution of $f(-1)$ and stops.A3 Replaces a and/or b by ± 1 or ± 2 .

Part (c)**15 (10, 5) marks****Att (3, 2)**

- (c) (i) Find the smallest natural number k such that
 $2x + 4(x+3) + 7(2x+4) < 20(x+k)$.
- (ii) The lengths of the sides of a triangle are
 $4\sqrt{x}$, $(x-4)$ and $(x+4)$ where $x > 4$.
 Prove that the triangle is right-angled.

(c)(i)**10 marks****Att 3**

$$2x + 4x + 12 + 14x + 28 < 20x + 20k \quad \dots 4m$$

$$\Rightarrow 20x + 40 < 20x + 20k \quad \text{or} \quad \Rightarrow 20k > 40 \quad \dots 7m$$

$$\Rightarrow k > 2 \quad \dots 9m$$

$$\Rightarrow k = 3 \quad \dots 10m$$

Blunders (-3)

- B1 Distribution error to max $2 \times B$.
 B2 Transposition error(s).
 B3 Algebraic errors.
 B4 Error in direction of inequality.

Slips (-1)

- S1 Finishes at $k > 2$.
 S2 Numerical errors.

Misreadings (-1)

- M1 Uses ' \leq ' (Answer $k = 2$).

Attempts (3 marks)

- A1 Any correct step in multiplication/transposition/simplification.

(c)(ii)**5 marks****Att 2**

$$(4\sqrt{x})^2 = 16x \quad \text{and} \quad (x-4)^2 = x^2 - 8x + 16 \quad \text{and} \quad (x+4)^2 = x^2 + 8x + 16 \quad [4m]$$

$$\text{Pythagoras: } 16x + x^2 - 8x + 16 = x^2 + 8x + 16 \quad [5m]$$


Blunders (-3)

- B1 Error in squaring.

Slips(-1)

- S1 Hypotenuse incorrectly or not identified or no correct conclusion.

Attempts (2 marks)

- A1 Any mention of Pythagoras' Theorem or draws  with some relevant labelling of side(s).
 A3 Substitutes a value > 4 for x and continues.
 A3 Effort at squaring any term.

Worthless (0marks)

- W1 States slope formula.
 W2 States 90° and stops.
 W3 Adds 2 (or 3) sides.

QUESTION 3

Part (a)	15 marks	Att 5
Part (b)	20 marks	Att 7
Part (c)	15 marks	Att 5

Part (a)	15 marks	Att 5
-----------------	-----------------	--------------

(a) Find the value of $\frac{ab-c}{2}$, when $a = 3$, $b = \frac{2}{3}$ and $c = 1$.

(a)	15 marks	Att 5
------------	-----------------	--------------

$$\frac{3\left(\frac{2}{3}\right)-1}{2} \dots\dots[9m] \dots\dots = \frac{2-1}{2} \dots\dots[12m] \dots\dots = \frac{1}{2} \dots\dots[15m]$$

*Correct answer with no work for full marks.

Blunders (-3)

B1 Mathematical error each time to max 2×B.

Slips (-1)

S1 Numerical slips.

Misreadings (-1)

M1 Incorrect substitution for a , b , c (once only) and continues (work must be shown).

Attempts (5 marks)

A1 Some correct substitution and stops.

Worthless (0 marks)

W1 Incorrect answer with no work.

Part (b)	20 marks	Att 7
-----------------	-----------------	--------------

(b) Solve for x and y

$$x - 2y = 10$$

$$x^2 + y^2 = 20$$

(b)	20 marks	Att 7
------------	-----------------	--------------

$x = 2y + 10$7m
$\Rightarrow (2y + 10)^2 + y^2 = 20$...8m
$\Rightarrow 5y^2 + 40y + 80 = 0$ or $y^2 + 8y + 16 = 0$11m
$\Rightarrow (y + 4)^2 = 0$...14m
$\Rightarrow y = -4$...17m
$x = [2(-4) + 10] = 2$...20m

*Apply similar structure if y isolated first.

* Set of coordinates found by trial and error, or without work:

(2, -4) verified in both equations, 20 marks; if not verified in both, Att 7.

Incorrect answer: no marks, whether tried to verify or not.

*No additional marks from the point where the equation becomes linear. But see A3 below.

*No penalty for excess answer (Use of quadratic to find 2nd variable).

Blunders (-3)

- B1 Distribution error.
- B2 Sign error, each time.
- B3 Quadratic formula error (in formula, substitution or simplification). Each time to a maximum of 2 blunders (equivalent to two steps).
- B4 Incorrect factors. Apply once.
- B5 Incorrect root(s) from candidate's factor(s). Apply once.
- B6 Fails to find value of second variable.
- B7 Finds x but substitutes back into y (or vice versa).
- B8 Transposition errors (Each time.)

Attempts (7 marks)

- A1 Effort at isolating x or y e.g. $2y = x - 10$ and stops.
- A2 Correct quadratic formula and stops.
- A3 An effort to find the *second* variable, having found the first variable with work of no value.

Worthless (0)

- W1 Incorrect values without work.
- W2 Invented values substituted, and continues, e.g. $y = 0 \Rightarrow x = 10$ or some such.
- W3 $x - 2y = 10 \Rightarrow x^2 + 4y^2$ and continues. But see A3.

Part (c)

15 (10, 5) marks

Att (3, 2)

(c) Solve for x

$$x = \frac{3 + 2x}{x - 2}, \quad x \neq 2$$

and give your solutions in the form $a \pm \sqrt{b}$, where $a, b \in \mathbf{N}$.

Write one of your solutions correct to two decimal places. Using this value, show that the difference in the values of the left hand side and the right hand side of the given equation is less than 0.1

(c)[(i)]

10 marks

Att3

$$x = \frac{3 + 2x}{x - 2} \Rightarrow x(x - 2) = 3 + 2x \quad [3m] \Rightarrow x^2 - 4x - 3 = 0. \quad [4m]$$

$$\Rightarrow x = \frac{4 \pm \sqrt{16 + 12}}{2} = \frac{4 \pm \sqrt{28}}{2} \quad [7m] = 2 \pm \sqrt{7}. \quad [10m]$$

*No additional marks from the point where the equation becomes linear.

Blunders (-3)

- B1 Quadratic formula error (in formula, substitution or simplification). Each time to a maximum of 2 blunders.
- B2 Transposition errors (each time).
- B3 Answer not written in surd form.

Attempts (3 marks)

- A1 Correct quadratic formula and stops.
- A2 Some correct attempt at eliminating fraction (even if incomplete or only partially correct).

Worthless (0 marks)

- W1 Unsuccessful trial and error approach.

(c)[(ii)]

5 marks

Att 2

$2 + \sqrt{7} = 2 + 2.645 = 4.65$2 marks....	$2 - \sqrt{7} = 2 - 2.645 = -0.65$
$\text{RHS} = \frac{3 + 2(4.65)}{4.65 - 2} = 4.64$4 marks.....	$\text{RHS} = \frac{3 + 2(-0.65)}{-0.65 - 2} = -0.64$
$ \text{LHS} - \text{RHS} =$ $ 4.65 - 4.64 = 0.01$ [< 0.1]5 marks....	$ \text{LHS} - \text{RHS} =$ $ -0.65 - (-0.64) = 0.01$ [< 0.1]

*Accept candidate's answer from(i)

Blunders (-3)

- B1 $\sqrt{7} = 3.5$.
- B2 Mathematical error.
- B3 Substitution into equations $x(x - 2) = 3 + 2x$ or $x^2 - 4x - 3 = 0$ and finishes.

Slips (-1)

- S1 $\sqrt{7}$ incorrectly rounded or not rounded.

Attempts (2 marks)

- A1 Some effort at expressing x as decimal.
- A2 Some effort at substitution using answers from (i).

Worthless (0marks)

- W1 Substitution of (incorrect) value(s) other than from (i).

QUESTION 4

Part (a)	10 marks	Att 3
Part (b)	20 marks	Att 6
Part (c)	20 marks	Att 6

Part (a)	10 marks	Att 3
-----------------	-----------------	--------------

Let $u = 3 - 6i$ where $i^2 = -1$.
Calculate $|u + 2i|$.

(a)	10 marks	Att 3
------------	-----------------	--------------

$ 3 - 6i + 2i $ 3m
$= 3 - 4i $ 4m
$=\sqrt{(3)^2 + (-4)^2}$ or $\sqrt{9+16}$ 7m
$=\sqrt{25}$ or 5	...10m

- * No penalty for substituting 4 for -4 in mod formula.
- * Accept distance formula applied to (0, 0) and (3, -4).

Blunders (-3)

- B1 Mathematical errors.
- B2 Error in modulus formula.
- B3 Incorrect substitution into formula e.g. $(-4i)^2$.
- B4 No $\sqrt{\quad}$ used.
- B5 Finds $|u| + |2i|$.

Attempts (3 marks)

- A1 Substitutes for u and stops.
- A2 $\sqrt{a^2 + b^2}$ or distance formula correct and stops.
- A3 $\sqrt{a^2 - b^2}$, distance formula with 1 error, or $a^2 + b^2$ with some correct substitution and stops.

Worthless (0 marks)

- W1 Incorrect formula (other than A3) with/without substitution.

Part (b)	20 (10, 10) marks	Att (3, 3)
-----------------	--------------------------	-------------------

- (i) Solve $z^2 - 4z + 29 = 0$.
Write your answers in the form $x + yi$ where $x, y \in \mathbf{R}$.
- (ii) Write in its simplest form $i(i^4 + i^5 + i^6)$.

(b)(i)

10 marks

Att 3

$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \dots 3m \dots$	$(x + iy) + (x - iy) = 4$ <p>and/or</p> $(x + iy)(x - iy) = 29$
$= \frac{4 \pm \sqrt{(-4)^2 - 4(1)(29)}}{2(1)} \quad \dots 7m \dots$	$\Leftrightarrow 2x = 4 \Rightarrow x = 2$
<p>or</p> $= \frac{4 \pm \sqrt{-100}}{2} \quad \dots 7m \dots$	$x^2 + y^2 = 29$
$= \frac{4 \pm 10i}{2} \quad \text{or} \quad 2 \pm 5i \quad \dots 10m \dots$	$(2)^2 + y^2 = 29$ $y = \pm 5$

*Correct answers by trial and error must be verified to get full marks otherwise Att 3.

Blunders (-3)

B1 Error in quadratic formula (formula/substitution) to max 2×B.

B2 Errors in simplification.

Attempts (3 marks)

A1 Effort at factorising e.g. $(z + \dots)(z - \dots)$.

A2 Effort at trial and error.

(b)(ii)

10 marks

Att 3

$i(i^4 + i^5 + i^6)$ $= i(1 + i - 1)$ $= i(i) = i^2$ $= -1. \quad [10m]$	or	$i(i^4 + i^5 + i^6)$ $= i^5 + i^6 + i^7$ $= i - 1 - i$ $= -1 \quad [10m]$
--	----	---

*Correct answer without work: Att 3.

*Care needed in distinguishing i and 1.

10 marks: Fully correct with work.

7 marks : (a) If a non trivial power of i is evaluated correctly (i.e. other than i or i^2)
Examples $i^6 = 1$; $i^3 = -i$ etc.

and

(b) $i \times$ (quantity with i) is correctly multiplied.
Examples: $i(i^4) = i^5$; $i(i) = i^2$; $i(i^{15}) = i^{16}$ (but e.g. $i(3) = 3i$ is not sufficient).

3 marks: One of the elements (a) or (b) above only

Attempts (3 marks)

A1 Any correct work with relevant indices e.g. $i^2 = -1$, $i^4 = i.i.i.i$ (but $i(i^{15})$ and stops W)

Worthless (0 marks)

W1 $1(4+5+6) = 15$ or such.

(i) Express $\frac{3-2i}{1-4i}$ in the form $x + yi$.

(ii) Hence, or otherwise, find the values of the real numbers p and q such that

$$p + 2qi = \frac{17(3-2i)}{1-4i}.$$

(c)(i)

10 marks

Att 3

(I)	3m...	(II)	$\frac{3-2i}{1-4i} = x + yi$
$\frac{3-2i}{1-4i} \times \frac{1+4i}{1+4i}$...3m...	$\Rightarrow 3-2i = (1-4i)(x+yi)$	
$3-2i+12i-8i^2$ <u>or</u> $1-16i^2$...4m..	$x-4xi+yi-4yi^2$ or $x+4y+(-4x+y)i$ (RHS)	
$= 3-2i+12i-8i^2$ <u>and</u> $1-16i^2$...7m...		
$= \frac{3-2i+12i-8i^2}{1-16i^2}$...7m	$x+4y = 3$	$-4x+y = -2$
$= \frac{11+10i}{17}$ or $\frac{11}{17} + \frac{10i}{17}$...10m	$x = \frac{11}{17}$	$y = \frac{10}{17}$

Blunders (-3)

B1 Incorrect conjugate.

B2 $i^2 \neq -1$. Once only if consistent.

B3 Each omitted or incorrect term when multiplying out. max of 2 (1 on num., 1 on denom.).

B4 Real and imaginary parts mixed up, e.g. when adding.

B5 Inverts in the last step.

Attempts (3 marks)

A1 Correct conjugate and stops.

A2 Any correct relevant multiplication indicated.

(c)(ii)

10 marks

Att 3

$p + 2qi = 17\left(\frac{11}{17} + \frac{10i}{17}\right)$ [3m]	$= 11 + 10i$ [4m]
$\Rightarrow p = 11$ and/or $2q = 10$ [7m]	$\Rightarrow p = 11$ and $q = 5$ [10m]

*Accept candidate's $x + iy$ from (i) unless oversimplified.

*Correct p and q found without work: if both verified, 10m, otherwise Att 3.

* If $(p+2qi)(1-4i) = 17(3-2i)$ used, mark as in part (i) (II)

Blunders (-3)

B1 $i^2 \neq -1$, once if consistent.

B2 Sign errors, each time.

B3 Real and imaginary parts confused, e.g. real \neq real, imaginary \neq imaginary.

Attempts (3 marks)

A1 Some correct multiplication of brackets.

QUESTION 5

Part (a)	10 marks	Att 3
Part (b)	20 marks	Att 7
Part (c)	20 marks	Att 7

Part (a) **10 marks** **Att 3**

The first term of an arithmetic sequence is 17 and the common difference is -8 .
Find, in terms of n , an expression for T_n , the n th term.

(a) **10 marks** **Att 3**

$a = 17$ or $d = -8$ or $T_n = a + (n-1)d$ [3m]
 $a = 17$ or $d = -8$ **with** $T_n = a + (n-1)d$ [4m]
 all 3 written [7m]
 $= 17 + (n-1)(-8)$ or $= 25 - 8n$. [10m].

- * Accept correct answer without work.
- * Ignore notation.

Blunders(-3)

- B1 Incorrect a .
- B2 Incorrect d but a and d interchanged $1 \times B$.
- B3 Error in formula (not more than 1 error, otherwise attempt at best).
- B4 Finds S_n of AP instead of T_n .

Attempts (3marks)

- A1 GP formula with any correct substitution.
- A2 S_n of AP formula written.
- A3 $T_1 = a$.

Case:	
17, 9,	4 m
17, 25,	3m
$a = 17$	3m
17 on its own	0 m

Worthless(0marks)

- W1 17 and /or -8 written.
- W2 GP formula(e) and stops.

(b) **20 (5, 5, 10) marks** **Att (2, 2, 3)**

The n th term of a geometric series is $T_n = 4\left(\frac{1}{2}\right)^n$.

- (i)** Find a , the first term.
- (ii)** Find r , the common ratio.
- (iii)** Write $4 - S_{10}$ in the form $\frac{1}{2^k}$, $k \in \mathbf{N}$, where S_{10} is the sum of the first ten terms.

(b)(i) **5 marks** **Att 2**

$$T_1 = a = 4\left(\frac{1}{2}\right)^1 \quad [2m] \quad = 2 \quad [5m]$$

- * Accept correct answer with no work.
- * Award 5 marks for ans = 2 even if $(4 \times \frac{1}{2})^1$ written.

Blunders (-3)

B1 Incorrect value of n used.

Attempts (2 marks)

A1 States $n = 1$ and stops.

A2 Writes T_1 and stops.

A3 Writes T_n for GP (ar^{n-1}).

A4 Formula for S_n of GP written correctly.

A5 $4 \div \frac{1}{2}$.

A6 T_n of AP with some correct substitution.

Worthless (0marks)

W1 $4 \pm \frac{1}{2}$

W2 T_n or S_n of AP and stops.

(b)(ii)

5 marks

Att 2

$$T_2 = 4\left(\frac{1}{2}\right)^2 = \left[4\left(\frac{1}{4}\right)\right] = 1 \quad [2m] \Rightarrow r = \frac{T_2}{T_1} = \frac{1}{2} \quad [5m] \quad (\text{or by inspection, or any } T_n \div T_{n-1})$$

* Accept correct answer with no work.

* Accept value of a from (i).

Blunders (-3)

B1 Mathematical errors

Attempts (2 marks)

A1 T_1 or $a = 2$

A2 Formula for T_n of GP written and stops.

A3 Formula for T_n of AP with some correct substitution

Worthless (0marks)

W1 T_n or S_n of AP and stops.

(b)(iii)

10 marks

Att 3

$$(I) \quad S_n = \frac{a(1-r^n)}{1-r} [3m] \quad S_{10} = \frac{2\left(1-\left(\frac{1}{2}\right)^{10}\right)}{1-\frac{1}{2}} \quad \text{or} \quad 4-4\left(\frac{1}{2}\right)^{10} \quad \text{or} \quad 3\frac{255}{256} [7m]$$

$$\Rightarrow 4 - S_{10} = 4 - 4 + 4\left(\frac{1}{2}\right)^{10} = \frac{4}{2^{10}} = \frac{1}{2^8} [10m]$$

$$(II) \quad 2 + 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \frac{1}{64} + \frac{1}{128} + \frac{1}{256} [7m] \Rightarrow 4 - (2+1+ \dots) = \frac{1}{256}$$

$$= \frac{1}{2^8} [10m]$$

* Accept a and r values from (i) and (ii) if not oversimplified.

* Accept $S_n = \frac{a(r^n - 1)}{r - 1}$ form of formula.

Blunders (-3)

- B1 Error in formula (not more than 1 error, otherwise attempt at best).
- B2 Error in substitution (once if consistent).
- B3 Mathematical error in calculation to max B×2.
- B4 Each term missing in (II).
- B5 Extra term(s) included in addition in last step in (II).

Slips (-1)

- S1 Gives answer as 2^{-8} or $1/256$ or 0.003906....
- S2 Numerical errors.

Attempts (3 marks)

- A1 Attempt at adding terms.
- A2 Writes out at least 2 correct terms.
- A3 Formula for S_n of AP with some correct substitution.

(c) **20 (10, 5, 5)** **Att (3, 2, 2)**

The first three terms of an arithmetic sequence are

$$h + 3, \quad 5h - 2, \quad 6h - 13$$

where h is a real number.

- (i) Find the value of h .
- (ii) Hence, write down the value of each of the first three terms.
- (iii) Find the value of the eleventh term.

(c)(i) **10 marks** **Att 3**

$$5h - 2 - (h + 3) = 6h - 13 - (5h - 2) \quad [3m]$$

$$\Rightarrow 5h - 2 - h - 3 = 6h - 13 - 5h + 2 \quad \Rightarrow 3h = -6 \quad [7m] \quad \Rightarrow h = -2 \quad [10m]$$

* Accept correct answer without work or by T + E.

Blunders (-3)

- B1 Mathematical errors.

Attempts (3 marks)

- A1 $T_3 - T_2 = T_2 - T_1$ or similar and stops.
- A2 a or $T_1 = h + 3$ and/or $T_2 = 5h - 2$ and/or $T_3 = 6h - 13$.
- A3 Effort at T+E.
- A4 $a, a + d,$ and/or $a + 2d$ written, or T_n formula written.

Worthless (0 marks)

- W1 $\frac{T_2}{T_1} = \frac{T_3}{T_2}$ or similar, and stops, but see A2.

(c)(ii)

5 marks

Att 2

$$h + 3 = -2 + 3 = 1$$

$$5h - 2 = 5(-2) - 2 = -12$$

$$6h - 13 = 6(-2) - 13 = -25$$

- * Accept h from part (i).
- * Accept correct answers with no work.

Blunders (-3)

B1 One term omitted.

B2 Sign error.

Slips (-1)

S1 Arithmetic error.

Attempts (2 marks)

A1 T_n of AP and stops.

A2 One term only calculated.

A3 Some relevant substitution.

A4 Two correct answers (with/without 3rd incorrect answer) with no work.

Worthless(0marks)

W1 Incorrect answers, or only 1 correct answer, with no work.

(c)(iii)

5 marks

Att 2

$$d = T_2 - T_1 = -12 - 1 = -13 \quad [2m]$$

$$T_{11} = a + 10d = 1 + 10(-13) \quad [\text{still } 2m] = 1 - 130 = -129 \quad [5m]$$

or

$$\text{List: } 1, -12, -25, \dots, -129 \quad [5m]$$

- * Accept terms from (ii).

Blunders (-3)

B1 Error in formula (not more than 1 error, otherwise attempt at best).

B2 Error in substitution.

B3 Mathematical error in calculation.

B4 $n = 11 \pm 1$ in listing method.

Slips (-1)

S1 Arithmetic errors.

Attempts (2 marks)

A1 Finds T_4 .

A2 Formula for T_n written.

A3 S_n formula with some correct substitution.

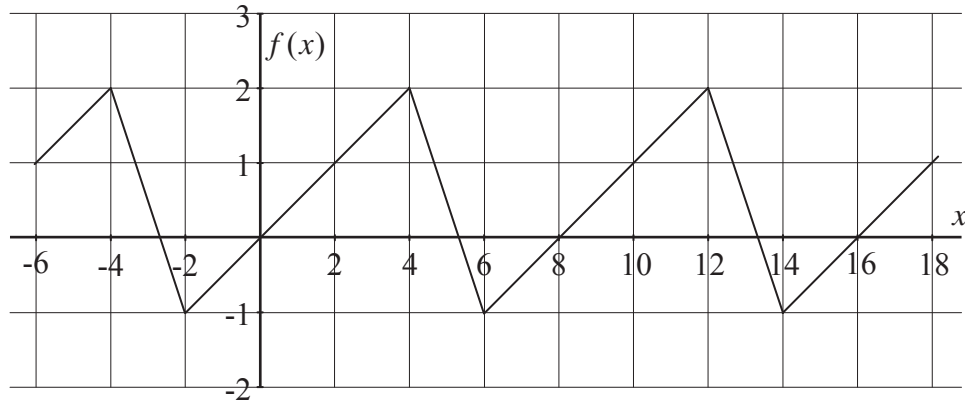
QUESTION 6

Part (a)	15 marks	Att 6
Part (b)	20 marks	Att 7
Part (c)	15 marks	Att 6

Part (a) **Att (2, 2, 2)**

15 (5, 5, 5) marks

- (a) $f : x \rightarrow f(x)$ is a periodic function defined for $x \in \mathbf{R}$.
The period is as indicated in the diagram



- (i) Write down the period and range of the function.
(ii) Find $f(44)$.

(a)(i) **Att (2, 2)**

10 (5, 5) marks

Period = 8, Range = [-1, 2]

- * Accept correct answers without work for all (a).
- * Accept -2, 6 or 6,-2 or equivalent for period.
- * Accept [2, -1] for range.

Blunders (-3)

- B1 One incorrect element in period (provided 2 elements given).
B2 One incorrect element in range.

Slips (-1)

- S1 Answers unlabelled in wrong order, or incorrectly labelled.

Attempts (2 marks)

- A1 Some idea of periodicity e.g. 'pattern repeats 3 times'.
A2 Range given as 3.

(a) (ii) **Att 2**

5 marks

$f(44) = f(4) = 2$

- * Accept correct answer without work
- * Accept candidate's value for period from (i) or correct period.

Blunders (-3)

- B1 Uses incorrect period.
B2 Incorrect reading using correct period.

Slips (-1)

- S1 Numerical slip (if work shown).

Attempts (2 marks)

A1 Shows some understanding of periodicity e.g. $f(0) = f(8)$ (in this part).

A2 Any incorrect answer in range $[-1, 2]$ with/without work.

Worthless (0 marks)

W1 Incorrect answer outside range $[-1, 2]$ without work.

Part (b)

20 (10, 5, 5) marks

Att (3, 2, 2)

The temperature, C , in degrees Celsius, of a liquid in an insulated container is related to time t , in hours, by

$$C = 86 - 6t.$$

(i) Draw the straight line graph of this relation, putting t on the horizontal axis, for $0 \leq t \leq 8$.

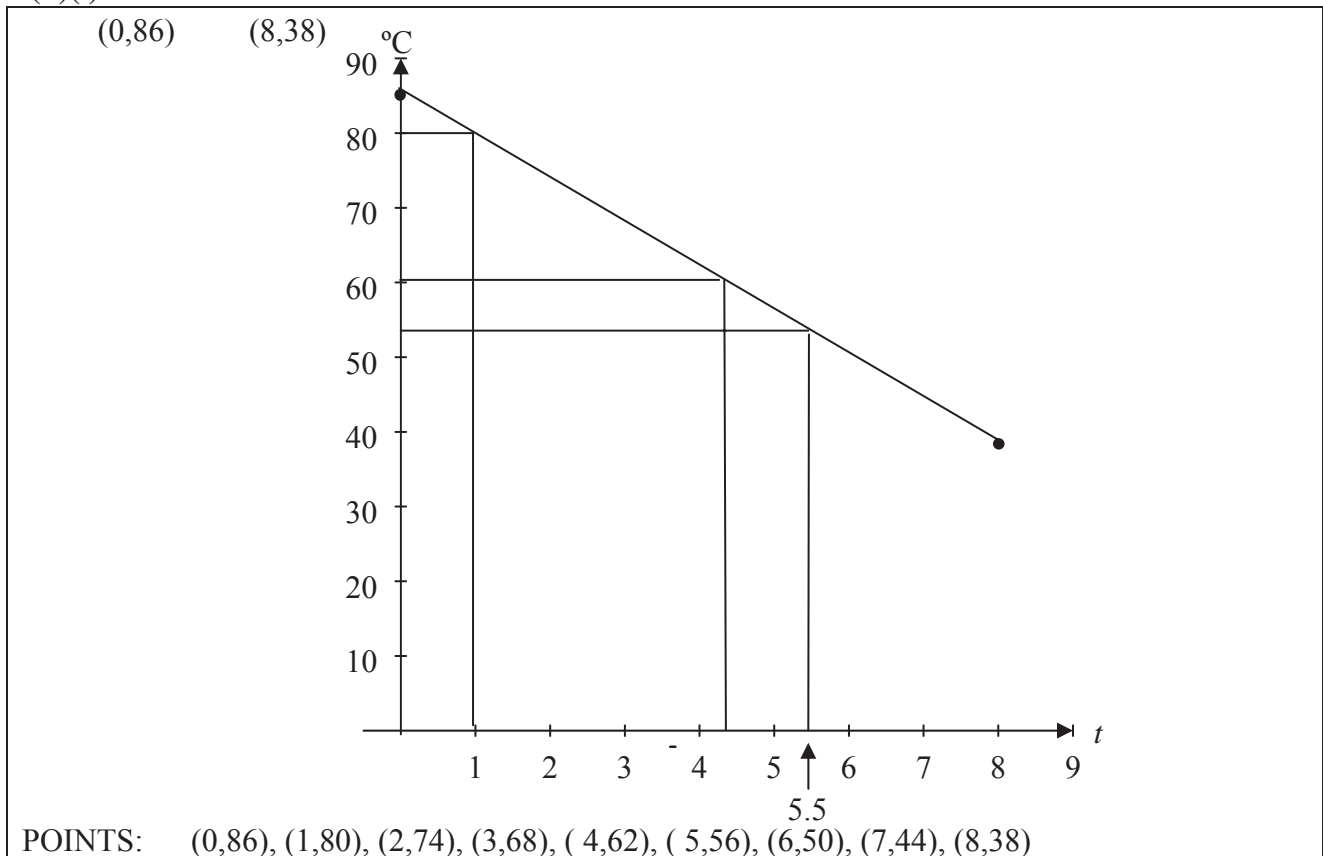
(ii) Use your graph to estimate the temperature when $t = 5.5$ hours.

(iii) Use your graph to estimate the time it takes for the temperature to fall from 80 degrees to 60 degrees.

(b)(i)

10 marks

Att3



* 2 points only needed to draw graph but graph must pass through (0, 86) and (8, 38).

* Do not penalise if graph extends beyond given domain.

* Range need only be $[f(0), f(8)]$[38, 86].

Blunders (-3)

B1 Points plotted but not joined.

B2 Scale error (once only).

Slips (-1)

S1 Each point calculated incorrectly to max -2.

Case:

Any 1 correct point plotted correctly and stops: 4 marks

No point from candidate's table plotted correctly, or no correct point plotted. Att 3

- S2 Each point plotted incorrectly to max -2.
 S3 Each end point $t = 0$, $t = 8$ omitted (i.e. full domain not used).

Misreadings (-1)

- M1 Axes reversed.

Attempts (3 marks)

- A1 Draws scaled axes and stops.
 A2 One or more points correctly calculated but not plotted – Table but no graph.

(b)(ii)

5 marks

Att 2

53°

- * Accept candidate's graph from (i).
- * Allow tolerance of $\pm 2^\circ$.
- * Units: S(-1) once if unit omitted/incorrect in either of parts (ii) or (iii). Deduct at first non-zero or non attempt mark section, where applicable.

Blunders (-3)

- B1 Value of C calculated (53°).

Attempts (2 marks)

- A1 $t = 5.5$ marked and stops.
 A2 Answer read from wrong axis.

(b)(iii)

5 marks

Att 2

$4.3 - 1 = 3.3$ hours

- * Use candidate's graph.
- * Allow tolerance of ± 0.2 hours.

Blunders (-3)

- B1 Calculates answer.
 B2 Subtraction not done.
 B3 Times indicated on graph.

Slips (-1)

- S1 Numerical error.

Attempts (2 marks)

- A1 80 and/or 60 indicated on graph and stops.

Part (c)**15 (5, 5, 5) marks****Att 6(2, 2, 2)**

Let $f(x) = 3 + 8x - 2x^2$, $x \in \mathbf{R}$.

- (i) Find the co-ordinates of the point at which the curve $y = f(x)$ cuts the y -axis.
- (ii) Find the value of x for which $f(x)$ is a maximum.
- (iii) For what range of values of x is $f'(x) > 4$?

(c)(i)**5 marks****Att 2**

$f(0) = 3$. or $(0, 3)$.

- * Accept solution from table or graph for parts (i) and (ii) if accurate and specific.
- * Accept correct answer without work.

Blunders (-3)

B1 Mathematical errors in calculation.

Slips (-1)

S1 Numerical slips.

S2 x -ordinate missing i.e. answer = 3 without work.

S3 $(3, 0)$ without work.

Attempts (2 marks)

A1 Some *correct* substitution of $x = 0$ and stops.

A2 $f(x) = 0$ and continues.

Worthless (0 marks)

W1 Uses derivative.

(c)(ii)**5 marks****Att 2**

$f'(x) = 8 - 4x$ [2m] $\Rightarrow 8 - 4x = 0$ [2m] $\Rightarrow 4x = 8 \Rightarrow x = 2$. [5m]

Table: Possible Domain $(0, 3)$ $(1, 9)$ $(2, 11)$, $(3, 9)$ $(4, 3)$

- * Correct answer without work Att 2m

Blunders (-3)

B1 Differentiation errors, once per term.

B2 Transposition errors.

Slips (-1)

S1 If using table/graph, value of $f(x)$ given.

Attempts (2 marks)

A1 Any mention of $f'(x)$ or similar, or any term differentiated correctly.

(c)(iii)

5 marks

Att 2

$8 - 4x > 4$ [2m] $\Rightarrow -4x > -4$ [2m] $\Rightarrow x < 1$ [5m]
--

* Accept candidate's value of $f'(x)$ from (ii).

Blunders (-3)

B1 Transposition errors.

B2 Error in handling inequality.

B3 $f'(x) = 4$ and continues ie inequality ignored.

Slips (-1)

S1 $x \in \mathbf{N}$ i.e. $x = 0$ (ignore if $x < 1$ written).

Attempts (2 marks)

A1 Value of $f'(x)$ stated or found again in this part.

Worthless (0 marks)

W1 $f(x) > 4$ whether solved or not.

QUESTION 7

Part (a)	10 marks	Att 3
Part (b)	20 marks	Att 6
Part (c)	20 marks	Att 7

Part (a) **10 marks** **Att 3**

(a) Differentiate $5x^3 - 4x + 7$ with respect to x .

(a) **10 marks** **Att3**

$$\left[\frac{dy}{dx} \right] = 15x^2 - 4$$

*Correct answer without work or notation: full marks, 10m.

*If done from first principles, ignore errors in procedure – just mark the answer.

*Only one term correctly differentiated, award 4 marks.

Blunders (-3)

B1 Differentiation error, once per term.

Attempts (3 marks)

A1 Unsuccessful effort at first principles, e.g. $y + \Delta y$ on L.H.S., or x replaced by $x + \Delta x$ on R.H.S., ‘limit’ mentioned, $\Delta x \rightarrow 0$, $f(x+h)$, etc.

A2 Writes down the notation ‘ dy/dx ’ or ‘ $f'(x)$ ’ and stops.

Worthless (0)

W1 No term differentiated correctly, but check attempts first.

Part (b) **20 (10, 10) marks** **Att 6**

(b) (i) Differentiate $\frac{x^2 - 1}{x + 1}$ with respect to x and write your answer in its simplest form.

(ii) Given that $y = (5 - x^2)^3$, find $\frac{dy}{dx}$ when $x = 2$.

(b)(i) **10 marks** **Att 3**

<p>(I)</p> $\frac{(x+1)2x - (x^2 - 1)1}{(x+1)^2} \quad 7 \text{ m}$ $= \frac{2x^2 + 2x - x^2 + 1}{(x+1)^2}$ $= \frac{x^2 + 2x + 1}{(x+1)^2} \quad 9 \text{ m}$ $= 1 \quad 10 \text{ m}$	<p>(II)</p> $\frac{(x+1)(x-1)}{x+1}$ $= x-1 \quad 3 \text{ m}$ $\Rightarrow \frac{dy}{dx} = 1 \quad 10 \text{ m}$
---	---

- * Apply penalties as in guidelines.
- * No penalty for omission of brackets if multiplication implied. (Decide by later work).
- * If $u.v$ used (even if u/v identified initially) i.e. $(x^2 - 1)(x + 1) \dots$ and continues: Att3.
- * No marks for writing down u/v or $u.v$ formula from Tables, and stopping.
- * Penalties for simplification part to max -3.

Blunders (-3)

B1 Differentiation errors, once per term.

Attempts (3 marks)

A1 u and/or v correctly identified and stops.

A2 Any correct differentiation.

A3 $x^2 - 1$ factorised correctly and stops.

(b)(ii)

10 marks

Att 3

(I)	(II)	(III)
	$y = 125 - 75x^2 + 15x^4 - x^6$	$u = 5 - x^2$
$\frac{dy}{dx} = 3(5 - x^2)^2(-2x)$ 7 marks	$\frac{dy}{dx} = -150x + 60x^3 - 6x^5$	$\frac{dy}{dx} = 3u^2(-2x)$ 7 marks
$= 3(5 - 4)^2(-4)$	$= -150(2) + 60(2)^3 - 6(2)^5$	$= 3(5 - 4)^2(-4)$
$= -12$.	-12	$= -12$
10 marks		10 marks

- * Treat $3(5-x^2)^2$ and $(-2x)$ as separate parts.
- * No penalty for omission of brackets if multiplication implied.
- * If differentiation correct accept -12 without work.

Blunders (-3)

B1 Differentiation errors, once per term.

B2 (II) Errors in expanding brackets to max $2 \times B$.

<u>Case:</u>	
$\frac{dy}{dx} = 3(5-(2)^2)^2(-2(2))$	7m
$= -12$	10m

Attempts (3 marks)

A1 If cube ignored...-2x even if finished correctlyoversimplification.

A2 $u = 5 - x^2$ and stops.

A3 Some correct element of chain rule eg index = 2 or coefficient =3.

A4 At least 1 correct term multiplied out.(II).

A5 Any correct differentiation.

Worthless (0marks)

W1 Substitutes $x = 2$ into $f(x)$ and stops.

W2 uv or u/v written and stops.

Part (c)

20 (10, 5, 5) marks

Att (3, 2, 2)

A missile is fired straight up in the air. The height, h metres, of the missile above the firing position is given by

$$h = t(200 - 5t)$$

where t is the time in seconds from the instant the missile was fired.

- (i) Find the speed of the missile after 10 seconds.
- (ii) Find the acceleration of the missile.
- (iii) One second before reaching its greatest possible height, the missile strikes a target. Find the height of the target.

(c)(i)

10 marks

Att 3

$$h = 200t - 5t^2 \text{ [3m]} \Rightarrow \frac{dh}{dt} = 200 - 10t \text{ [7m]} = 200 - 10(10) = 100 \text{m/s [10m]}$$

- * Correct answer without work Att 3.
- * No retrospective marking.
- * No penalty for incorrect notation.
- * If the parts of (c) are unlabelled, and the context doesn't identify which part is which, assume the questions were answered in sequence from (c)(i) to (c)(iii).
- * Units: S(-1) once if unit omitted/incorrect in any of parts (i) (ii) or (iii). Deduct at first non-zero or non attempt mark section, where applicable.

Blunders (-3)

- B1 Differentiation errors, once per term.
- B2 Incorrect or no value for t substituted into dh/dt equation.

Slips (-1)

- S1 Numerical slips.

Attempts (3 marks)

- A1 dh/dt or dy/dx or $f'(x)$ mentioned.
- A2 $\frac{d^2h}{dt^2} = -10$ (candidate may rectify error in this part).

Worthless (0 marks)

- W1 $t = 10$ substituted into h equation.
- W2 Incorrect answer without work.
- W3 States speed = d^2h/dt^2 and stops.
- W4 Effort to use Speed = Distance \div Time.

(c)(ii)

5 marks

Att 2

$$\frac{d^2h}{dt^2} = -10 \text{ m/s}^2$$

- * Accept correct answer without work.
- * Accept $v = u + at$ or equivalent with correct values for u , v and corresponding t calculated and used.
- * Accept use of dh/dt from (i) provided expression contains 't'. Otherwise Att at most.

Blunders (-3)

B1 Differentiation errors, once per term.

Attempts (2 marks)

A1 $\frac{d^2h}{dt^2}$ or $\frac{dv}{dt}$ or similar written.

A2 Finds or attempts to find dh/dt in this part.

Worthless (0 marks)

W1 Incorrect answer without work.

(c)(iii)

5 marks

Att 2

$$\frac{dh}{dt} = 200 - 10t \quad [2m] \quad 200 - 10t = 0 \quad [2m] \quad \Rightarrow 10t = 200 \Rightarrow t = 20 \quad [2m]$$

$$\text{At } t = 19, h = 200(19) - 5(19)^2 = 3800 - 1805 = 1995m \quad [5m]$$

- * Accept use of dh/dt from (i) provided expression contains 't'. Otherwise Att at most.
- * Ignores 'maximum height' element: Att2 at best
- * Correct answer without work: Att2

Blunders (-3)

B1 Differentiation errors, once per term.

B2 Mathematical errors.

B3 Incorrect equation substituted.

Slips (-1)

S1 Numerical slips.

S2 Ignores '1 second before' and continues.

Attempts (2 marks)

A1 Mentions dh/dt or similar or attempts to differentiate (even partially correctly).

A2 Makes some use of $h = t(200 - 5t)$.

Worthless (0 marks)

W1 Incorrect answer without work.

QUESTION 8

Part (a)	10 marks	Att 3
Part (b)	20 marks	Att 6
Part (c)	20 marks	Att 7

Part (a) **10 marks** **Att 3**

$$\text{Let } g(x) = \frac{3}{x+1}, \quad x \in \mathbf{R}, \quad x \neq -1.$$

Evaluate $g(0.5) - g(-0.5)$.

(a) **10 marks** **Att 3**

$g(0.5) = \frac{3}{0.5+1} \text{ or } = \frac{3}{1.5} \text{ or } = 2 \quad \dots\dots 4\text{m}$ $g(-0.5) = \frac{3}{-0.5+1} \text{ or } = \frac{3}{0.5} \text{ or } = 6 \quad \dots\dots 7\text{m}$ $2 - 6 = -4 \quad \dots\dots 10\text{m}$	}	interchangeable
--	---	-----------------

* Correct answer without work: Att3.

Blunders(-3)

- B1 Precedence errors
- B2 Mathematical error e.g. $g(0.5) - g(-0.5) = g(0.5) + g(0.5)$ provided work is not oversimplified e.g. $g(0.5 - (-0.5)) = g(1) \dots\dots$
- B3 Decimal error e.g. 0.5 written as 5.

Misreading(-1)

- M1 $g(0.5) + g(-0.5)$ gives 8.

Attempts (3 marks)

- A1 $g(\text{any number})$, substituted.
- A2 $g(x) = 0.5$ or -0.5 and continues.

Part (b) **20 (10, 10) marks** **Att (3, 3)**

$$\text{Let } h(x) = x^2 + 2x - 1, \quad x \in \mathbf{R}.$$

(i) Simplify $h(x - 5)$.

(ii) Find the value of x for which $h(x - 5) = h(x) - 5$.

(b)(i) **10 marks** **Att 3**

$$h(x - 5) = (x - 5)^2 + 2(x - 5) - 1 \quad [3\text{m}] = x^2 - 10x + 25 + 2x - 10 - 1 \quad [7\text{m}] = x^2 - 8x + 14. \quad [10\text{m}]$$

Blunders(-3)

- B1 Mathematical errors.

Attempts(3marks)

- A1 Some correct substitution and stops.
- A2 $h(5)$ and /or $h(-5)$ substituted.

Worthless (0 marks)

- W1 $hx - 5h$ or similar.

(b)(ii)

10 marks

Att 3

$$h(x-5) = h(x) - 5 \Rightarrow x^2 - 8x + 14 = x^2 + 2x - 1 - 5 \quad [4m]$$
$$\Rightarrow -10x = -20 \quad \text{or similar} \quad [7m] \Rightarrow x = 2. \quad [10m]$$

* Accept candidate's answer from (i).

Blunders(-3)

B1 Error in finding $h(x) - 5$.

B2 Mathematical errors.

B3 If work from (i) leads to a non-algebraic equation i.e. x terms cancel e.g. $0 = -20$.

Attempts (3marks)

A1 Candidate's answer from (i) transferred to this part.

A2 $x^2 + 2x - 1 - 5$ written and stops.

Part (c)

20 (10, 5, 5) marks

Att (3, 2, 2)

$$\text{Let } f(x) = \frac{1}{x-2}, \quad x \in \mathbf{R}, \quad x \neq 2.$$

(i) Find $f'(x)$, the derivative of $f(x)$.

(ii) Find the values of x for which $f'(x) = -1$.

(iii) Find the co-ordinates of the two points on the curve $y = f(x)$ at which the slope of the tangent is -1 .

(c)(i)

10 marks

Att 3

$$f'(x) = -1(x-2)^{-2} = \left[\frac{-1}{(x-2)^2} \right] \quad \text{or} \quad f'(x) = \frac{(x-2)(0) - (1)(1)}{(x-2)^2}$$

* Apply penalties as in guidelines.

* No penalty for omission of brackets if multiplication implied. (Decide by later work).

* Answer need not be simplified. Penalise, if necessary, in (ii).

Attempts (3 marks)

A1 $(x-2)^{-1}$ and stops.

A2 Any correct derivative and stops.

(c)(ii)

5 marks

Att 2

$$\frac{-1}{(x-2)^2} = -1 \Rightarrow (x-2)^2 = 1 \text{ [2m]} \Rightarrow x-2 = \pm 1 \Rightarrow x=3, x=1 \text{ [5m]}$$

or

$$\frac{-1}{(x-2)^2} = -1 \Rightarrow (x-2)^2 = 1 \text{ [2m]} \Rightarrow x^2 - 4x + 3 = 0 \text{ [2m]} \Rightarrow (x-3)(x-1) = 0 \Rightarrow x=3, x=1 \text{ [5m]}$$

or uses quadratic formula.

*Allow candidate's $f'(x)$ from previous part, unless it oversimplifies the question.

*Apply blunders to quadratic formula as in previous questions.

Blunders (-3)

B1 Mathematical errors.

B2 One root only given.

Attempts (2marks)

A1 Answer from(i) = -1 and stops.

A2 Simplifies answer to (i) in this part.

A3 Finds $f'(-1)$.

(c)(iii)

5 marks

Att 2

$$x=3 \Rightarrow f(3) = \frac{1}{3-2} = 1 \quad \text{or} \quad (3, 1)$$

$$x=1 \Rightarrow f(1) = \frac{1}{1-2} = -1 \quad \text{or} \quad (1, -1)$$

*Accept candidate's answers from (ii).

*If only one point available from (ii) then Att2 at best.

*Accept correct answer without work from candidate's answer to (ii).

Blunders (-3)

B1 Precedent error

B2 Sign errors

B3 Only one set of coordinates found

Attempts (2marks)

A1 One x -value substituted and stops.

A2 $f'(x)$ or $\frac{dy}{dx}$ mentioned.

MARKING SCHEME

LEAVING CERTIFICATE EXAMINATION 2006

MATHEMATICS – ORDINARY LEVEL – PAPER 2

GENERAL GUIDELINES FOR EXAMINERS – PAPER 2

1. Penalties of three types are applied to candidates' work as follows:
 - Blunders - mathematical errors/omissions (-3)
 - Slips - numerical errors (-1)
 - Misreadings (provided task is not oversimplified) (-1).

Frequently occurring errors to which these penalties must be applied are listed in the scheme. They are labelled: B1, B2, B3,..., S1, S2,..., M1, M2,...etc. These lists are not exhaustive.

2. When awarding attempt marks, e.g. Att(3), note that
 - any *correct, relevant* step in a part of a question merits at least the attempt mark for that part
 - if deductions result in a mark which is lower than the attempt mark, then the attempt mark must be awarded
 - a mark between zero and the attempt mark is never awarded.
3. Worthless work is awarded zero marks. Some examples of such work are listed in the scheme and they are labelled as W1, W2,...etc.
4. The phrase “hit or miss” means that partial marks are not awarded – the candidate receives all of the relevant marks or none.
5. The phrase “and stops” means that no more work is shown by the candidate.
6. Special notes relating to the marking of a particular part of a question are indicated by an asterisk. These notes immediately follow the box containing the relevant solution.
7. The sample solutions for each question are not intended to be exhaustive lists – there may be other correct solutions. Any examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his/her advising examiner.
8. Unless otherwise indicated in the scheme, accept the best of two or more attempts – even when attempts have been cancelled.
9. The *same* error in the *same* section of a question is penalised *once* only.
10. Particular cases, verifications and answers derived from diagrams (unless requested) qualify for attempt marks at most.
11. A serious blunder, omission or misreading results in the attempt mark at most.
12. Do not penalise the use of a comma for a decimal point, e.g. €5.50 may be written as €5,50.

APPLYING THE GUIDELINES

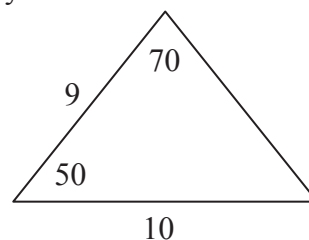
Penalties are subject to marks already secured.

Blunders - examples of blunders are as follows:

- Algebraic errors: $8x + 9x = 17x^2$ or $5p \times 4p = 20p$
- Sign error: $-3(-4) = -12$ or $(-3)^2 = 6$.
- Fraction error: Incorrect fraction inversion etc. apply once
- Cross-multiplication error.
- Error in misplacing the decimal point.
- Transposing error: $-2x - k + 3 = 0 \Rightarrow -2x = 3 + k$ or $-3x = 6 \Rightarrow x = 2$
or $4x = 12 \Rightarrow x = 8$ each time.
- Distributive law errors (once per term, unless otherwise directed)
 $\frac{1}{2}(3 - x) = 6 \Rightarrow 6 - 2x = 6$ or $-(4x + 3) = -4x + 3$ or $3(2x + 4) = 6x + 4$
- Expanding brackets incorrectly (apply once unless directed otherwise)
 $(2x - 3)(x + 4) = 8x^2 - 12x$
- Omission, if work not oversimplified, unless directed otherwise.
- Index error, each time unless directed otherwise.
- Factorisation: error in one or both factors of a quadratic, apply once
 $2x^2 - 2x - 3 = (2x - 1)(x + 3)$.
- Root errors from candidate's factors, error in one or both roots, apply once
- Incorrect substitution into formulae (where not an obvious slip):

e.g. $2x^2 + 3x + 4 = 0 \Rightarrow x = \frac{-3 \pm \sqrt{9 - 4(2)(4)}}{2(2)}$

or $\frac{10}{\sin 70} = \frac{9}{\sin 50}$



- Incorrectly treating co-ordinates as (x_1, x_2) and (y_1, y_2) when using co-ordinate geometry formula.
- Errors in formula for example: $\frac{y_2 + y_1}{x_2 + x_1}$ or $A = P\left(1 + \frac{n}{100}\right)^r$ or $a^2 = b^2 + c^2 + bc \cos A$ or $\sqrt{(x_2 - x_1)^2 - (y_2 - y_1)^2}$, except as indicated in scheme.

Slips – examples are as follows:

- Numerical slips such as: $4 + 7 = 10$ or $3 \times 6 = 24$ but $5 + 3 = 15$ is a blunder.
- An omitted round-off to a required level of accuracy or an incorrect round-off to either the incorrect accuracy or an early round-off are penalised as a slip once in each section. This applies to **Q1** (b) (ii); **Q5** (b) (i) and (ii), (c) (i) and (ii); **Q10** (c) (ii) unless otherwise directed.
- However, an early round-off which has the effect of simplifying the work is at least a blunder.
- The omission of the units of measurement in an answer or giving the incorrect units of measurement is treated as a slip once per part (a), (b) and (c) of each question where appropriate and at the first place where it matters. This applies to **Q1** (a),(b), (c); and **Q4** (c).

Misreadings

- Examples such as 436 for 346 will not alter the nature of the question and are penalised -1.
- However, writing 5026 as 5000 would alter the work and is penalised as at least a blunder.

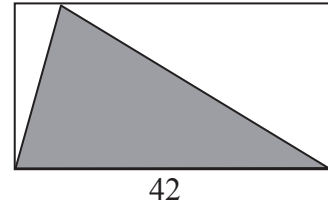
Note: A correct relevant formula isolated and stops: award the attempt mark if the formula is not in the Tables.

QUESTION 1

Part (a)	10 marks	Att 4
Part (b)	25 marks	Att 9
Part (c)	15 marks	Att 5

Part (a)	10 (5, 5) marks	Att (2, 2)
-----------------	------------------------	-------------------

The diagram shows a rectangle of length 42 cm .
The area of the rectangle is 966 cm² .



- (i) Find the height of the rectangle.
(ii) Find the area of the shaded triangle.

a(i)	5 marks	Att 2
-------------	----------------	--------------

$\text{Area of rectangle} = 42 \times h \downarrow = 966 \Rightarrow h = 966 \div 42 = 23 \text{ cm} \downarrow$ <p style="text-align: center;">[2 marks]</p>		<p style="text-align: center;">[5 marks]</p>
---	--	--

- * Accept correct answer without work.
- * Each step presupposes the previous ones.
- * Not more than 1 mark may be deducted for errors in calculations, if correct operation shown.
- * Any error other than an obvious slip merits the attempt mark at most.

Attempts (2 marks)

- A1 Some relevant work, e.g. draws vertical height on diagram or $42 = \text{base}$ and stops.
- A2 Labels vertical edge or uses variable to represent vertical edge.
- A3 Correct relevant formula not transcribed from the tables.
- A4 Substitution of the given dimension into a reasonable formula.
- A5 Writes 966×42 with or without work.

Worthless (0)

- W1 Incorrect answer without work, except A5.
- W2 966 ± 42 with or without work.

a(ii)	5 marks	Att 2
--------------	----------------	--------------

$\begin{aligned} \text{Area of triangle} &= \frac{1}{2}(966) \\ &= 483 \text{ cm}^2 \end{aligned}$	<p style="text-align: center;">or</p> $\begin{aligned} \text{Area of triangle} &= \frac{1}{2} \times 42 \times 23 \\ &= 483 \text{ cm}^2 \end{aligned}$	<p style="text-align: right;">[2 marks]</p> <p style="text-align: right;">[5 marks]</p>
--	--	---

- * Accept correct answer without work.
- * Accept answers consistent with section (i).
- * Any error other than an obvious slip merits the attempt mark at most.

Slips (-1)

- S1 Each slip, including units penalty, to a maximum of 3.

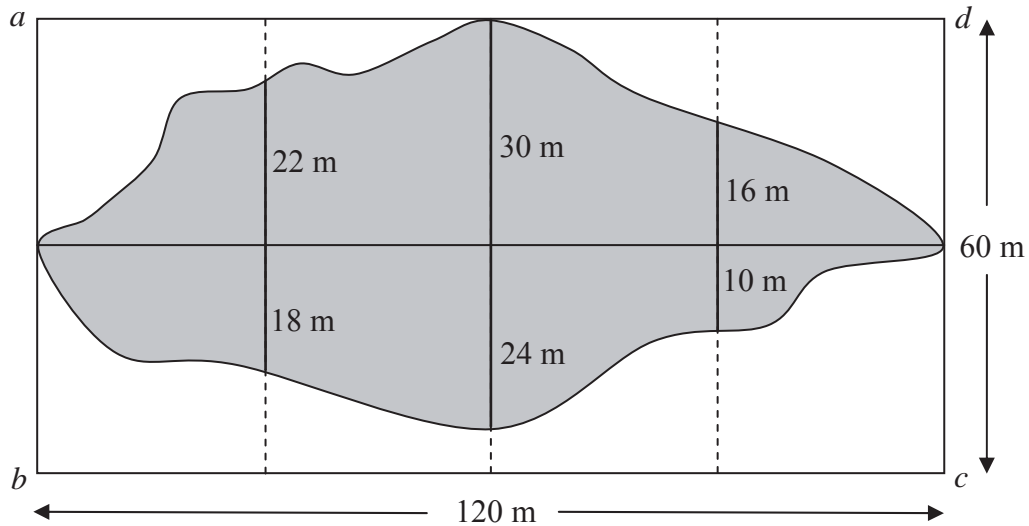
Attempts (2 marks)

- A1 Statement of, or correct use of, any relevant result e.g. rectangle comprises two triangles.

Worthless (0 marks)

- W1 966 or 23 only.
- W2 Incorrect answer without work.

Archaeologists excavating a rectangular plot $abcd$ measuring 120 m by 60 m divided the plot into eight square sections as shown on the diagram. At the end of the first phase of the work the shaded area had been excavated. To estimate the area excavated, perpendicular measurements were made to the edge of the excavated area, as shown.



- (i) Use Simpson's Rule to estimate the area excavated.
- (ii) Express the excavated area as a percentage of the total area, correct to the nearest whole number.

(b) (i) Use of formula
Calculations

15 marks
5 marks

Att 5
Att 2

$$h = 120 \div 4 = 30$$

$$\begin{aligned} \text{Area} &= \frac{h}{3} \{F + L + 2(\text{odds}) + 4(\text{evens})\} && [5 \text{ marks}] \\ &= \frac{30}{3} \{0 + 0 + 2(30) + 4(22 + 16)\} + \frac{30}{3} \{0 + 0 + 2(24) + 4(18 + 10)\} && [15 \text{ marks}] \\ &= 10(60 + 152) + 10(48 + 112) \\ &= 10(212) + 10(160) = 2120 + 1600 = 3720 \text{ m}^2. && [20 \text{ marks}] \end{aligned}$$

or

$$\begin{aligned} \text{Area} &= \frac{30}{3} \{0 + 0 + 2(54) + 4(40 + 26)\} && [15 \text{ marks}] \\ &= 10(108 + 264) = 10(372) = 3720 \text{ m}^2. && [20 \text{ marks}] \end{aligned}$$

- * Allow $\frac{h}{3} = \{F + L + \text{TOFE}\}$ and penalise in calculations if formula not used correctly.
- * Accept correct TOFE or TOFE consistent with candidates F and L.
- * Marks for section (i) may be awarded for work done in section (ii).

Blunders (-3)

- B1 Incorrect $\frac{h}{3}$ (once).
- B2 Incorrect F and / or L or extra terms with F and / or L (once).
- B3 Incorrect TOFE (once), if not consistent with candidates F and L.
- B4 E or O omitted (once).

Slips (-1)

- S1 Each slip to a maximum of 3.

Attempts (5 marks for substituting into formula, 2 marks for calculations)

- A1 Some relevant step, e.g. identifies F and / or L or odds or evens and stops. [5 m]
A2 Statement of Simpson's Rule not transcribed from tables. [5 m]
A3 E and O omitted (candidate may be awarded attempt(s) at most). [Max. 5 m and/or 2 m]
A4 Area of rectangle or of one square. [5 m]
A5 Correct answer without work. [5 m + 2 m]
A6 Some correct calculation only. [2 m]

Worthless (0)

- W1 Incorrect answer without work.
W2 Formula transcribed from tables and stops.

b(ii) **5 marks** **Att 2**

$$\text{Area of } abcd = 120 \times 60 = 7200 \text{ m}^2.$$

$$\text{Percentage excavated} = \frac{3720}{120 \times 60} \times 100 = 51.66 \text{ or } \frac{3720}{7200} \times 100 = 51.66 = 52\%.$$

* Accept correct or consistent answer without work, subject to round-off penalty.

Blunders (-3)

- B1 Mathematical error in calculation, e.g. divides instead of multiplies by 100.

Attempts (2 marks)

- A1 Some relevant step, e.g. work with candidates area or 7200 or $3720 \div 7200$.

Part (c) **15 (10, 5) marks** **Att (3, 2)**

- (i) The volume of a hemisphere is $486\pi \text{ cm}^3$.
Find the radius of the hemisphere.

- (ii) Find the volume of the smallest rectangular box that the hemisphere will fit into.

c(i) **10 marks** **Att 3**

$$\text{Volume of hemisphere} = \frac{2}{3}\pi r^3 = 486\pi \downarrow \Rightarrow r^3 = \frac{486 \times 3}{2} \text{ or } 729 \Rightarrow r = 9 \text{ cm} \downarrow$$

[4 marks] [7 marks] [10 marks]

- * Accept volume of hemisphere read as $\frac{2}{8}\pi r^3$.
* Accept a correct answer based on $3 \leq \pi \leq 3.2$, substituted.

Blunders (-3)

- B1 Incorrect relevant hemisphere formula e.g. πr^3 , $\frac{1}{3}\pi r^3$, and continues.

Slips (-1)

- S1 Each slip to a maximum of 3.

Misreading (-1)

- M1 $\frac{2}{3}\pi r^3 = 486$ and continues.

Attempts (3 marks)

- A1 Some relevant step, e.g. diagram with a correct dimension shown.
A2 Correct answer without work.

QUESTION 2

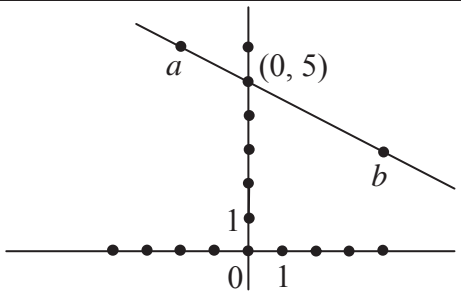
Part (a)	30 marks	Att 10
Part (b)	20 marks	Att 7

Part (a) **30 (10, 5, 10, 5) marks** **Att (3, 2, 3, 2)**

$a(-2, 6)$ and $b(4, 3)$ are two points.

- (i) Plot a and b on a co-ordinate diagram.
- (ii) From your diagram, write down the co-ordinates of the point at which the line ab cuts the y -axis.
- (iii) Find the slope of ab .
- (iv) Calculate the area of the triangle abc , where the co-ordinates of c are $(1, -3)$.

(a) (i) **10 marks** **Att 3**



Axes	[3 marks]
One correct point	[7 marks]
Two correct points	[10 marks]

- * Accept a vertical x -axis and a horizontal y -axis, or different scales on the x and y axes.
- * Intervals should be indicated or implied.

Blunders (-3)

- B1 Scales unreasonably inconsistent (to the eye).
- B2 $(-2, 6)$ plotted as $(6, -2)$ and $(4, 3)$ plotted as $(3, 4)$ if not consistent.

Attempts (3 marks)

- A1 Plots axes and stops.

(a) (ii) **5 marks** **Att 2**

$(0, 5)$

- * No penalty if the line ab is not drawn.

Blunders (-3)

- B1 Point written as $(5, 0)$.
- B2 Co-ordinates read are incorrect, to the eye, from candidates diagram.

Slips (-1)

- S1 Point given as y co-ordinate, e.g. 5 or $y = 5$.
- S2 Reads incorrect axis, $(10, 0)$.
- S3 Finds equation of ab without plotting point(s) and continues.

Attempts (2 marks)

- A1 Line ab drawn but no point indicated.
- A2 States $x = 0$.

(a) (iii)

10 marks

Att 3

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 6}{4 + 2} = \frac{-3}{6} \text{ or } -\frac{1}{2}$ <div style="display: flex; justify-content: space-around; font-size: small;"> [3 marks] [7 marks] [10 marks] </div>
--

* Accept correct answer without work.

Blunders (-3)

B1 Incorrect relevant formula e.g. $\frac{y_2 + y_1}{x_2 + x_1}$ or $\frac{y_2 - y_1}{x_1 - x_2}$ or $\frac{x_2 - x_1}{y_2 - y_1}$ and continues.

B2 Two or more incorrect substitutions, if formula is written.

Slips (-1)

S1 One incorrect sign in $(x_2 - x_1)$ or $(y_2 - y_1)$ part of formula.

S2 One incorrect substitution.

Attempts (3 marks)

A1 Some relevant step, e.g. $(-2, 6)$ with x_1 and y_1 identified.

A2 Incorrect formula, partially substituted.

(a) (iv)

5 marks

Att 2

$(-2, 6) \rightarrow (0, 0); \quad (4, 3) \rightarrow (6, -3); \quad (1, -3) \rightarrow (3, -9).$	[2 marks]
$\text{Area} = \frac{1}{2} x_1 y_2 - x_2 y_1 = \frac{1}{2} 6 \times -9 - 3 \times -3 = \frac{1}{2} -54 + 9 = \frac{1}{2} -45 $	or 22.5
[4 marks]	[5 marks]

or

$\text{Area} = \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$	[2 marks]
$= \frac{1}{2} -2(3 + 3) + 4(-3 - 6) + 1(6 - 3) $	[4 marks]
$= \frac{1}{2} -12 - 36 + 3 = \frac{1}{2} -45 $	or 22.5
[5 marks]	

or

$\text{Area} = \frac{1}{2} [x_1 y_2 + x_2 y_3 + x_3 y_1 - x_1 y_3 - x_3 y_2 - x_2 y_1]$	[2 marks]
$= \frac{1}{2} [-2 \times 3 + 4 \times -3 + 1 \times 6 + 2 \times -3 - 1 \times 3 - 4 \times 6]$	[4 marks]
$= \frac{1}{2} [-6 - 12 + 6 - 6 - 3 - 24] = \frac{1}{2} -45 $	or 22.5
[5 marks]	

or

$\text{Area} = \frac{1}{2} ab \cdot bc = \frac{1}{2} \sqrt{45} \cdot \sqrt{45} = 22.5$
--

* $\frac{1}{2} |-45| = -22.5$ incurs no penalty.

Blunders (-3)

B1 Incorrect relevant formula and continues e.g. $\frac{1}{2} |x_1 y_2 + x_2 y_1|$ or omits the $\frac{1}{2}$.

B2 Two or more incorrect substitutions.

B3 No translation or error in translation.

Slips (-1)

S1 One incorrect substitution.

S2 Obvious misreading of one co-ordinate e.g. if translation used.

Attempts (2 marks)

A1 Correct answer without work.

Worthless (0 marks)

W1 Irrelevant formula and stops e.g. $\frac{1}{2}$ on its own.

Blunders (-3)

- B1 Any blunder in arriving at slope of K , once.
- B2 Switches x and y in substituting.
- B3 Incorrect relevant formula, e.g. $y + y_1 = m(x + x_1)$. [Both signs incorrect.]
- B4 Two or more incorrect substitutions or signs in formula.
- B5 Uses an arbitrary point.

Slips (-1)

- S1 One incorrect sign in substitution.

Attempts (2 marks)

- A1 Incorrect relevant formula with some substitution.

b(iii)

5 marks

Att 2

$x = 0 \cap 3x + 2y - 7 = 0 \Rightarrow 2y = 7 \Rightarrow y = \frac{7}{2}$	Vertex $(0, \frac{7}{2})$
$x = 0 \cap 3x + 2y - 4 = 0 \Rightarrow 2y = 4 \Rightarrow y = 2$	Vertex $(0, 2)$
$x = 3 \cap 3x + 2y - 4 = 0 \Rightarrow 2y = -5 \Rightarrow y = -\frac{5}{2}$	Vertex $(3, -\frac{5}{2})$
$x = 3 \cap 3x + 2y - 7 = 0 \Rightarrow 2y = -2 \Rightarrow y = -1$	Vertex $(3, -1)$

- * Award 2 marks for one correct vertex, 3 marks for 2 correct, 4 marks for 3 correct and 5 marks for 4 correct vertices, subject to S1 and blunders in simplifying K .
- * Accept correct or consistent answers without work.

Blunders (-3)

- B1 Incorrect substitution of x -value for y , applied once.
- B2 Uses an arbitrary x or y value to find a pair of co-ordinates.

Slips (-1)

- S1 Answer not expressed as co-ordinates, applied once.
- S2 Each numerical slip to a maximum of 3.

Attempts (2 marks)

- A1 Relevant work by trial and error.
- A2 Line(s) plotted (no points named).
- A3 Work towards simplifying K , even if done in (ii).

Worthless (0 marks)

- W1 Tests an arbitrary point in one line.

QUESTION 3

Part (a)	30 marks	Att 10
Part (b)	20 marks	Att 7

Part (a)	30 (5, 10, 10, 5) marks	Att (2, 3, 3, 2)
-----------------	--------------------------------	-------------------------

The circle C has equation $x^2 + y^2 = 25$.

The line L is a tangent to C at the point $(-3, 4)$.

- (i) Verify that the point $(-3, 4)$ is on C .
- (ii) Find the slope of L .
- (iii) Find the equation of L .
- (iv) The line T is another tangent to C and is parallel to L . Find the co-ordinates of the point at which T touches C .

a (i)	5 marks	Att 2
--------------	----------------	--------------

	$x^2 + y^2 = 25 \Rightarrow (-3)^2 + 4^2 \downarrow = 9 + 16 \downarrow = 25 \downarrow$ <div style="display: flex; justify-content: space-around; width: 100%;"> [2 marks] [4 marks] [5 marks] </div>	
or	$\sqrt{(0+3)^2 + (0-4)^2} = \sqrt{25}$ or $5 \downarrow$ and $r = \sqrt{25}$ or $5 \downarrow$ or $(0+3)^2 + (0-4)^2 \downarrow = 25 \downarrow$ <div style="display: flex; justify-content: space-around; width: 100%;"> [2 marks] [5 marks] [2 marks] [5 marks] </div>	

- * Accept “distance from $(-3, 4)$ to $(0, 0)$ is 5 which is length of the radius”.
- * Any error other than an obvious slip merits the attempt mark at most.

Attempts (2 marks)

- A1 Some relevant step e.g. mentions or indicates origin or 5 or $(-3)^2$.
- A2 Correct relevant formula and stops, e.g. $x^2 + y^2 = r^2$ or $(x-h)^2 + (y-k)^2 = r^2$.
- A3 Any formula with $(x_2 - x_1)$ or $(y_2 - y_1)$ and some correct substitution.
- A4 Accurate diagram drawn with $(-3, 4)$ shown on circle, at most.
- A5 Statement of Theorem of Pythagoras.

(a) (ii)	10 marks	Att 3
-----------------	-----------------	--------------

	$\text{Slope of radius} = \frac{y_2 - y_1}{x_2 - x_1} \downarrow = \frac{4 - 0}{-3 - 0} = -\frac{4}{3} \downarrow$ <div style="display: flex; justify-content: space-around; width: 100%;"> [3 marks] [7 marks] </div>	
	$\text{Slope of } L = m_1 \Rightarrow m_1 \times -\frac{4}{3} = -1 \Rightarrow m_1 = \frac{3}{4} \downarrow$ <div style="text-align: right; margin-right: 20px;">[10 marks]</div>	

- * Accept slope = $\frac{\text{vertical}}{\text{horizontal}} = \frac{4}{-3}$ for 7 marks.
- * Accept correct answer without work.

Blunders (-3)

- B1 Error in more than one sign when substituting.
- B2 Uses one arbitrary point in finding slope of radius.

Slips (-1)

- S1 One error in sign in slope formula or substitution.

(a) (iii)

10 marks

Att 3

	Equation of L : $y - 4 = \frac{3}{4}(x + 3)$ $\Rightarrow 4y - 16 = 3x + 9 \Rightarrow 3x - 4y + 25 = 0$	[10 marks]
<i>or</i>	Equation of L : $y = \frac{3}{4}x + c$ $4 = \frac{3}{4}(-3) + c \Rightarrow c = 4 + \frac{9}{4} = \frac{25}{4}$ $y = \frac{3}{4}x + \frac{25}{4} \Rightarrow 4y = 3x + 25$	[3 marks] [10 marks]
<i>or</i>	Equation of L : $x_1x + y_1y = r^2$ $\Rightarrow -3x + 4y = 25$ or $3x - 4y + 25 = 0$	[3 marks] [10 marks]

- * Accept an equation consistent with earlier work.
- * Accept a correct answer without work shown.
- * The equation of a circle given for the equation of a line merits the attempt mark at most.

Blunders (-3)

- B1 Uses an arbitrary point for the line.
- B2 Uses an incorrect slope, e.g. of radius if not penalised above.

Slips (-1)

- S1 One incorrect sign in line formula.
- S2 One incorrect sign in substitution.

Attempts (3 marks)

- A1 Gives a correct relevant formula and stops.
- A2 Draws a rough diagram.

(a) (iv)

5 marks

Att 2

	$(-3, 4) \rightarrow (0, 0) \rightarrow (3, -4)$	[5 marks]
<i>or</i>	$\frac{-3+x}{2} = 0 \Rightarrow x = 3; \frac{4+y}{2} = 0 \Rightarrow y = -4$	[5 marks]

- * Accept a correct answer with no work shown.

Blunders (-3)

- B1 Uses point $(4, -3)$ or $(-4, 3)$.
- B2 Blunder in use of translation e.g. $(0, 0) \rightarrow (-3, 4) \rightarrow (-6, 8)$.

Slips (-1)

- S1 Obvious misreading of a co-ordinate.

Attempts (2 marks)

- A1 Diagram showing parallel tangents.

Part (b)

20 (5, 10, 5) marks

Att (2, 3, 2)

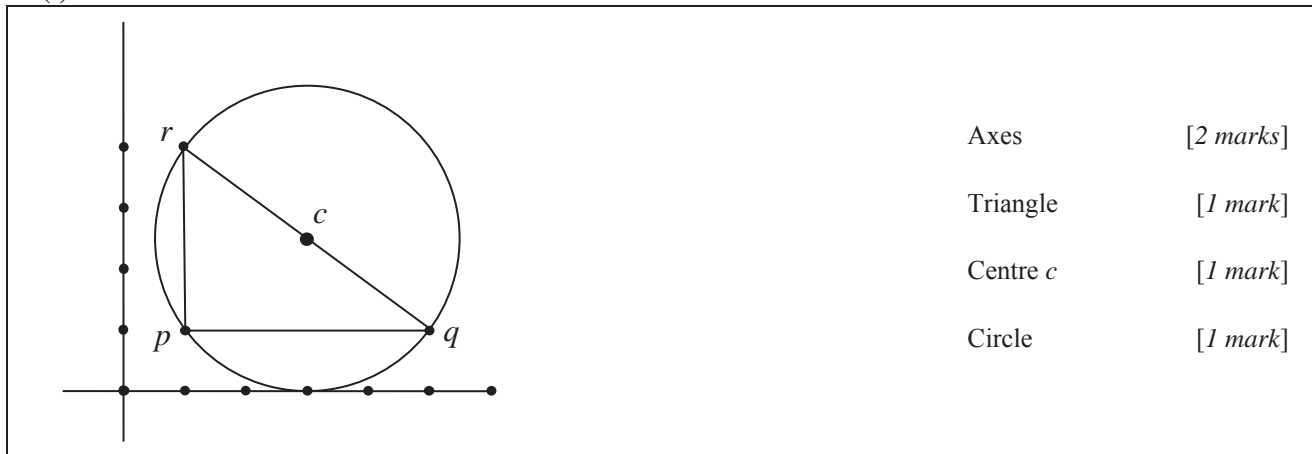
The vertices of a right-angled triangle are $p(1, 1)$, $q(5, 1)$ and $r(1, 4)$.
The circle K passes through the points p , q and r .

- (i) On a co-ordinate diagram, draw the triangle pqr .
Mark the point c , the centre of K and draw K .
- (ii) Find the equation of K .
- (iii) Find the equation of the image of K under the translation $(5, 1) \rightarrow (1, 4)$.

b(i)

5 marks

Att 2



- * Accept horizontal y axis and vertical x axis.
- * Scale must be indicated or implied for full marks.
- * Accept reasonable freehand drawing of circle.

Blunders (-3)

- B1 Confuses x and y co-ordinates, e.g. $(5, 1)$ plotted as $(1, 5)$, if not consistent with 1st *.
- B2 Scale unreasonably inconsistent.

(ii)

10 marks

Att 3

Centre c = mid-point of $[qr] = \left(\frac{5+1}{2}, \frac{1+4}{2} \right) = (3, 2\frac{1}{2})$ [3 marks]

Radius = $|cr| = \sqrt{(3-1)^2 + (2\frac{1}{2}-4)^2} = \sqrt{4 + \frac{9}{4}} = \sqrt{\frac{25}{4}}$ or $\frac{5}{2}$ or

Radius = $|cq| = \sqrt{(3-5)^2 + (2\frac{1}{2}-1)^2} = \sqrt{4 + \frac{9}{4}} = \sqrt{\frac{25}{4}}$ or $\frac{5}{2}$ or

Radius = $\frac{1}{2}|qr| = \frac{1}{2}\sqrt{(5-1)^2 + (1-4)^2} = \frac{1}{2}\sqrt{16+9} = \frac{1}{2}\sqrt{25} = \frac{5}{2}$ [7 marks]

Equation of K : $(x-3)^2 + (y-2\frac{1}{2})^2 = (\frac{5}{2})^2$ or $\frac{25}{4}$ [10 marks]

or

Equation of K : $x^2 + y^2 + 2gx + 2fy + c = 0$ [3 marks]

Substitute $(1, 1)$: $1 + 1 + 2g + 2f + c = 0$

Substitute $(5, 1)$: $25 + 1 + 10g + 2f + c = 0$

Substitute $(1, 4)$: $1 + 16 + 2g + 8f + c = 0$ [7 marks]

Solving: $g = -3, f = -2\frac{1}{2}, c = 9$ [10 marks]

Equation of K : $x^2 + y^2 - 6x - 5y + 9 = 0$

- * Equation of a line given for equation of a circle merits the attempt mark at most in (ii) & (iii).
- * If $(3, 2\frac{1}{2})$ marked on diagram, award 3 marks here if there is no further work.

Blunders (-3)

- B1 Any blunder in radius, once.
B2 Any blunder in centre, e.g. midpoint of an incorrect side used, once.

Slips (-1)

- S1 One incorrect substitution.
S2 One incorrect non-central sign in formula e.g. $(x-h)^2 + (y-k)^2 = r^2$.

Attempts (3 marks)

- A1 States $[rq]$ is diameter or centre is midpoint of $[rq]$ or similar.
A2 States result that angle in a semi-circle is a right angle.
A3 Gives correct relevant formula for equation of circle and stops.
A4 Correct answer without work shown.

b(iii)

5 marks

Att 2

By $(5, 1) \rightarrow (1, 4)$

$(3, 2\frac{1}{2}) \rightarrow (3-4, 2\frac{1}{2}+3) = (-1, 5\frac{1}{2})$, the centre of the image. [2 marks]

Equation of image of K : $(x+1)^2 + (y-5\frac{1}{2})^2 = (\frac{5}{2})^2$ or $\frac{25}{4}$. [5 marks]

- * Accept r^2 from (ii).
* Accept centre from (ii).

Blunders (-3)

- B1 Incorrect centre used e.g. $(5, 1)$ or $(1, 4)$.
B2 Error in use of translation, unless an obvious slip.

Slips (-1)

- S1 Obvious misreading of a co-ordinate.

Attempts (2 marks)

- A1 Effort at a relevant diagram.
A2 Attempt to use the given translation.
A3 Correct answer without work shown.
A4 Gives equation $(x-h)^2 + (y-k)^2 = r^2$ or $x^2 + y^2 + 2gx + 2fy + c = 0$.

Worthless (0 marks)

- W1 Gives equation $x^2 + y^2 = r^2$.

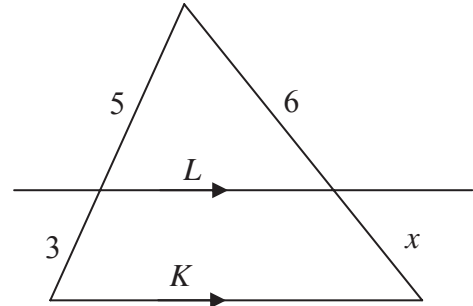
QUESTION 4

Part (a)	10 marks	Att 3
Part (b)	20 marks	Att 7
Part (c)	20 marks	Att 7

Part (a) **10 marks** **Att 3**

In the diagram, $L \parallel K$.

Find the value of x .



(a) **10 marks** **Att 3**

$$\frac{5}{3} = \frac{6}{x} \downarrow \Rightarrow 5x = 18 \downarrow \Rightarrow x = \frac{6 \times 3}{5} \downarrow = \frac{18}{5} \text{ or } 3.6 \downarrow$$

[3 marks]
[6 marks]
[9 marks]
[10 marks]

* Accept correct answer without work shown.

Blunders (-3)

- B1 Blunder in ratio e.g. inverting one side.
- B2 Incorrect application of theorem.

Slips (-1)

- S1 Each numerical slip to a maximum of 3.

Attempts (3 marks)

- A1 Some relevant step or statement, e.g. $5/6$ and stops.

Worthless (0 marks)

- W1 Incorrect answer without work.

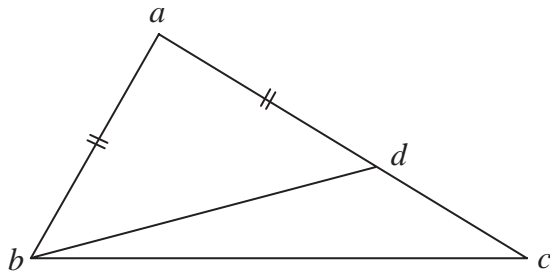
Prove that if the lengths of two sides of a triangle are unequal, then the degree-measures of the angles opposite to them are unequal, with the greater angle opposite to the longer side.

Part b

20 marks

Att 7

In the triangle abc , $|ac| > |ab|$.



To prove: $|\angle abc| > |\angle acb|$.

Construction:

Let d be a point on $[ac]$ such that

$|ad| = |ab|$. Join bd .

[7 marks]

Proof:

Δabd is isosceles

$\Rightarrow |\angle abd| = |\angle adb|$.

[10 marks]

$$|\angle adb| = |\angle dbc| + |\angle bcd| \quad (\text{Exterior angle})$$

[13 marks]

$$|\angle abd| = |\angle dbc| + |\angle bcd|$$

[16 marks]

$$|\angle abd| + |\angle dbc| = 2|\angle dbc| + |\angle bcd|$$

[19 marks]

$$\text{Hence, } |\angle abc| > |\angle bcd|.$$

[20 marks]

or

$$|\angle adb| = |\angle dbc| + |\angle bcd| \quad (\text{Exterior angle})$$

[13 marks]

$$\Rightarrow |\angle bcd| = |\angle adb| - |\angle dbc|$$

$$|\angle abc| = |\angle abd| + |\angle dbc|$$

[16 marks]

$$= |\angle adb| + |\angle dbc|$$

[19 marks]

$$\text{Hence, } |\angle abc| > |\angle bcd|.$$

[20 marks]

or

$$|\angle abc| > |\angle abd|$$

[13 marks]

$$\Rightarrow |\angle abc| > |\angle adb|$$

[16 marks]

$$\text{But } |\angle adb| > |\angle dcb|$$

[19 marks]

$$\Rightarrow |\angle abc| > |\angle acb|.$$

[20 marks]

* Proof without a diagram merits att 7, if proof can be reconciled with a diagram.

Blunders (-3)

B1 Each step omitted or incorrect, except the last.

B2 Steps written in an illogical order. [Penalise once only.]

[Note: Some of the steps above may be interchanged.]

Attempts (7 marks)

A1 Any relevant step stated or indicated, e.g. triangle with additional relevant information.

A2 States or illustrates a special case, e.g. measuring the line segments and angles.

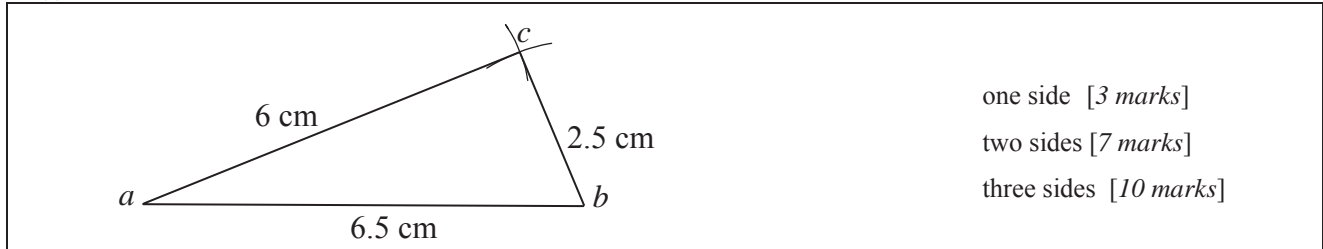
Worthless (0 marks)

W1 Any irrelevant theorem, subject to the attempt mark.

W2 Triangle only.

Part (c)**20 (10, 5, 5) marks****Att (3, 2, 2)**

- (i) Construct a triangle abc in which $|ab| = 6.5$ cm, $|bc| = 2.5$ cm and $|ac| = 6$ cm.
- (ii) Construct the image of the triangle abc under the enlargement of scale factor 1.8 and centre c .
- (iii) Given that the area of triangle abc is 7.5 cm², find the area of the image triangle.

c(i)**10 marks****Att 3**

- one side [3 marks]
 two sides [7 marks]
 three sides [10 marks]

- * Allow tolerance of ± 5 mm in measurements, in each section.
 * It is not necessary to label the vertices, or write the measurements on diagram, in each section.

Blunders (-3)

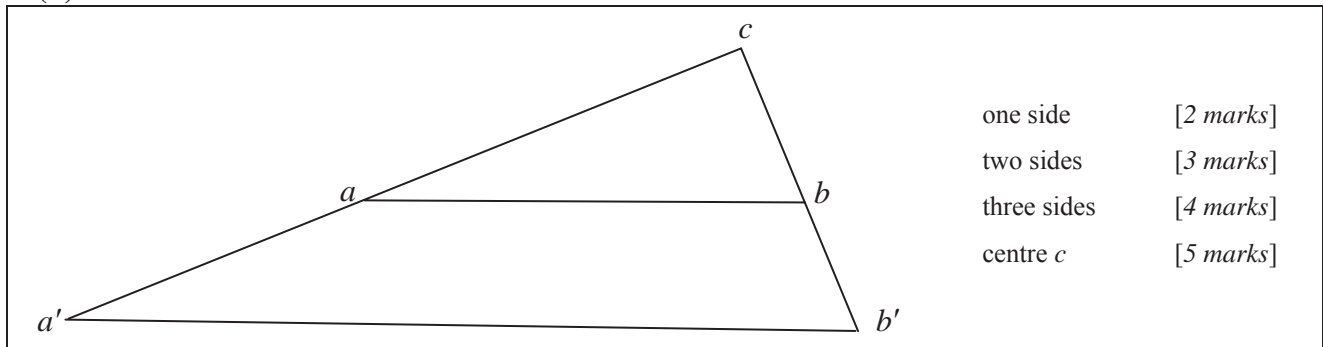
B1 Draws the required triangle to scale.

Slips (-1)

- S1 Side within tolerance but not straight i.e. no straight edge used (once).
 S2 Points a , b and c not joined.
 S3 Arc not indicated (once).

Attempts (3 marks)

- A1 Relevant step, e.g. one side drawn or a rough sketch.
 A2 A triangle, with no side within tolerance.

c(ii)**5 marks****Att 2**

- one side [2 marks]
 two sides [3 marks]
 three sides [4 marks]
 centre c [5 marks]

- * Allow tolerance of ± 5 mm in measurements, in each section.

Slips (-1)

- S1 Scale factor 0.8 or -1.8 .
 S2 Centre at a or b .
 S3 Sides within tolerance but not straight.
 S4 Vertices not joined.

Attempts (2 marks)

- A1 Some relevant step, e.g. centre clearly indicated.

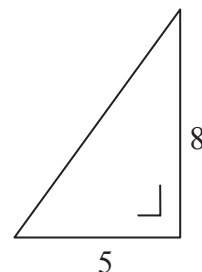
QUESTION 5

Part (a)	10 marks	Att 4
Part (b)	20 marks	Att 6
Part (c)	20 marks	Att 6

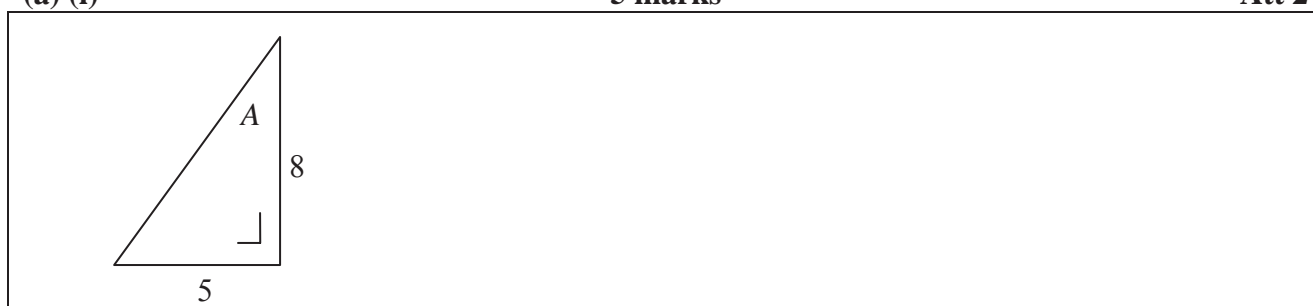
Part (a) **10 (5, 5) marks** **Att (2, 2)**

The lengths of two sides of a right-angled triangle are shown in the diagram.

- (i) Copy the diagram into your answer book
and on it mark the angle A such that $\tan A = \frac{5}{8}$.
- (ii) Find the area of the triangle.



(a) (i) **5 marks** **Att 2**



* Accept any clear indication of correct angle.

Attempts (2 marks)

- A1 Diagram copied correctly.
A2 Writes $\tan A = \text{opposite/adjacent}$ or says top angle (without diagram drawn).
A3 Incorrect angle marked or both marked.

(a) (ii) **5 marks** **Att 2**

Area of triangle = $\frac{1}{2}$ base \times perpendicular height = $\frac{1}{2} \times 5 \times 8 = 20$. <i>or</i> Area of triangle = $\frac{1}{2} ab \sin C = \frac{1}{2} \times 5 \times 8 \sin 90^\circ = 20$.
--

* Accept correct answer without work shown.

Slips (-1)

- S1 Answer not multiplied out or maximum of one slip in working.

Attempts (2 marks)

- A1 Some correct substitution into incorrect relevant formula e.g. $\frac{1}{2} a \sin C$.
A2 Finds length of hypotenuse.

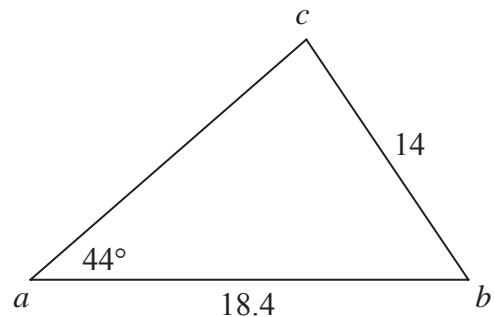
Worthless (0 marks)

- W1 Incorrect answer without work.

Part (b)**20 (10, 10) marks****Att (3, 3)**

In the triangle abc ,
 $|ab| = 18.4$, $|bc| = 14$ and $|\angle cab| = 44^\circ$.

- (i) Find $|\angle bca|$, correct to the nearest degree.
 (ii) Find the area of the triangle abc , correct to the nearest whole number.

**b(i)****10 marks****Att 3**

$$\frac{14}{\sin 44} = \frac{18.4}{\sin |\angle bca|} \quad [3 \text{ marks}]$$

$$\Rightarrow \sin |\angle bca| = \frac{18.4 \times \sin 44}{14} \quad \text{or} \quad \frac{18.4 \times 0.6947}{14} \quad \text{or} \quad 0.9130 \quad [6 \text{ marks}]$$

$$\Rightarrow |\angle bca| = 65.9^\circ \quad [9 \text{ marks}]$$

$$= 66^\circ \quad [10 \text{ marks}]$$

Blunders (-3)

- B1 Incorrect substitution into formula and continues.
 B2 Incorrect function read e.g. cosine instead of sine and continues.
 B3 Error in use of inverse function.
 B4 Error in use of sine rule (once).

The above blunders are to be applied in each of sections (b) (i), (b) (ii), (c) (i) and c(ii).

- B5 Uses radians (or gradient) mode incorrectly, applied once in part (b) and once in (c).

Slips (-1)

- S1 Numerical slips to a maximum of 3.

Attempts (3 marks)

- A1 Incorrect relevant formula (e.g. area of triangle) with some correct substitution.
 A2 Correct answer without work shown.

b(ii)**10 marks****Att 3**

$$|\angle abc| = 180^\circ - (44^\circ + 66^\circ) = 70^\circ \quad [3 \text{ marks}]$$

$$\text{Area } \Delta abc = \frac{1}{2} |ab| \cdot |bc| \cdot \sin |\angle abc| \quad \text{or} \quad \frac{1}{2} |ab| \times \text{perp. height}$$

$$= \frac{1}{2} (18.4)(14) \sin 70^\circ \quad \text{or} \quad \frac{1}{2} (18.4)(14)(0.9400) \quad [6 \text{ marks}]$$

$$= 121.072 \quad [9 \text{ marks}]$$

$$= 121. \quad [10 \text{ marks}]$$

* May use area = $\sqrt{s(s-a)(s-b)(s-c)}$.

* Where the only work shown is a correct formula transcribed from the Tables applied to the question with the answer award 10 marks for 121; 9 marks for 121.07; otherwise 0.

Blunders (-3)

- B6 Sum of angles in a triangle $\neq 180^\circ$.

Attempts (3 marks)

- A1 Some correct use of sine rule or cosine rule.
- A2 Perpendicular drawn from c to ab .
- A3 Answer of 121.07 or 121 without work.
- A4 Treats triangle as right-angled even if finishes e.g. $\frac{1}{2} \times 14 \times 18.4$.

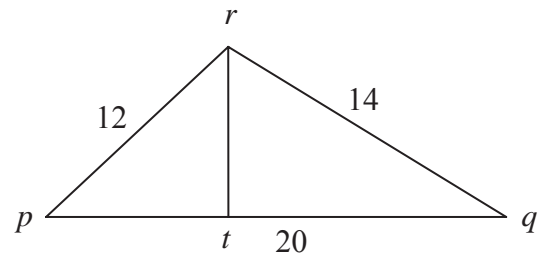
Part (c)

20 (10, 10) marks

Att (3, 3)

The lengths of the sides of the triangle pqr are
 $|pq| = 20$, $|qr| = 14$ and $|pr| = 12$.

- (i) Find $|\angle rpq|$, correct to one decimal place.
- (ii) Find $|rt|$, where $rt \perp pq$. Give your answer correct to the nearest whole number.



c(i)

10 marks

Att 3

$$a^2 = b^2 + c^2 - 2bc \cos A \Rightarrow 14^2 = 12^2 + 20^2 - 2(12)(20) \cos |\angle rpq| \quad [3 \text{ marks}]$$

$$\Rightarrow 196 = 144 + 400 - 480 \cos |\angle rpq| \Rightarrow \cos |\angle rpq| = \frac{348}{480} \quad [6 \text{ marks}]$$

or

$$\cos |\angle rpq| = \frac{b^2 + c^2 - a^2}{2bc} \text{ or } \frac{12^2 + 20^2 - 14^2}{2(12)(20)} \quad [3 \text{ marks}]$$

$$= \frac{144 + 200 - 196}{480} = \frac{348}{480} \quad [6 \text{ marks}]$$

$$\Rightarrow \cos |\angle rpq| = 0.725 \Rightarrow |\angle rpq| = 43.53^\circ \downarrow = 43.5^\circ \downarrow$$

[9 marks] [10 marks]

Blunders (-3)

- B7 Error in use of cosine rule (once).
- B8 Angle other than $|\angle rpq|$ found correctly if not obvious misreading.

Attempts (3 marks)

- A1 Correct formula with some correct substitution.
- A2 Incorrect relevant formula (e.g. area of triangle) with some correct substitution.
- A3 Correct answer without work shown.

Worthless (0 marks)

- W1 Measurement from diagram.

QUESTION 6

Part (a)	10 marks	Att 3
Part (b)	20 marks	Att 8
Part (c)	20 marks	Att 8

Part (a)	10 marks	Att 3
-----------------	-----------------	--------------

Evaluate $5\binom{8}{3} - 4\binom{8}{4}$.

(a)	10 marks	Att 3
------------	-----------------	--------------

$$5\binom{8}{3} - 4\binom{8}{4} = 5 \times \frac{8 \times 7 \times 6}{1 \times 2 \times 3} - 4 \times \frac{8 \times 7 \times 6 \times 5}{1 \times 2 \times 3 \times 4} = 0$$

[3 marks]
[7 marks]
[10 marks]

or

$$5\binom{8}{3} - 4\binom{8}{4} = 5 \times 56 - 4 \times 70 = 280 - 280 = 0$$

[3 marks]
[7 marks]
[10 marks]

* Accept correct answer without work shown.

Blunders (-3)

- B1 Treats combination as a permutation, once only.
- B2 Blunder in evaluating or expanding a term.

Slips (-1)

- S1 Numerical slips to a maximum of 3.

Attempts (3 marks)

- A1 Any indicated integer from solution in first method.
- A2 Writes $\binom{8}{5}$ or $\frac{8!}{3!5!}$ or $\frac{{}^8P_3}{3!}$ or 8P_3 or 8P_4 or 8C_3 or 8C_4 and stops.
- A3 Writes $8 - 3$ or $8 - 4$ and stops.
- A4 Works $5\binom{8}{3} - 4\binom{8}{4}$

Part (b)	20 (5, 5, 5, 5) marks	Att (2, 2, 2, 2)
-----------------	------------------------------	-------------------------

Niamh uses a password formed from one letter of her name followed by four of the digits 1 to 9, inclusive .

- (i) How many such passwords can be formed?
- (ii) How many of the passwords begin with N?
- (iii) How many of the passwords end in an even digit?
- (iv) How many of the passwords begin with N and use only odd digits?

(b) (i)

5 marks

Att 2

$$5 \times 9 \times 8 \times 7 \times 6 \downarrow = 15\,120 \downarrow$$

[4 marks] [5 marks]

or

$$\binom{5}{1} \times {}^9P_4 \downarrow = 5 \times 3024 \downarrow = 15\,120 \downarrow$$

[2 marks] [4 marks] [5 marks]

- * If sections of (b) are not identified and it is not obvious which section is being attempted, treat each section in order.
- * Accept correct answer without work in each section.

Attempts (2 marks)

- A1 Multiplication replaced by addition.
- A2 Any indicated integer from the solution in first method and stops.
- A3 An incomplete list with at least one element correct.
- A4 One incorrect digit in the list.

The above attempts apply to each section.

Worthless (0 marks)

- W1 Statement of Probability Theorem in each section.
- W2 An incorrect answer with no work shown.

(b) (ii)

5 marks

Att 2

$$1 \times 9 \times 8 \times 7 \times 6 \downarrow = 3024 \downarrow$$

[4 marks] [5 marks]

or

$$1 \times {}^9P_4 \downarrow = 1 \times 3024 \downarrow = 3024 \downarrow$$

[2 marks] [4 marks] [5 marks]

or

$$\frac{15120}{5} \downarrow = 3024 \downarrow$$

[4 marks] [5 marks]

- * Accept answer from (i) $\div 5$.

(b) (iii)

5 marks

Att 2

$$5 \times 8 \times 7 \times 6 \times 4 \downarrow = 6720 \downarrow$$

[4 marks] [5 marks]

Attempts (2 marks)

- A5 Attempt to apply condition i.e. answer from (i) $\div 4$.

(b) (iv)

5 marks

Att 2

$$1 \times 5 \times 4 \times 3 \times 2 \downarrow = 120 \downarrow$$

[4 marks] [5 marks]

Attempts (2 marks)

- A6 Attempt to apply condition i.e. answer from (ii) $\div 5$.
- A7 5!.

Three coins are tossed. Each coin gives either a head or a tail.

- (i) Write down all the possible outcomes. For example, “H, T, H” or “head, tail, head” is one possible outcome.
- (ii) Find the probability that the result is three tails.
- (iii) Find the probability that the result includes no more than one head.
- (iv) Find the probability that the result has at least one head.

(c) (i)

5 marks

Att 2

H, H, H;
 H, H, T; H, T, H; T, H, H;
 H, T, T; T, H, T; T, T, H;
 T, T, T.

- * Penalise –1 for each incorrect or omitted entry, subject to the attempt mark for at least one correct entry other than H, T, H.

(c) (ii)

5 marks

Att 2

$$P(\text{three tails}) = \frac{1}{8}$$

- * Accept correct answer without work in each of the following sections.
- * Accept an answer consistent with section (i) in each of the following sections.

Attempts (2 marks)

- A1 Relevant step such as relevant entries in the sample space listed or indicated.
- A2 Any incorrect relevant ratio in the form a/b or $a:b$ but $0 \leq a/b \leq 1$. ($a:b$ may be in decimal or percentage form).
- A3 Statement of Probability Theorem specifically adapted to each section.
- The above attempts apply to each section.*
- A4 Answer of 1 or 8 or $\frac{1}{8}$.

(c) (iii)

5 marks

Att 2

$$P(\leq \text{one head}) = \frac{4}{8} \quad \text{or} \quad \frac{1}{2}$$

Attempts (2 marks)

- A5 Answer of 4 or 8 or $\frac{8}{4} = 2$.

Worthless (0 marks)

- W1 1 or 2

(c) (iv)

5 marks

Att 2

$$P(\geq \text{one head}) = \frac{7}{8}$$

or

$$1 - P(\text{no head}) = 1 - \frac{1}{8} = \frac{7}{8}$$

Attempts (2 marks)

- A6 Answer of 7 or 8 or $\frac{8}{7}$ or $\frac{1}{8}$ or 1.

QUESTION 7

Part (a)	10 marks	Att 3
Part (b)	40 marks	Att 13

Part (a)	10 marks	Att 3
-----------------	-----------------	--------------

The mean of the five numbers 2, 4, 7, 8, 9 is 6.
 Calculate the standard deviation of the five numbers, correct to one decimal place.

(a)	10 marks	Att 3
------------	-----------------	--------------

Deviations from mean: $2 - 6, 4 - 6, 7 - 6, 8 - 6, 9 - 6$ [3 marks]

$$\sigma = \sqrt{\frac{(2-6)^2 + (4-6)^2 + (7-6)^2 + (8-6)^2 + (9-6)^2}{5}}$$
[7 marks]

$$= \sqrt{\frac{(-4)^2 + (-2)^2 + 1^2 + 2^2 + 3^2}{5}} = \sqrt{\frac{16 + 4 + 1 + 4 + 9}{5}}$$

$$= \sqrt{\frac{34}{5}} = \sqrt{6.8} = 2.60 = 2.6$$
[10 marks]

- * Accept correct answer without work i.e. uses calculator.
- * Accept either positive or negative deviations.

10 marks: Answer of 2.6, apply (-1) if not rounded off.

9 marks: 2.60 or 34/5 or 6.8 or $\sqrt{6.8}$ or $\sqrt{34/5}$.

7 marks: Answer of 34 and 5 without fraction or fraction written as 5/34 or 34/? or ?/5.

4 marks: 34 or 5 or $\sqrt{30/5}$ or incorrect fraction with work.

[Apply maximum of one blunder for numerator and one blunder for denominator].

3 marks Some relevant step e.g work to find mean.

0 marks Worthless work.

Slips (-1)

- S1 Each numerical error to a maximum of 3.
- S2 Each incorrect deviation to a maximum of 3.
- S3 Candidate uses sample standard deviation on calculator.

Attempts (3 marks)

- A1 Any relevant step, e.g. arbitrary fraction.
- A2 Any correct deviation.
- A3 A correct squaring and stops.
- A4 Gives a reasonable estimate, without work, e.g. $2 \leq \sigma \leq 3$.
- A5 1+1+1+1+1 and stops.

Part (b)

40 (10, 10, 5, 10, 5) marks

Att (3, 3, 2, 3, 2)

The number of new cars in various price ranges sold by a retailer in one month is recorded in the following table:

Price (€1000's)	10 – 15	15 – 20	20 – 25	25 – 30	30 – 50
Number sold	5	15	25	15	20

[Note: 10 – 20 means at least 10 but less than 20 etc.]

- (i) Draw a histogram to represent the data.
- (ii) By taking the data at the mid-interval values, calculate the mean price per car.
- (iii) Copy and complete the following cumulative frequency table:

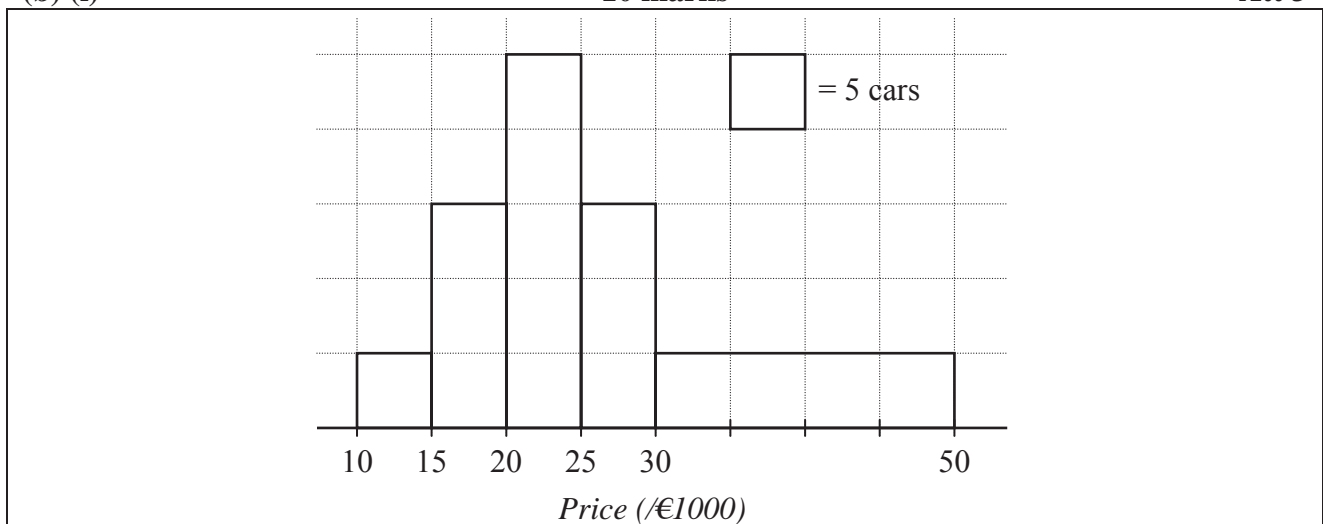
Price (€1000's)	< 15	< 20	< 25	< 30	< 50
Number sold					

- (iv) Draw the cumulative frequency curve (ogive).
- (v) Using your curve, estimate how many of the cars sold were priced between the mean and the median.

(b) (i)

10 marks

Att 3



- * Each rectangle may be blundered only once.
- * Accept areas of rectangles proportional to frequencies; (i.e. no penalty for omitting the area key, and no penalty for including a vertical scale, even if labelled).
- * If the only error in scale is in the 30-50 rectangle, penalise this rectangle once only.

- 10 marks Correct histogram
- 7 marks Scale(s) incorrect, rectangles subsequently correct
or scales correct, one rectangle incorrect or omitted
or scales correct, rectangles correct but spaces put between rectangles.
- 4 marks Scale(s) incorrect, one rectangle incorrect or omitted
or scales correct, two rectangles incorrect or omitted
or scales correct, one rectangle incorrect or omitted and spaces between rectangles.
- 3 marks Attempt at answer as below.

Attempts (3 marks)

- A1 Draws axes and stops, even without labels or scales.
- A2 Treats 0 – 5, 5 – 15 etc. as intervals and 15, 20 etc as frequencies.

(b) (ii)

10 marks

Att 3

Mid-interval values 12.5, 17.5, 22.5, 27.5, 40 [3 marks]

$$\text{Mean } \bar{x} = \frac{12.5 \times 5 + 17.5 \times 15 + 22.5 \times 25 + 27.5 \times 15 + 40 \times 20}{5 + 15 + 25 + 15 + 20} \quad [4 \text{ marks}]$$

$$= \frac{62.5 + 262.5 + 562.5 + 412.5 + 800}{80} = \frac{2100}{80} \text{ or } 26.25 \downarrow$$

[7 marks] [10 marks]

or

Interval	Mid-interval (x)	f	fx
10 – 15	12.5	5	62.5
15 – 20	17.5	15	262.5
20 – 25	22.5	25	562.5
25 – 30	27.5	15	412.5
30 – 50	40	20	800
		80	2100

[7 marks]

$$\text{Mean } \bar{x} = \frac{\sum fx}{\sum f} = \frac{2100}{80} \text{ or } 26.25 \text{ or } €26\,250 \quad [10 \text{ marks}]$$

- * Accept correct answer without work i.e. uses calculator.
- * All answers must be consistent with written mid-interval values and frequency values, otherwise incorrect answer without work merits zero, subject to A6.

Award marks as follows:

10 marks: Answer of 2100/80 or 26.25.

7 marks: Answer of 2100 and 80 without fraction or fraction written as 80/2100 or 2100/? or ?/80.

4 marks: 2100 or 80 or incorrect fraction with work.
[Apply maximum of one blunder for numerator and one blunder for denominator].

3 marks: Some relevant step.

0 marks: Worthless work.

Slips (-1)

S1 Each numerical error to a maximum of 3.

S2 Each incorrect mid-interval value to a maximum of 3.

Attempts (3 marks)

A1 Some relevant step e.g. finds the median or modal class or draws a cumulative frequency curve or arbitrary fraction.

A2 Writes formula for mean and stops.

A3 A correct multiplication and stops.

A4 Addition of frequencies indicated and stops.

A5 One or more correct mid-interval values and stops.

A6 Gives a reasonable estimate of $25 \leq \bar{x} \leq 27$.

(b) (iii)

5 marks

Att 2

Price (€1000's)	< 15	< 20	< 25	< 30	< 50
Number sold	5	20	45	60	80

* Award 1 mark for each correct or (correct) consistent entry, where an error has been made, subject to blunders or attempt mark.

Blunders (-3)

B1 Subtracts instead of adds.

Attempts (2 marks)

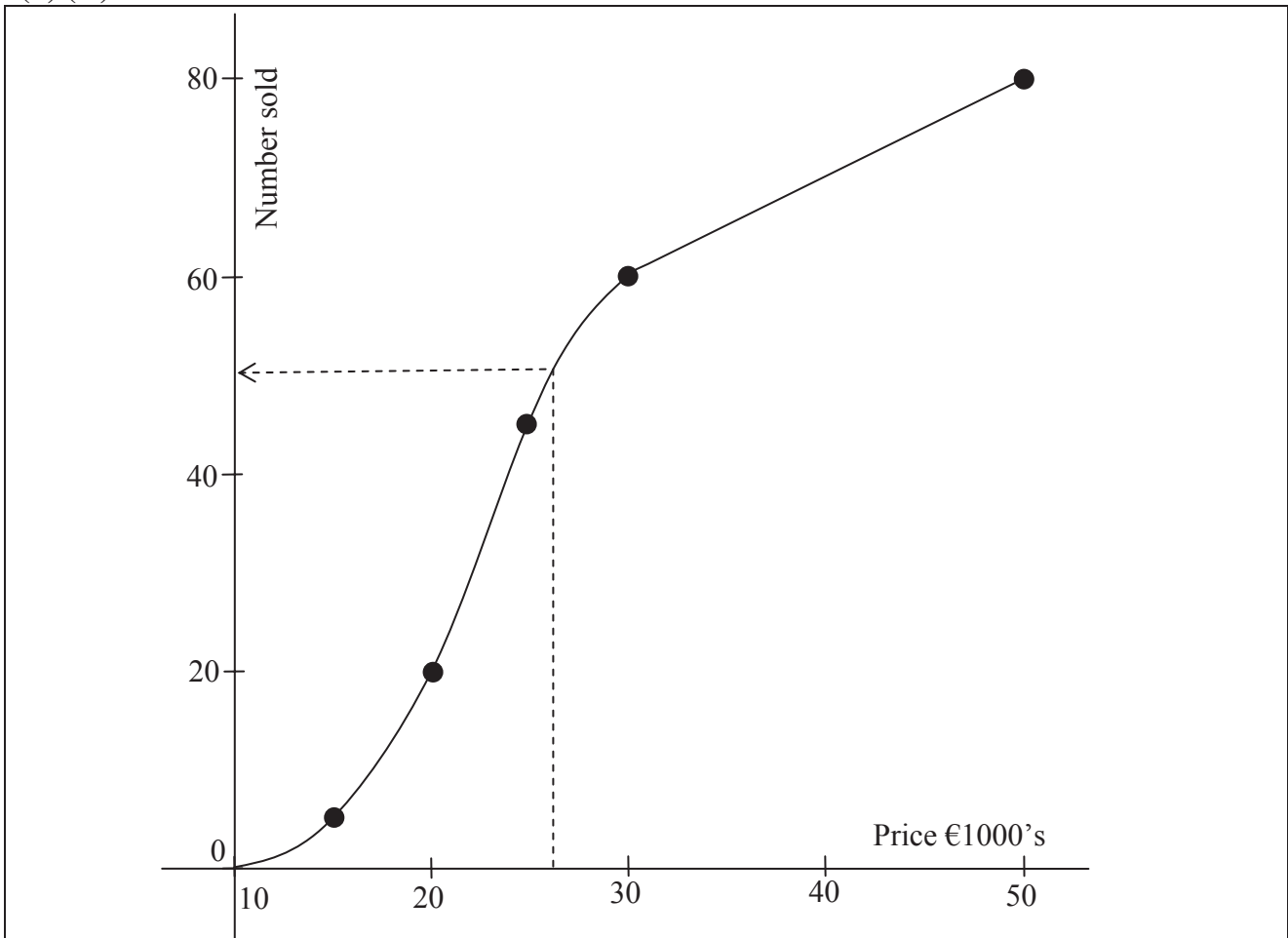
A1 One correct frequency and stops.

A2 Effort at copying the given table and stops.

(b) (iv)

10 marks

Att 3



* A correct ogive presupposes a correct table. If table not given award 5 + 10.

* Accept frequency on the horizontal.

* Accept a cumulative frequency polygon.

* Do not penalise for not joining (10, 0) to (15, 5).

* Accept a curve consistent with the table offered in section (iii).

* If section (iii) is not attempted and a curve consistent with the original table is drawn award 0 + Att 3.

Blunders (-3)

- B1 Scale irregular (once).
- B2 Draws “histogram” of cumulative frequencies instead of ogive – apply slip and blunder.
- B3 Draws “cumulative cumulative” curve, if not already penalised – apply slip and blunder.

Slips (-1)

- S1 Each point omitted or incorrectly plotted (to the eye). Blunders also apply.
- S2 Each pair of points not joined - to a maximum of 3.

Note: Award 10 marks for 5 points plotted and joined,
8 marks for 4 points plotted and joined,
6 marks for 3 points plotted and joined,
4 marks for 2 points plotted and joined,
3 marks for 1 point plotted.

Attempts (3 marks)

- A1 Draws a frequency curve or polygon not consistent with the table in (iii) and incorrect.
- A2 Draws the axes and stops.

(b) (v)

5 marks

Att 2

Number of cars priced between mean and median = $50 - 40 = 10$
--

- * Allow a tolerance of ± 2 in reading candidate’s answer from candidate’s graph.
- * If a histogram or frequency polygon is drawn in (iv), the attempt mark at most is awarded in this part.
- * Accept correct answer without work.
- * Any error other than an obvious slip merits the attempt mark at most.

Slips (-1)

- S1 Writes the difference but does not do the subtraction.

Attempts (2 marks)

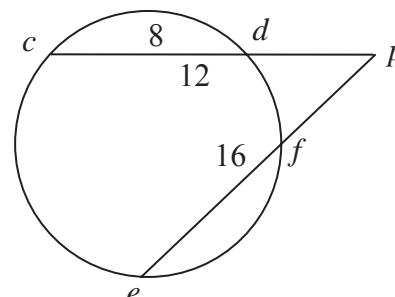
- A1 Draws a relevant vertical or horizontal line.
- A2 Attempt at an answer without reference to the graph.
- A3 Relevant step e.g. 40 either indicated or stated.

QUESTION 8

Part (a)	10 marks	Att 4
Part (b)	20 marks	Att 7
Part (c)	20 marks	Att 8

Part (a)	10 (5, 5) marks	Att (2, 2)
-----------------	------------------------	-------------------

$[cd]$ and $[ef]$ are chords of a circle which, when produced, intersect at a point p outside the circle.
 $|cd| = 8$, $|cp| = 12$ and $|ep| = 16$.



(i) Find $|pd|$.

(ii) Find $|pf|$.

(a) (i)	5 marks	Att 2
----------------	----------------	--------------

$$|pd| = |cp| - |cd| \downarrow = 12 - 8 = 4 \downarrow$$

[2 marks] [5 marks]

* Accept correct answers without work or answer clearly indicated on a diagram.

Attempts (2 marks)

A1 States $|pd| = x$ and indicates on diagram.

A2 States $|pd| = 12 - x$ or $|pd| = 8 - x$.

Worthless (0 marks)

W1 Incorrect answer without work shown.

(a) (ii)	5 marks	Att 2
-----------------	----------------	--------------

$$|cp| \times |pd| = |ep| \times |pf|$$

$$\Rightarrow 12 \times 4 = 16 \times |pf| \downarrow \Rightarrow |pf| = \frac{12 \times 4}{16} \downarrow = \frac{48}{16} \text{ or } 3 \downarrow$$

[2 marks] [4 marks] [5 marks]

* Accept candidates answer from section (i).

Slips (-1)

S1 Each numerical slip to a maximum of 3.

Misreading (-1)

M1 Taking $|ef| = 16$ and $|fp| = x$ leads to the quadratic $x^2 + 16x - 48 = 0$ giving $x = 2.83$.

Attempts (2 marks)

A1 Geometrical result indicated on a diagram or stated without numerical data.

A2 Some relevant step, e.g. begins substitution into result.

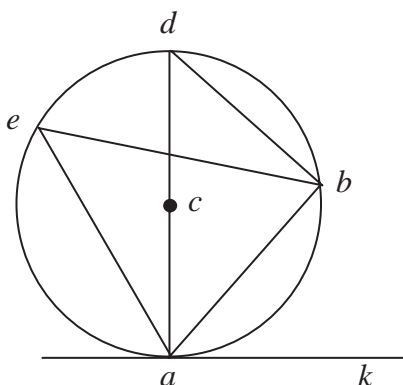
Prove that an angle between a tangent ak and a chord $[ab]$ of a circle has degree measure equal to that of any angle in the alternate segment.

(b)

20 marks

Att 7

ak is a tangent to the circle at a . $[ab]$ is a chord of the circle. To prove: $|\angle bak| = |\angle bea|$



Construction:

Let $[ad]$ be a diameter of the circle. Join db . [10 marks]

Proof:

$|\angle abd| = 90^\circ$ (Angle in a semi-circle)

$\Rightarrow |\angle bda| + |\angle dab| = 90^\circ$. [13 marks]

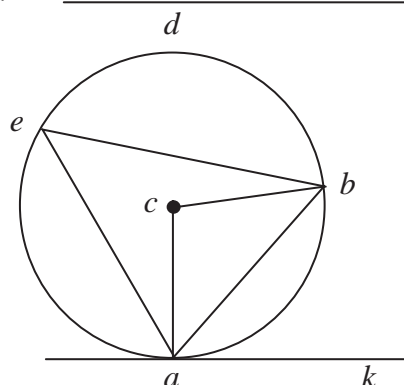
$|\angle dab| + |\angle bak| = 90^\circ$ (tangent \perp diameter)

Hence, $|\angle bak| = |\angle bda|$. [16 marks]

But $|\angle bda| = |\angle bea|$. (angles on same arc) [19 marks]

Hence, $|\angle bak| = |\angle bea|$. [20 marks]

or



Construction:

Join c to a and to b . [10 marks]

Proof:

$|\angle bca| + |\angle cab| + |\angle abc| = 180^\circ$

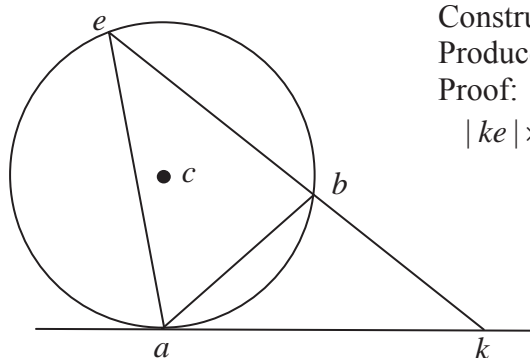
$\Rightarrow \frac{1}{2} |\angle bca| + |\angle cab| = 90^\circ$. [13 marks]

$|\angle cab| + |\angle bak| = 90^\circ$. (tangent \perp diameter) [16 marks]

Hence, $|\angle bak| = \frac{1}{2} |\angle bca|$ [17 marks]

$= |\angle bea|$. (angles at the circle) [20 marks]

or



Construction:

Produce eb to meet the tangent at k . [10 marks]

Proof:

$|ke| \times |kb| = |ka|^2$ [13 marks]

$\Rightarrow \frac{|kb|}{|ka|} = \frac{|ka|}{|ke|}$. [16 marks]

Hence, Δakb and Δake are equiangular. [19 marks]

Hence, $|\angle bak| = |\angle bea|$. [20 marks]

* Accept steps clearly marked on a diagram, subject to B2.

Blunders (-3)

B1 Incorrect step or step omitted where appropriate.

B2 Steps written in an illogical order. [Penalise once only.]

B3 Incomplete steps in proof.

Attempts (7 marks)

A1 Outline diagram and steps. (Minimum required circle and tangent or chord).

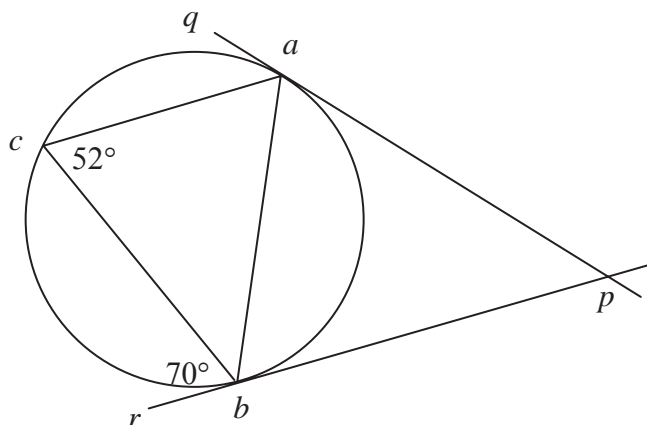
A2 Particular case e.g. diagram with angles measured.

A3 States a relevant result e.g. tangent perpendicular to diameter.

The lines pq and pr are tangents to the circle at the points a and b , respectively. c is a point on the circle.

$|\angle acb| = 52^\circ$ and $|\angle cbr| = 70^\circ$.

- (i) Find $|\angle abp|$.
 (ii) Find $|\angle bac|$.
 (iii) Find $|\angle caq|$.
 (iv) Find $|\angle bpa|$.



(c) (i)

5 marks

Att 2

$$|\angle abp| = |\angle acb| = 52^\circ$$

- * Accept answer written on a diagram in each section.
 * Accept correct or consistent answer without work in each section.

Blunders (-3)

B1 $|\angle abp| = \frac{1}{2}(180^\circ - 70^\circ)$ or similar.

Attempts (2 marks)

A1 Relevant step e.g. an indication of equal angles.

(c) (ii)

5 marks

Att 2

$$|\angle bac| = |\angle rbc| = 70^\circ.$$

Attempts (2 marks)

A1 Relevant statement e.g. angle between tangent and chord equals angle in alternate segment.

(c) (iii)

5 marks

Att 2

$$|\angle caq| = |\angle cba| = 180^\circ - (|\angle rbc| + |\angle abp|) = 180^\circ - (70^\circ + 52^\circ) = 58^\circ.$$

or

$$|\angle caq| = |\angle cba| = 180^\circ - (|\angle bac| + |\angle acb|) = 180^\circ - (70^\circ + 52^\circ) = 58^\circ.$$

- * Accept correct work based on candidate's previous answers.

Blunders (-3)

B1 $180^\circ - (52^\circ + 52^\circ) = 76^\circ$ or $180^\circ - (70^\circ + 70^\circ) = 40^\circ$.

Attempts (2 marks)

A1 Relevant statement eg angle sum in a triangle is 180° or sum of angles in quadrilateral is 360° .

(c) (iv)

5 marks

Att 2

$$|\angle bpa| = 180^\circ - (|\angle abp| + |\angle pab|) = 180^\circ - (52^\circ + 52^\circ) = 76^\circ.$$

Attempts (2 marks)

A1 Indication of isosceles triangles e.g. $|pa| = |pb|$.

QUESTION 9

Part (a)	10 marks	Att 4
Part (b)	20 marks	Att 7
Part (c)	20 marks	Att 8

Part (a) **10 (5, 5) marks** **Att (2, 2)**

Let $\vec{v} = 3\vec{i} - 5\vec{j}$.

- (i) Express \vec{v}^\perp in terms of \vec{i} and \vec{j} .
- (ii) Express $\vec{v} + \vec{v}^\perp$ in terms of \vec{i} and \vec{j} .

(a) (i) **5 marks** **Att 2**

$$\vec{v}^\perp = 5\vec{i} + 3\vec{j}.$$

* Accept answer without arrows.

Blunders (-3)

B1 Sign error, e.g. $\vec{v}^\perp = -5\vec{i} + 3\vec{j}$.

Slips (-1)

S1 Answer given is $-\vec{v}^\perp$.

Attempts (2 marks)

A1 Draws \vec{v} , with or without, \vec{v}^\perp on a diagram.

A2 Image under an axial symmetry or central symmetry in the origin.

(a) (ii) **5 marks** **Att 2**

$$\vec{v} + \vec{v}^\perp = 3\vec{i} - 5\vec{j} + 5\vec{i} + 3\vec{j} \downarrow = 8\vec{i} - 2\vec{j} \downarrow$$

[2 marks] [5 marks]

* Accept correct answer or an answer consistent with (i) without work shown.

Blunders (-3)

B1 Mixes up the \vec{i} and \vec{j} components. *Apply this blunder in each section of the question.*

Attempts (2 marks)

A1 Effort at grouping or adding the components.

Part (b)**20 (10, 5, 5) marks****Att (3, 2, 2)**

Let $\vec{p} = 3\vec{i} - \vec{j}$ and $\vec{q} = 4\vec{i} + 2\vec{j}$.

- (i) Express $5\vec{p} - 2\vec{q}$ in terms of \vec{i} and \vec{j} .
- (ii) Calculate $\vec{p} \cdot \vec{q}$, the dot product of \vec{p} and \vec{q} .
- (iii) Verify that $|\vec{q}| > |\vec{pq}|$.

(b) (i)**10 marks****Att 3**

$$5\vec{p} - 2\vec{q} = 5(3\vec{i} - \vec{j}) - 2(4\vec{i} + 2\vec{j})$$

[3 marks]

$$= 15\vec{i} - 5\vec{j} - 8\vec{i} - 4\vec{j} \downarrow = 7\vec{i} - 9\vec{j} \downarrow$$

[7 marks]

[10 marks]

* Accept correct answer without work shown in sections (i) and (ii).

Slips (-1)

S1 Numerical slips to a maximum of 3.

S2 Interchanges \vec{p} with \vec{q} .*Attempts (3 marks)*A1 $7\vec{i}$ or $-9\vec{j}$ without work shown and stops.

A2 Either bracket multiplied out correctly and stops.

A3 Plots one or more relevant vectors.

Worthless (0 marks)

W1 Incorrect answer without work.

(b) (ii)**5 marks****Att 2**

$$\vec{p} \cdot \vec{q} = (3\vec{i} - \vec{j}) \cdot (4\vec{i} + 2\vec{j}) \downarrow = 12 - 2 \downarrow = 10 \downarrow$$

[2 marks]

[4 marks]

[5 marks]

*Blunders (-3)*B1 $\vec{i}^2 \neq 1$ or $\vec{j}^2 \neq 1$ or $\vec{i} \cdot \vec{j} \neq 0$, applied once.B2 Incorrect relevant formula e.g. $|\vec{x}| |\vec{y}| \sin \theta$ or $|\vec{x}| = \sqrt{a^2 - b^2}$.*Attempts (2 marks)*

A1 Correct relevant formula and stops.

A2 Finds the length of one vector and stops.

A3 Some correct work in multiplication.

Worthless (0 marks)

W1 Incorrect answer without work.

(b) (iii)

5 marks

Att 2

$$|\vec{q}| = |4\vec{i} + 2\vec{j}| = \sqrt{4^2 + 2^2} = \sqrt{20} \quad [2 \text{ marks}]$$

$$|\vec{pq}| = |\vec{q} - \vec{p}| = |4\vec{i} + 2\vec{j} - 3\vec{i} + \vec{j}| = |\vec{i} + 3\vec{j}| \begin{matrix} [4 \text{ marks}] \\ \downarrow \\ \sqrt{1^2 + 3^2} = \sqrt{10} \\ [5 \text{ marks}] \end{matrix}$$

Blunders (-3)

B1 Incorrect modulus formula.

B2 $\vec{pq} = \vec{q} + \vec{p}$ or $\vec{p} - \vec{q}$ or $\vec{p} \cdot \vec{q}$.

Attempts (2 marks)

A1 Relevant work on a diagram.

A2 Work towards calculating the length of either vector.

Worthless (0 marks)

W1 Incorrect answer without work, subject to attempts.

Part (c)

20 (5, 5, 5, 5) marks

Att (2, 2, 2, 2)

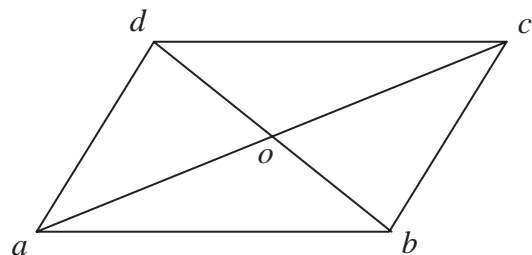
$abcd$ is a parallelogram. The diagonals intersect at o , the origin.

(i) Express $\vec{ab} + \vec{bc}$ in terms of \vec{c} .

(ii) Express $\vec{ad} - \vec{bd}$ in terms of \vec{a} and \vec{b} .

(iii) Show that $\vec{ad} - \vec{ac} + \vec{ab} = \vec{o}$.

(iv) Write $\vec{a} + \vec{b} + \vec{c} + \vec{d}$ in its simplest form.



Each part

5 marks

Att 2

(i)	$\vec{ab} + \vec{bc} = \vec{ac} = \vec{c} - \vec{a} = \vec{c} - (-\vec{c}) = 2\vec{c}$.
(ii)	$\vec{ad} - \vec{bd} = \vec{ad} + \vec{db} = \vec{ab} = \vec{b} - \vec{a}$.
(iii)	$\vec{ad} - \vec{ac} + \vec{ab} = \vec{ad} + \vec{ab} - \vec{ac} = \vec{ac} - \vec{ac} = \vec{o}$. or $\vec{ad} - \vec{ac} + \vec{ab} = \vec{d} - \vec{a} - \vec{c} + \vec{a} + \vec{b} - \vec{a} = \vec{d} + \vec{b} - \vec{a} - \vec{c} = \vec{o}$.
(iv)	$\vec{a} + \vec{b} + \vec{c} + \vec{d} = \vec{a} + \vec{b} - \vec{a} - \vec{b} = \vec{o}$.

* Allow \vec{oa} for \vec{a} etc.

* Accept correct answers without work in (i) and (ii).

* Do not penalise for the omission of arrows.

* Accept answer to (iii) shown on a diagram but not in (iv).

Apply the following to each section, as applicable.

Blunders (-3)

- B1 Incorrect direction.
- B2 Error in triangle law or parallelogram law.
- B3 Failure to reduce the vector to the required components.
- B4 Incorrect use of the origin e.g. $\vec{ac} = \vec{a}$.

Attempts (2 marks)

- A1 Relevant work on a diagram.
- A2 Correct relevant work e.g. $\vec{a} = -\vec{c}$ or $\vec{ab} = \vec{b} - \vec{a}$.

Worthless (0 marks)

- W1 Diagram reproduced without modifications.

QUESTION 10

Part (a)	10 marks	Att 3
Part (b)	20 marks	Att 7
Part (c)	20 marks	Att 7

Part (a)	10 marks	Att 3
-----------------	-----------------	--------------

Expand $(1-x)^5$ fully.

(a)	10 marks	Att 3
------------	-----------------	--------------

$(1-x)^5 = \binom{5}{0} + \binom{5}{1}(-x) + \binom{5}{2}(-x)^2 + \binom{5}{3}(-x)^3 + \binom{5}{4}(-x)^4 + \binom{5}{5}(-x)^5$ $= 1 + 5(-x) + 10(x^2) + 10(-x^3) + 5(x^4) + 1(-x^5)$ $= 1 - 5x + 10x^2 - 10x^3 + 5x^4 - x^5.$	<p>[4 marks]</p> <p>[7 marks]</p> <p>[10 marks]</p>
--	---

- * Accept long multiplication or Pascal's triangle and penalise each incorrect term -1, subject to attempt.
- * Accept correct answer without work.

Blunders (-3)

- B1 Incorrect power in a term.
- B2 Incorrect treatment of power e.g. power treated as a denominator.
- B3 Blunder in working out binomial coefficients.
- B4 Treats binomial coefficients as fractions e.g. $\frac{5}{3}(-x)^3$.
- B5 Puts a + sign between coefficient and power of x.
- B6 Expands $(1+x)^5$ or $(x-1)^5$ or $(1-x)^4$.

Slips (-1)

- S1 Numerical slips to a maximum of 3.

Attempts (3 marks)

- A1 Any term, including first term, written down correctly.
- A2 Answer of $1-x^5$ is attempt mark at most.
- A3 Gives part of Pascal's triangle or effort at Pascal's triangle.
- A4 Gives coefficients only.
- A5 Any step towards getting a binomial coefficient e.g. $\binom{5}{2}$.
- A6 Any correct step towards long multiplication.

Worthless (0 marks)

- W1 Writes $5(1-x)^4$ or $5(1-x)^4(-1)$.

Part (b)**20 (10, 5, 5) marks****Att (3, 2, 2)**

A geometric series is $1 + \frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \dots$

- (i) Find the sum of the first 20 terms of the series.
 (ii) Find S , the sum to infinity of the series.
 (iii) The sum to infinity of another geometric series is also S .
 The first term of this series is 2. Find its common ratio.

(b) (i)**10 marks****Att 3**

$$1 + \frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \dots \Rightarrow a = 1 \text{ and } r = \frac{1}{3} = \frac{1}{3} \quad [3 \text{ marks}]$$

$$S_{20} = \frac{a(1-r^n)}{1-r} = \frac{1(1-(\frac{1}{3})^{20})}{1-\frac{1}{3}} \downarrow = \frac{3}{2} \left(1 - \frac{1}{3^{20}}\right) \text{ or } \frac{3}{2} - \frac{1}{2 \times 3^{19}} \text{ or } 1.5 \downarrow$$

[7 marks] [10 marks]

* Note: Answer from calculator is 1.5.

Blunders (-3)

- B1 Incorrect a .
 B2 Incorrect r .
 B3 Blunder in fractions.

Slips (-1)

- S1 Numerical slips to a maximum of 3.

Attempts (3 marks)

- A1 Correct relevant formula and stops.
 A2 Some relevant step e.g. states the value for a or the value for r .
 A3 Adds 2 or more of the given terms e.g. $S_2 = \frac{4}{3} = 1.3333$ or $S_3 = \frac{13}{9} = 1.4444$.
 A4 One correct step in adding relevant fractions.
 A5 Treats as arithmetic series with further work, e.g. identifies a .
 A6 Writes $T_n = ar^{n-1}$ or $1(\frac{1}{3})^{n-1}$.
 A7 Gives $T_5 = \frac{1}{3^4}$ or $\frac{1}{81}$.
 A8 Gives answer of 1.5 without work.
 A9 Incorrect relevant formula e.g. $\frac{a}{1+r}$ giving answer $\frac{3}{4}$ or T_{20} .

Worthless (0 marks)

- W1 Formula for arithmetic series and stops.
 W2 $1 + \frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} = 1\frac{3}{3^6}$ or similar.

A company invests $\text{€}P$ in new machinery.
The machinery depreciates at the rate of $r\%$ per annum.

- (i) Write down, in terms of P and r , the value of the machinery after 8 years.
(ii) If the machinery depreciates to one-quarter of its original value after 8 years, find r , correct to the nearest whole number.

(c) (i)

5 marks

Att 2

$$A = P \left(1 - \frac{r}{100} \right)^8$$

* Award 5 marks for correct answer, 2 marks for partially correct answer, otherwise 0 marks.

$$A = P \left(1 + \frac{r}{100} \right)^8 \text{ or } A = P \left(1 \pm \frac{8}{100} \right)^n \text{ or } A = P \left(\frac{100+r}{100} \right)^8 \text{ or } A = P \left(\frac{100 \pm 8}{100} \right)^n \text{ or } P \text{ and } A \text{ switched.}$$

(c) (ii)

15 marks

Att 5

$$P \left(1 - \frac{r}{100} \right)^8 = \frac{P}{4} \quad [5 \text{ marks}]$$

$$\Rightarrow \left(1 - \frac{r}{100} \right)^8 = 0.25 \Rightarrow 1 - \frac{r}{100} = \sqrt[8]{0.25} \quad [8 \text{ marks}]$$

$$\Rightarrow 1 - \frac{r}{100} = 0.8409 \downarrow \Rightarrow \frac{r}{100} = 0.1591 \downarrow \Rightarrow r = 16\% \downarrow$$

[11 marks] [14 marks] [15 marks]

* Accept candidate's answer from (i), if used where it does not simplify the work. Otherwise mark on work shown.

Blunders (-3)

B1 Mathematical error e.g. $\sqrt[8]{0.25} = 0.25 \div 8$.

Slips (-1)

S1 Numerical slips to a maximum of 3.

S2 Answer not rounded-off to nearest percentage.

S3 Premature rounding that affects accuracy of final answer.

Attempts (5 marks)

A1 Mention of $\frac{1}{4}P$ or similar relevant statement.

A2 Correct answer without work.

QUESTION 11

Part (a)

15 marks

Att 6

Part (b)

35 marks

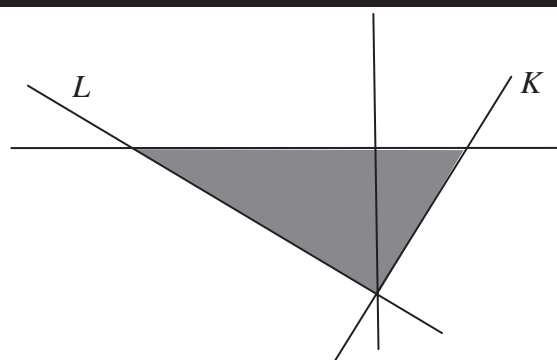
Att 14

Part (a)

15 (5, 5, 5) marks

Att (2, 2, 2)

The equation of the line L is $5x + 8y + 40 = 0$.
 The equation of the line K is $10x - 7y - 35 = 0$.



Write down the 3 inequalities that together define the shaded region in the diagram.

(a)

15 (5, 5, 5) marks

Att (2, 2, 2)

$L: \quad 5x + 8y + 40 \geq 0$

[5 marks]

$K: \quad 10x - 7y - 35 \leq 0$

[5 marks]

$x\text{-axis:} \quad y \leq 0$

[5 marks]

- * Accept correct inequalities without work shown.
- * Accept $<$ for \leq and $>$ for \geq without penalty.
- * Accept $5x + 8y + 40 = 0 \Rightarrow 0 + 0 + 40 \geq 0$ for 5 marks.
- * It is possible to award only 2 marks for part (a).

Blunders (-3)

- B1 Incorrect inequality symbol, each inequality.
- B2 Mathematical errors (e.g. sign errors) when testing points, each inequality.
- B3 Incorrect or no conclusion given e.g. $0 + 0 + 40 = 0$.
- B4 Both directions of inequality given, each inequality.

Slips (-1)

- S1 Numerical slips e.g. $5 \times 0 = 5$, to a maximum of 3 for each inequality.

Attempts (2 marks for each inequality)

- A1 Substitutes point e.g. $(0, 0)$ and stops without conclusion.
- A2 Finds or plots one or more correct points on given line (apply to each line).
- A3 $x \leq 0$ or $x \geq 0$ for $y \leq 0$.
- A4 $y = 0$ and stops.
- A5 $L \geq 0$.
- A6 $K \leq 0$.
- A7 Some relevant step, e.g. adding half-plane arrows to the diagram.

Worthless (0 marks)

- W1 Diagram copied with nothing relevant added to it.

Part (b)**35 (20, 10, 5) marks****Att (8, 4, 2)**

Due to a transport disruption, a bus company is contracted at short notice to carry up to 1500 passengers to complete their journey. Passengers not carried by this company will be carried by a taxi company.

The bus company has available standard buses and mini-buses. Each standard bus carries 60 passengers and each mini bus carries 30 passengers.

Each bus is operated by one driver and there are at most 30 drivers available.

- (i) Taking x as the number of standard buses and y as the number of mini-buses, write down two inequalities in x and y and illustrate these on graph paper.
- (ii) The operating profit for the journey is €80 for a standard bus and €50 for a mini-bus. How many of each type of bus should be used in order to maximise the profit?
- (iii) If the bus company paid each driver a bonus for working at short notice, the operating profit for each bus would be reduced by €30. By how much would this decrease the maximum profit available to the company?

(b) (i) Inequalities**10 (5, 5) marks****Att (2, 2)**

Passengers: $60x + 30y \leq 1500$

Drivers: $1x + 1y \leq 30$

Also accept

	Standard buses	Mini-buses	Maximum
Passengers	60	30	1500
Drivers	1	1	30

or

60	30	1500
1	1	30

Award 10 marks but penalise in graph if link-up is incorrect

* Accept correct multiples or fractions of inequalities or use of different letters.

* Do not penalise here for an incorrect or for no inequality sign. Penalise in graph if used.

Blunders (-3)

B1 Mixes up x 's and y 's (once if consistent error).

B2 Confuses rows and columns in table, e.g. $6x + y \leq 150$ (once if consistent).

B3 Misplaced decimal point, e.g. $6x + 3y \leq 15$, once.

Attempts (2 marks for each inequality)

A1 Incomplete relevant data in table and stops (each inequality).

A2 Any other correct inequality, e.g. $x \geq 0$, $y \geq 0$, (each time).

A3 Some variable ≤ 1500 or ≤ 30 (each time).

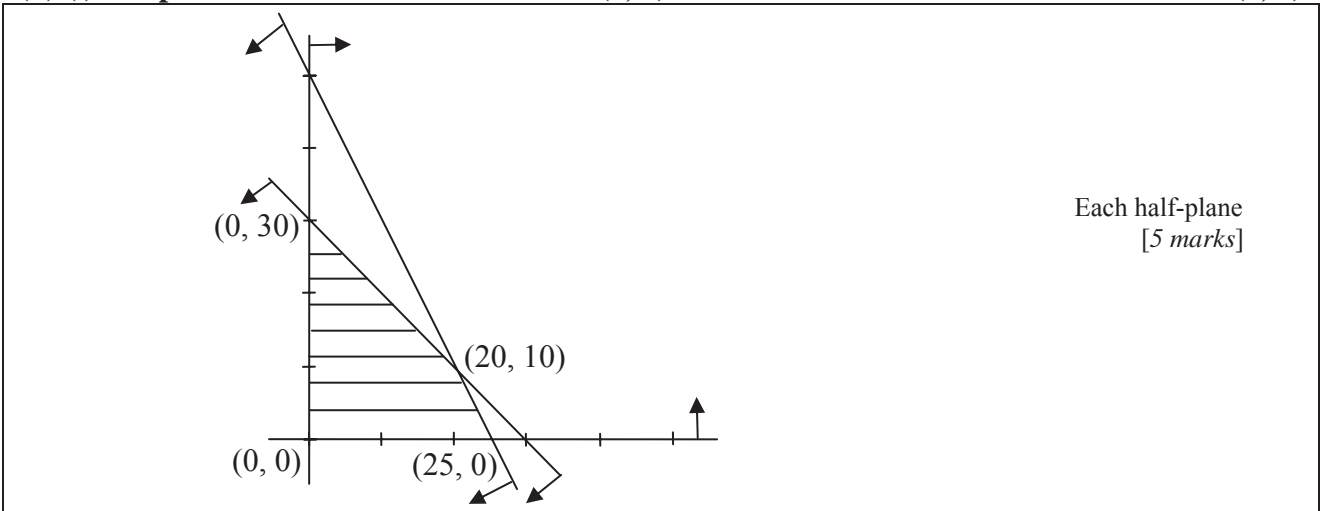
A4 $60x$ and/or $30y$ and stops ($1 \times$ Att 2).

A5 x and/or y and stops ($1 \times$ Att 2).

(b) (i) Graph

10 (5, 5) marks

Att (2, 2)



- * Points or scales required.
- * Half-planes required but no penalty for not indicating intersection if half-planes are indicated.
- * If half-planes are indicated correctly, do not penalise for incorrect shading.
- * Accept correct shading of intersection for half-planes but candidates may shade out areas that are not required and leave intersection blank.
- * Correct shading over-rides arrows.
- * Two lines drawn and **no shading** indicated, only one of the following applies :

Case 1:	Two sets of arrows in expected direction	10 marks
Case 2:	Two sets of arrows in unexpected direction	10 marks
Case 3:	One set of arrows "correct" and the other "incorrect"	7 marks (5+Att 2)
Case 4:	One line with and the other without arrows	7 marks (5+Att 2)
Case 5:	No arrows	4 marks (Att 2, Att 2)

Blunders (-3)

- B1 No half-plane indicated (each time).
- B2 Blunder in plotting a line or calculations (each line).
- B3 Incorrect shading (once) , e.g. one or both of the small triangles shaded.

Attempts (2 marks each half-plane)

- A1 Some relevant work towards a point on a line, i.e. 2 marks for each line attempted.
- A2 Draws axes or axes and one line (1 × Att 2).
- A3 Draws axes and two lines reasonably accurately (award Att 2 + Att 2).

(b) (ii) Intersection of lines

5 marks

Att 2

$ \begin{array}{lcl} 6x + 3y = 150 & \Rightarrow & 6x + 3y = 150 \\ x + y = 30 & & \underline{3x + 3y = 90} \\ & & 3x = 60 \Rightarrow x = 20 \Rightarrow y = 10 \end{array} $
--

- * Accept candidate's own equations from previous sections.
- * If x is calculated, accept consistent value for y without further work and vice versa.

Blunders (-3)

- B1 Fails to multiply / divide both sides of equation(s) correctly when eliminating variable.
- B2 x or y value only found.

Slips (-1)

- S1 Numerical slips to a maximum of 3.

Attempts (2 marks)

- A1 Correct or consistent answer without work or from a graph.
[Should get the *exact same* values from graph as if they had been found algebraically.]
- A2 Any relevant step towards solving equations.

Worthless (0 marks)

- W1 Incorrect answer without work and inconsistent with graph.

(b) (ii) Profit

5 marks

Att 2

Step 1	Vertices	$80x + 50y$	Profit
Step 2	(25, 0)	$2000 + 0$	2000
Step 3	(20, 10)	$1600 + 500$	2100
Step 4	(0, 30)	$0 + 1500$	1500
Step 5	20 standard buses and 10 mini-buses to maximise profit.		

- * Accept point of intersection from previous work.
- * Accept work on a feasible set of points formed by axes and one line without further penalty.
- * Information does not have to be in table form.
- * Award 1 mark for each consistent step, subject to the attempt mark, using $80x + 50y$.
- * Accept only vertices consistent with previously accepted work, not arbitrary ones. If (30, 0) or (0, 50) is tested and result is used to give maximum income, apply (-1), otherwise ignore.
- * Accept correct vertices or vertices from candidate's indicated area on non-simplified graph.
- * Accept any correct multiple or fraction of $80x + 50y$ here.
- * If no marks have been awarded for intersection of lines and this point is written here award Att 2 for the previous work and also reward it here if the step is correct.
- * Step 5 must be explicitly written to gain the final mark. Otherwise (-1).
- * Testing only (20, 10) to get 2100 merits Att 2 even if the candidate writes 20 standard buses and 10 mini-buses i.e. no comparison means the attempt mark at most.

Slips (-1)

- S1 Each arithmetic slip to a maximum of 3.
- S2 Each step of the solution omitted, subject to the attempt mark [Step 1 may be implied].

Attempts (2 marks)

- A1 Any relevant work involving x or y and / or 80, 50 or similar.
- A2 Any attempt at substituting co-ordinates into some expression.
- A3 States 20 standard buses and / or 10 mini-buses with no other work.

(b) (iii) Profit**5 marks****Att 2**

Step 1	Vertices	$50x + 20y$	Profit
Step 2	(25, 0)	$1250 + 0$	1250
Step 3	(20, 10)	$1000 + 200$	1200
Step 4	(0, 30)	$0 + 600$	600

Decrease in maximum profit = $2100 - 1250 = 850$.

* Accept candidate's vertices and maximum profit from previous sections.

Blunders (-3)

B1 Fails to calculate profit for new expression and uses (20, 10) to get 900.

Slips (-1)

S1 Numerical slips to a maximum of 3.

Attempt (2)

A1 Any correct step e.g. $80 - 30 = 50$.

A2 Some work with $50x$ and / or $20y$.

Worthless (0)

W1 Simply writing down €30 or 30 buses.

BONUS MARKS FOR ANSWERING THROUGH IRISH

Bonus marks are applied separately to each paper as follows:

If the mark achieved is less than 226, the bonus is 5% of the mark obtained, rounding *down*.
(e.g. 198 marks \times 5% = 9.9 \Rightarrow bonus = 9 marks.)

If the mark awarded is 226 or above, the following table applies:

Marks obtained	Bonus
226 – 231	11
232 – 238	10
239 – 245	9
246 – 251	8
252 – 258	7
259 – 265	6
266 – 271	5
272 – 278	4
279 – 285	3
286 – 291	2
292 – 298	1
299 – 300	0

