2012. M329 S

Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate Examination, 2012
Sample Paper

Mathematics
(Project Maths – Phase 3)

Paper 1

Higher Level

Time: 2 hours, 30 minutes

300 marks

Examination number

Centre stamp

Running total	

For examiner	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
Total	

Grade

Instructions

There are **two** sections in this examination paper:

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	3 questions

Answer all nine questions.

Write your answers in the spaces provided in this booklet. There is space for extra work at the back of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the booklet of *Formulae and Tables*. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

Marks will be lost if all necessary work is not clearly shown.

Answers should include the appropriate units of measurement, where relevant.

Answers should be given in simplest form, where relevant.

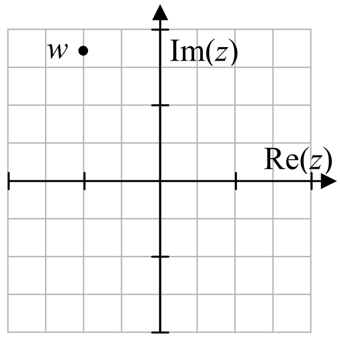
Write the make and model of your calculator(s) here:

Section A	Concepts and Skills	150 marks

Answer **all six** questions from this section.

Question 1 **(25 marks)**

(a) $w = -1 + \sqrt{3}i$ is a complex number, where $i^2 = -1$.



(i) Write w in polar form.

(ii) Use De Moivre's theorem to solve the equation $z^2 = -1 + \sqrt{3}i$, giving your answer(s) in rectangular form.

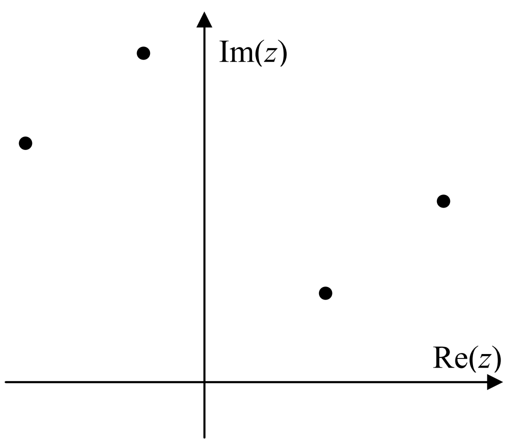
(b) Four complex numbers z_1, z_2, z_3 and z_4 are shown on the Argand diagram. They satisfy the following conditions:

$z_2 = iz_1$

$z_3 = kz_1$, where $k \in \mathbb{R}$

$z_4 = z_2 + z_3$.

The same scale is used on both axes.



(i) Identify which number is which, by labelling the points on the diagram.

(ii) Write down the approximate value of k.

Answer: ______________

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Question 2 **(25 marks)**

(a) **(i)** Prove by induction that, for any n, the sum of the first n natural numbers is $\frac{n(n+1)}{2}$.

(ii) Find the sum of all the natural numbers from 51 to 100, inclusive.

(b) Given that $p = \log_c x$, express $\log_c \sqrt{x} + \log_c (cx)$ in terms of p.

Question 3 **(25 marks)**

A cubic function *f* is defined for $x \in \mathbb{R}$ as

$$f : x \mapsto x^3 + (1-k^2)x + k \,, \quad \text{where } k \text{ is a constant.}$$

(a) Show that $-k$ is a root of *f*.

(b) Find, in terms of *k*, the other two roots of *f*.

(c) Find the set of values of *k* for which *f* has exactly one real root.

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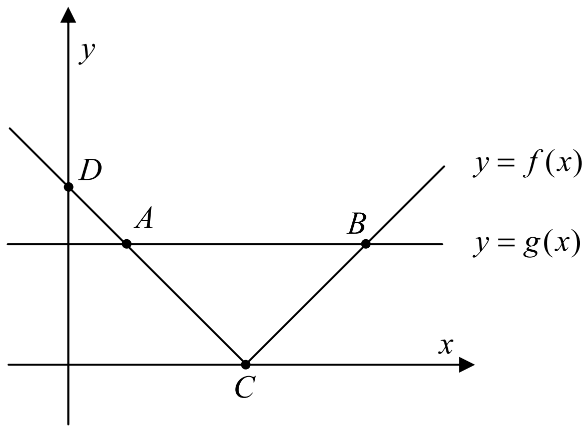
Question 4 **(25 marks)**

(a) Solve the simultaneous equations,

$$2x+8y-3z=-1$$
$$2x-3y+2z=2$$
$$2x+y+z=5.$$

(b) The graphs of the functions $f:x\mapsto|x-3|$ and $g:x\mapsto 2$ are shown in the diagram.

(i) Find the co-ordinates of the points A, B, C and D.



$A=(\quad,\quad)$ $B=(\quad,\quad)$

$C=(\quad,\quad)$ $D=(\quad,\quad)$

(ii) Hence, or otherwise, solve the inequality $|x-3|<2$.

Question 5 **(25 marks)**

A is the closed interval $[0,5]$. That is, $A = \{x \mid 0 \le x \le 5, x \in \mathbb{R}\}$.

The function f is defined on A by:

$$f : A \to \mathbb{R} : x \mapsto x^3 - 5x^2 + 3x + 5 .$$

(a) Find the maximum and minimum values of f.

(b) State whether f is injective. Give a reason for your answer.

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Question 6 **(25 marks)**

(a) **(i)** Write down three distinct anti-derivatives of the function

$$g : x \mapsto x^3 - 3x^2 + 3, \qquad x \in \mathbb{R}.$$

1. ______________________________

2. ______________________________

3. ______________________________

(ii) Explain what is meant by the indefinite integral of a function *f*.

(iii) Write down the indefinite integral of *g*, the function in part **(i)**.

Answer: ______________________________

(b) **(i)** Let $h(x) = x \ln x$, for $x \in \mathbb{R}$, $x > 0$.
Find $h'(x)$.

(ii) Hence, find $\int \ln x \, dx$.

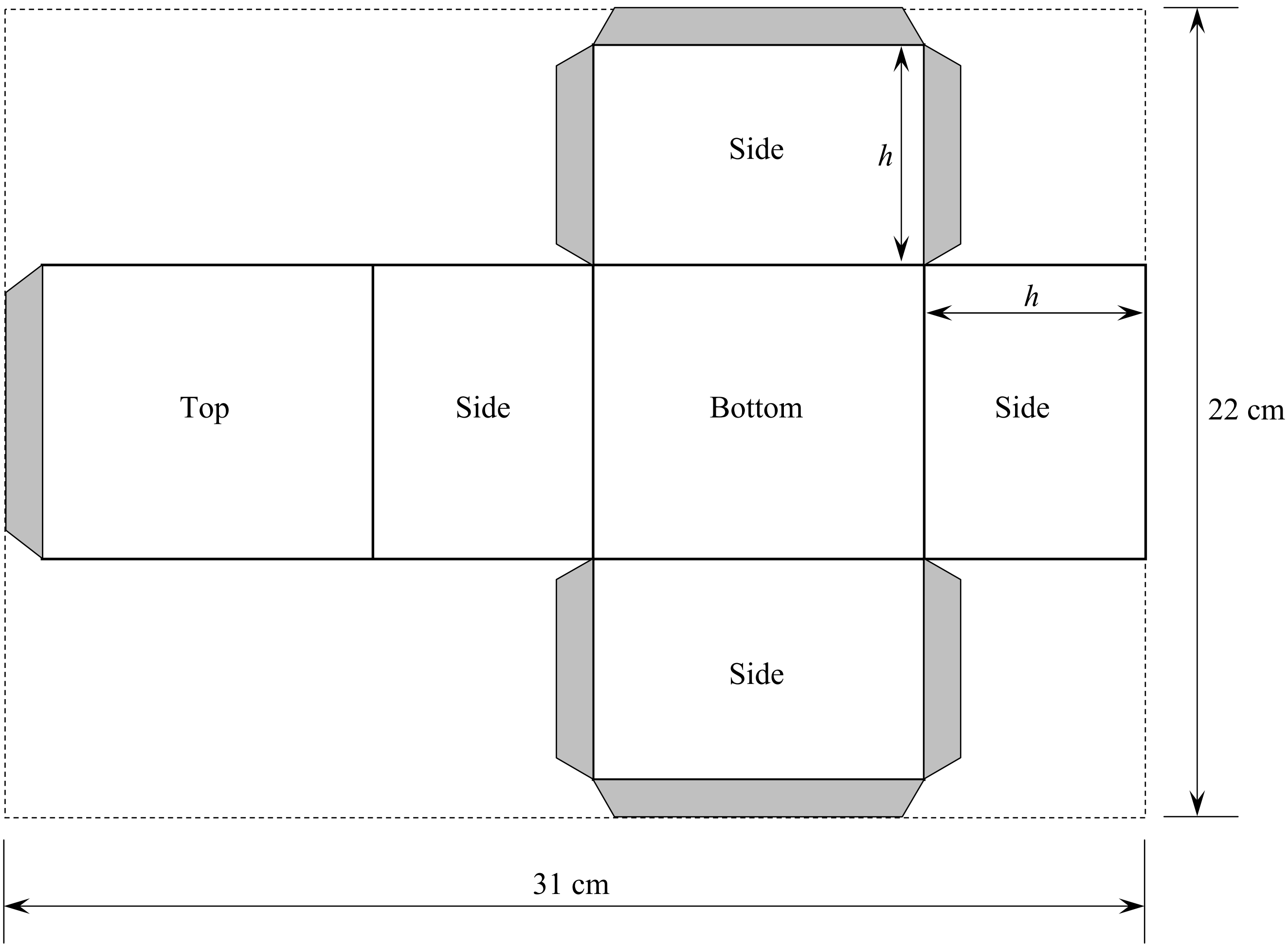
Section B — Contexts and Applications — 150 marks

Answer **all three** questions from this section.

Question 7 **(50 marks)**

A company has to design a rectangular box for a new range of jellybeans. The box is to be assembled from a single piece of cardboard, cut from a rectangular sheet measuring 31 cm by 22 cm. The box is to have a capacity (volume) of 500 cm^3.

The net for the box is shown below. The company is going to use the full length and width of the rectangular piece of cardboard. The shaded areas are flaps of width 1 cm which are needed for assembly. The height of the box is h cm, as shown on the diagram.



(a) Write the dimensions of the box, in centimetres, in terms of h.

height = h cm

length = ________________ cm

width = ________________ cm

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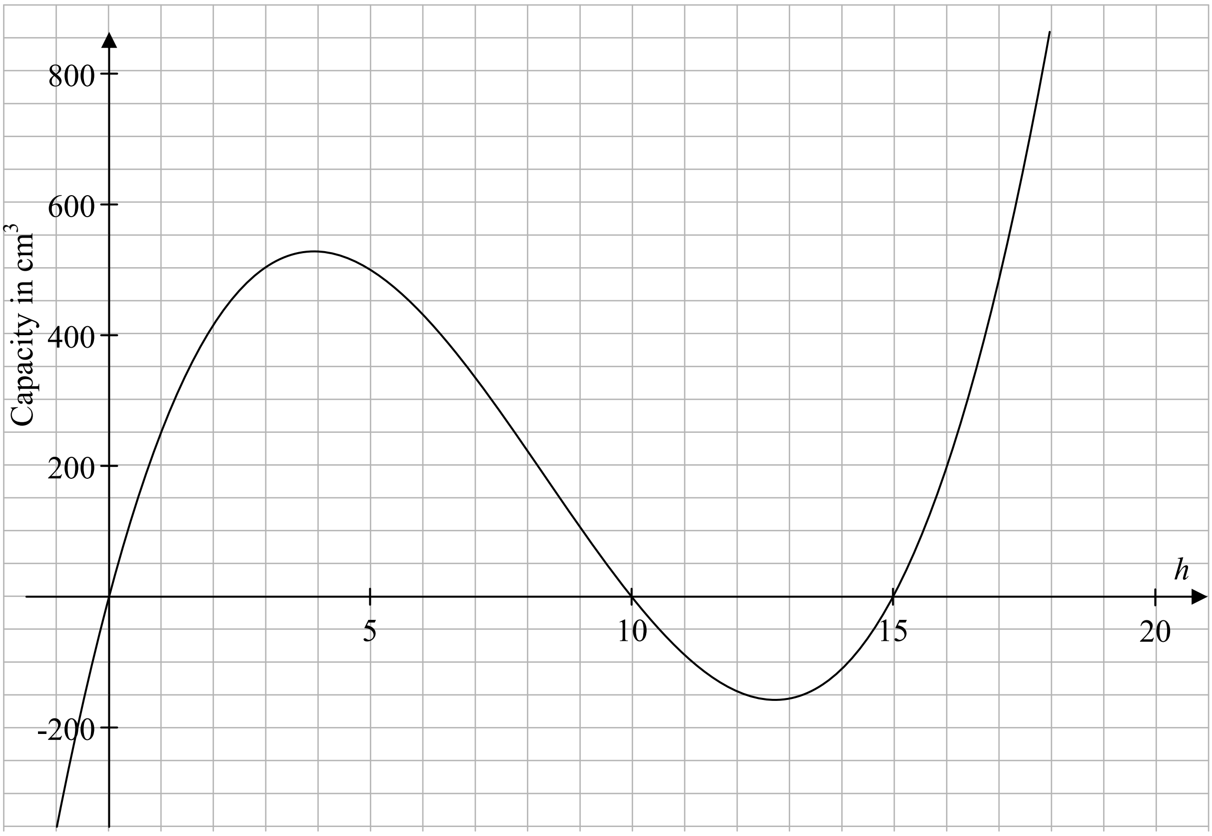
(b) Write an expression for the capacity of the box in cubic centimetres, in terms of h.

(c) Show that the value of h that gives a box with a square bottom will give the correct capacity.

(d) Find, correct to one decimal place, the other value of h that gives a box of the correct capacity.

(e) The client is planning a special "10% extra free" promotion and needs to increase the capacity of the box by 10%. The company is checking whether they can make this new box from a piece of cardboard the same size as the original one (31 cm × 22 cm). They draw the graph below to represent the box's capacity as a function of h. Use the graph to explain why it is *not* possible to make the larger box from such a piece of cardboard.



Explanation:

Question 8 **(50 marks)**

Pádraig is 25 years old and is planning for his pension. He intends to retire in forty years' time, when he is 65. First, he calculates how much he wants to have in his pension fund when he retires. Then, he calculates how much he needs to invest in order to achieve this. He assumes that, in the long run, money can be invested at an inflation-adjusted annual rate of 3%. Your answers throughout this question should therefore be based on a 3% annual growth rate.

(a) Write down the present value of a future payment of €20,000 in one years' time.

(b) Write down, in terms of t, the present value of a future payment of €20,000 in t years' time.

(c) Pádraig wants to have a fund that could, from the date of his retirement, give him a payment of €20,000 at the start of each year for 25 years. Show how to use the sum of a geometric series to calculate the value on the date of retirement of the fund required.

(d) Pádraig plans to invest a fixed amount of money every month in order to generate the fund calculated in part **(c)**. His retirement is $40 \times 12 = 480$ months away.

(i) Find, correct to four significant figures, the rate of interest per month that would, if paid and compounded monthly, be equivalent to an effective annual rate of 3%.

(ii) Write down, in terms of n and P, the value on the retirement date of a payment of €P made n months before the retirement date.

(iii) If Pádraig makes 480 equal monthly payments of €P from now until his retirement, what value of P will give the fund he requires?

(e) If Pádraig waits for ten years before starting his pension investments, how much will he then have to pay each month in order to generate the same pension fund?

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Question 9 **(50 marks)**

(a) Let $f(x) = -0{\cdot}5x^2 + 5x - 0{\cdot}98$, where $x \in \mathbb{R}$.

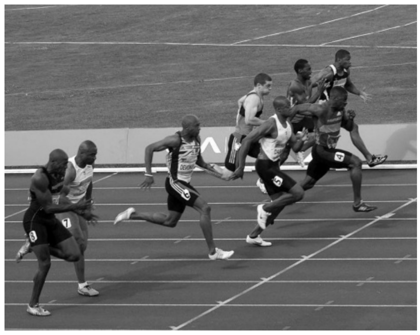
(i) Find the value of $f(0{\cdot}2)$

(ii) Show that f has a local maximum point at $(5,\ 11{\cdot}52)$.

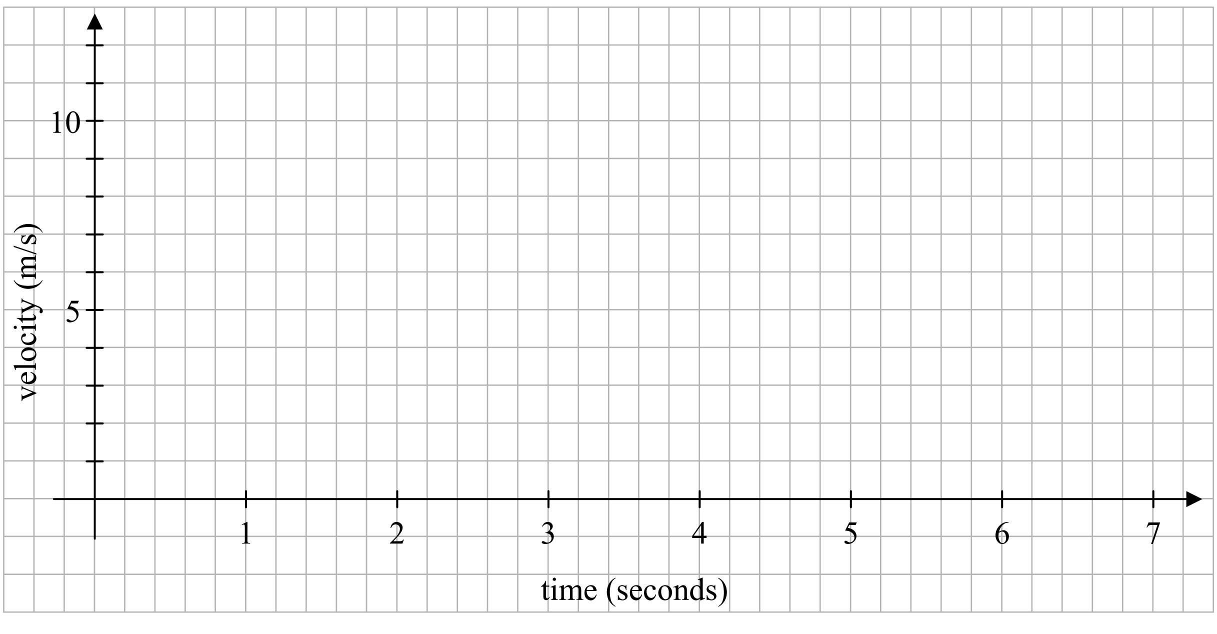
(b) A sprinter's velocity over the course of a particular 100 metre race is approximated by the following model, where v is the velocity in metres per second, and t is the time in seconds from the starting signal:

$$v(t) = \begin{cases} 0, & \text{for } 0 \le t < 0{\cdot}2 \\ -0{\cdot}5t^2 + 5t - 0{\cdot}98, & \text{for } 0{\cdot}2 \le t < 5 \\ 11{\cdot}52, & \text{for } t \ge 5 \end{cases}$$

Note that the function in part **(a)** is relevant to $v(t)$ above.



(i) Sketch the graph of v as a function of t for the first 7 seconds of the race.



(ii) Find the distance travelled by the sprinter in the first 5 seconds of the race.

(iii) Find the sprinter's finishing time for the race. Give your answer correct to two decimal places.

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(c) A spherical snowball is melting at a rate proportional to its surface area. That is, the rate at which its volume is decreasing at any instant is proportional to its surface area at that instant.

(i) Prove that the radius of the snowball is decreasing at a constant rate.

(ii) If the snowball loses half of its volume in an hour, how long more will it take for it to melt completely?

Give your answer correct to the nearest minute.

You may use this page for extra work.

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Note to readers of this document:

This sample paper is intended to help teachers and candidates prepare for the June 2012 examination in the *Project Maths* initial schools. The content and structure do not necessarily reflect the 2013 or subsequent examinations in the initial schools or in all other schools.

Leaving Certificate 2012 – Higher Level

Mathematics (Project Maths – Phase 3) – Paper 1

Sample Paper

Time: 2 hours 30 minutes