Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate Examination 2014
Sample Paper

Mathematics
(Project Maths – Phase 3)

Paper 2

Higher Level

Time: 2 hours, 30 minutes

300 marks

Examination number

Centre stamp

Running total	

For examiner	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
Total	

Grade

Instructions

There are **two** sections in this examination paper.

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	3 questions

Answer all nine questions, as follows:

In Section A, answer:

Questions 1 to 5 and

either Question 6A **or** Question 6B.

In Section B, answer Questions 7, 8, and 9.

Write your answers in the spaces provided in this booklet. You may lose marks if you do not do so. There is space for extra work at the back of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

You will lose marks if all necessary work is not clearly shown.

Answers should include the appropriate units of measurement, where relevant.

Answers should be given in simplest form, where relevant.

Write the make and model of your calculator(s) here:

Section A — Concepts and Skills — 150 marks

Answer **all six** questions from this section.

Question 1 (25 marks)

The random variable X has a discrete distribution. The probability that it takes a value other than 13, 14, 15 or 16 is negligible.

(a) Complete the probability distribution table below and hence calculate $E(X)$, the expected value of X.

x	13	14	15	16
$P(X = x)$	0·383	0·575		0·004

(b) If X is the age, in complete years, on 1 January 2013 of a student selected at random from among all second-year students in Irish schools, explain what $E(X)$ represents.

(c) If ten students are selected at random from this population, find the probability that exactly six of them were 14 years old on 1 January 2013. Give your answer correct to three significant figures.

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Question 2 **(25 marks)**

(a) Explain what is meant by *stratified sampling* and *cluster sampling*. Your explanation should include:

- a clear indication of the difference between the two methods
- one reason why each method might be chosen instead of simple random sampling.

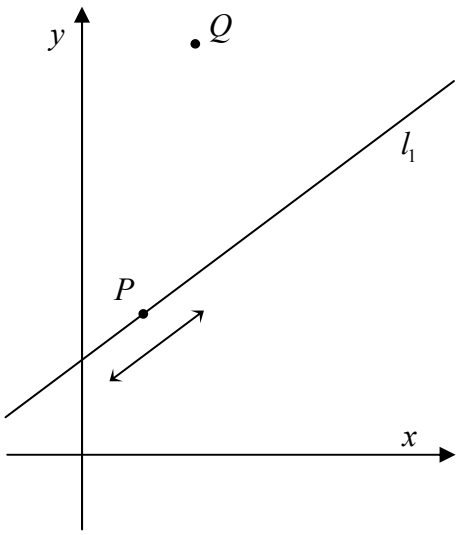
(b) A survey is being conducted of voters' opinions on several different issues.

(i) What is the overall margin of error of the survey, at 95% confidence, if it is based on a simple random sample of 1111 voters?

(ii) A political party had claimed that it has the support of 23% of the electorate. Of the voters in the sample above, 234 stated that they support the party. Is this sufficient evidence to reject the party's claim, at the 5% level of significance?

Question 3 **(25 marks)**

(a) Show that, for all $k \in \mathbb{R}$, the point $P(4k-2, 3k+1)$ lies on the line $l_1 : 3x - 4y + 10 = 0$.



(b) The line l_2 passes through P and is perpendicular to l_1. Find the equation of l_2, in terms of k.

(c) Find the value of k for which l_2 passes through the point $Q(3, 11)$.

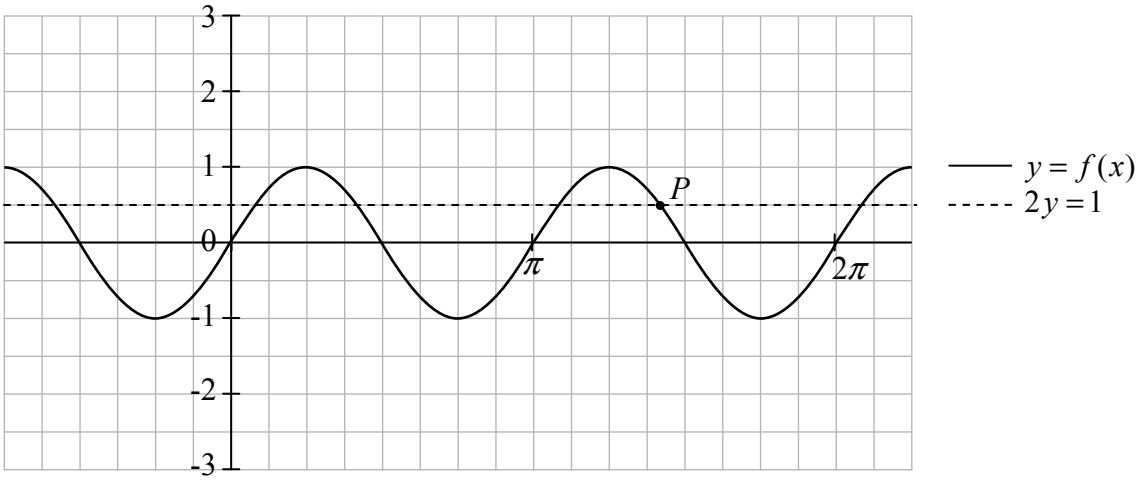
(d) Hence, or otherwise, find the co-ordinates of the foot of the perpendicular from Q to l_1.

Question 4 **(25 marks)**

The centre of a circle lies on the line $x + 2y - 6 = 0$. The x-axis and the y-axis are tangents to the circle. There are two circles that satisfy these conditions. Find their equations.

Question 5 **(25 marks)**

The diagram below shows the graph of the function $f : x \mapsto \sin 2x$. The line $2y = 1$ is also shown.



(a) On the same diagram above, sketch the graphs of $g : x \mapsto \sin x$ and $h : x \mapsto 3\sin 2x$. Indicate clearly which is g and which is h.

(b) Find the co-ordinates of the point P in the diagram.

Question 6 **(25 marks)**

Answer **either** 6A **or** 6B.

Question 6A

Explain, with the aid of an example, what is meant by *proof by contradiction*.

Note: you do not need to provide the full proof involved in your example. Give sufficient outline to illustrate how contradiction is used.

Explanation:

Example:

OR

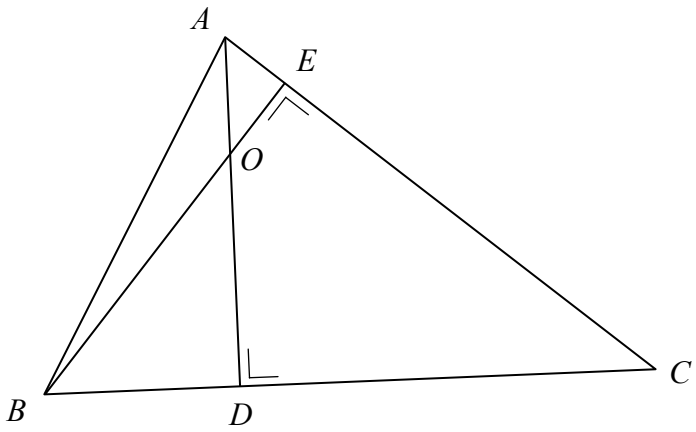
Question 6B

ABC is a triangle.

D is the point on BC such that $AD \perp BC$.
E is the point on AC such that $BE \perp AC$.

AD and BE intersect at O.

Prove that $|\angle DOC| = |\angle DEC|$.



Section B — Contexts and Applications — 150 marks

Answer Question 7, Question 8, and Question 9.

Question 7 (75 marks)

The *King of the Hill* triathlon race in Kinsale consists of a 750 metre swim, followed by a 20 kilometre cycle, followed by a 5 kilometre run.

The questions below are based on data from 224 athletes who completed this triathlon in 2010.

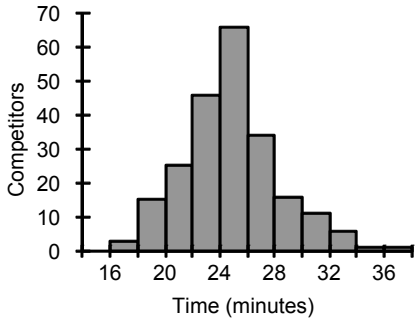
Máire is analysing data from the race, using statistical software. She has a data file with each competitor's time for each part of the race, along with various other details of the competitors.

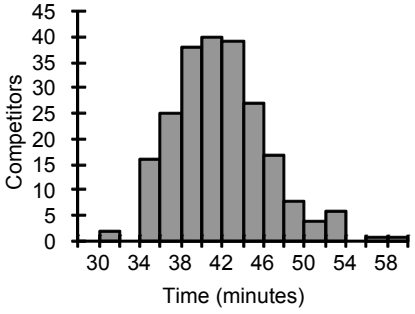
Lizzie Lee, winner of the women's event

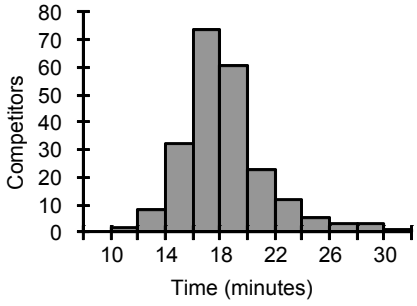
Máire gets the software to produce some *summary statistics* and it produces the following table. Three of the entries in the table have been removed and replaced with question marks (?).

	Swim	*Cycle*	*Run*
Mean	18.329	41.927	**?**
Median	17.900	41.306	**?**
Mode	#N/A	#N/A	#N/A
Standard Deviation	**?**	4.553	3.409
Sample Variance	10.017	20.729	11.622
Skewness	1.094	0.717	0.463
Range	19.226	27.282	20.870
Minimum	11.350	31.566	16.466
Maximum	30.576	58.847	37.336
Count	224	224	224

Máire produces histograms of the times for the three events. Here are the three histograms, without their titles.







(a) **(i)** Use the summary statistics in the table to decide which histogram corresponds to each event. Write the answers above the histograms.

(ii) The mean and the median time for the run are approximately equal. Estimate this value from the corresponding histogram.

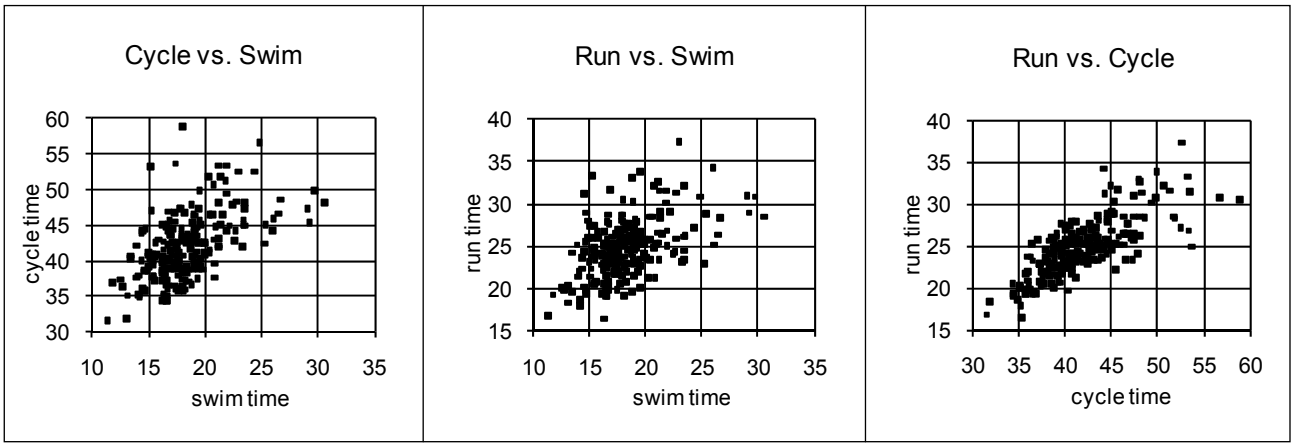
mean ≈ median ≈ _____

(iii) Estimate from the relevant histogram the standard deviation of the times for the swim.

standard deviation ≈ _____

(iv) When calculating the summary statistics, the software failed to find a *mode* for the data sets. Why do you think this is?

Máire is interested in the relationships between the athletes' performance in the three different events. She produces the following three scatter diagrams.



(b) Give a brief summary of the relationship between performance in the different events, based on the scatter diagrams.

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(c) The best-fit line for run-time based on swim-time is $y = 0 \cdot 53x + 15 \cdot 2$. The best-fit line for run-time based on cycle-time is $y = 0 \cdot 58x + 0 \cdot 71$. Brian did the swim in 17·6 minutes and the cycle in 35·7 minutes. Give your best estimate of Brian's time for the run, and justify your answer.

The mean finishing time for the overall event was 88·1 minutes and the standard deviation was 10·3 minutes.

(d) Based on an assumption that the distribution of overall finishing times is approximately normal, use the *empirical rule* to complete the following sentence:

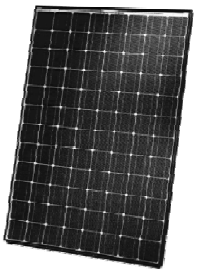
"95% of the athletes took between _____ and _____ minutes to complete the race."

(e) Using normal distribution tables, estimate the number of athletes who completed the race in less than 100 minutes.

(f) After the event, a reporter wants to interview two people who took more than 100 minutes to complete the race. She approaches athletes at random and asks them their finishing time. She keeps asking until she finds someone who took more than 100 minutes, interviews that person, and continues until she finds a second such person. Assuming the athletes are cooperative and truthful, what is the probability that the second person she interviews will be the sixth person she approaches?

Question 8 **(50 marks)**

A stand is being used to prop up a portable solar panel. It consists of a support that is hinged to the panel near the top, and an adjustable strap joining the panel to the support near the bottom.



By adjusting the length of the strap, the angle between the panel and the ground can be changed.

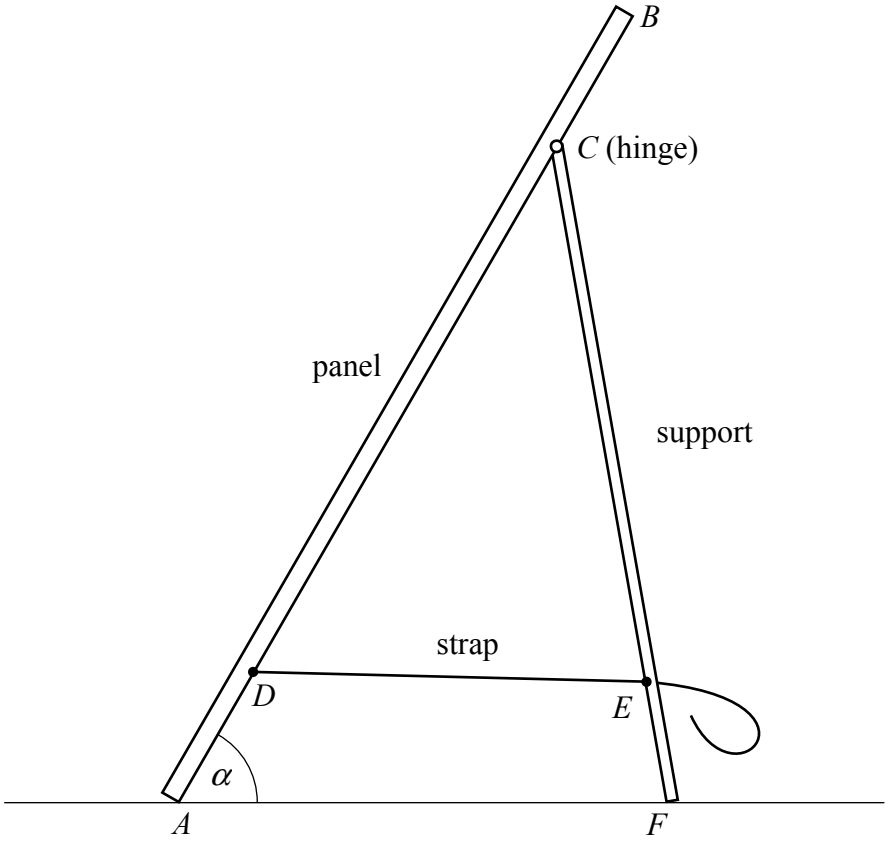
The dimensions are as follows:

$|AB| = 30$ cm

$|AD| = |CB| = 5$ cm

$|CF| = 22$ cm

$|EF| = 4$ cm.



(a) Find the length of the strap [DE] such that the angle α between the panel and the ground is 60°.

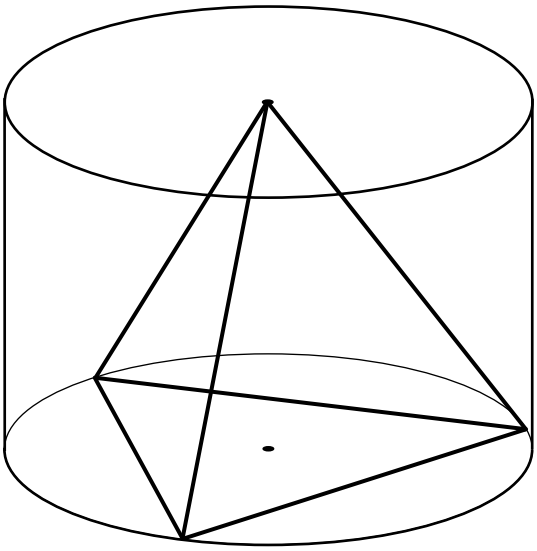
(b) Find the maximum possible value of α, correct to the nearest degree.

Question 9 **(25 marks)**

A regular tetrahedron has four faces, each of which is an equilateral triangle.

A wooden puzzle consists of several pieces that can be assembled to make a regular tetrahedron. The manufacturer wants to package the assembled tetrahedron in a clear cylindrical container, with one face flat against the bottom.

If the length of one edge of the tetrahedron is $2a$, show that the volume of the smallest possible cylindrical container is $\left(\frac{8\sqrt{6}}{9}\right)\pi a^3$.



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Note to readers of this document:

This sample paper is intended to help teachers and candidates prepare for the June 2014 examination in *Mathematics* under Phase 3 of *Project Maths*. The content and structure do not necessarily reflect the 2015 or subsequent examinations.

Section A of the examination paper will consist of six questions, each carrying 25 marks.
In accordance with the footnote in the syllabus in relation to geometry, there will be a choice within Question 6, as illustrated here.

Section B will consist of two, three, or four questions. These questions will not necessarily carry equal marks. The number of marks for each will be stated on the examination paper. The total number of marks for Section B will be 150.

Leaving Certificate 2014 – Higher Level

Mathematics (Project Maths – Phase 3) – Paper 2

Sample Paper
Time: 2 hours 30 minutes