Coimisiún na Scrúduithe Stáit

State Examinations Commission

LEAVING CERTIFICATE EXAMINATION 2015

MATHEMATICS

CHIEF EXAMINER'S REPORT

Contents

1. Introduction

1.1 Context

Over the last number of years there has been a phased implementation of new mathematics syllabuses at both Junior and Leaving Certificate, as part of an initiative called *Project Maths*. In 2008, 24 initial schools began a phased introduction of the new Leaving Certificate syllabus. Based on their feedback, the phased implementation of an updated syllabus began in all schools nationwide in September 2010. This syllabus has been under constant review as it has been implemented, with parts of the Leaving Certificate syllabus deferred for study until September 2013. Thus, 2015 is the first year in which all candidates, both at Junior and Leaving Certificate, have been assessed in all parts of the new Mathematics syllabus. Due to the phased nature of the introduction of the new syllabuses, it should be borne in mind that it will be 2017 before a full cohort of candidates who have studied all of the strands of both the Junior and Leaving Certificate programmes will be examined.

While there is significant overlap between the old and new syllabuses, the new syllabus is different from the previous one both in terms of content and in terms of skills. In terms of content, among the changes at Leaving Certificate are: an increase in the proportion of the syllabus dealing with statistics and probability, the removal of vectors and matrices, and changes to the material on functions and calculus. In terms of skills, the new syllabus has an increased emphasis on problem-solving, as well as on the skills of explanation, justification, and communication.

It was mentioned above that implementation of the new syllabus has been subject to ongoing review. One outcome of this review was the identification of some difficulties with the course for Foundation level. Accordingly, further substantial changes were made both to the course and to the structure of the examination at that level. This course is no longer nested within the Ordinary-level course. It is also now more context based, and some of the more abstract topics and less straightforward mathematical notation have been removed. These were substantial revisions, and the revised course at Foundation level was examined for the first time in 2015.

The syllabus changes were only one aspect of the systemic change to maths education that was intended by the Project Maths initiative. The initiative also sought to engender widespread changes to the way mathematics learning is experienced by students in schools. Accordingly, an extensive programme of in-career development for teachers was put in place to support such changes. A national scheme was also put in place to provide a suitable postgraduate qualification to out-of-field mathematics teachers (teachers of mathematics whose main qualification is in a different subject

area) so as to allow them to become qualified teachers of the subject. Further information about the Project Maths initiative can be found on the website of the National Council for Curriculum and Assessment (NCCA) at:
http://www.ncca.ie/en/Curriculum_and_Assessment/Post-Primary_Education/Project_Maths/
Resources for teachers and students, along with information on other supports available to teachers, can be found at the website of the Project Maths Development Team at www.projectmaths.ie.

This is the first full report from the Chief Examiner for Leaving Certificate Mathematics since Project Maths was initiated. In issuing this report at this time, the State Examinations Commission hopes that the information and analysis it contains will assist teachers in their ongoing efforts to continually improve the quality of the learning experiences of their students, and thereby assist the students in preparing for the examination. The SEC also hopes that the report helps to inform ongoing policy formation regarding mathematics education at this level, including the forthcoming NCCA review of Leaving Certificate Mathematics.

Readers should remain fully cognisant of the fact that Project Maths laid out an ambitious programme of change for mathematics education in Ireland. Such change takes time to embed fully, and one cannot expect all of its objectives to be fully achieved in the first few years. To serve its purpose, this report must of necessity identify aspects of candidate achievement that currently fall short of the challenging expectations that have been set out. This should not be taken to imply any criticism of teachers or students, or to suggest that curriculum designers should shy away from setting such challenging goals. Delivering mathematics education of the highest quality is a national endeavour that all countries grapple with and none finds easy. Identifying as clearly as we can what is being done well and not so well is a crucial part of this endeavour, and the SEC hopes that this report can make a contribution in that respect.

1.2 Syllabus Structure

The Leaving Certificate Mathematics syllabus comprises five strands: Statistics and Probability; Geometry and Trigonometry; Number; Algebra; and Functions. Topics and learning outcomes are specified for each strand, with the Ordinary level material a subset of the Higher level material. The syllabus emphasises that topics and strands should not be studied in isolation but that, where appropriate, connections should be made within and across the strands and with other areas of learning.

The syllabus is offered at three levels: Higher, Ordinary and Foundation. Differentiation between the three levels is achieved through three main channels: the content and learning outcomes of the syllabus; the processes of teaching and learning; and the assessment arrangements associated with the examinations. At Foundation level learners are assessed mostly by means of questions set in meaningful contexts.

This report should be read in conjunction with the examination papers, the published marking schemes, and the syllabus. The examination papers and marking schemes are available on the State Examination Commission's website www.examinations.ie and the syllabus is available at www.curriculumonline.ie.

1.3 Assessment Specification

The syllabus is assessed at three levels: Higher, Ordinary, and Foundation. Assessment at each level is by means of a terminal written examination. All examination papers are presented as combined question-and-answer booklets, and candidates must answer all questions. The examinations at Higher level and Ordinary level each consist of two papers. Each paper is marked out of 300 marks and is of two and a half hours' duration. Each of these papers is split into two sections: Section A: Concepts and Skills (6 questions at 25 marks each); and Section B: Contexts and Applications (2 or 3 or 4 questions, of varying mark allocations and with a combined total of 150 marks). In 2015, Section B of all four of these papers contained three questions.

The examination at Foundation level consists of one paper, which is marked out of 300 marks and is of two and a half hours' duration. It is split into two sections: Section A (8 questions at 25 marks each); and Section B (2 or 3 or 4 questions, of varying mark allocations and with a combined total of 100 Marks). In 2015, there were 2 questions in section B.

1.4 Participation Trends

The breakdown of the Leaving Certificate Mathematics cohort in terms of participation at Higher, Ordinary, and Foundation levels over the last five years is given in **Table 1**, and shown in **Figure 1**.

Year	Total Mathematics Candidature	Number at Higher level	Number at Ordinary level	Number at Foundation level	% Higher	% Ord.	% Found.
2011	51 991	8237	37 505	6249	15·8	72·1	12·0
2012	50 440	11 131	33 916	5393	22·1	67·2	10·7
2013	50 856	13 014	32 165	5677	25·6	63·2	11·2
2014	52 382	14 326	32 428	5628	27·3	61·9	10·7
2015	53 570	14 691	33 266	5613	27·4	62·1	10·5

Table 1: Number and percentage of candidates at each level, 2011 to 2015.

Figures for 2011, 2012, 2013, and 2014 include candidates in the initial *Project Maths* schools.

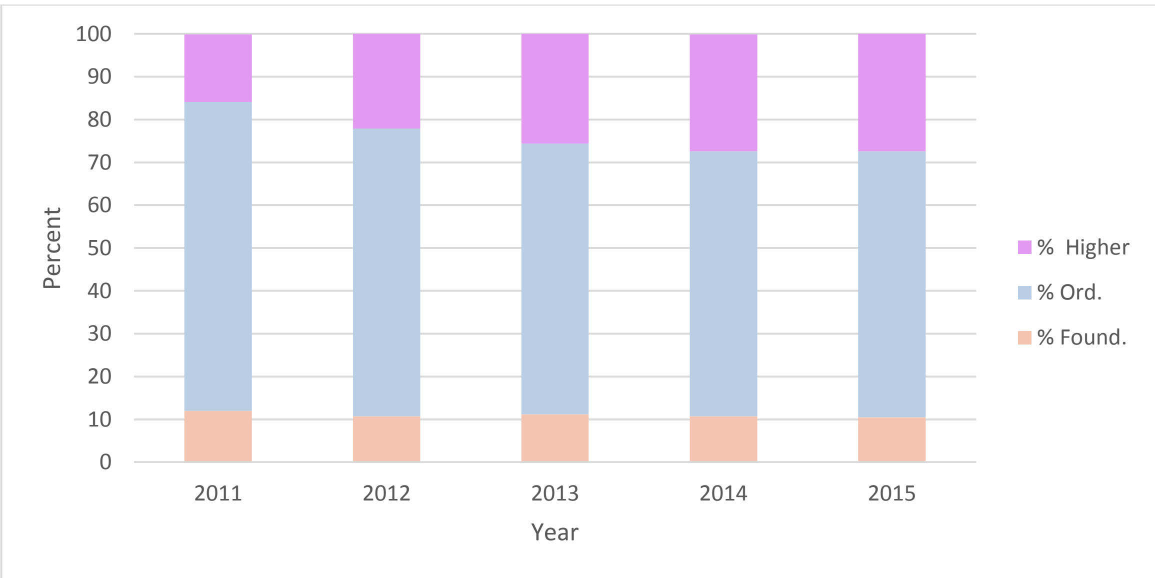

Figure 1: Percentage of candidates at each level, 2011 to 2015.

Percentages are based on the total mathematics candidature and include candidates in the initial *Project Maths* schools.

The table and graph show that from 2011 to 2015, the Higher level cohort increased from 15·8% to 27·4%, an increase of over 73%. Over the same period of time, the Ordinary level cohort fell from 72·1% to 62·1%, a decrease of almost 14%, while the Foundation level cohort has remained relatively constant at just over 10%.

The breakdown of the female and male Mathematics cohorts by level (Higher, Ordinary, and Foundation) are given in **Table 2**.

Year	Female			Male		
	% Higher	% Ordinary	% Foundation	% Higher	% Ordinary	% Foundation
2011	14·5	74·5	11·0	17·2	69·8	13·0
2012	20·8	69·2	10·0	23·3	65·3	11·4
2013	24·2	65·0	10·8	26·9	61·6	11·5
2014	26·0	63·7	10·3	28·7	60·1	11·2
2015	26·0	64·3	9·8	28·9	59·9	11·2

Table 2: Composition of female and male Mathematics cohorts by level, 2011 to 2015.
Figures are for school-based candidates only.
Percentages are based on the total number of candidates of that gender.

The figures for both female and male candidates show the same trends as the overall data, i.e. an increase in the percentage of candidates taking Higher level, and a related decrease in the percentages taking Ordinary with a slight decrease at Foundation level.

The gender breakdown has also been relatively stable over the last five years, with a slightly lower percentage of female than male candidates taking Higher level and Foundation level, and a higher percentage taking the Ordinary level.

2. Performance of Candidates

Care should be exercised when interpreting the grade distributions in this chapter. The syllabus and related changes that have occurred are such that neither the subject content nor the nature of the cognitive skills being tested is constant over the five years for which the statistics are presented. Furthermore, the percentage of the candidature opting for each level has changed very significantly, as seen in Section 1.4 above. The effect that these factors might be expected to have on the grade distribution at each level is dealt with in Section 2.1.1 below.

2.1 Higher Level

2.1.1 Overall performance

The distribution of grades awarded at Leaving Certificate Higher level Mathematics in each of the last five years is given in **Table 3** (lettered grades) and **Table 4** (sub-grades).

Year	A	B	C	A/B/C	D	E	F	NG	E/F/NG
2011	13·4	33·9	33·6	80·9	16·0	2·2	0·7	0·1	3·0
2012	9·8	37·3	36·2	83·3	14·3	1·8	0·5	0·0	2·3
2013	11·2	25·1	36·9	73·2	23·6	2·9	0·3	0·1	3·3
2014	10·4	28·9	33·4	72·7	23·0	3·5	0·7	0·0	4·2
2015	10·7	25·4	34·7	70·8	24·2	4·3	0·7	0·0	5·0

Table 3: Percentage of candidates awarded each lettered grade at Higher level Mathematics, 2011 to 2015.

Year	A1	A2	B1	B2	B3	C1	C2	C3	D1	D2	D3	E	F	NG
2011	5·9	7·5	10·0	11·5	12·4	12·9	11·7	9·0	7·3	4·9	3·8	2·2	0·7	0·1
2012	3·4	6·4	10·4	12·7	14·2	14·0	12·5	9·8	6·8	4·4	3·1	1·8	0·5	0·0
2013	5·6	5·6	6·5	8·5	10·1	12·2	12·8	11·9	10·3	7·3	6·0	2·9	0·3	0·1
2014	4·1	6·3	8·0	9·6	11·3	11·5	11·1	10·8	8·9	7·3	6·8	3·5	0·7	0·0
2015	5·2	5·5	7·1	8·3	10·0	11·4	11·7	11·6	9·5	8·1	6·6	4·3	0·7	0·0

Table 4: Percentage of candidates awarded each sub-grade at Higher level Mathematics, 2011 to 2015.

In general, these results show a decrease in the A-rate and A/B/C-rate from 2011 to 2015, as well as a slight rise in the E/F/NG-rate. This is not surprising given the changes in syllabus and the change in the composition of the cohort over the course of this period. In particular, the cohort now taking Higher level includes a large number of candidates that formerly would have taken Ordinary level.

The increase in the number of candidates taking Higher level from 2010 to 2015 has been substantial. This increase, which is often attributed to the roll-out of the *Project Maths* initiative and to the “bonus points” awarded for Higher level Mathematics in the CAO system, is a stated policy aim – *Literacy and Numeracy for Learning and Life*[1] specifies a goal of 30% of Leaving Certificate candidates taking Higher level Mathematics by 2020. However, such a steep change over such a short timeframe necessarily has a very significant impact on the grade distributions that might be expected at the various levels. The candidates whose choice of level is least certain are those near the overlap of standards between the levels – they are among the lower achieving candidates at Higher level and the higher achieving candidates at Ordinary level. When the proportion of such candidates opting for Higher level increases, an increase can be expected in the percentage of low grades awarded at Higher level, along with a decrease in the percentage of high grades awarded at Ordinary level.

Furthermore, in the case of the current syllabus change in Mathematics, there has been a deliberate attempt to increase the emphasis on higher-order thinking skills. These are skills that students find difficult to master and teachers may find difficult to instil. The syllabus expectations are ambitious at all levels. Accordingly one might expect candidates to fall short of syllabus expectations to a greater degree than before, and certainly more than if the changes were solely or primarily related to content.

Grade distribution data at all three levels need to be considered in the context of the above issues, which clearly had an impact over the period from 2011 to 2014. However, 2015 appears to have seen a slight recovery, or at least a stabilisation in many of these rates. This may, at least in part, be attributable to the ‘bedding-in’ of the new syllabus. Teachers and students are becoming more familiar with the revised syllabus content and the expectations of the assessment process. It is also reasonable to consider that the extensive programme of continuing professional development and the retraining of large numbers of out-of-field teachers through the postgraduate programme are bearing fruit in relation to candidate achievement. At this stage, there is also a significant amount of material available as an aid to examination preparation (previous examination papers, support service materials and resources, text books, etc.). In this regard, it should be noted that material from section 1.7 of the syllabus on inferential statistics, which had been deferred for a number of years, was examined for the first time in 2015 at Higher level and Ordinary level. At Higher level,

[1] Department of Education and Skills, *Literacy and Numeracy for Learning and Life: The National Strategy to Improve Literacy and Numeracy among Children and Young People, 2011-2020*, Dublin, 2011. Available at: www.education.ie/en/Publications/Policy-Reports/lit_num_strategy_full.pdf

in many instances, this material was competently dealt with. However, in the case of certain centres, the majority of the candidates seemed to be unfamiliar with this material. Another cause for concern was that, at Ordinary level, the concepts of margin of error, the creation of 95% confidence intervals, the use of the Empirical Rule, and understanding of standard deviation, were poorly dealt with. It may be anticipated that, as familiarity with these topics increases, the outcomes in relation to them in future years will improve at both levels.

2.1.2 Performance by Gender

The distribution of grades by gender over the last five years is given in **Table 5** (female candidates) and **Table 6** (male candidates).

Year	A1	A2	B1	B2	B3	C1	C2	C3	D1	D2	D3	E	F	NG
2011	4·6	6·8	10·9	12·1	13·3	13·5	12·5	8·9	7·5	4·4	3·3	1·8	0·5	0·0
2012	1·7	5·3	9·4	13·2	15·2	14·2	13·6	10·8	6·7	4·7	3·2	1·6	0·3	0·0
2013	3·3	4.0	5·2	8·3	10·4	13·1	13·4	12·9	11·7	8·2	6·5	2·6	0·3	0·1
2014	2·5	5·6	7·5	9·8	11·7	11·9	11·3	11·5	9·1	7·6	7·3	3·5	0·6	0·1
2015	3·2	4·4	6·5	8·0	10·0	11·8	12·4	12·5	9·9	9·0	7·0	4·5	0·7	0·0

Table 5: Percentage of female candidates awarded each sub-grade at Higher level Mathematics, 2011 to 2015.

Year	A1	A2	B1	B2	B3	C1	C2	C3	D1	D2	D3	E	F	NG
2011	7·0	8·1	9·4	11·1	11·6	12·3	11·1	9·1	7·2	5·3	4·2	2·7	0·8	0·2
2012	4·8	7·5	11·3	12·3	13·4	13·7	11·6	8·9	6·9	4·0	2·9	1·9	0·7	0·1
2013	7·5	7·0	7·6	8·6	9·9	11·5	12·2	11·0	9·1	6·5	5·6	3·1	0·4	0·1
2014	5·5	6·9	8·5	9·5	10·9	11.2	10·9	10·1	8·8	7·0	6·5	3·6	0·7	0·0
2015	7·1	6·5	7·6	8·6	9·9	11·1	11·0	10·8	9·0	7·3	6·3	4·1	0·6	0·1

Table 6: Percentage of male candidates awarded each sub-grade at Higher level Mathematics, 2011 to 2015.

The results of both female and male candidates have followed the overall Higher level trends described above. A higher percentages of male candidates have achieved A-grades and A/B/C grades over this period. These patterns have been reasonably consistent since the introduction of the new syllabus.

2.2 Ordinary Level

2.2.1 Overall Performance

The distribution of grades awarded at Leaving Certificate Ordinary level Mathematics in each of the last five years is given in **Table 7** (lettered grades) and **Table 8** (sub-grades).

Year	A	B	C	A/B/C	D	E	F	NG	E/F/NG
2011	11·4	30·4	27·7	69·5	20·7	6·8	2·6	0·4	9·8
2012	4·7	28·4	33·0	66·1	24·7	7·2	2·0	0·2	9·4
2013	5·4	25·3	33·6	64·3	26·6	7·1	1·9	0·3	9·3
2014	6·8	28·0	31·9	66·7	24·7	6·6	1·8	0·2	8·6
2015	5·5	32·3	35·9	73·3	20·4	4·6	1·1	0·1	5·8

Table 7: Percentage of candidates awarded each lettered grade at Ordinary level Mathematics, 2011 to 2015.

Year	A1	A2	B1	B2	B3	C1	C2	C3	D1	D2	D3	E	F	NG
2011	4·1	7·3	9·4	10·4	10·6	9·9	9·3	8·5	7·5	6·4	6·8	6·8	2·6	0·4
2012	1·0	3·7	7·2	9·9	11·3	11·3	11·2	10·5	9·2	7·5	8·0	7·2	2·0	0·2
2013	1·6	3·8	6·3	8·9	10·1	11·0	11·6	11·0	9·4	8·4	8·5	7·1	1·9	0·2
2014	2·0	4·8	7·6	9·6	10·8	10·8	11·0	10·1	9·1	7·6	8·0	6·6	1·8	0·2
2015	1·3	4·2	8·2	11·3	12·8	13·2	12·1	10·6	8·4	6·2	5·8	4·6	1·1	0·1

Table 8: Percentage of candidates awarded each sub-grade at Ordinary level Mathematics, 2011 to 2015.

These results show a substantial decrease in the A-rate from 2011 to 2015. Potential reasons for this are as already outlined in the commentary on the Higher level outcomes in Section 2.1.1 above. There has also been a decrease in the E/F/NG rate.

2.2.2 Performance by Gender

The distribution of grades by gender over the last five years is given in **Table 9** (female candidates) and **Table 10** (male candidates).

Year	A1	A2	B1	B2	B3	C1	C2	C3	D1	D2	D3	E	F	NG
2011	4·8	8·4	10·2	11·2	11·2	10·0	9·5	8·0	7·0	5·7	5·9	6·0	1·9	0·2
2012	1·1	3·8	7·4	10·6	11·8	11·6	11·4	10·5	9·1	7·0	7·7	6·5	1·5	0·1
2013	1·7	4·1	6·7	9·7	10·8	11·9	11·8	10·9	9·1	7·8	7·8	6·1	1·3	0·1
2014	2·1	5·0	8·0	9·6	10·7	10·5	11·0	10·2	9·2	7·5	8·0	6·4	1·5	0·1
2015	1·2	4·4	8·5	11·7	13·2	13·3	12·1	10·5	8·3	6·1	5·6	4·3	0·9	0·1

Table 9: Percentage of female candidates awarded each sub-grade at Ordinary level Mathematics, 2011 to 2015.

Year	A1	A2	B1	B2	B3	C1	C2	C3	D1	D2	D3	E	F	NG
2011	3·4	6·1	8·5	9·5	10·0	9·9	9·1	9·0	8·1	7·0	7·7	7·8	3·3	0·6
2012	1·0	3·6	6·9	9·1	10·8	10·9	11·1	10·4	9·2	8·0	8·3	7·8	2·5	0·3
2013	1·6	3·5	6·0	8·1	9·4	10·0	11·4	11·1	9·8	8.9	9·1	8·2	2·5	0·4
2014	1·9	4·6	7·2	9·5	10·8	11·1	11·0	10·0	8·9	7·7	8·0	6·9	2·0	0·2
2015	1·4	4·0	7·9	10·9	12·4	13·1	12·1	10·7	8·4	6·4	6·1	5·0	1·3	0·1

Table 10: Percentage of male candidates awarded each sub-grade at Ordinary level Mathematics, 2011 to 2015.

The results of both female and male candidates have followed the overall Ordinary level trends described above. Female candidates have outperformed their male counterparts with respect to A-rates, A/B/C-rates, and E/F/NG-rates in each of the last five years. These patterns have been reasonably consistent over this period, although the gap has narrowed with respect to the A/B/C and E/F/NG-rates more recently.

2.3 Foundation Level

2.3.1 Overall Performance

The distribution of grades awarded at Leaving Certificate Foundation level Mathematics in each of the last five years is given in **Table 11** (lettered grades) and **Table 12** (sub-grades).

Year	A	B	C	A/B/C	D	E	F	NG	E/F/NG
2011	9·7	34·6	33·5	77·8	17·1	3·8	1·2	0·2	5·2
2012	6·1	30·2	36·1	72·4	20·3	5·7	1·6	0·1	7·4
2013	4·5	29·8	39·9	74·2	20·6	3·8	1·2	0·2	5·2
2014	5·3	32·6	37·6	75·5	19·0	4·2	1·3	0·1	5·6
2015	9·2	32·3	34·4	75·9	18·7	4·2	1·0	0·1	5·3

Table 11: Percentage of candidates awarded each lettered grade at Foundation level Mathematics, 2011 to 2015.

Year	A1	A2	B1	B2	B3	C1	C2	C3	D1	D2	D3	E	F	NG
2011	3·5	6·2	8·7	12·3	13·6	13·0	11·7	8·8	6·8	5·4	4·9	3·8	1·2	0·2
2012	2·2	4·0	6·9	10·6	12·2	12·8	12·8	10·5	7·6	6·5	6·2	5·7	1·6	0·1
2013	1·6	2·9	6·0	10·2	13·6	14·8	14·3	10·8	9·2	5·9	5·5	3·8	1·2	0·2
2014	1·6	3·7	7·6	10·8	14·2	14·4	13·0	10·2	8·0	5·7	5·3	4·2	1·3	0·1
2015	3·5	5·7	7·6	11·1	13·6	11·8	12·5	10·1	7·1	5·8	5·8	4·2	1·1	0·1

Table 12: Percentage of candidates awarded each sub-grade at Foundation level Mathematics, 2011 to 2015.

As with the Ordinary level, these results show a substantial decrease in the A-rate from 2011 to 2014. As dealt with in the introduction above, a significantly revised Foundation level syllabus was examined for the first time in 2015, and the increase in the percentage of candidates achieving an A-grade may be an early indication that this syllabus is more suited to the cohort taking the examination. Neither the E/F/NG-rate nor the A/B/C-rate shows any particular pattern of movement over this period.

2.3.2 Performance by Gender

The distribution of grades by gender over the last five years is given in **Table 13** (female candidates) and **Table 14** (male candidates).

Year	A1	A2	B1	B2	B3	C1	C2	C3	D1	D2	D3	E	F	NG
2011	3·4	6·3	9·7	12·5	15·0	13·1	11·7	8·3	6·2	5·3	3·8	3·4	1·0	0·3
2012	1·7	3·8	6·9	10·2	13·0	13·3	13·2	11·4	8·0	5·8	6·1	4·9	1·5	0·0
2013	1·5	2·7	6·6	11·6	14·2	16·2	14·2	10·2	8·7	5·3	4·8	2·9	1·0	0·1
2014	1·3	3.3	8.4	10.8	15·0	14·5	13·7	9·5	7·8	5·5	5·3	4·0	0·9	0·1
2015	2·7	4·7	7·4	9·9	13·9	11·4	12·9	10·9	7·5	5·9	7·1	4·9	0·9	0·0

Table 13: Percentage of female candidates awarded each sub-grade at Leaving Certificate Foundation level Mathematics, 2011 to 2015.

Year	A1	A2	B1	B2	B3	C1	C2	C3	D1	D2	D3	E	F	NG
2011	3·5	6·2	7·8	12·1	12·5	12·9	11·6	9·2	7·3	5·4	5·8	4·2	1·5	0·1
2012	2·4	4·1	6·9	10·9	12·3	12·4	12·3	9·8	7·3	7·0	6·2	6·4	1·7	0·2
2013	1·7	3·0	5·5	9·0	13·1	13·5	14·4	11·4	9·7	6·4	6·0	4·5	1·3	0·3
2014	1·9	4·1	6·9	10·7	13·5	14·2	12·5	10·9	8·1	5·8	5·4	4·3	1·6	0·2
2015	4·3	6·5	7·7	12·1	13·4	12·2	12·2	9·5	6·8	5·7	4·6	3·6	1·0	0·2

Table 14: Percentage of male candidates awarded each sub-grade at Leaving Certificate Foundation level Mathematics, 2011 to 2015.

Following the shift previously mentioned towards more context-based questions in 2015, male candidates outperformed their female counterparts with respect to A-rates, A/B/C-rates, and E/F/NG-rates. From 2011 to 2014 female candidates outperformed their male counterparts with respect to A/B/C-rates and E/F/NG-rates.

3. Analysis of Candidate Performance

There were two main ways in which information on candidate performance was gathered.

First, examiners recorded the item-level marks for each of the candidates in a random sample of scripts. These results were collated, and are presented in Section 3.1 below.

Second, examiners submitted reports towards the end of the marking process that included observations on areas such as: topics where candidates showed strength or weakness, different approaches to questions that were in evidence, and common misconceptions. This feedback was used as the basis of the commentary in Section 3.2 below, where the success of candidates in meeting each of the syllabus objectives is examined.

3.1 General Commentary on Engagement and Performance

3.1.1 Higher Level

Table 15 is a summary based on an analysis of a random sample of 720 scripts (≈ 4·9% of all scripts) from Higher level Mathematics candidates in 2015.

In the Higher level examination, topics from Strand 3 (Number), Strand 4 (Algebra), and Strand 5 (Functions) are generally assessed in Paper 1, while topics from Strand 1 (Statistics and Probability), Strand 2 (Geometry and Trigonometry), and Topic 3.4 (Applied measure) are generally assessed on Paper 2. That said, this division of topics is not absolute. As mentioned above, the syllabus emphasises that topics and strands should not be studied in isolation but that, where appropriate, connections should be made within and across the strands. In order to adequately assess candidates' proficiency in the syllabus, it is thus necessary to have questions which require candidates to use skills from a range of different topics and strands.

On Higher level Paper 1, the overall mean mark was 190·8, (63·6%) with a standard deviation of 50·4 (16·8%). All but one question on this paper had a mean mark of at least 50% of the available marks, the exception being Question 4, (complex numbers / fractions), where the mean mark was 40%). The highest mean mark was 88% in Question 3 (functions). Candidates also performed particularly well on Question 1 (sequences), Question 2 (algebra), and Question 6 (financial applications of sequences).

On Higher level Paper 2, the overall mean mark was 189 (63%), with a standard deviation of 50·2 (16·7%). All questions on this paper had a mean mark of at least 50%, with the highest mean mark of 84% in Question 1 (probability). Candidates also performed well on Question 3 and Question 4 (co-ordinate geometry, line and circle).

Paper 1					
Section	Q	Mean Mark	Mean Mark (%)	Mark Ranking (Paper)	Main Topic
A	1	18·0	73	3	Sequences
A	2	17·4	70	4	Algebra
A	3	22·0	88	1	Functions
A	4	13·3	40	9	Complex numbers in fraction form
A	5	15·5	62	5	Functions (calculus)
A	6	18·3	73	2	Sequences (financial applications)
B	7	29·5	59	7	Algebra/functions
B	8	26·6	53	8	Functions/rates of change
B	9	30·7	61	6	Functions/trigonometry/calculus

Paper 2					
Section	Q	Mean Mark	Mean Mark (%)	Mark Ranking (Paper)	Main Topic
A	1	21·6	84	1	Probability
A	2	14·8	59	6	Inferential statistics
A	3	17·5	70	3	Co-ordinate geometry - line
A	4	18·6	75	2	Co-ordinate geometry - circle
A	5	14·2	57	7	Trigonometry
A	6	16·0	64	4	Geometry
B	7	22·8	57	8	Geometry/area
B	8	35·3	54	9	Probability
B	9	28.8	64	5	Trigonometry

Table 15: Question level data including mean mark per question, Higher level Mathematics 2015.

3.1.2 Ordinary Level

In the Ordinary level examination, as in the Higher level one, topics from Strand 3 (Number), Strand 4 (Algebra), and Strand 5 (Functions) are generally assessed in Paper 1, while topics from Strand 1 (Statistics and Probability), Strand 2 (Geometry and Trigonometry), and Topic 3.4 (Applied measure) are generally assessed on Paper 2. As at Higher level, this division of topics is not absolute.

On Paper 1, the overall mean mark was 162·5 (54·2%) with a standard deviation of 79·0 (26·3%).

Three questions on this paper had a mean mark of less than 50%: Question 4 (algebra), Question 5 (functions) and Question 9 (functions/graphs). The highest mean mark,71%, was in Question 1 (Number).

On Paper 2, the overall mean mark was 186·4 (62·1%), with a standard deviation of 50·6 (16·9%).

There was less variation in the marks per question on this paper than on Paper 1. All but one question on this paper had a mean mark of at least 50%, the exception being Question 7 (geometry/enlargements), where the mean mark was 46%. The highest mean mark, 82%, was in Question 4 (geometry). Candidates also performed well on Question 3 (co-ordinate geometry of the circle) and Question 8 (applied measure).

Overall at this level, candidates struggled noticeably with questions that involved any significant amount of algebra.

Table 16 is a summary based on an analysis of a random sample of 1120 scripts (≈ 4·6% of all scripts) from Ordinary level Mathematics candidates in 2015.

Paper 1					
Section	Q	Mean Mark	Mean Mark (%)	Mark Ranking (Paper)	Main Topic
A	1	17·7	71	1	Arithmetic
A	2	15·3	61	2	Arithmetic
A	3	14·9	60	3	Algebra
A	4	11·5	46	7	Algebra
A	5	9·4	38	9	Functions
A	6	14·6	58	5	Complex Numbers
B	7	37·1	53	6	Patterns
B	8	29·8	60	4	Functions/graphs
B	9	13·4	45	8	Functions/graphs

Paper 2					
Section	Q	Mean Mark	Mean Mark (%)	Mark Ranking (Paper)	Main Topic
A	1	13·8	55	7	Principle of counting
A	2	14·4	58	5	Co-ordinate geometry - line
A	3	17·7	71	4	Co-ordinate geometry - circle
A	4	20·4	82	1	Geometry
A	5	13·1	52	8	Trigonometry
A	6	18·8	75	2	Inferential statistics
B	7	16·2	46	9	Geometry/enlargements
B	8	33·0	73	3	Applied measure
B	9	40·6	58	6	Statistics

Table 16: Question level data including mean mark per question, Ordinary level Mathematics 2015.

3.1.3 Foundation Level

In the Foundation level examination, all strands of the syllabus were assessed in a single paper for the first time in 2015. The overall mean mark was 193·3 (64·4%), with a standard deviation of 45·7 (15·2%). There were many similarities between candidates' performances at Foundation level and the other two levels. Two questions on this paper had a mean mark of less than 50%: Question 4 (enlargements) and Question 10 (applied measure / geometry). Four questions had a mean mark greater than 80%: Question 7 (number patterns), Question 1 (arithmetic), Question 3 (arithmetic), and Question 9 (functions and graphs).

Table 17 is a summary based on an analysis of a random sample of 260 scripts (≈ 4·6% of all scripts) from Foundation level Mathematics candidates in 2015.

Section	Q	Mean Mark	Mean Mark (%)	Mark Ranking (Paper)	Main Topic
A	1	20·6	82	2	Arithmetic
A	2	14·3	57	8	Arithmetic
A	3	20·3	81	3	Arithmetic
A	4	9·4	38	10	Geometry/enlargements
A	5	17·7	71	5	Probability
A	6	15·3	61	6	Statistics
A	7	22·3	89	1	Patterns/Problem solving
A	8	14·8	59	7	Geometry/Trigonometry
B	9	40·4	81	4	Functions/graphs
B	10	21·5	43	9	Applied measure/Geometry

Table 17: Question level data including mean mark per question, Foundation level Mathematics 2015.

3.2 Meeting of Specific Syllabus Objectives

The stated objective of the Leaving Certificate Mathematics syllabus is that learners develop mathematical proficiency, which it breaks down into five areas:

1. **conceptual understanding**: comprehension of mathematical concepts, operations, and relations
2. **procedural fluency**: skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
3. **strategic competence**: ability to formulate, represent, and solve mathematical problems in both familiar and unfamiliar contexts
4. **adaptive reasoning**: capacity for logical thought, reflection, explanation, justification and communication
5. **productive disposition**: habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence, perseverance and one's own efficacy.

The success of candidates in meeting each of the first four of these syllabus objectives can be evaluated by considering the quality of candidate responses to specific parts of questions on the examination papers. As stated earlier, this is made possible by information captured by examiners during their work and the examiners' observations in their reports.

There are no examination questions that specifically assess the fifth objective on the list, candidates' productive disposition. Examiners reported that candidates are more inclined to attempt all question parts than in previous years which could be taken as an indication of an increase in candidates' productive disposition. At Higher level in both papers it was apparent that candidates made a determined effort to complete the entire examination paper. They were prepared to make a number of attempts in many questions and to persevere in solving problems even when, because of errors, the numerical values were not user-friendly. However, at Ordinary level, there was little evidence of the same diligence and perseverance when problems arose. Candidates at this level generally abandoned the work as soon as difficulty was encountered, rather than trying different ideas.

While these impressions, as reported by the examining teams, do give some indication of candidates' productive disposition, it is not possible to provide a fully robust analysis of this objective on the basis of candidate responses captured in the examination.

3.2.1 Conceptual understanding

Most candidates demonstrated good levels of knowledge and comprehension of basic mathematical concepts and relations, which is fundamental to the successful development of mathematical proficiency. Candidates struggled at times when more involved understanding was required, or when the concepts were slightly less standard. It is recommended that candidates give due attention to understanding the mathematical terms contained in the syllabus, including being able to differentiate between them where appropriate, and being able to explain what they mean in an accurate and coherent fashion.

At **Higher level**, candidates showed that they had a good grasp of many mathematical operations. Question 5(a) on Paper 1 involved an equation involving a square root. Its solution required squaring both sides, which may introduce an extraneous root. Most candidates successfully squared both sides and solved the resulting quadratic equation. That is, this much was generally procedurally correct. However, most candidates did not test the resulting roots, assuming instead that both solutions would satisfy the original equation, which was not the case. In part (c) of the same question, many candidates did not know that solving $f'(x) = 0$ would yield the *x*-coordinates of the turning point of the function. In many instances, candidates sought to solve $f''(x) = 0$ instead, which illustrated a lack of comprehension of the concept of a turning point and its relationship to the derivative.

Most candidates at this level demonstrated a conceptual understanding of basic probability (Paper 2, Question 1(b)(i)). The concept of slopes was clearly well understood by the majority of candidates (Paper 2, Question 3 (a)) although final answers in this question were often incorrect in the case of less able candidates who incurred penalties for arithmetic error(s).

At **Ordinary level**, the completion of a table based on patterns shown in a diagram (Paper 1, Question 7 (b)) was very well handled, indicating a high standard and quality of comprehension across the full cohort of candidates. Furthermore, in a question that required both knowledge of basic angle relationships in triangles and parallelograms and the application of such knowledge (Question 4(a)), most candidates correctly worked out all three of the angles sought and were able to give correct reasons for at least two of them. In contrast, in a question that required a basic application of the *fundamental principle of counting* to a scenario involving passwords (Paper 2, Question 1(a)) most candidates were not able to deal with the possible repetition of digits or the fact that the password consisted of six digits. Many students simply worked with a factorial (6!), while others worked with nPr, neither of which displays any appreciable level of understanding.

At **Foundation level**, most candidates displayed a thorough knowledge of place value by providing correct answers in Question 3(a), which required them to rearrange some digits so as to maximise and then minimise the resulting number. They also successfully managed to calculate the volumes of two boxes and to draw a correct conclusion based on those answers (Question 10(a)(ii)). However, they were not as successful in the calculations of the surface areas of the same shapes (Question 10(a)(i))

3.2.2 Procedural fluency

Many of the questions in Section A of the examination papers relate to procedural fluency; that is, the ability of candidates to carry out mathematical procedures accurately and appropriately. For example, at **Higher level**, candidates were asked to solve a cubic equation in Paper 1, Question 2. There were extremes in answering this question. Many candidates, who were presumably well prepared for such a question, had no difficulty with it. However, a significant minority (almost 20%) were clearly unfamiliar with the procedure, as they were not able to produce work of sufficient merit to be awarded any marks. Most candidates demonstrated a knowledge of the procedure for rationalising complex fractions (Paper 1, Question 4(a)). However, the majority of candidates struggled to deal with the fractions involved, and a lot of the work produced here was of very poor quality.

Where candidates were asked to find the co-ordinates of the centre of a circle (Paper 2 Question 4(b)(i)), the cohort was divided between those who used the ratio method, and those who used a translation. Many of the latter demonstrated their steps to solution on a diagram. Both methods were generally successfully completed by the majority of those candidates who chose them.

Similarly, in finding the solutions to a trigonometric equation in the different quadrants (Paper 2, Question 5(b)), many candidates used the general form $60° + n(360°)$. However, there was also evidence of drawings and general rough work successfully being used instead by some candidates.

At **Ordinary level**, a high quality of answering was evident when candidates were asked to divide money in a given ratio (Paper 1, Question 2(a)) and also when they were asked to simplify a linear expression (Paper 1, Question 3(a)). When given both an original and an image triangle, many candidates struggled with the construction of the centre of enlargement and the subsequent determination and application of scale factor (Paper 2, Question 7(a)). Most candidates showed understanding of the procedure used for showing that a point is on a given circle (Paper 2, Question 3(c)). That is, most candidates wrote the equation of the circle, substituted in the co-ordinates of the point, calculated and brought the work to a conclusion. However, some candidates did not draw any conclusion from their work. In Paper 2, Question 5(a)(i) and (ii), procedural fluency in trigonometric procedures was in evidence among the more able candidates, who presented the relevant formula, substituted correctly, did the calculation, and gave the answer correct to two decimal places with the relevant unit in both cases, as required. However, less able candidates showed little procedural fluency in part (a)(i). Treating the triangle as right-angled or trying to apply the Cosine Rule were common approaches.

At **Foundation level**, the calculation of profit was well managed in Question 1(c), but the conversion of this to a percentage profit proved difficult for many candidates. Similarly, in Question 2(a), the application of the tax credit was not handled well. Frequently, it was subtracted from the total income at the start instead of from the gross tax due, (i.e., treated like the tax-free allowances of old). Also in this question calculations involving percentages caused difficulty.

3.2.3 Strategic competence

A number of questions in the examination papers assessed candidates' strategic competence; that is, their ability to formulate, represent, and solve mathematical problems in both familiar and unfamiliar contexts. While these questions are more common on the Higher level paper, they do appear on the examination papers at all levels.

At Higher level, examiners reported that candidates were generally willing to engage with non-routine questions with varying degrees of success. However, at Ordinary level, the majority of candidates seemed unable to deal with problems presented in an unfamiliar context, even where these questions were relatively easy to solve if candidates had attempted them.

While problem-solving in unfamiliar contexts is an important skill, it cannot be achieved unless students are competent in the basics of the syllabus and in particular routine skills and procedural operations. Some examiners observed that candidates demonstrated a greater capacity to apply their knowledge than in previous years. However, there are still many struggling with this learning objective.

At **Higher level**, candidates struggled with a problem-solving question that involved finding the time it took an oil slick to reach land (Paper 1, Question 8(d)). While the question was a challenging one, there were a number of different possible approaches to solving the problem. However, candidates showed little initiative in coming up with a solution.

In another question, when candidates were required to recognise the presence of a right-angled triangle and find the length of one side of it (Paper 2, Question 7(a)), they did, as envisaged, use

Pythagoras Theorem. However, many errors resulted from incorrect recognition of the right angle and an inability to successfully interpret the information on lengths provided in the text and diagram.

Further, in Paper 2, Question 9(c)(ii), where candidates were required to apply knowledge to an unfamiliar context, many did so incorrectly – they had clearly linked the concept of speed with calculus in their minds, and they therefore attempted to differentiate the function given in the question, which was not an appropriate approach in the context. Some examiners reported that this practice was more likely to occur in centre batches than individually; that is, candidates from some centres clearly had the competence to complete the problem correctly while, in other centres, all candidates immediately attempted to find a derivative. This suggests that candidates in these latter centres, rather than considering and seeking to understand the question presented, were following a direction such as "always differentiate when you see any question about speed". This raises questions about the approach to developing strategic competence in the schools or classes concerned.

At **Ordinary level**, it was evident that a majority of candidates were unable to formulate and represent information in mathematical form and hence solve problems. In Paper1, Question 8(e), only a minority of the candidates demonstrated an ability to successfully understand the information presented, create an equation and solve it. Testing problem solving requires that in some cases candidates are asked to solve problems with little or no scaffolding (preliminary parts of questions that direct candidates towards a correct approach or step them through one). The lack of scaffolding in Paper 1, Question 9(b) led to a lack of progress for the majority of candidates. Those who resorted to trial and improvement achieved some limited success. Candidates displayed more competence in topics that they were probably more familiar with. In Paper 2, Question 8(c)(ii), for example, more than half of the candidates were able to use their answers from the preceding parts (c)(i) and (a)(ii) to formulate the correct equation and hence solve the problem. A minority of candidates were not able to formulate the required equation correctly and just substituted into a formula from the *Formulae and Tables* booklet or put the volume equal to the answer to part (c)(ii).

At **Foundation level**, when required to calculate and compare the unit prices for a product sold in two different quantities (Question 2(b)), candidates struggled to complete the task successfully. However, when asked to complete, based on some given information, two tables of fees for a service offered by different companies (Question 9(a)) and make cost comparisons from the tables, the majority of candidates did so competently.

3.2.4 Adaptive reasoning

A particular feature of mathematics examination papers since the introduction of the new mathematics syllabus is the presence of more questions that assess candidates' adaptive reasoning; that is, their capacity for logical thought, reflection, explanation, justification, and communication. As mentioned above, there has been a general improvement in candidates' performance in this type of question since it was introduced, although weaker candidates often continue to struggle with this.

At **Higher level**, in a question where candidates needed to realise that the sum to infinity of a geometric series was required (Paper 1, Question 1(c)), it was clear from the work presented that candidates understood the fundamentals of the problem. However, in too many instances, subsequent mistakes – either through adding the "rises" only or omitting the original two-metre drop – resulted in marks being lost.

Question 9 on Paper 2 dealt with the use of a trigonometric function to model the hours of daylight throughout the year. When asked to find the length of the longest day, candidates did show a capacity for logical reasoning. Many saw the connection between the derivative and finding the length of the longest day. Interestingly, others used a totally different but very logical approach. They realised (based on their prior knowledge) that the longest day in Galway would occur on 21 June, counted the days to that date, and substituted this into the original function. In Paper 2, Question 3(c), many candidates introduced, unnecessarily, the formula for the angle between two lines in terms of their slopes. This may have resulted from them latching on to the words 'angles between lines' in the question, without giving due thought to the actual question at hand. While it is possible to resolve the problem by this method, it is by no means straightforward, and most candidates did not appear to consider other possible methods, especially that of perpendicular distances as used in the previous section. Better answers might well have been provided if a sketch had been produced first, but in almost all cases this did not happen.

In Paper 2, Question 8 (f)(ii), candidates were asked to explain why a certain sequence of events (free throws in a basketball game) could not be considered to be a sequence of Bernoulli trials. While they may have encountered similar contexts in the past, it is common in such contexts that such a sequence *is* taken to be a sequence of Bernoulli trials. Accordingly, this was probably, for most candidates, somewhat unfamiliar territory. The question therefore required understanding of the relevant concepts and adaptive reasoning to apply it and explain. Nonetheless, the question was reasonably well attempted, with many candidates using the phrase 'not independent'. A small number demonstrated their awareness of the influence of the player's previous shot on his success, being justified in doing so by the introductory section of the question. Also, in this question, many of the candidates displayed a poor understanding of the significance of subscripts in mathematical notation. The expression p_{n+1} was, in many instances, treated as $p_n + 1$, leading to incorrect work. A majority of candidates struggled to make progress in part Question 8 (e)(i). This part involved subscript notation as well as the manipulation of fractions. However, it may well be that many of the candidates also had difficulty with it not just because of the notation but also because it involved a learning outcome from a different strand of the syllabus integrated into a question on probability.

At **Ordinary level**, candidates performed well when asked to convert one currency to another (Paper 1. Question 1 (a) and (b)). However, when asked to convert using two exchange rates (Part(c) of the same question), the majority were unable to successfully complete this conversion. Logical thought was more successfully applied elsewhere on the same paper, in extending a graph and reading a value outside the initial range (Paper 1, Question 9 (a) (ii)).

When asked to explain why two triangles were congruent (Paper 2, Question 2(c)) the majority of candidates were able to use previous parts of the question to help explain their answer.

Similarly, when asked to comment on a statistical statement (Paper 2, Question 9(c)(iii)), excellent use was made of the histogram from the previous part. Even less able candidates did quite well, recognising that the height of one bar was double the other. Good logical explanations were given here. However when asked for a more difficult explanation (part (c)(iv) of the same question) the explanations were not as good, with many candidates focusing on the tallest bar rather than on tall students.

At **Foundation level**, candidates showed a very good understanding of the value of numbers presented in different forms (decimal, fraction, and percentage – Question 1(b)). Similarly, in Question 5(c), candidates generally knew what information was relevant in order to assess the truth of a given probabilistic statement. Likewise, in Question 8(a)(ii), candidates could clearly express the practical implication of the relevant geometrical work just done.

4. Conclusions

The change in the proportion of the cohort presenting at Higher level and Ordinary level in recent years was dealt with in Section 2.1. There is no doubt that the shift of many of the more able candidates from Ordinary level to Higher level has had consequences for both levels. At Higher level, as would be expected, the numbers achieving a grade C or below has shown a marked increase. When such candidates achieve a C or D grade, one might regard this choice of level as having been a good one, as these candidates have clearly benefitted from being exposed to a higher level of mathematical reasoning and problem solving, which will stand to them in the future. However, such benefit is much less clear in the case of candidates achieving an E grade or below, and an E/F/NG rate of about 5% does indicate that not all of those who opt for the Higher level are necessarily making the optimum choice. At Ordinary level, the proportion of candidates achieving a grade B or above has declined significantly. It is clear that many of the candidates who might previously have fallen into this category are now opting to study Mathematics at Higher level.

Candidates showed a great variety of achievement across all three levels of the examination. The highest-achieving candidates at Higher level showed a good depth of understanding of the whole syllabus, along with an ability to be both flexible and accurate in their work, and to bring knowledge and skills from a number of different strands to bear on a given question. These candidates were generally able to express themselves clearly and coherently, and to engage with questions with which they were not likely to have been familiar. They clearly invested considerable time and effort in engaging with the syllabus objectives and achieved high grades in the examination as a result.

The overall performance of some Higher level candidates with respect to their ability to apply basic skills appropriately and accurately is a cause for concern. It is clear that the proportion of the candidature for whom this is a significant difficulty has increased since 2011, and that a significant minority of candidates now struggle to complete multi-step procedures accurately. Candidates who in previous years might not have studied Mathematics at this level need to be better prepared in these areas.

At Ordinary level, many candidates displayed a lack of knowledge of standard procedures, a lack of basic competence in algebra (and in algebraic manipulation in particular) and a lack of perseverance. In light of the migration of candidates from Ordinary level to Higher level, these observations are not necessarily surprising, but they are a cause for concern. An appropriate balance needs to be struck between developing and consolidating candidates' basic skills on the one hand, and developing their capacity to apply, mathematize and reason in less familiar contexts on the other. Candidates who cannot complete basic arithmetical and algebraic procedures are unlikely to make much progress in questions where they first have to mathematize the problem. They are also unable to perform well in the section of the paper that directly tests these basic skills. Many candidates at this level also had difficulty with understanding and working with functions. This is one of the strands that the majority of the cohort would not have studied as part of the new syllabus at junior cycle, as referred to in section 1.1. The idea of a function cuts across all of the syllabus strands, and might be profitably approached in an integrated way rather than as a stand-alone strand in itself.

It could be pointed out that the majority of these skills and concepts should be acquired at junior cycle and that all that should be needed as one progresses through the Leaving Certificate course is that they be consolidated and improved. Nonetheless, it is clear that, for many candidates at Ordinary level and Foundation level, this has clearly not occurred. These skills need to be reintroduced and reinforced as candidates progress towards their examination.

Candidates at all levels were often successful at engaging in problems that were not of a routine kind, and many examiners commented positively that candidates performed well on many questions that involved reading the context for the problem involved.

While candidates were usually successful in moving from one area to another on the examination papers without knowing in advance the order of topics in the papers – something which would not have been required to the same extent in the past – they had more difficulty with questions which required them to draw on multiple strands of the syllabus at once, so there is clearly still a sense in which their knowledge and skills are compartmentalised.

In many instances, candidates showed an improvement over preceding years in their answers to questions that required an explanation or justification. It would appear that candidates are becoming more used to explaining and justifying their reasoning and understanding, which reflects good classroom practice and is a very positive development.

5. Recommendations to Teachers and Students

The following advice is offered to teachers and students preparing for Leaving Certificate Mathematics examinations.

5.1 In advance of the examination

Many of the points below are good habits that should be developed over the course of the students' studies in mathematics. It is unlikely that candidates will be successful at checking over work effectively, or at performing algebraic manipulations accurately, on the day of the examination if these skills and habits have not been developed over a period of time before the examination.

- Teachers and students should cover the full syllabus. This is of particular importance as there is no choice on any of the examination papers.
- Teachers should use the support material produced by the Project Maths Development Team and the National Council for Curriculum and Assessment. It has been developed specifically to support the kind of learning envisaged in the current mathematics syllabus.
- Close to the time of examination, questions from past and sample examination papers provided by the State Examinations Commission should be used for practice. However, examination papers should not be relied on excessively during any extended period of learning, as this might unnecessarily restrict the range of student learning and limit candidates' ability to deal with questions set in unfamiliar contexts.
- Students should get into the habit of showing supporting work at all times. This will help them tackle more difficult problems, and will allow them to check back for mistakes in their work.
- Students should develop strategies for checking their answers. In addition to techniques for identifying that an error has been made, techniques for finding those errors quickly and calmly, including getting to know one's own weaknesses, should also be developed.
- Teachers should provide frequent opportunities for students to gain comfort and accuracy in the basic skills of computation, algebraic manipulation, and calculus. Students should be particularly careful with signs, powers, and the order of operations. When the basic skills are being attended to properly it is much easier to engage also with context-based questions.

- Students should always round their answers to the required level of accuracy, and include the appropriate unit where relevant. These are skills that are not conceptually challenging, and they should be developed to a high standard through regular practice.
- Students should get used to describing, explaining, justifying, giving examples, etc. These are skills that are worth practising, as they will improve understanding, as well as being skills that may be directly assessed in the examination. Students do not need to be able to reproduce learned off statements of results or definitions, but they do need to be able to state or explain these reasonably clearly.
- Students should make sure that they have geometric instruments and should practise using them accurately. In particular, students should be familiar with the various 'centres' of a triangle and be able to construct them, as appropriate to the syllabus at the level concerned.
- Teachers should provide opportunities for students to apply the skills and knowledge from one strand to material from another strand. Mathematics is not a list of discrete rules and definitions to be learned but rather a series of interconnected principles that can be understood and then applied in a wide variety of contexts. While compartmentalising knowledge may help keep it organised, it will restrict the ability to cope with unfamiliar questions, particularly those requiring the synthesis of knowledge and skills from several strands.
- Students should practise different ways of solving problems; building up their arsenal of techniques on familiar problems will help them to tackle unfamiliar ones. Students at Higher level and Ordinary level should pay particular attention to algebraic methods of solving problems, as such methods are directly examinable at these levels.
- Teachers should provide students with opportunities to practise solving problems involving real-life applications of mathematics, and to get used to dealing with "messy data" in such problems. Students should also be encouraged to construct algebraic expressions or equations to model these situations, and / or to draw diagrams to represent them.
- Teachers should provide students with opportunities to solve unfamiliar problems and to develop strategies to deal with questions for which a productive approach is not immediately apparent. Students should be encouraged to persevere in these types of question – if the initial attempt does not work, they should be prepared to try the question a different way.

- Students should have a good knowledge of the information available in the *Formulae and Tables* booklet, and the location of relevant material within the booklet.

5.2 During the examination

- Candidates should attempt all of the questions the examination paper.
- Candidates should read each question carefully, paying particular attention to key words. They should also examine carefully any diagram that is provided.
- Candidates should make sure that they answer the exact question asked, and give the answer in the required form with the appropriate unit. Many candidates fail to do this.
- The question often contains clues regarding the nature of the answer. For instance, if it asks to *find the value of x*, there should only be one answer, while if it asks to *find the values of x*, one might expect more than one. Similarly, attention should be paid to whether the question indicates that the answer a natural number, integer, real number, etc.
- Candidates should concentrate when answering, as careless errors will result in marks being needlessly lost. Errors can also disadvantage candidates as they can make the work that follows more difficult. Further, in the event that an error in a particular section oversimplifies follow-on work in a subsequent section, then marks will be lost in that following section as well.
- Candidates should present their work as neatly and tidily as possible. This will help when checking back over it, and will help the examiner to find relevant work for which marks can be awarded.
- Candidates should show all of their work. In some questions, full marks will not be awarded unless candidates show supporting work. Furthermore, marks are generally not awarded for an incorrect answer without any supporting work, whereas if candidates show a procedure which would lead to a solution, then they may get credit for this.
- If possible, candidates should have an estimate of the answer in advance. They should check that the answer makes sense, including the appropriate unit – if it does not, they should review their work.
- In many topics, including coordinate geometry and trigonometry, drawing sketches or diagrams may aid candidate in understanding how to solve the problem.
- In instances where graphing or a co-ordinate geometry diagram is required, take care when drawing these. Use a ruler and compass and scale correctly (if the scale is not given),

- Plot points carefully, and understand when to join points in a straight line and when a smooth freehand curve is required.
- When candidates believe that they have made a mistake, it is often most beneficial to start again. They should draw a single line through the incorrect work. They should not use corrector fluid or otherwise make the work illegible – if there is valid work presented and still visible, it may be awarded some marks.
- Candidates should avoid trying to force the correct answer when this is given in the question. They should instead state their conclusions based on their own work.
- Candidates should be prepared for the unexpected. The syllabus states that they should be able to solve problems in familiar and unfamiliar contexts, so the examination paper would not be fit for purpose if the candidates recognised everything on it. Candidates should expect that there will be questions on the examination paper that, at first glance, they will not know how to complete. They should stay calm, and use the problem-solving skills that they have developed throughout their studies. The less familiar the territory, the more credit is likely to be awarded for attempts at applying appropriate strategies. It is always advisable to make an attempt at these questions.
- Candidates should use all of the time in the examination. If they finish the examination paper early, they should review their work, checking as many answers as possible.