Coimisiún na Scrúduithe Stáit
State Examinations Commission

JUNIOR CERTIFICATE EXAMINATION 2015

MATHEMATICS

CHIEF EXAMINER'S REPORT

Contents

1. Introduction

1.1 Context

In 2008, twenty four schools began a phased piloting of a new mathematics syllabus at Junior Certificate, under an initiative called *Project Maths*. Based on their feedback, the phased implementation of an updated syllabus began in all schools nationwide in September 2010. This syllabus has been under constant review as it has been implemented, with 2015 being the first year in which all Junior Certificate candidates have been assessed on the new Mathematics syllabus.[1]

While there is significant overlap between the old and new syllabuses, the new syllabus is different from the previous one both in terms of content and in terms of skills. In terms of content, the biggest change is an increase in the proportion of the syllabus dealing with statistics and probability. In terms of skills, the new syllabus has an increased emphasis on problem-solving, as well as on the skills of explanation, justification, and communication.

The syllabus changes were only one aspect of the systemic change to maths education that was intended by the Project Maths initiative. The initiative also sought to engender widespread changes to the way mathematics learning is experienced by students in schools. Accordingly, an extensive programme of in-career development for teachers was put in place to support such changes. A national scheme was also put in place to provide a suitable postgraduate qualification to out-of-field mathematics teachers (teachers of mathematics whose main qualification is in a different subject area) so as to allow them to become qualified teachers of the subject. Further information about the Project Maths initiative can be found on the website of the National Council for Curriculum and Assessment (NCCA) at:
http://www.ncca.ie/en/Curriculum_and_Assessment/Post-Primary_Education/Project_Maths/
Resources for teachers and students, along with information on other supports available to teachers, can be found at the website of the Project Maths Development Team at www.projectmaths.ie.

[1] In fact, the syllabus has been subject to some further minor changes following its implementation, so that the examinations in 2016 and subsequent years will be based on a slightly different syllabus to that examined in 2015. The following changes have been made to the syllabus for examination from 2016 onwards: the objectives (page 6) have been rephrased, and are now the same as those in the Leaving Certificate Mathematics syllabus (see Section 3.2 below); there is an expanded section on what teachers and students can expect in a problem-solving environment (pages 10 and 11); rotations have been added to transformation geometry (page 20); the order of the topics in Strand 2 has been changed (page 20); an extra learning outcome has been added to Section 3.1 ("use the number line to order numbers in $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$ (and $\mathbb{R}$ for HL)", page 22); and expectations for students taking the Foundation level examination have been further clarified (page 32).

This is the first report from the Chief Examiner for Junior Certificate Mathematics since Project Maths was initiated. In issuing this report at this time, the State Examinations Commission hopes that the information and analysis it contains will assist teachers in their ongoing efforts to continually improve the quality of the learning experiences of their students, and thereby assist the students in preparing for the examination. The SEC also hopes that the report helps to inform ongoing policy formation regarding mathematics education at this level, including the work that the NCCA currently has underway in developing a subject specification for mathematics in the reformed Junior Cycle.

Readers should remain fully cognisant of the fact that Project Maths laid out an ambitious programme of change for mathematics education in Ireland. Such change takes time to embed fully, and one cannot expect all of its objectives to be fully achieved in the first few years. To serve its purpose, this report must of necessity identify aspects of candidate achievement that currently fall short of the challenging expectations that have been set out. This should not be taken to imply any criticism of teachers or students, or to suggest that curriculum designers should shy away from setting such challenging goals. Delivering mathematics education of the highest quality is a national endeavour that all countries grapple with and none finds easy. Identifying as clearly as we can what is being done well and not so well is a crucial part of this endeavour, and the SEC hopes that this report can make a contribution in that respect.

1.2 Syllabus Structure

The Junior Certificate Mathematics syllabus comprises five strands: Statistics and Probability; Geometry and Trigonometry; Number; Algebra; and Functions. Topics and learning outcomes are specified for each strand, with the Ordinary level material a subset of the Higher level material. The syllabus emphasises that topics and strands should not be studied in isolation but that, where appropriate, connections should be made within and across the strands.

The syllabus is offered at two levels: Higher and Ordinary. There is no separate course for Foundation level. Differentiation between Higher level and Ordinary level is achieved through three main channels: the content and learning outcomes of the syllabus; the processes of teaching and learning; and the assessment arrangements associated with the examinations. Differentiation between Foundation and Ordinary level is achieved through the second and third of these.

This report should be read in conjunction with the examination papers, the published marking schemes, and the syllabus. The examination papers and marking schemes are available on the State

Examination Commission's website (www.examinations.ie) and the syllabus is available at www.curriculumonline.ie.

1.3 Assessment Specification

The syllabus is assessed at three levels: Higher, Ordinary, and Foundation. Assessment at each level is by means of a terminal written examination. All examination papers are presented as combined question-and-answer booklets, and candidates must answer all questions. The total number of questions on any examination paper may vary from year to year. A suggested maximum time which candidates should devote to each question is shown on the paper.

The examination at Higher level consists of two papers. Each paper is marked out of 300 marks and is of two and a half hours' duration. In 2015, there were 14 questions on each examination paper. Topics from Strand 3 (Number), Strand 4 (Algebra), and Strand 5 (Functions) are generally assessed in Paper 1, while topics from Strand 1 (Statistics and Probability), Strand 2 (Geometry and Trigonometry), and Topic 3.4 (Applied measure) are generally assessed on Paper 2. That said, this division of topics is not absolute. In order to adequately assess candidates' proficiency in the syllabus, it is necessary to have questions which require candidates to use skills from a range of different topics and strands.

The examination at Ordinary level consists of two papers. Each paper is marked out of 300 marks and is of two hours' duration. In 2015, there were 12 questions on each examination paper. As at Higher level, topics from Strand 3 (Number), Strand 4 (Algebra), and Strand 5 (Functions) are generally assessed in Paper 1, while topics from Strand 1 (Statistics and Probability), Strand 2 (Geometry and Trigonometry), and Topic 3.4 (Applied measure) are generally assessed on Paper 2. Again, this division of topics is not absolute.

The examination at Foundation level consists of one paper, which is marked out of 300 marks and is of two hours' duration. In 2015, there were 14 questions on the examination paper.
In the Foundation level examination, all strands of the syllabus are assessed in the single paper.

1.4 Participation Trends

The breakdown of the Junior Certificate Mathematics cohort in terms of participation at Higher, Ordinary, and Foundation levels over the last five years is given in **Table 1** below. In order to put these figures in context, **Figure 1** below shows the breakdown of the cohort by level from 1996 to 2015.

Year	Total Mathematics Candidature	Number at Higher level	Number at Ordinary level	Number at Foundation level	% Higher	% Ord.	% Found.
2011	56 025	25 554	26 064	4407	45·6	46·5	7·9
2012	58 069	27 913	25 945	4211	48·1	44·7	7·3
2013	59 088	30 500	24 687	3901	51·6	41·8	6·6
2014	59 620	32 041	24 047	3532	53·7	40·3	5·9
2015	58 874	32 534	22 856	3484	55·3	38·8	5·9

Table 1: Number and percentage of candidates at each level, 2011 to 2015.
Figures for 2011, 2012, 2013, and 2014 include candidates in the initial *Project Maths* schools.

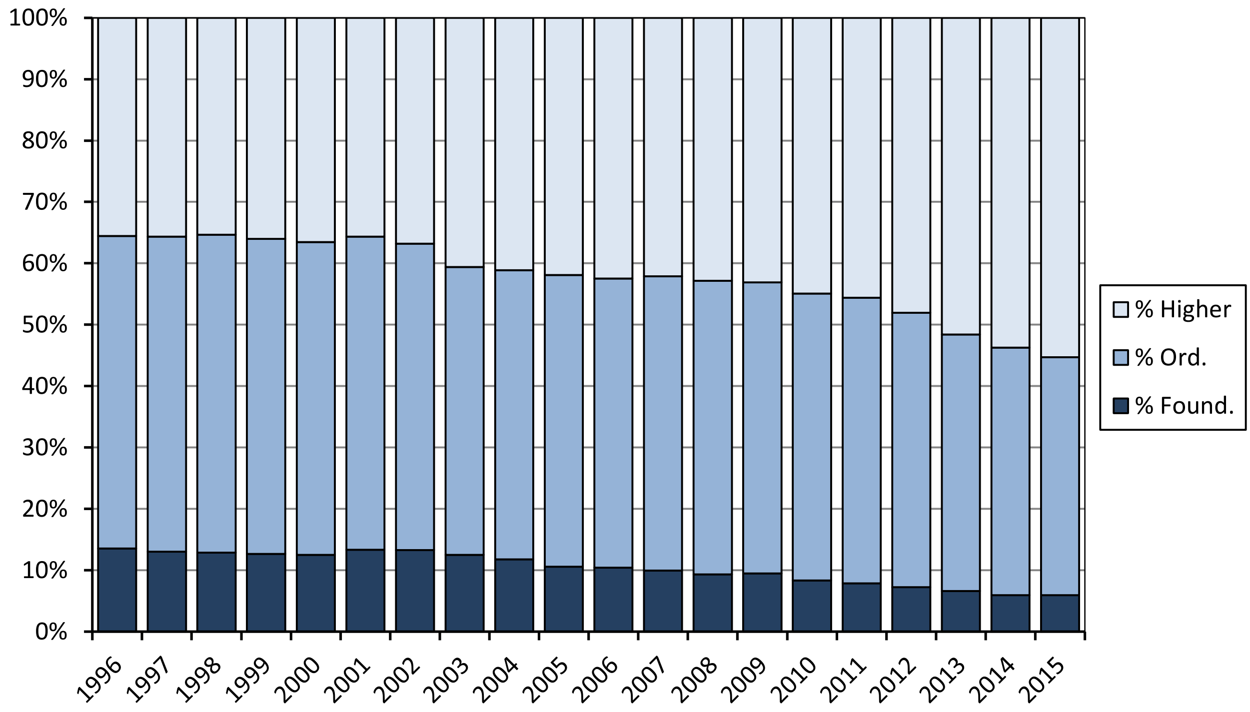

Figure 1: Percentage of candidates at each level, 1996 to 2015.
Figures for 2011, 2012, 2013, and 2014 include candidates in the initial *Project Maths* schools.

The graph shows that the percentage of candidates at each level remained fairly constant for the period from 1996 to 2002. There was a visible increase in the percentage of candidates taking Higher level from 2002 to 2003, following the introduction of a new Junior Certificate Mathematics syllabus which was first examined in 2003. This was followed by a period of relative stability in the Higher level rate until 2009. There was a more gradual decrease in the percentage taking Foundation level over the period from 2002 to 2009.

However, more recently there has been a significant change in the proportions of the cohort taking the different levels . From 2011 to 2015, the Higher level cohort increased from 45·6% to 55·3%, an increase of over 20%. Over the same period of time, the Ordinary level cohort fell from 46·5% to 38·8%, a decrease of over 15%, while the Foundation level cohort fell from 7·9% to 5·9%, a decrease of 25%.

The breakdown of the female and male Mathematics cohorts by level (Higher, Ordinary, and Foundation) is given in **Table 2** below.

Year	Female			Male		
	% Higher	% Ordinary	% Foundation	% Higher	% Ordinary	% Foundation
2011	47·2	46·0	6·9	44·2	47·1	8·8
2012	49·7	43·9	6·4	46·5	45·4	8·1
2013	53·1	41·2	5·7	50·2	42·4	7·5
2014	55·1	39·6	5·4	52·5	41·0	6·5
2015	56·9	37·9	5·2	53·7	39·7	6·6

Table 2: Composition of female and male Mathematics cohorts by level, 2011 to 2015.
Figures are for school-based candidates only.

The figures for both female and male candidates show the same trends as the overall data, i.e. an increase in the percentage of candidates taking Higher level, and a concomitant decrease in the percentages taking Ordinary and Foundation levels.

The gender breakdown has also been relatively stable over the last five years, with a higher percentage of female than male candidates taking Higher level, and a slightly lower percentage of female than male candidates taking Ordinary and Foundation levels. This difference in uptake complicates any analysis of differences in grade achievement at the different levels, as when comparing males and females one is not comparing equivalent proportions of the gender cohort.

2. Performance of Candidates

Care should be exercised when interpreting the grade distributions in this chapter. The syllabus and related changes that have occurred are such that neither the subject content nor the nature of the cognitive skills being tested is constant over the five years for which the statistics are presented. Furthermore, the percentage of the candidature opting for each level has changed very significantly, as seen in Section 1.4 above. The effect that these factors might be expected to have on the grade distribution at each level is dealt with in Section 2.1 below.

2.1 Higher Level

The percentage of grades awarded at Junior Certificate Higher level Mathematics in each of the last five years is given in **Table 3** below. The distribution of grades by gender over the last five years are given in **Table 4** (female candidates) and **Table 5** (male candidates) below.

Year	A	B	C	A/B/C	D	E	F	NG	E/F/NG
2011	17·5	31·8	30·4	79·7	16·8	3·0	0·5	0·1	3·6
2012	15·1	31·4	32·7	79·2	18·0	2·5	0·3	0·0	2·8
2013	12·0	32·3	34·1	78·4	18·4	2·8	0·5	0·0	3·3
2014	10·7	28·4	33·9	73·0	22·5	3·9	0·5	0·0	4·4
2015	11·3	31·2	32·2	74·7	21·1	3·7	0·5	0·0	4·2

Table 3: Percentage of candidates awarded each lettered grade at Higher level Mathematics, 2011 to 2015.

Year	A	B	C	A/B/C	D	E	F	NG	E/F/NG
2011	17·1	32·8	30·9	80·8	16·4	2·5	0·3	0·0	2·8
2012	14·4	33·8	33·1	81·2	16·5	2·1	0·2	0·0	2·3
2013	11·1	32·8	35·3	79·2	18·1	2·3	0·3	0·0	2·7
2014	10·6	29·9	34·4	74·9	21·5	3·2	0·4	0·0	3·6
2015	10·6	31·8	32·6	75·0	21·4	3·3	0·4	0·0	3·7

Table 4: Percentage of female candidates awarded each lettered grade at Higher level Mathematics, 2011 to 2015.

Year	A	B	C	A/B/C	D	E	F	NG	E/F/NG
2011	17·8	30·8	29·9	78·5	17·2	3·6	0·6	0·1	4·3
2012	15·9	29·0	32·3	77·2	19·5	2·9	0·4	0·0	3·3
2013	12·8	31·8	32·9	77·5	18·7	3·3	0·6	0·0	3·9
2014	10·9	27·0	33·4	71·2	23·4	4·7	0·7	0·0	5·4
2015	12·0	30·7	31·7	74·4	20·8	4·2	0·6	0·1	4·8

Table 5: Percentage of male candidates awarded each lettered grade at Higher level Mathematics, 2011 to 2015.

The overall Higher level results in **Table 3** show a decrease in the A-rate and A/B/C-rate from 2011 to 2015, as well as a slight rise in the E/F/NG-rate in the same period. This is not surprising given the changes in the composition of the cohort over the course of this period. In particular, the cohort now taking Higher level includes a large number of candidates that formerly would have taken Ordinary level. The increase in the number of candidates taking Higher level from 2010 to 2015 has been substantial, and mirrors the increase that has been observed at Leaving Certificate. This increase at both Junior and Leaving Certificate level, which is often attributed to the roll-out of the *Project Maths* initiative and to the "bonus points" awarded for Higher level Mathematics in the CAO system, is a stated policy aim – *Literacy and Numeracy for Learning and Life* specifies a goal of 60% of Junior Certificate candidates taking Higher level Mathematics by 2020.

However, such a steep change over such a short timeframe necessarily has a very significant impact on the grade distributions that might be expected at the various levels. The candidates whose choice of level is least certain are those near the overlap of standards between the levels – they are among the lower achieving candidates at Higher level and the higher achieving candidates at Ordinary level. When the proportion of such candidates opting for Higher level increases, an increase can be expected in the percentage of low grades awarded at Higher level, along with a decrease in the percentage of high grades awarded at Ordinary level.

Furthermore, in the case of the current syllabus change in Mathematics, there has been a deliberate attempt to increase the emphasis on higher-order thinking skills. These are skills that students find difficult to master and teachers may find difficult to instil. The syllabus expectations are ambitious at all levels. Accordingly one might expect candidates to fall short of syllabus expectations to a greater degree than before, and certainly more than if the changes were solely or primarily related to content.

Grade distribution data at all three levels need to be considered in the context of the above issues.

While these issues clearly had an impact over the period from 2011 to 2014, there appears to have been some stabilisation of these rates in 2015. This may, at least in part, be attributable to the 'bedding-in' of the new syllabus. Teachers and students are becoming more familiar with the revised syllabus content and the expectations of the assessment process. It is also reasonable to consider that the extensive programme of continuing professional development and the retraining of large numbers of out-of-field teachers through the postgraduate programme are bearing fruit in relation to candidate achievement.

The results of both female and male candidates have followed the overall Higher level trends from 2011 to 2015. Notwithstanding the caveat in Section 1.4 above regarding comparisons of grade achievement for male and female candidates, at Higher level, a higher percentage of male candidates than female candidates were awarded A grades, while female candidates outperformed their male counterparts at all other points of the achievement spectrum over the last five years.

2.2 Ordinary Level

The percentage of grades awarded at Ordinary level Mathematics in each of the last five years is given in **Table 6** below. The distribution of grades by gender over the last five years are given in **Table 7** (female candidates) and **Table 8** (male candidates) below.

Year	A	B	C	A/B/C	D	E	F	NG	E/F/NG
2011	12·4	33·4	29·2	75·0	17·9	5·2	1·6	0·2	7·0
2012	14·3	33·9	28·0	76·2	17·0	5·0	1·5	0·2	6·7
2013	9·9	35·5	31·8	77·2	17·7	4·1	1·0	0·1	5·2
2014	6·3	33·2	35·3	74·8	20·6	3·6	0·9	0·1	4·6
2015	7·4	28·3	34·5	70·2	23·8	4·9	0·9	0·1	5·9

Table 6: Percentage of candidates awarded each lettered grade at Ordinary level Mathematics, 2011 to 2015.

Year	A	B	C	A/B/C	D	E	F	NG	E/F/NG
2011	14·1	35·7	28·0	77·7	16·7	4·4	1·2	0·1	5·7
2012	16·5	35·5	27·3	79·4	15·4	4·1	1·0	0·1	5·2
2013	11·0	37·6	31·1	79·7	16·1	3·5	0·7	0·1	4·3
2014	7·4	35·8	34·2	77·4	18·5	3·2	0·7	0·1	4·1
2015	8·9	29·9	33·5	72·3	22·3	4·5	0·8	0·1	5·4

Table 7: Percentage of female candidates awarded each lettered grade at Ordinary level Mathematics, 2011 to 2015.

Year	A	B	C	A/B/C	D	E	F	NG	E/F/NG
2011	10·9	31·3	30·4	72·7	19·1	6·0	2·0	0·2	8·2
2012	12·3	32·4	28·5	73·2	18·6	5·9	2·0	0·3	8·2
2013	8·9	33·5	32·5	74·8	19·2	4·7	1·2	0·2	6·0
2014	5·4	30·7	36·2	72·3	22·5	4·0	1·0	0·1	5·1
2015	6·0	26·9	35·4	68·2	25·2	5·3	1·1	0·1	6·6

Table 8: Percentage of male candidates awarded each lettered grade at Ordinary level Mathematics, 2011 to 2015.

The overall Ordinary level results in **Table 6** show a substantial decrease in the A-rate from 2011 to 2015. There has also been a decrease in both the A/B/C-rate and the E/F/NG rate over this period. As at Higher level, there has been a stabilisation of the A- and E/F/NG-rates (although not of the A/B/C-rate) in 2015. Potential reasons for this are as already outlined in the commentary on the Higher level outcomes in Section 2.1 above.

The results of both female and male candidates have followed the overall Ordinary level trends from 2011 to 2015. Notwithstanding the caveat in Section 1.4 above regarding comparisons of grade achievement for male and female candidates, female candidates have consistently outperformed their male counterparts at all points of the achievement spectrum at Ordinary level over the last five years.

2.3 Foundation Level

The percentage of grades awarded at Foundation level Mathematics in each of the last five years is given in **Table 9** below. The distribution of grades by gender over the last five years is given in **Table 10** (female candidates) and **Table 11** (male candidates) below.

Year	A	B	C	A/B/C	D	E	F	NG	E/F/NG
2011	19·6	37·8	26·9	84·3	12·9	2·3	0·5	0·0	2·8
2012	17·1	34·2	30·1	81·4	15·6	2·3	0·7	0·0	3·0
2013	12·5	35·9	35·8	84·2	13·0	2·2	0·3	0·2	2·7
2014	10·9	34·5	35·0	80·4	16·5	2·4	0·5	0·1	3·0
2015	14·6	35·9	30·2	80·7	16·2	2·7	0·4	0·1	3·2

Table 9: Percentage of candidates awarded each lettered grade at Foundation level Mathematics, 2011 to 2015.

Year	A	B	C	A/B/C	D	E	F	NG	E/F/NG
2011	18·2	39·6	26·9	84·8	12·8	1·9	0·5	0·1	2·4
2012	14·7	35·0	31·4	81·0	16·7	1·6	0·7	0·0	2·3
2013	11·9	38·7	34·9	85·4	12·1	2·1	0·3	0·1	2·5
2014	10·3	36·7	35·1	82·1	15·1	2·4	0·4	0·1	2·8
2015	13·2	35·1	31·3	79·6	17·4	2·7	0·3	0·0	3·0

Table 10: Percentage of female candidates awarded each lettered grade at Foundation level Mathematics, 2011 to 2015.

Year	A	B	C	A/B/C	D	E	F	NG	E/F/NG
2011	20·7	36·4	26·9	83·9	13·0	2·6	0·4	0·0	3·1
2012	18·9	33·7	29·1	81·7	14·8	2·8	0·7	0·0	3·5
2013	13·0	33·8	36·5	83·4	13·7	2·3	0·4	0·2	2·9
2014	11·4	32·9	34·9	79·2	17·7	2·5	0·6	0·2	3·2
2015	15·6	36·5	29·3	81·3	15·3	2·7	0·6	0·1	3·4

Table 11: Percentage of male candidates awarded each lettered grade at Foundation level Mathematics, 2011 to 2015.

The overall Foundation level results in **Table 9** show a decrease in the A-rate from 2011 to 2015, with a stabilisation in 2015, as at both Ordinary level and Higher level. Neither the E/F/NG-rate nor the A/B/C-rate shows any particular pattern of movement over this period.

The results of both female and male candidates have followed the overall Foundation level trends from 2011 to 2015. Notwithstanding the caveat in Section 1.4 above regarding comparisons of grade achievement for males and females, high-performing male candidates at Foundation level have outperformed their female counterparts, while low-performing female candidates have outperformed their male counterparts, over the last five years. There has been no consistent difference between male and female candidates with respect to those in the middle of the achievement spectrum at Foundation level over the last five years.

3. Analysis of Candidate Performance

The data and analysis in this section are based on the marks for each candidate in a random sample of scripts (presented in Section 3.1), and on the observations of examiners with respect to candidate performance, which were collected near the end of the marking process (presented in Section 3.2).

3.1 General Commentary on Performance

3.1.1 Higher Level

Table 12 below is a summary based on an analysis of a random sample of 1440 scripts (≈ 4·4% of all scripts) from Higher level Mathematics candidates in 2015.

On Paper 1, the overall mean mark per question was 67·3%, with a standard deviation of 13·0%.[2] Candidates performed particularly well in questions involving sets (Questions 1 and 10), as well as questions involving fairly straightforward arithmetic and algebra (Question 5) and a slightly more unusual inequalities question (Question 8). Candidates had more difficulty with graphing functions (Question 13) and income tax, particularly when asked to "work backwards" (Question 3).

On Paper 2, the overall mean mark per question was 65·2%, with a standard deviation of 12·2%.[3] Candidates performed very well in questions involving Strand 1 (Statistics and Probability), but struggled with topics from Strand 2 (Geometry and Trigonometry) and Topic 3.4 (Applied measure), with trigonometry causing particular difficulty (Questions 8 and 13).

[2] These figures are calculated by weighting the mean mark for each question (as a percentage) by the total number of marks available for that question. The unweighted figures for Higher level Paper 1, just using the mean mark for each question (as a percentage), are a mean of 68·2% and a standard deviation of 13·8%.

[3] As above, these are the 'weighted' figures. The unweighted figures for Higher level Paper 2 are a mean of 66·0% and a standard deviation of 13·4%.

Paper	Q	Mean Mark / Total Mark	Mean Mark (%)	Mark Ranking (Examination)	Main Topic[4]
1	1	12·0 / 15	80	5	3.5 Number
1	2	10·9 / 15	73	11	3.3 Number
1	3	11·8 / 25	47	27	3.3 Number
1	4	6·0 / 10	60	20	5.1 Functions
1	5	12·0 / 15	80	4	4.6, 4.7 Algebra
1	6	22·9 / 30	76	9	5.2, 5.3 Functions
1	7	15·3 / 20	77	8	4.6 Algebra
1	8	12·3 / 15	82	3	4.7, 4.8 Algebra
1	9	14·4 / 20	72	12	4.6, 4.7 Algebra
1	10	19·9 / 25	80	6	3.5 Number
1	11	25·3 / 40	63	17	2.1 Geom. & Trig. 4.2, 4.7 Algebra 5.2 Functions
1	12	13·7 / 20	69	14	4.6, 4.7 Algebra
1	13	6·9 / 20	35	28	5.1, 5.2, 5.3 Functions
1	14	18·3 / 30	61	19	3.1, 3.6 Number
2	1	14·1 / 15	94	1	1.6, 1.8 Stats & Prob.
2	2	17·6 / 20	88	2	1.1, 1.6, 1.8 Stats & Prob.
2	3	12·7 / 20	64	16	1.4, 1.5 Stats & Prob.
2	4	18·5 / 30	62	18	3.4 Applied measure
2	5	18·7 / 25	75	10	2.2, 2.3 Geom. & Trig.
2	6	11·9 / 20	60	21	2.3 Geom. & Trig.
2	7	7·8 / 15	52	25	2.1 Geom. & Trig.
2	8	7·7 / 15	51	26	2.1, 2.4 Geom. & Trig.
2	9	19·6 / 30	65	15	1.3, 1.6, 1.8 Stats & Prob.
2	10	10·6 / 15	71	13	1.6, 1.8 Stats & Prob.
2	11	11·4 / 20	57	22	2.1 Geom. & Trig.
2	12	15·4 / 20	77	7	2.1 Geom. & Trig.
2	13	18·3 / 35	52	24	2.4 Geom. & Trig. 3.4 Applied measure
2	14	11·1 / 20	56	23	3.2 Number 3.4 Applied measure

Table 12: Mean mark for each question, Higher level Mathematics 2015.

[4] The numbers of the topics in Tables 12, 13, and 14 refer to the syllabus for examination in 2015 only.

3.1.2 Ordinary Level

Table 13 below is a summary based on an analysis of a random sample of 1120 scripts (≈ 4·9% of all scripts) from Ordinary level Mathematics candidates in 2015.

On Paper 1, the overall mean mark per question was 64·7%, with a standard deviation of 20·3%.[5] There was very large variation in the mean marks per question on this paper, and while candidates performed well in a broad number of areas, they struggled noticeably with questions that involved substantial amounts of algebra (Questions 11, 7, and 9).

On Paper 2, the overall mean mark per question was 65·2%, with a standard deviation of 10·4%.[6] As at Higher level, candidates performed relatively well on questions involving Strand 1 (Statistics and Probability), in particular those involving statistics (Questions 4, 5, and 8), but struggled with questions from other strands, in particular co-ordinate geometry (Questions 6 and 12) and the construction of a triangle (Question 7).

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[5] As above, these are the 'weighted' figures. The unweighted figures for Ordinary level Paper 1 are a mean of 69·3% and a standard deviation of 20·5%.

[6] As above, these are the 'weighted' figures. The unweighted figures for Ordinary level Paper 2 are a mean of 65·9% and a standard deviation of 11·3%.

Paper	Q	Mean Mark / Total Mark	Mean Mark (%)	Mark Ranking (Examination)	Main Topic
1	1	20·3 / 25	81	6	3.1, 3.2 Number
1	2	13·7 / 20	69	14	3.5 Number
1	3	21·0 / 25	84	3	3·3, 3.6 Number
1	4	16·4 / 25	66	15	3.3, 3.4 Number
1	5	9·4 / 10	94	1	3·5 Number
1	6	18·7 / 20	94	2	3.3, 3.6 Number
1	7	8·3 / 20	42	23	5.2 Functions
1	8	14·3 / 20	72	11	4.5 Algebra 3.4 Number
1	9	12·7 / 40	32	24	4.6 Algebra
1	10	24·8 / 35	71	12	5.2, 5.3 Function 4.2 Algebra
1	11	17·0 / 40	43	22	4.7 Algebra
1	12	16·6 / 20	83	4	3.1, 3.6 Number 4.6 Algebra
2	1	12·3 / 20	62	16	1.1, 1.3 Stats & Prob.
2	2	16·0 / 20	80	7	3.4 Number
2	3	18·4 / 30	61	17	2.1, 2.2, 2.4 Geom. & Trig.
2	4	20·4 / 25	82	5	1.6 Stats & Prob.
2	5	11·6 / 15	77	8	1.6, 1.8 Stats & Prob.
2	6	23·8 / 40	60	18	2.3 Geom. & Trig.
2	7	12·0 / 25	48	21	2.1 Geom. & Trig.
2	8	21·5 / 30	72	9	1.6 Stats & Prob.
2	9	17·9 / 25	72	10	3.4 Number
2	10	17·3 / 30	58	19	3.4 Number
2	11	14·0 / 20	70	13	2.4 Geom. & Trig.
2	12	9·8 / 20	49	20	2.3 Geom. & Trig.

Table 13: Mean mark for each question, Ordinary level Mathematics 2015.

3.1.3 Foundation Level

Table 14 below is a summary based on an analysis of a random sample of 180 scripts (≈ 5·2% of all scripts) from Foundation level Mathematics candidates in 2015.

The overall mean mark per question was 69·3%, with a standard deviation of 15·8%.[7] Foundation level candidates' performance showed many similarities to those at the other levels, with strengths in Strand 1 (Statistics and Probability), especially in relation to graphical displays of data (Questions 4 and 5), and high mean marks in questions on arithmetic involving money (Questions 2 and 7). They struggled with algebraic manipulation (Question 10) and co-ordinate geometry (Question 11), and had much more difficulty reading a distance-time graph (Question 9) than their Ordinary level peers.

Q	Mean Mark / Total Mark	Mean Mark (%)	Mark Ranking	Main Topic
1	26·3 / 30	88	4	3.1, 3.2, 3.4 Number
2	9·0 / 10	90	2	3.3 Number
3	8·5 / 15	57	10	3.5 Number 1.6 Stats & Prob.
4	9·2 / 10	92	1	1.6 Stats & Prob.
5	18·0 / 20	90	3	1.6 Stats & Prob.
6	22·4 / 40	56	11	2.1, 2.2 Geom. & Trig. 3.4 Number
7	17·3 / 20	87	5	3.3 Number
8	15·2 / 25	61	9	3.4 Number
9	7·9 / 15	53	13	4.5 Algebra
10	18·5 / 35	53	12	4.6, 4.7 Algebra
11	15·3 / 30	51	14	2.3, 2.5 Geom. & Trig.
12	12·8 / 15	85	6	1.3 Stats & Prob.
13	11·4 / 15	76	8	4.6 Algebra
14	15·5 / 20	78	7	4.2, 4.8 Algebra

Table 14: Mean mark for each question, Foundation level Mathematics 2015.

[7] As above, these are the 'weighted' figures. The unweighted figures for Foundation level are a mean of 72·6% and a standard deviation of 16·4%.

3.2 Meeting of Specific Syllabus Objectives

The objectives listed in the syllabus for examination in 2015 are the same as those in the previous Junior Certificate mathematics syllabus, *viz*. to develop: the ability to recall relevant mathematical facts; instrumental understanding ("knowing how"); relational understanding ("knowing why"); the ability to apply mathematical knowledge and skills to solve problems; analytical and creative powers in mathematics; and an appreciation of and positive disposition towards mathematics.

As mentioned in Section 1.1 above, these objectives have been rephrased in the syllabus for examination from 2016 onwards. This rephrasing essentially consists of a regrouping of the objectives listed above – they do not contain any genuinely new objectives, nor do they omit any of the above objectives, but rather constitute a reorganisation of those same objectives into slightly different categories. They are also the same objectives as those in the Leaving Certificate syllabus for examination from 2015 onwards. Given all of this, it would seem appropriate – both for the future usefulness of the following analysis and for consistency with the Leaving Certificate – to refer in the following discussion to the objectives as categorised in the Junior Certificate syllabus for examination from 2016 onwards in the analysis of candidate performance.

These objectives are that learners develop mathematical proficiency, characterised in the syllabus as:

1. **conceptual understanding**: comprehension of mathematical concepts, operations, and relations
2. **procedural fluency**: skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
3. **strategic competence**: ability to formulate, represent, and solve mathematical problems in both familiar and unfamiliar contexts
4. **adaptive reasoning**: capacity for logical thought, reflection, explanation, justification and communication
5. **productive disposition**: habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence, perseverance and one's own efficacy.

The success of candidates in meeting each of the first four of these syllabus objectives is examined by an analysis of the standard of candidate responses to specific parts of questions on the examination papers. As stated above, such analysis is made possible by information captured by examiners during their work.

There are no examination questions that specifically assess candidates' productive disposition. Examiners have commented that candidates attempted more question parts than in previous years, which could be taken as an indication of an increase in candidates' productive disposition. However, it would not be appropriate to attempt to conduct an analysis of the level of candidates' productive disposition on the basis of candidate responses captured in the examination.

3.2.1 Conceptual Understanding

Most candidates demonstrated good levels of knowledge and comprehension of basic mathematical concepts and relations, which is fundamental to the successful development of mathematical proficiency. Candidates struggled at times when more involved understanding was required, or when the concepts were slightly less standard. It was observed that there has been an improvement in the standard of explanations and descriptions given by candidates in examinations over the last number of years, although weaker candidates often continue to struggle with this, particularly at Higher level.

At **Higher level**, candidates were well able to correctly identify the shape of a water tank (Paper 2, Question 13(c)) and one possible length of the third side of an isosceles triangle, given the lengths of two of the sides (Paper 2, Question 12(a)). However, they struggled when asked to complete an identity relating to the distribution of set union over intersection (Paper 1, Question 1(c)) and had great difficulty explaining how someone might take a simple random sample (Paper 2, Question 3(a)). Furthermore, while most candidates were able to correctly distinguish between numerical and categorical data, they had more difficulty distinguishing between nominal and ordinal data (Paper 2, Question 3(a)).

At **Ordinary level**, candidates were very successful at identifying $A \setminus B$ from its definition (Paper 1, Question 2(d)); reading values from a bar chart (Paper 2, Question 4(a) – (d)); co-ordinating the plane (Paper 2, Question 6(a) & (c)); and identifying the number of faces in a cube (Paper 2, Question 9(b)). Candidates had some difficulties identifying which side in a right-angled triangle was opposite a particular angle, often confusing it with the adjacent side (Paper 2, Question 3(c)(i)); and identifying whether a given triangle was isosceles, scalene, or equilateral (Paper 2, Question 7(a)). Most candidates struggled with a question assessing their understanding of the graphs of inequalities (Paper 1, Question 7).

At **Foundation level**, candidates generally did well on questions that assessed their knowledge and basic understanding. As at Ordinary level, candidates at Foundation level were very successful at reading values from a bar chart (Question 5(b), Question 4(a) & (b)). Almost all candidates were able to correctly list the elements of A from a given Venn diagram, with most able to also identify the element in $A \cap B$ (Question 3(a)). Candidates at this level struggled more than their Ordinary level counterparts at co-ordinating the plane (Question 11(a)). In algebra, candidates were reasonably well able to match algebraic terms with their description (Question 13), although $2x$ was often confused with x^2. (This difficulty with squaring was also seen among Ordinary level candidates, as discussed in Section 3.2.2 below).

3.2.2 Procedural Fluency

Many of the questions on the examination papers relate to procedural fluency, i.e. the ability of candidates to carry out mathematical procedures accurately and appropriately.

At **Higher level**, candidates performed well at a number of these questions, particularly when they did not involve too many steps. In Paper 1, these included questions involving filling in Venn diagrams and performing set operations on two sets (Questions 1 and 10); basic percentages, including calculating net income given a gross income above the standard-rate cut-off point (Questions 2(a) and 3(a)); substituting values into a function (Question 7(a) and (b)); and continuing a quadratic pattern (Question 11(c)). Many candidates showed proficiency in performing basic algebraic manipulations when specifically instructed to do so, for instance: multiplying out brackets (Question 7(a)); factorising four terms by grouping (Question 7(b)); and factorising a quadratic expression (Question 9(a)(i))). Of particular note was the fact that examiners reported seeing more use of different methods, for instance the array method to multiply out brackets or perform algebraic long division, which often led to correct answers.

In Paper 2, questions on which candidates performed well included using the fundamental principle of counting (Question 2(a)); finding the volume of a rectangular swimming pool (Question 4(c)); finding probabilities from a grouped frequency distribution (Question 9 (a) and (b)); and finding the volume of a sphere, given its radius (Question 14(a)). Once again, examiners saw evidence of these questions being approached in different ways – for example, many candidates used a tree diagram in Question 2(a) instead of explicitly using the fundamental principle of counting.

Candidates struggled when the procedures involved became more complex. Instances of this in Paper 1 were when candidates were asked to list the elements of $A \cup (B \cap C)$ (Question 1(b)) or to compound two percentage increases (Question 2(a)). Candidates also struggled in some reasonably standard algebra questions on this paper, for example when asked to add two algebraic fractions (Question 7(c)) or to find the irrational roots of a quadratic equation (Question 9(b)). In both of these questions, examiners reported that many candidates were unable to carry out the procedures accurately, with most having errors (sign, distribution, transposition, rounding, etc.) of some kind. Many candidates also appeared to conflate methods related to simplifying expressions with those related to solving equations, as evidenced by the fact that they multiplied each of the algebraic fractions in Question 7(c) by 12 (the common denominator) rather than expressing each one as a fraction with 12 as the denominator. This conflation may also have been the reason why, when asked to factorise $x^2 + 7x - 30$ in Question 9(a)(i), candidates often continued on past "$(x + 10)(x - 3)$", the correct answer, to write down "$x = -10$ or $x = 3$", i.e. the solutions to $x^2 + 7x - 30 = 0$. It is recommended that teachers and candidates give due attention to distinguishing between equations and expressions, and understanding why some procedures may validly be applied to one and not the other.

In Paper 2, candidates continue to find geometry and trigonometry challenging. Candidates struggled with "traditional" co-ordinate geometry questions (in particular, Question 6(a) and 6(c)), continuing a trend observed in 2015, and only performed slightly better when asked to match four graphs of lines to their equations, given in the form $y = mx + c$ (Question 5(a)). In trigonometry, while candidates were reasonably successful at finding the height of a right-angled triangle, given the length of its base and the angle of elevation (Question 13(a)), many candidates had difficulty correctly identifying which side was the hypotenuse in Question 8(b) – this may have been partly due to the fact that the triangle was oriented with its hypotenuse facing downwards, an arrangement candidates may not have been familiar with.

While candidates generally performed well on questions involving statistics and probability, candidates continued to have difficulty when asked to find the mean of a grouped frequency distribution (Question 9(c)). Many candidates failed to multiply each mid-interval value by the corresponding frequency, and many others divided by the number of groups instead of the sum of the frequencies. Interestingly, candidates performed just as well in the subsequent part of the question (Question 9(d)), which asked them to use the grouped frequency distribution to find the smallest possible value that a particular total could take. This was a question with which candidates would be less familiar, and required them to think more about what the values in the table meant. Some candidates who had neglected to multiply each mid-interval value by the corresponding frequency in part (c) were successfully able to multiply the lower endpoint by the corresponding frequency in part (d). It may be that these candidates were trying to remember a procedure to find the mean in part (c) without fully understanding it. It is recommended that, as far as possible, students are encouraged to understand why the procedures they are using actually work, rather than simply learning to follow them mechanically.

As a general note, candidates were usually able to identify the importance of units when asked to explain what was wrong with a given answer that had incorrect units (Question 4(a)). However, they often failed to apply this understanding of units to their own work, either omitting them entirely (in particular the degree symbol in Question 7(a)), or confusing units in their work and / or answers. Question 4(c) on Paper 2 in particular showed that units are something which may profitably be dealt with at the beginning of, or throughout, a question, rather than left until the end. It would also be beneficial for candidates to check whether a given answer makes sense in the context of the question asked, and units can often play a central role in this. For example, whether or not "55" is a plausible weight for a teenager will depend on whether this is 55 grams, 55 kilograms, or 55 tons. Problems relating to incorrect or (more usually) omitted units were also seen at Ordinary and Foundation levels.

One further general point: while examiners across all levels generally felt that the amount of supporting work provided had improved from previous years, many candidates still produced answers with inadequate or no supporting work, which may have resulted in them not achieving as highly as they might otherwise have.

At **Ordinary level**, candidates often carried out straightforward procedures successfully, particularly when they did not involve too many steps. In Paper 1, such questions included filling in a table to show a series of operations described in English being applied to given starting numbers (Question 12(a)); using a graph to estimate the maximum heart rate of someone given their age (Question 10(a)); and completing a linear pattern given the first two terms (Question 10(d)). In Question 3(a) – (c), candidates performed reasonably well when asked to find the special offer price of a bike under three different offers (involving "10% off", "one quarter off", and an offer involving paying in instalments), although a number of candidates found the amount of the reduction (i.e. 10% and one quarter) but failed to find the special offer price itself. This may have reflected a lack of care in reading the question rather than a lack of understanding of what was being asked.

In Paper 2, questions on which candidates performed well included finding basic probabilities (Question 1(b) & (c)); finding the size of the third angle in a triangle (Question 3(a)); and drawing the net of a cube (Question 9(c)), with most candidates presenting their net in a "crucifix" shape. Many candidates correctly evaluated sine and cosine of two angles (Question 11(a)), but rounded their answers incorrectly, or failed to round it at all.

Candidates struggled to show procedural fluency in a number of areas that have caused difficulty for students at this level in the past. In Paper 1, while candidates were generally able to find 10% of a given price (Question 3(a), mentioned above), they struggled to write 60 as a percentage of 80 (Question 4(c)). They also had great difficulty splitting 4000 in the ratio 3:5, with most candidates dividing 4000 by 3 and by 5 (Question 4(d)). As discussed in Section 3.1.2 above, questions involving algebraic manipulation were generally those with the lowest mean mark at this level. Many candidates left these questions completely blank, or had great difficulty in presenting relevant work. Most candidates struggled with factorising (Question 9), and in particular when asked to factorise a quadratic expression (part (c)). Some students showed an awareness of the form the answer should take in this part, but were unable to find the actual answer.

Most candidates also had difficulty solving equations (Question 11(a) and (c)). When required to solve a linear equation in part (a), many simply moved terms, unchanged, from one side of the equation to the other. Candidates also had difficulty solving the simultaneous equations in part (c). Many candidates used trial and improvement here rather than algebraic manipulation, but generally only substituted the correct values into one of the equations rather than into both of them. Those who did use algebraic manipulation often displayed a general idea of what to do, but were unable to carry out the procedures accurately and usually stopped when they had found a value for x only.

In Paper 2, candidates struggled to draw the graph of the line $y = x + 4$, having been given the graph of $y = x + 1$ (Question 12(c)). Many candidates left this part of the question blank, while some drew a line parallel to the given line but with an incorrect y-intercept. As at Higher level (Paper 2, Question 5(a), mentioned above) many Ordinary level candidates appeared to have difficulty understanding the significance of m and c when a line is written in the form $y = mx + c$.

Candidates also had difficulty when questions involved squaring, for instance when asked to use the theorem of Pythagoras to find the length of the hypotenuse of a right-angled triangle (Question 3(c)(ii)), or to find the length of a given line segment (Question 6(b)). They also had difficulty accurately finding the volume of a cylinder of given dimensions (Question 10(c)). Many candidates used an incorrect formula, with some treating the cylinder as a rectangle and finding its area, while others treated it as a rectangular box. Those who did use the correct formula often had difficulty evaluating the r^2 component, and a substantial number of candidates who had the correct value rounded it inaccurately.

Ordinary level candidates struggled much more than their Higher level counterparts with the construction that was asked (construct a triangle with given side lengths – Question 7(b)). Even those candidates who produced a triangle that was accurate to within tolerance often failed to show construction arcs. It is recommended that candidates have all of the required equipment with them in the examination, and that they are able to use it appropriately and accurately.

At **Foundation level**, most of the questions on the examination paper assessed candidates' procedural fluency. There was great variation in the levels of competence in evidence here. Candidates were successful at using given values to finish a bar chart (Question 5(a)); drawing the next pattern in a sequence of tile patterns (Question 14(a)); and working out the cost of a bike for which a deposit and 24 monthly instalments would be paid (Question 7(a)).

Performance was more mixed when candidates had to read the time from an analogue clock (Question 1(e)(i)), with many candidates mixing up the hour and minute hands, or giving an answer of 2:45 instead of 1:45. Many candidates struggled to find the probabilities of two different events from a frequency table (Question 12(c) & (d)), although most showed at least some understanding of the principles involved.

Candidates had difficulty with questions involving co-ordinate geometry. When asked to find the slope of a line (Question 11(b)), many candidates mentioned the rise and / or run, but were unable to proceed further with the question; very few candidates successfully completed this question.

Candidates also struggled to construct the bisector of an angle. As at other levels, solutions that were correct within tolerance were seen without any supporting work. However, there was a far greater proportion of candidates at this level who did not appear to understand what was being asked of them in their construction.

Finally, candidates had great difficulty with algebraic manipulation, with many unable to present any valid work when asked to simplify $7x + 2y + x + 3y$ (Question 10(b)). Some candidates combined the terms to give an x^2 and a y^2 term, while many candidates who successfully arrived at $5y$ had difficulty dealing with the lone x, presumably because it did not appear with an explicit coefficient. Candidates also struggled to solve the linear equation $3x – 1 = 11$; many used trial and improvement here, or simply solved the equation by inspection, rather than performing algebraic manipulation. Candidates fared better when the algebra involved substitution rather than manipulation, and were generally able to evaluate $3x + 2$ when $x = 5$ (Question 10(a)).

3.2.3 Strategic Competence

A number of questions in the examination papers assessed candidates' strategic competence, i.e. their ability to formulate, represent, and solve mathematical problems in both familiar and unfamiliar contexts. While these questions are more common on the Higher level paper, they do appear on the examination papers at all levels.

At all levels, examiners reported that candidates were generally more willing to engage with non-routine questions than they had been in the past. Much of this engagement took the form of trial and improvement. However, it was noticeable for most candidates that if they did not hit upon the correct answer immediately, they showed little purpose in their further attempts. Furthermore, while trial and improvement is a suitable method for some questions (particularly at Ordinary and

Foundation level), and an increase in engagement with these questions is a positive development, some non-routine questions (particularly at Higher level) require candidates to engage accurately and effectively with algebra in order to solve them, and candidates tended to struggle with these questions. On the Leaving Certificate course, there is a greater requirement for candidates at both Ordinary and Higher level to solve problems of the latter sort. It is therefore recommended that, in order to lay a solid foundation for senior cycle, candidates give this type of question due attention in their studies.

At **Higher level**, the vast majority of candidates were successful at drawing a set of trend graphs from data presented in a stacked bar chart (Paper 2, Question 1(b)), despite the fact that this is a less common graphical representation and they would therefore have been unlikely to have been asked to perform a task identical to this in class. It may be that they have encountered such displays in other places, either in or outside of school, and were bringing their outside experience to bear on their performance in this question. If this is the case, then it is a very positive development. Candidates also performed very well in Paper 1, Question 14(a) – (c). While the technical level of mathematics required to do this question was not very high, the question was unusual in comparison to typical examination or textbook mathematics questions, and required candidates to read a (relatively) large amount of text.

When required to "work backwards" in a question, candidates had more mixed results. In Paper 1, Question 5(c), candidates were generally successful at converting 400 kelvin into degrees Fahrenheit, having been given a formula for converting from Fahrenheit to kelvin. However, they had great difficulty working out someone's gross income, given their net income (Paper 1, Question 3(b)). While most of those who got the latter question correct did so by working backwards through the normal procedure for calculating net income, it was often unsuccessfully attempted by trial and improvement. Most candidates who attempted it this way were not able to systematically improve their estimate of the gross income, based on the values they had already tried. A very small number of candidates used a variable to represent one of the unknown values (usually the gross income), and proceeded to form an equation and solve it.

In Paper 1, Question 12(a) candidates were asked to factorise $n^2 - 1$, and (hence or otherwise) find two consecutive odd natural numbers whose product is 399. Candidates generally performed better in the second part of this question than in the first, and generally did it using non-algebraic methods. These typically involved: trial and improvement; finding the square root of 399 and looking for odd numbers either side of it; and factorising 399. Given the particular question asked here, all of these are acceptable alternatives to algebraic methods, and may even have been preferable to them.

There is certainly a place for the judicious use of non-algebraic methods in solving mathematics questions.

A question involving non-algebraic methods with which candidates had more difficulty was Paper 2, Question 4(b), in which candidates were asked to find the number of tiles needed to cover the inside of a swimming pool. While this was not as conceptually challenging as some of the other questions mentioned in this section, many candidates had difficulty correctly finding the surface area of the inside of the pool. Most candidates also had issues with units in this question, making errors in converting from one unit to another, or ignoring them altogether. Many candidates presented incorrect answers that made little sense, and showed no evidence of having reviewed their work.

Candidates struggled when algebra became more central to the solution of problems, for instance when they were required to find the value of n if the sides of a right-angled triangle are 9, n, and $n + 1$. By far the most common mistake here was to write that $(n + 1)^2 = n^2 + 1$. Candidates who used only trial and improvement in this question were generally unsuccessful. Candidates also struggled to find the radius of a sphere when told that its volume was three times a particular value (Paper 2, Question 14(b)). Many candidates struggled to successfully set up the required equation, and those who did set it up successfully had difficulty expressing the radius in the required (surd) form.

Candidates also struggled with questions that involved understanding functions. For example, a large number of candidates were unable to successfully form and solve two simultaneous equations in b and c when given the function $h(x) = 2x^2 + bx + c$, and told that $h(1) = 5$ and $h(2) = 13$ (Paper 1, Question 11(d)). Candidates were generally better at solving the two incorrect equations that they found (which does not involve an understanding of functions) than they were at forming the two correct equations in the first place (which does).

Paper 1, Question 4(c) also involved functions, and the latter stages posed difficulties for candidates. While candidates were generally able to accurately evaluate $f(7)$ when told that $f(x) = 3x + 5$, they struggled to solve $f(k) = k$ for the same function. Part of the difficulty that arose here was that in part (b) of this question, where candidates were asked to write $f(k)$ in terms of k, many candidates wrote "$3k + 5 = 0$" and solved this equation for k. This is a similar issue to the one seen in Paper 1, Question 9(a)(i), described in Section 3.2.2 above. As noted in that section, this error is clearly attributable to candidates failing to note the difference between an expression and an equation.

Candidates had great difficulty when required to make connections between a function and its graph in Paper 1, Question 13. Here, candidates were asked to draw the graphs of $y = f(x) + 2$ and $y = -f(x)$, given the graph of $y = f(x)$. A large proportion of candidates did not successfully engage with this question at all; and those who wrote co-ordinates of points on the graph did not associate the y values they had written with the values of $f(x)$ and manipulate them as required.

Finally, many candidates had difficulty with Paper 2, Question 13. This was one of the most challenging questions on the examination paper, involving a practical application of trigonometry and scaling to estimate the volume of a cylindrical water tank shown in a photograph, and required candidates to use skills and knowledge from a number of different strands of the syllabus. While candidates were reasonably successful with part (a), they struggled in part (b), which required them to connect the photograph to the diagram given and to draw their own diagram (or adapt substantially the one given) before using trigonometry to find the height of the water tank itself. It is worth mentioning that, in examination papers under the previous syllabus, candidates would generally have been aware what questions on the examination paper required, say, trigonometry. This was not the case here, so candidates first had to identify that trigonometry was required to complete the question before they were able to start it at all.

Part (c) of this question (Paper 2, Question 13) required candidates to take measurements from the photograph, scale these up to estimate the actual radius of the water tank, and then estimate its volume in m^3. While most candidates were able to perform some of these steps correctly, very few candidates successfully completed all of them, with candidates most often struggling with the scaling. It may be that candidates were more used to mathematics questions where the figures are reasonably straightforward, and found the relatively messy figures in this question off-putting. It is recommended that candidates become used to dealing with such messy data in the course of their studies, especially when it comes to real-life applications. (Ordinary level candidates also struggled with a question involving a similar type of scaling (Paper 2, Question 10(b)), although these candidates were given much more explicit instructions, and the figures were much cleaner.)

At **Ordinary level**, candidates showed good levels of strategic competence in many questions, particularly those that were more amenable to a trial and improvement approach, which was the most common approach adopted by candidates. At this level, none of the questions requiring strategic competence had to be solved using algebraic manipulation (although some could have been), and candidates generally avoided the algebraic manipulation route.

Candidates performed very well when they had to find a combination of smoothies that could be bought from a menu for €7 or less, given certain conditions (Paper 1, Question 6(b)). Examiners were generally surprised at how well candidates performed on this question, given that candidates had to read quite an amount of text, then try different combinations of smoothies until they found one that worked, then use that combination to complete a sentence and find the total cost of the smoothies involved. None of these tasks require a sophisticated level of mathematics, but nonetheless examiners expected candidates to struggle more. However, this was one of the best-answered items by candidates at this level.

Candidates were also generally successful at working out the starting number, given a list of operations and the outcome (Paper 1, Question 12(b)). Again, most candidates here worked by trial and improvement, trying different starting numbers until one gave the required outcome. A small number of candidates worked backwards, reversing the given operations to arrive at the correct starting number. A very small number of candidates used algebra here; those who did often struggled to successfully complete the question.

Candidates at Ordinary level struggled to fill in the correct constant in the box in the formula: "Maximum heart rate = □ minus your age", having been given a graph of maximum heart rate against age (Paper 1, Question 10(b)). Usually the value given by a candidate in this part of the question was not consistent with the values read from the graph (usually correctly) in part (a).

Candidates also struggled when asked to find the mean number of cars sold per month, having been given quarterly data (Paper 2, Question 4(f)). Of the candidates who appeared to understand the concept of the mean, the majority divided the total number by four (presumably because the data was presented in quarterly form) rather than twelve, which may indicate that they did not read the question carefully enough, rather than not understanding the concept of the mean.

At **Foundation level**, there were relatively few questions that assessed candidates' strategic competence. Results were mixed when candidates were asked to scale up the perimeter of a pitch in a photograph to the actual perimeter of the pitch, given the scale of the photograph (Question 8(c)). While many candidates appeared to understand the concept of scaling and were able to correctly scale up the length and / or the width of the pitch, most were unable to successfully complete the question by finding the new perimeter. As mentioned above, scaling is a topic that also caused problems at the other two levels.

As in previous sections, Foundation level candidates had difficulty with co-ordinate geometry here, with only a minority of candidates able to draw a line through a point *A* with a slope greater than that of the given line *AB* (Question 11(c)). Many candidates drew a line that did not go through *A*, while more drew a line through *A* perpendicular to the line *AB* (which did not meet the required criteria).

Finally, when given a sequence of tile patterns, each consisting of white and shaded tiles, candidates struggled to work out the number of shaded tiles in a particular pattern when given the number of white tiles (Question 14(c)). Many candidates presented an incorrect answer with no supporting work, while some candidates drew the relevant pattern from the sequence but did not extract the required answer from it.

3.2.4 Adaptive Reasoning

A particular feature of mathematics examination papers since the introduction of the new mathematics syllabus is the presence of more questions that assess candidates' adaptive reasoning, i.e. their capacity for logical thought, reflection, explanation, justification, and communication. As mentioned above, there has been a general improvement in candidates' performance in this type of question since it was introduced, although weaker candidates often continue to struggle with this, particularly at Higher level.

At **Higher level**, candidates performed well when asked to find, with justification, whether an extra starter, main course, or dessert would lead to the biggest total number of possible meals that could be ordered (Paper 2, Question 2(c)). The vast majority of candidates successfully picked the correct answer (i.e. dessert), and most candidates were able to justify their answer satisfactorily. Candidates also performed well when asked to decide, given a pair of functions and matching graphs, which of two companies charged no fixed monthly fee, giving a reason for their answer (Paper 1, Question 6(b)). The reasons given were generally based on the graphs rather than the functions.

Candidates had more difficulty later in the same question when asked a more involved question, *viz*. to explain how the point of intersection could help someone choose between the two companies (Paper 1, Question 6(d)). While candidates were generally able to make some sense of the graph in relation to one aspect of the question asked (e.g. by saying that the companies charge the same at the point of intersection, or that one company is more expensive for a particular value of x), they struggled to explain that one company would be cheaper for x values to the left of the point of intersection, while the other would be cheaper for x values to the right.

As with other syllabus objectives, candidates struggled with adaptive reasoning relating to geometry. Candidates had difficulty proving a geometric cut, *viz*. that if the diagonal of a parallelogram bisects the angle, then the four sides of the parallelogram must be equal in length (Paper 2, Question 11(b)). Many candidates used the diagram given in the examination paper to prove that the opposite sides of the parallelogram were equal, and then stopped. Very few candidates were able to successfully prove the required result and give a reason for each step in the proof. Candidates also struggled to state a result (a theorem or corollary) that shows that a particular angle in a diagram must measure 90° (Paper 2, Question 8(a)). A small number of candidates stated the converse of an appropriate result, while a large number of candidates had difficulty expressing the required result in reasonably accurate language.

The most challenging question on the Higher level paper was Paper 1, Question 14(d), which asked candidates to explain why $T_{100} = T_{99} + T_{98}$, for a sequence T_n which candidates had explored earlier in the question. As mentioned above, this whole question was a less familiar application of mathematics, and also required candidates to read a (relatively) large amount of text. Part (d), the final part of this question, required candidates to reflect on the work they had done in parts (a) to (c), to identify a pattern in the terms T_n which they had already found, and to explain why this pattern exists. Only a minority of candidates managed to refer to the existence of this pattern, while only a small number of those managed to explain the reason for its existence. These candidates were generally very strong performers on the paper as a whole.

At **Ordinary level**, candidates generally showed improvement in questions dealing with this aspect of the syllabus. The tasks they were asked to do were, appropriately, less demanding than those asked of Higher level candidates, and Ordinary level candidates performed better than expected on them, with a number of examiners expressing that they were pleasantly surprised with the standard of answering in these question parts.

When asked to give a reason for something, candidates at this level were often able to get across the central point required. For instance, candidates were very successful at explaining why the set $\{a, b, w\}$ is not a subset of $\{a, b, c, d, e\}$ (Paper 1, Question 5(b)), and were also generally able to identify which one of three special offers someone should choose, giving a reason for their answer (Paper 1, Question 3(d)). In the latter case, candidates did not always pick the cheaper option – instead, a number of candidates chose the option that could be paid in instalments, giving the valid reason that the money did not have to be paid all at once.

Candidates also performed well when asked to give an example of a list that has a mode, and identify the mode of their list (Paper 2, Question 5(b)). It was nice to see many non-numerical lists given here, with many candidates giving lists of colours, makes of car, etc. Candidates' answering in these questions certainly represents an improvement on their answering to similar questions in the very recent past.

At **Foundation level**, candidates were not asked this year to provide written explanations or examples. The question that did assess their adaptive reasoning asked them to match parts of a time-distance graph to a description (Question 9). As mentioned in Section 3.1.3 above, this question was also asked at Ordinary level. Candidates at Foundation level found this to be a much more difficult task than their Ordinary level peers, with examiners commenting that many of the incorrect answers may have been arrived at through guesswork rather than reasoning or reflection.

4. Conclusions

Candidates showed a great variety of achievement across all three levels of the examination.
The highest-achieving candidates at Higher level showed a good depth of understanding of the whole syllabus; they also demonstrated an ability to be both flexible and accurate in their work, and to bring knowledge and skills from a number of different strands to bear on a given question. These candidates were generally able to express themselves clearly and coherently, and to engage with questions with which they were not likely to have been familiar. These candidates and their teachers had clearly invested considerable time and effort in engaging with the syllabus content and objectives, and the candidates consequently achieved high grades in the examination.

There were, however, a number of concerns raised over the course of the marking. At both Ordinary and Foundation level, concerns were expressed regarding candidates' lack of basic competency in algebra, and in particular in algebraic manipulation. A number of examiners observed that many of these candidates would struggle with aspects of the Leaving Certificate Ordinary level Mathematics syllabus as a result of this. In light of the migration of candidates from Ordinary to Higher level and from Foundation to Ordinary level, this state of affairs is not necessarily surprising. Nonetheless it is a cause for concern.

At Higher level, the standard demonstrated by candidates in basic algebraic manipulation shows some decline, with most candidates struggling to complete multi-step procedures accurately. Unless rectified, this will undoubtedly cause problems for these candidates in Leaving Certificate Mathematics.

Candidates at all levels were often successful at engaging in problems that were not of a routine kind, and many examiners commented positively that candidates performed well on many questions in spite of the relatively large amount of reading involved. At Ordinary and Foundation levels, such questions were amenable to solution by methods that did not involve algebraic manipulation (such as trial and improvement), and many candidates were successful in using these methods. However, when these non-routine problems required algebraic manipulation to solve them, as they often did at Higher level, candidates tended to struggle.

While candidates were usually successful in moving from one area to another on the examination papers without knowing in advance the order of topics in the papers – something which would not have been required to the same extent in the past – they had more difficulty with questions which required them to draw on multiple strands of the syllabus at once. A related point is the difficulty Higher level candidates had with functions. The idea of a function cuts across all of the syllabus strands, and might be profitably learned in this way rather than as a stand-alone strand in itself.

Candidates showed an improvement in questions that required an explanation or other (somewhat extended) verbal response. It would appear that candidates are becoming more used to explaining and justifying their reasoning and understanding, which reflects good classroom practice and is a very positive development.

Finally, candidate performance would indicate that a small proportion of candidates are studying mathematics and taking the examination at a higher level than is appropriate to their current level of achievement. It remains to be seen how such candidates will fare as the new syllabus 'beds in', with teachers and students becoming more accustomed to the expectations of the syllabus and the examination in terms of the content knowledge and the skills that students are expected to be able to demonstrate.

5. Recommendations to Teachers and Students

The following advice is offered to teachers and students preparing for Junior Certificate Mathematics examinations.

5.1 In advance of the examination

Many of the points below are good habits that should be developed over the course of the students' studies in mathematics. It is unlikely that candidates will be successful at checking over work effectively, or at performing algebraic manipulations accurately, on the day of the examination if these skills and habits have not been developed over a period of time before the examination.

- Teachers and students should cover the full syllabus. This is of particular importance as there is no choice on any of the examination papers.
- Teachers should use the support material produced by the Project Maths Development Team and the National Council for Curriculum and Assessment. It has been developed specifically to support the kind of learning envisaged in the current mathematics syllabus.

- Close to the time of examination, questions from past and sample examination papers provided by the State Examinations Commission should be used for practice. However, examination papers should not be relied on excessively during the main period of learning, as this might unnecessarily restrict the range of student learning.
- Students should get into the habit of showing supporting work at all times. This will help them tackle more difficult problems, and will allow them to check back for mistakes in their work.
- Students should develop strategies for checking their answers. One of these is to have an estimate of the answer in advance. In real-life problems, check if the answer makes sense. For example, it is unlikely that someone's net income will be greater than their gross income, or only a tiny fraction of it. In addition to techniques for identifying that an error has been made, techniques for finding those errors quickly and calmly, including getting to know one's own weaknesses, should also be developed.
- Teachers should provide frequent opportunities for students to gain competence and accuracy in basic skills of computation and algebraic manipulation. Students should be particularly careful with signs, powers, and the order of operations.
- Students should understand the difference between an expression and an equation, and what operations can be validly done to each.
- Students should always round their answers to the required level of accuracy, and include the appropriate unit where relevant. These are skills that are not conceptually challenging, and they should be developed to a high standard through regular practice.
- Students should get used to describing, explaining, justifying, giving examples, etc. These are skills that are worth practising, as they will improve understanding, as well as being skills that may be assessed in the examination. Students do not need to be able to produce word-perfect statements of results or definitions, but they do need to be able to state or explain reasonably clearly what these are.
- Students should make sure that they have geometric instruments and should practise using them accurately. This applies particularly to students at Ordinary and Foundation level, where there was evidence that many candidates either did not have the requisite geometric instruments with them in the examination, or were unable to use them appropriately.
- Teachers should provide opportunities for students to apply the skills and knowledge from one strand to material from another strand. Mathematics is not a list of discrete rules and

definitions to be learned but rather a series of interconnected principles that can be understood and then applied in a wide variety of contexts. While compartmentalising knowledge may help keep it organised, it will restrict the ability to cope with unfamiliar questions, particularly those requiring the synthesis of knowledge and skills from several strands.

- Students should practise different ways of solving problems – building up their arsenal of techniques on familiar problems will help them to tackle unfamiliar ones. Students at Higher level should pay particular attention to algebraic methods of solving problems, as there may well be questions that require such methods in the examination.
- When using trial and improvement, students should develop methods for systematically improving the answer. For example, does an increase in the input lead to an increase in the output? If so, this may allow the problem to be solved more quickly.
- Teachers should provide students with opportunities to practise solving problems involving real-life applications of mathematics, and to get used to dealing with messy data in such problems. Students should also be encouraged to construct algebraic expressions or equations to model these situations, and / or to draw diagrams to represent them.
- Teachers should provide students with opportunities to solve unfamiliar problems and to develop strategies to deal with questions for which a productive approach is not immediately apparent. Students should be encouraged to persevere with these types of question – if the initial attempt does not work, they should be prepared to try the question a different way.
- Students at Higher level should learn the five examinable formal geometric proofs. They should understand the logic of each proof and the meaning of each result. They should also pay attention to the method of proof: statements built up one after the other, with a reason given for each one based on previous statements or previous results. Many candidates struggle when asked to apply the results of axioms and theorems to prove a result with which they are not familiar.

5.2 During the examination

- It is intended that Junior Certificate Mathematics examination papers will have the easier questions near the start, and the more difficult questions near the end. However, there may be some areas in which a particular candidate is more confident than others. It is generally a good idea to try to start with one's better questions. However, candidates need to remember to answer all of the questions the examination paper.

- Candidates should read each question carefully, paying particular attention to key words. For example, *calculate* and *measure* are asking for different things, as are *solve* and *factorise*. Giving an answer *correct to the nearest metre* is not the same as giving it *correct to the nearest centimetre*. Candidates should make sure that they answer the exact question asked, and give the answer in the required form with the appropriate unit – many candidates fail to do this.
- The question often contains clues regarding the nature of the answer. For instance, if it asks to *find the value of x*, there should only be one answer, while if it asks to *find the values of x*, one might expect more than one. Similarly, attention should be paid to whether the question indicates that the answer is a natural number, integer, real number, etc.
- Candidates should concentrate when answering, as careless errors will result in marks being needlessly lost. Errors can further disadvantage candidates as they can make the work that follows more difficult.
- Candidates should present their work as neatly and tidily as possible. This will help when checking back over it, and will help the examiner to find relevant work for which marks can be awarded.
- Candidates should show all of their work. In some questions, full marks will not be awarded unless candidates show supporting work. Furthermore, marks are generally not awarded for an incorrect answer without any supporting work. However, if candidates show what they are thinking, then they may get credit for this.
- If possible, candidates should have an estimate of the answer in advance. They should check that the answer makes sense, including the appropriate unit – if it does not, they should review their work.
- When candidates believe that they have made a mistake, it is often most beneficial to start again. They should draw a single line through the incorrect work. They should not use corrector fluid or otherwise make the work illegible – if there is valid work presented and still visible, it may be awarded some marks.
- Candidates should be prepared for the unexpected. The syllabus states that they should be able to solve problems in familiar and unfamiliar contexts, so the examination paper would

not be fit for purpose if the candidates recognised everything on it. Candidates should expect that there will be questions on the examination paper that, at first glance, they will not know how to complete. They should stay calm, and use the problem-solving skills that they have developed throughout their studies. The less familiar the territory, the more credit is likely to be awarded for attempts at applying appropriate strategies. It is always advisable to make an attempt at these questions.

- Candidates should use all of the time in the examination. If they finish the examination paper early, they should review their work, checking as many answers as possible.