Coimisiún na Scrúduithe Stáit
State Examinations Commission

LEAVING CERTIFICATE EXAMINATION 2004

APPLIED MATHEMATICS

ORDINARY LEVEL CHIEF EXAMINER'S REPORT

HIGHER LEVEL CHIEF EXAMINER'S REPORT

CONTENTS

1. GENERAL INTRODUCTION

1.1 The Syllabus

The syllabus for Higher Level and Ordinary Level in Applied Mathematics is set out in the *Rules and Programme for Secondary Schools*. This syllabus has been in operation for the past number of years. Those parts of the syllabus which are printed in italics belong to the Higher Level course only. The Higher Level course includes the Ordinary Level course treated in greater depth. Knowledge of the relevant parts of the Mathematics course is assumed.

1.2 The Examination

Structure of Examination for Higher Level.

The examination consists of one paper, comprising ten questions.
Six questions must be answered correctly to obtain full marks.
Each question carries 50 marks.
The duration of the paper is $2\frac{1}{2}$ hours.

Structure of Examination for Ordinary Level

The examination consists of one paper, comprising nine questions.
Six questions must be answered correctly to obtain full marks.
Each question carries 50 marks.
The duration of the paper is $2\frac{1}{2}$ hours.

2. ORDINARY LEVEL

2.1 Introduction

The examination consists of one paper, comprising nine questions.
Six questions must be answered correctly to obtain full marks. Each question carries 50 marks. The duration of the paper is $2\frac{1}{2}$ hours.
The number of candidates sitting Applied Mathematics for each of the last three years is given in Table 1 below.

Year	Applied Mathematics Candidates	Higher Level		Ordinary Level	
		Candidates	%	Candidates	%
2002	1293	1199	92.7	94	7.3
2003	1394	1286	92.3	108	7.7
2004	1456	1358	93.3	98	6.7

Table 1: Candidates sitting Applied Mathematics for the years 2002 - 2004

2.2 Performance of Candidates

Table 2 shows how the candidates performed with a summary of the results of the examination for 2004.

Applied Mathematics – Ordinary Level - 2004								
Grade	A	B	C	D	E	F	NG	Total Candidates
Number	27	27	15	16	9	3	1	98
% of Total	27.6	27.5	15.3	16.4	9.2	3.1	1.0	

Table 2: Summary of Results in 2004

Table 3 shows the percentage of candidates who obtained A grades, A+B+C grades, D grades and E+F+NG grades in the years 2002, 2003, and 2004.

	Applied Mathematics – Ordinary Level			
	% A Grades	% High Grades (A, B, C)	% D Grade	% Low Grades (E, F, NG)
2002	41.5	78.7	10.7	10.6
2003	19.4	77.7	14.8	7.4
2004	27.6	70.4	16.4	13.3

Table 3: Percentages of A, A+B+C, D, E+F+NG Grades Awarded in 2002, 2003 and 2004

Table 4 ranks each question in two ways: firstly, according to candidates' performance in the question and secondly, according to the question's popularity among candidates. The Table gives the average mark per question and corresponding rank order, the response rate and rank for the question and its topic. These figures are derived from an analysis of a random sample of 40.8% of the scripts marked.

Performance			**Popularity**		
Question	**Average Mark**	**Rank Order**	**Response Rate (%)**	**Rank Order**	**Topic of Question**
1	46	1	100.0	1	Linear Motion
2	37	2	97.5	2	Relative Velocity
3	35	3	72.5	5	Projectiles
4	33	6	90.0	3	Connected Particles
5	35	4	82.5	4	Collisions
6	33	5	25.0	9	Centre of Gravity
7	27	8	40.0	7	Statics
8	24	9	27.5	8	Circular Motion
9	27	7	45.0	6	Hydrostatics

Table 4: Ranking of Questions according to Average Mark and Response Rate

2.3 Analysis of Candidate Performance

Question 1 **Average mark 46**

This question was very well answered by the majority of candidates. Candidates had no difficulty in finding the uniform retardation of the car or in calculating the time it took the car to travel from *a* to *b*. However some candidates failed to give the answers in the final two parts of the question in the correct format. A common error was $\sqrt{50} = 2\sqrt{5}$ or $\sqrt{50} = 5\sqrt{3}$. In part (iv) a number of candidates got the total distance the car travelled before coming to rest and forgot to subtract the distance travelled before the relevant point.

Question 2 **Average mark 37**

(a) Most candidates had no difficulty in writing down the velocity of ship A, the velocity of ship B and the velocity of ship A relative to ship B in the correct format. A small number of candidates had difficulty with the components of the velocity of ship B.

Finding the time at which ship A intercepted ship B in part **(iii)** caused much difficulty. A number of candidates who solved the rest of the question failed to attempt this part. A common error was equating 0.366 hours to 36 minutes.

(b) Part **(i)** was very well answered. In part **(ii)** some candidates added the difference in displacement of the two vehicles to 100 metres rather than subtracting it. A common error was $100 - 35 = 75$.

Question 3 **Average mark 35**

(a) The vast majority of candidates found the value of u correctly. Many candidates had difficulty in finding the time taken for the ball to strike the horizontal ground. A common error was to include acceleration due to gravity in the expression for horizontal displacement.

(b) This part was well answered by most candidates. Some candidates did not read the question correctly and failed to find the initial velocity of the ball in the form requested.

Question 4 **Average mark 33**

(a) Most candidates were able to show on separate diagrams all the forces acting on each particle. Some candidates did not read the question correctly and introduced a frictional force. A common error was to omit the normal reaction of the table.

In part **(ii)** some candidates used weight instead of mass in the equations of motion. A common error was to write the equation of motion for the 8 kg mass as $T - 8g = 8f$ rather than $T = 8f$.

(b) The vast majority of candidates were able to show on a diagram the forces acting on the particle. Some candidates had poor diagrams. Many inserted both the weight and its components on the diagram. A common error on the force diagram was the inclusion of the frictional force as an accelerating force. While part **(ii)** was well answered by most candidates, part **(iii)** proved too difficult for most.

Question 5 **Average mark 35**

(a) Many candidates used the wrong direction of motion for the 3 kg mass in the equations and as a result found an incorrect value for u. This part of the question was well answered by most candidates. However, a number of candidates had an incorrect expression for the coefficient of restitution even though this expression is given in the tables.

(b) Finding the value of v, was very well answered by all candidates who attempted this question. There were many excellent attempts at part **(ii)** but finding the rebound velocity of the ball did cause problems for some candidates. A common error was $h = (1.25)e = 1$.

Question 6 **Average mark 33**

(a) Most candidates had no difficulty in calculating the areas of the different sections of the shape *abcd* or in finding the coordinates of the centre of gravity of the rectangular section. However, many candidates had difficulty in finding the coordinates of the centre of gravity of the triangular sections.

(b) While this part was well answered by many candidates, a number of candidates did not attempt this part at all. A common error was not finding the position of the centre of gravity of the remainder with respect to the centre of the circle of radius r as specified in the question.

Question 7 **Average mark 27**

Most candidates were able to show on a diagram all the forces acting on the ladder. The following were common errors:

- Incorrect direction for the frictional force at *a*
- Frictional force at *b*
- Weight of man acting at the centre of the ladder
- Weight of man was omitted.

Part **(ii)** was well answered, but many candidates had difficulty in taking moments about the point *b* in part **(iii)**. Very few candidates were able to find the value of the coefficient of friction between the ladder and the floor in part **(iv)**.

Question 8 **Average mark 24**

This was the lowest scoring question on the paper. No candidate achieved full marks for this question.

(a) Many candidates did not read the question correctly and did not have the mass describing a horizontal circle. Some candidates found the angular velocity of the mass rather than the greatest speed of the particle as specified in the question.

(b) **(i)** Many candidates successfully attempted this part.

(ii) Very few candidates were able to show on a diagram the forces acting on the mass M.

(iii) This part proved too difficult for candidates.

Question 9 **Average mark 27**

(i) While most candidates were able to state the Principle of Archimedes correctly, a number of candidates had confused statements of the principle.

(ii) While well answered generally, some candidates used the relative density rather than the actual density of the liquid when calculating the pressure at a point in the liquid.

(iii) and **(iv)** A common error in parts **(iii)** and **(iv)** was

$$\text{Thrust} = \text{Pressure} \times \text{Volume of cylinder}$$

(v) This part depended on understanding the Principle of Archimedes which many candidates did not fully have.

2.4 CONCLUSIONS

- In general, candidates showed a good level of ability to extract from the text of the given problems the mathematical equations necessary to lead to successful solutions.

- Where the answering was such that the final grade awarded was lower than D there seemed to be two main causes. Firstly, candidates had not attempted the required number of questions. Secondly, their answers did not demonstrate an understanding of what was required.

- Analysis of patterns in answering suggests that the choice of questions was often made in advance of seeing the examination paper. This is a practice that is not recommended. It is, of course, unavoidable if all parts of the syllabus have not been studied. There was some evidence of the latter happening. For example, all candidates in some centres omitted the questions on circular motion and hydrostatics.

- In general, the drawing of force diagrams posed difficulties. It should be noted that, when force diagrams are asked for, the component parts of forces are not required.

2.5 RECOMMENDATIONS TO TEACHERS AND STUDENTS

- Students should read questions thoroughly and repeatedly in order to extract the equations necessary to solve the problems posed.
- Practising problems regularly is an essential part of preparation for this examination.
- Complete coverage of the syllabus is strongly recommended in order to ensure that the grade achieved reflects the ability of the student.
- Do not decide in advance the questions you will answer – wait until you have read the paper.
- Start each new question at the beginning of a new page.
- Beware of careless mistakes particularly in addition, subtraction etc.
- Double check formulae you write down.
- Draw neat diagrams with clear labels.

3. HIGHER LEVEL

3.1 Introduction

The examination consists of one paper, comprising ten questions.
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The number of candidates sitting Applied Mathematics for each of the last three years is given in Table 1 below.

Year	Applied Mathematics Candidates	Higher Level		Ordinary Level	
		Candidates	%	Candidates	%
2002	1293	1199	92.7	94	7.3
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2004	1456	1358	93.3	98	6.7

Table 1: Candidates sitting Applied Mathematics for the years 2002 - 2004

3.2 Performance of Candidates

Table 2 shows how the candidates performed with a summary of the results of the examination for 2004.

Applied Mathematics – Higher Level - 2004								
Grade	**A**	**B**	**C**	**D**	**E**	**F**	**NG**	**Total Candidates**
Number	398	345	287	204	74	35	15	1358
% of Total	29.3	25.4	21.2	15.0	5.5	2.6	1.1	

Table 2: Summary of Results in 2004

Table 3 shows the percentage of candidates who obtained A grades, A+B+C grades, D grades and E+ F + NG grades in the years 2002, 2003 and 2004.

	Applied Mathematics – Higher Level			
	% A Grades	**% A+B+C Grades**	**% D Grade**	**% E+F+NG Grades**
2002	26.8	75.3	16.5	8.2
2003	28.8	79.5	11.5	9.1
2004	29.3	75.9	15.0	9.2

Table 3: Percentages of A, A+B+C, D, E+F+NG Grades Awarded in 2002, 2003 and 2004

Table 4 ranks each question in two ways: firstly, according to candidates' performance in the question and secondly, according to the question's popularity among the candidates. The Table gives the average mark per question and corresponding rank order, the response rate and rank order for the question and its topic. These figures are derived from an analysis of a random sample of 9.6% of the scripts marked.

Performance			Popularity		
Question	**Average Mark**	**Rank Order**	**Response Rate (%)**	**Rank Order**	**Topic of Question**
1	38	3	85.0	4	Linear Motion
2	37	4	70.0	6	Relative Velocity
3	40	2	91.7	2	Projectiles
4	40	1	95.8	1	Connected Particles
5	37	5	85.8	3	Collisions
6	36	6	17.5	9	Circular & Simple Harmonic Motion
7	26	10	18.3	8	Statics
8	33	9	27.5	7	Rigid Body Motion
9	35	7	11.7	10	Hydrostatics
10	33	8	81.7	5	Differential Equations

Table 4: Ranking of Questions according to Average Mark and Response Rate

3.3 Analysis of Candidate Performance

Question 1 **Average mark 38**

(a) **(i)** The majority of candidates initially had the times of the particles before collision in terms of a variable *t*. However, most candidates later realised that the required times were given and then proceeded to find the initial velocity of the second particle.

(ii) This part was poorly answered. Many candidates confused distance with displacement and gave $s_1 = 15.9$ and $s_2 = 15.9$. Very few candidates realised that the second ball was on its way down when the collision occurred. A small number of candidates having found a negative value for the velocity of the second ball after 2 seconds realised that the second ball was falling and then proceeded to get the correct values for the distances travelled by each ball before the collision.

(b) **(i)** This part was well answered. Most candidates were able to calculate the acceleration of the car along the horizontal. Many combined the motion of the car and the motion of the caravan into one equation of motion and had little difficulty in finding the value of *f*.

(ii) This part was also well answered. However, some omitted the *g* in $2100g\sin\alpha$, the component of the weight down the inclined plane and as a result could not solve $\sin\alpha = \frac{2310}{2100}$. A significant number of candidates failed to round α to the nearest degree.

Question 2 **Average mark 37**

(a) **(i)** The candidates who drew a correct diagram for the flight of the bird had little difficulty with this part of the question. Some candidates confused the velocity of the bird with the velocity of the bird relative to the wind. Many candidates failed to get the direction in which the bird must fly to reach its nest to the nearest degree and left the answer as $35.3°$.

(ii) Many candidates failed to include $18\cos 45°$ when finding *v*. In general, the candidates who used the sine rule to find *v* were the most successful. A common error was taking the time taken to reach the nest as $\frac{250}{22}$.

(b) **(i)** This part was answered correctly by nearly all candidates.

(ii) Many candidates had a poor diagram and there was much confusion in finding the angle required to enable candidates to find the shortest distance between P and Q.

(iii) This part was well answered by those candidates that had successfully found the shortest distance in part **(ii)**. A common error in part **(iii)**

was to take the time that P and Q were closest together as the shortest distance divided by the velocity of Q relative to P.

Question 3 **Average mark 40**

(a) This part of the question caused difficulty for many candidates. There was a tendency to focus on the maximum range and neglect the maximum height of 8 metres. Many candidates found it difficult to solve the simultaneous equations $r_j = 8$ and $v_j = 0$ to find α. Some candidates took the incorrect approach of putting $\alpha = 45°$ for the maximum range. A common error was using the time to reach the maximum height as the time of flight.

(b) This part was well answered by most candidates. Those candidates who found two expressions for time *t* and then equated them seemed to have less difficulty in solving the question. Some candidates did not take a general approach to the solution and as a result made the work difficult. Having found an expression for *t* by putting $v_i = 0$, *t* was then substituted into the expression that resulted by putting $r_j = 0$. The trigonometric functions were then expanded. This often resulted in candidates having difficulties in solving for $\tan\alpha$. Recognition of $\frac{\sin(\alpha-\theta)}{\cos(\alpha-\theta)} = \tan(\alpha-\theta)$ and expanding $\tan(\alpha-\theta)$ rather than expanding $\sin(\alpha-\theta)$ and $\cos(\alpha-\theta)$ separately led to significantly less errors.

Question 4 **Average mark 40**

(a) **(i)** This part of the question was very well answered. Candidates had no difficulty in getting the equations of motion and solving to find the acceleration of the system.

(ii) Many candidates found the distance travelled by each particle but failed to double this distance to find the vertical distance between the particles when the speed of each particle was v. Candidates who drew a diagram or read the question carefully remembered to double the distance.

(b) **(i)** The vast majority of candidates were able to show, on separate diagrams, the forces acting on the wedge and on the particle. Some candidates had poor diagrams. Many inserted both the forces and their components on the diagrams. A common error on the force diagram for the wedge was the inclusion of a normal force 3g on the face of the wedge rather than a force R.

(ii) This part was well answered by those candidates who had practised this type of question. Those who had not been prepared quite often had irrelevant equations of motion e.g. $R = 3g\cos 45°$. In general, candidates had little difficulty with algebraic manipulation or the manipulation of surds.

(iii) Although most candidates were able to find how far the wedge moved in this part, some candidates were confused by the acceleration of the particle relative to the wedge.

Question 5 **Average mark 37**

(a) **(i)** Most candidates were able to find the speed, in terms of *u* and *e*, of each sphere after the collision.

(ii) Candidates overall difficulties with inequalities surfaced in this part of the question. Common errors were $-v_1 = v_2$ or $v_1 = -ev_2$. Some of those who made arithmetical slips ended up with positive values for v_1 and v_2 and as a result were unable to answer this part.

(b) Part **(b)** was well answered by candidates who had practised this type of question. The majority of candidates found the coefficient of restitution before calculating the value of θ. Candidates who used the method on page 12 of the marking scheme were more successful than candidates who used alternative methods. Very few candidates confused speed and velocity by putting $v_1 = v_2$ after first finding v_1 and v_2 in terms of *e* and *u*.

Question 6 **Average mark 36**

(a) This part of the question was generally well answered. Candidates were able to use the principle of conservation of energy correctly and also recognised the condition satisfied by the particle when it lost contact with the sphere. A common error was to put $\frac{mv^2}{r} = mg\sin\alpha - R$ with $\sin\alpha$ as a constant, usually as $\sin\alpha = \frac{4}{5}$.

(b) **(i)** This part was very well answered. Candidates had no difficulty in verifying that the motion of the particle was a simple harmonic motion.

(ii) A common error was to put $a\cos(\pi - \beta) = +5$ rather than $a\cos(\pi - \beta) = -5$.

Question 7 **Average mark 26**

This was the lowest scoring question on the paper. It was attempted by 18.3% of the candidates.

(a) This was well attempted by most candidates. However, some candidates omitted the particle of weight W, making the question meaningless. The vast majority of candidates had no difficulty in taking moments about a suitable point and proceeding to calculate the required distance from *p* to the point from which the particle can be suspended without disturbing equilibrium.

(b) **(i)** This was poorly answered. There seemed to be much confusion with candidates resolving forces horizontally and vertically and taking moments about the point *c* and *b* without much purpose or direction. Very few candidates found the inclination of the rod to the vertical.

(ii) This part was solved successfully by very few candidates.

Question 8 **Average mark 33**

(a) This was very well answered by the vast majority of candidates. A number of candidates attempted only this part of the question.

(b) **(i)** This part caused difficulty for most candidates. Those who were familiar with the formula for kinetic energy of rotation did well. A common error for candidates not familiar with the concept of rotational kinetic energy was to give the kinetic energy gained by the wheel as $\frac{1}{2}(0.3)(1.2)^2$.

(ii) A number of candidates successfully attempted this part using the principle of Angular Momentum. However, some candidates were confused by various methods and made very little progress in finding the distance descended by the particle. A common error was to treat the question as a simple pulley and ignore the rotation of the pulley.

Question 9 **Average mark 35**

This was the least popular question on the paper. This is consistent with other years. It was attempted by 11.7% of the candidates.

(a) This was very well answered by the majority of candidates. Those who selected this question were very well prepared. Many candidates started from

$$950g(0.12) = 925gh + 1000g(0.12 - h)$$

and correctly found the height of the column of liquid B. A small number of candidates omitted $1000g(0.12 - h)$ from the above equation.

(b) Candidates were very well prepared for this part of the question and answered it confidently. A number of candidates, after taking moments about the point where the line of action of the buoyancy force crosses the diagonal [*pr*], found $T = \frac{W}{2}$ and then easily found the relative density by resolving forces.

A common error was expressing the buoyancy force as $\frac{W}{s}$ rather than $\frac{W}{2s}$.

Question 10 **Average mark 33**

(a) This was well answered by most candidates. The most common errors were a failure to separate the variables properly and incorrect integration of x^{-2} as $\frac{x^{-3}}{-3}$. A small number of candidates failed to isolate *y* and left their answer as $\ln(y-4) = -\frac{1}{x} + 1$.

(b) **(i)** There were many excellent attempts at this part of the question. The vast majority of candidates were aware of the properties of logs and the use of logs was in general very good. However, this question did cause difficulty for a number of candidates. A typical solution that caused difficulty with signs was

$$\frac{vdv}{-g-kv^2} = dx \quad \Rightarrow \quad \frac{1}{2k}\ln\left(-g-kv^2\right) = x + C$$

A common error was writing the equation of motion as

$$\frac{vdv}{dx} = -kv^2 \quad \text{or} \quad \frac{vdv}{dx} = 2g - kv^2.$$

(ii) A solution that sometimes caused problems was

$$1 + 4kg = \left(\frac{g+4kg^2}{g+kg^2}\right)^2 \quad \Rightarrow \quad (g+kg)^2(1+4kg) = \left(g+4kg^2\right)^2$$

with no prior simplification.

3.4 CONCLUSIONS

- Candidates showed a good level of ability to extract from the text of the given problems the mathematical equations necessary to lead to successful solutions.

- Where the answering was such that the final grade awarded was lower than D there seemed to be two main causes. Firstly, candidates had not attempted the required number of questions. Secondly, their answers did not demonstrate an understanding of what was required.

- Analysis of patterns in answering suggests that the choice of questions was often made in advance of seeing the examination paper. This is a practice that is not recommended. It is, of course, unavoidable if all parts of the syllabus have not been studied. There was some evidence of the latter happening. For example, all candidates in some centres omitted the questions on circular motion, simple harmonic motion, statics and hydrostatics.

- Marks were often lost as a direct result of failing to read questions with due care. For example, misreading distance as displacement, not rounding or giving the answer to the correct degree of accuracy frequently incurred penalties for candidates.

- In general, the drawing of force diagrams posed difficulties. It should be noted that, when force diagrams are asked for, the component parts of forces are not required. There was much mutation of figures caused by rushing and unclear labelling which the candidate then misread. While the penalties for such slips are low it must be remembered that the knock-on effects commonly result in subsequent work being more difficult, or even impossible, to complete.

3.5 RECOMMENDATIONS TO TEACHERS AND STUDENTS

- Students should read questions thoroughly and repeatedly in order to extract the equations necessary to solve the problems posed.

- Practising problems regularly is an essential part of preparation for this examination.

- Complete coverage of the syllabus is strongly recommended in order to ensure that the grade achieved reflects the ability of the student.

- Do not decide in advance the questions you will answer – wait until you have read the paper.

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