



Leaving Certificate Examination

2001

Mathematics Ordinary Level

Chief Examiner's Report

Mathematics

Leaving Certificate - Ordinary Level, 2001

1. Introduction

A comprehensive report was published on the Leaving Certificate Mathematics examinations of 2000. However, as concern has been expressed in relation to the level of unsatisfactory performance of candidates at Ordinary Level in the 2001 examinations, the Minister asked that a further report be published this year. Readers are urged to refer to the 2000 report for a comprehensive treatment of all aspects of the examination, comments on the performance of candidates, and recommendations to teachers and students, as the content of this report continues to be highly relevant to many of the issues to be addressed here.

It should be noted that in awarding grades to candidates, the Department of Education and Science does not designate such grades as pass or fail grades. However, the grades E, F and NG are regarded as indicative of an unsatisfactory level of performance. Throughout this report, the term "low grades awarded" refers to the grades E, F and NG.

2 Performance of Candidates

2.1 Summary of 2001 Results

There are two papers in the examination, each lasting 2½ hours. On paper 1, candidates must answer any *six* of eight questions on core syllabus material. Paper 2 has two sections, A and B. Section A consists of seven questions on core syllabus material and Section B consists of one question on each of four optional topics. Candidates must answer any *five* questions from Section A and any *one* question from Section B.

Each question on each paper merits 50 marks, giving a total of 300 marks for each paper and hence an overall total of 600 marks.

The two papers test similar cognitive domains, although the more visual topics on the course (geometry, trigonometry, area and volume) appear on paper 2. All questions are designed to be of similar standard, with an internal grading of difficulty. This is typically achieved by questions with a three-part structure. The first part usually tests recall and basic understanding. The second usually tests application of routine procedures in relatively familiar contexts. The third tests less familiar applications or problem solving. All but four of the 19 questions on the two examination papers this year had such a three-part structure.

Table 1 below provides a summary of the results of the examination for all candidates, including the breakdown by gender.

Table 1: Summary of Results in 2001

Mathematics – Ordinary Level – 2001								
Grade	A	B	C	D	E	F	NG	Total
Number	5656	9974	9218	8513	4056	2228	330	39975
%	14.1	24.9	23.1	21.3	10.1	5.6	0.8	
Total Female	3403	5776	4877	4346	2016	1096	117	21631
% Female	15.7	26.7	22.5	20.1	9.3	5.1	0.5	
Total Male	2253	4198	4341	4167	2040	1132	213	18344
% Male	12.3	22.9	23.7	22.7	11.1	6.2	1.2	

Table 2 shows the percentages of candidates who obtained A grades, along with the breakdown into high (A, B, C) grades, D grades and low (E, F or NG) grades in 2001 and the two preceding years. A detailed breakdown of the examination statistics for 1999 to 2001 is provided in Appendix 1 at the end of this report.

Table 2: Percentage of A/ High/ D/ Low Grades awarded in 1999, 2000 and 2001

	Mathematics – Ordinary Level			
	% A Grades	% High Grades (A, B, C)	% D Grade	% Low Grades (E, F, NG)
1999	18.8	67.5	20.2	12.2
2000	14.4	65.8	21.5	12.7
2001	14.1	62.1	21.3	16.5

2.2 Low Grades

A widely remarked upon feature of the results profile of the Leaving Certificate Ordinary Level Mathematics examination in 2001 was the increase in the percentage of low grades awarded. This, coming on top of a rate that is already higher than desirable, is rightly viewed as a cause for concern.

Table 3 gives the percentage of low grades awarded in Leaving Certificate Ordinary Level Mathematics for each of the last five years.

Table 3: Percentage of Low Grades awarded from 1997 to 2001

Year	1997	1998	1999	2000	2001
% Low Grades	13.9	14.2	12.2	12.7	16.5

The nature of the examination is such that a certain amount of variation is inevitable each year. The process by which the examination papers are drawn up means that the exact equivalence of standard from one year to the next cannot be objectively guaranteed. For this reason, following the examination, adjustments are made to the marking scheme on the basis of consultation with experienced examiners and teachers, comments from relevant parties and a wide sampling of scripts. In this way, the impact of minor variations in the standard of the examination is minimised. Nonetheless, a certain amount of fluctuation in the results is likely to remain.

The consensus of the comments reported by the examining team is that the examination and marking scheme, taken together, do not constitute a different standard of examination than that of previous years. However, the team, which consists mostly of examiners with considerable experience in examining at this level, reported a noticeable increase in the incidence of the difficulties generally experienced by some candidates. These difficulties include poor computational skills, poor algebraic skills and difficulty in solving equations, and are outlined in detail in the reports on the 1995 and 2000 Leaving Certificate examinations, as well as in the reports on the 1998 and 1999 Junior Certificate examinations. Relevant extracts from some of these reports can be seen in Appendix 2 and Appendix 3.

Gender Factors

It should be noted that no gender-specific factors are apparent in the increase in low performance in the 2001 examination, with no unusual difference between male and female candidates regarding low grades awarded. Table 4 below gives the percentage of low grades awarded in Leaving Certificate Ordinary Level Mathematics, by gender, for each of the last five years.

Table 4: Percentage of Low Grades by Gender, 1997 - 2001

Year	1997	1998	1999	2000	2001
% Low Grades Female	11.3	12.6	10.8	11.4	14.9
% Low Grades Male	13.8	16.0	14.0	14.3	18.5
% Difference	22.1	27.0	29.6	25.4	24.2

Thus it can be seen that the percentages of males and females being awarded low grades has remained broadly in proportion, with males, on average, having a 26% higher level of low grades than females.

2.2.2 Suggested Causes of Low Grades

It must be stressed that while the examination may serve as an indicator of changing levels of performance, the answering does not provide evidence as to what causes such changes. It would therefore be inappropriate to suggest that the marking process enables the increased percentage of low grades to be confidently attributed to any particular source. The following have been suggested as possible explanations:

- **Follow-on Impact of Difficulties at Junior Cycle**

It should be noted that students' difficulties with mathematics at Junior Cycle have been apparent for some time (see Chief Examiners' reports of 1996, 1998 and 1999). Indeed, this was largely the stimulus for the revision of the Junior Certificate Mathematics syllabus, now in its second year of implementation. It is expected that the students coming through on the revised programme will have a deeper understanding of mathematics than their predecessors. Furthermore, the current In-Career Development programme for teachers of mathematics at Junior Cycle should have the effect of improving the teaching and learning of mathematics at all levels, as teacher skills are enhanced and the range of classroom teaching strategies broadened.

- **Increased Levels of Student Part-time Work**

There is considerable anecdotal evidence from teachers that the amount of time and energy being devoted by students to part-time work has increased dramatically over a short period of time. Furthermore, increases in discretionary income resulting from engaging in paid employment have impacted on the nature of students' social life. As a consequence of this, teachers are reporting increased levels of:

- absenteeism from class;
- failure to complete or even attempt homework;
- lack of energy and concentration, due to long hours of work and late-night activities.

It is worth exploring why such factors might impact more heavily on mathematics than on other subjects. Mathematics is, by its nature, a hierarchical subject. The ability to handle any particular part of the material is highly dependent on having grasped the earlier material upon which it is based. Thus, being regularly absent from a class, failing to consolidate material through homework, or failure to grasp a new concept through inability to concentrate, do not simply affect the work immediately to hand. Rather, there are serious medium and long-term implications for what is achievable. Hence, although the above problems are likely to have a negative effect on achievement in any subject, it is not at all surprising to see an accentuated impact on progress in mathematics.

If substantial numbers of second level students are becoming more and more engaged in part-time work to an inappropriate degree, or if their social activities are impacting negatively on their school commitments, these are serious causes for concern for all involved in education. It is recommended that research be carried out to determine accurately the extent of these phenomena.

- **Disruption of Learning Due to Industrial Action during the 2000/2001 School Year**

The Department of Education and Science has no quantitative evidence either to support or refute the suggestion that industrial action contributed, directly or indirectly, to the increase in low grades awarded in the 2001 examination.

- **Other Possible Factors**
There may be other factors operating at school level—time-tabling issues, a reduction in time allocation for mathematics, students not attending classes in the weeks prior to examinations, to list some possibilities—that are militating against good performance of senior cycle students at mathematics examinations and that should now be specifically identified. In the light of public interest in this matter and in the interests of partnership, it is recommended that a sample of school principals and teachers be surveyed to ascertain their perspectives regarding poor performance and their suggestions for remediation.

3 Analysis of 2001 Paper

3.1 Popularity, Uptake and Average Marks

Tables 5, 6 and 7 which follow, show the ranking of questions in order of decreasing popularity. They give the percentage uptake and the average mark awarded to those who chose each question. These figures are derived from an analysis of a random sample of scripts.

Table 5: Order of popularity, percentage uptake and average marks – Paper 1

Order of Popularity	Question Number	Topic(s)	Percentage Uptake	Average Mark (out of 50)
1 st	7	Calculus	93%	29
2 nd	1	Arithmetic	92%	38
3 rd	2	Algebra	86%	27
4 th	4	Complex Numbers	85%	36
5 th	6	Functions and Calculus	83%	30
6 th	3	Algebra	77%	28
7 th	5	Sequences and Series	66%	28
8 th	8	Functions and Calculus	61%	22

Table 6: Order of popularity, percentage uptake and average marks – Paper 2, section A.

Order of Popularity	Question Number	Topic(s)	Percentage Uptake	Average Mark (out of 50)
1 st	1	Arithmetic (Area and Volume)	98%	30
2 nd	7	Statistics	95%	38
3 rd	2	Co-ordinate Geometry (Line)	94%	35
4 th	3	Co-ordinate Geometry (Circle)	84%	27
5 th	6	Discrete mathematics	72%	34
6 th	5	Trigonometry	48%	32
7 th	4	Geometry	21%	19

Table 7: Order of popularity, percentage uptake and average marks – Paper 2, section B

Order of Popularity	Question Number	Topic(s)	Percentage Uptake	Average Mark (out of 50)
8 th	11	Linear Programming	64%	29
9 th	9	Vectors	18%	25
10 th	10	Further Sequences and Series	14%	21
11 th	8	Further Geometry	4%	14

An analysis of the answering on individual questions is presented below.

3.2 Analysis of Questions

The purpose of this analysis is to provide insights that may be helpful in preparing candidates for improved performance in future examinations. Therefore, rather than reporting on all aspects of answering to all questions as is usual in Chief Examiners' Reports, the following, although mentioning strengths, focuses on the deficiencies in candidates' work. In the light of this, it is important to stress that there were areas in which the overall standard of answering was high. These include calculations (with and without the use of a calculator), the factor theorem, complex numbers, probability, statistics and trigonometry.

Paper 1 Question 1

With a 92% uptake this question was the second most popular on this paper. It was the best-answered question on paper 1. The mean mark was 38 and full marks were obtained by 14% of candidates.

(a) Cooking time: Although some candidates had difficulty interpreting the extra 15 minutes cooking time, this was well answered. The following incorrect work often appeared: $315 \div 60 = 5.25 \Rightarrow 5$ hours 25 minutes. Other incorrect statements included 315 minutes = 3 hours 15 minutes and 1 kg = 100g.

(b)(i) Percentage error: This was well answered. However, many stopped after finding the error was 0.45. Others failed to round the answer or rounded incorrectly.

(b)(ii) Scientific notation: This was reasonably well answered. The candidates who retained the indices tended to mishandle them: $3.1 \times 10^5 - 1.5 \times 10^4 = 1.6 \times 10^9$, for example. The most popular approach was to convert the scientific notation and work with large numbers. For these candidates, an incorrect number of zeros in one or more terms and invalid cancellations were common causes of loss of marks. Those who used the calculator often failed to show any intermediate work.

(c)(i) Investment: This was well answered, often yielding full marks. The only common error was a failure to add the interest.

(c)(ii) Rate: Most candidates had a good idea of what to do here and the question was fairly well answered. However, many stopped before arriving at a value of r , for example, after adding the IR£4000, or after finding IR£377.36 or after IR£9811.36 – IR£4000. Others used guesswork or trial and error to solve $94.34 \times ? = 377.36$.

Paper 1 Question 2

This question was the third most popular (86% uptake) on paper 1. The average mark awarded was 27.

(a) Inequality: **Very few arrived at $\{0,1,2,3\}$, the correct solution. While most reached $-2n > -8$ or equivalent, the vast majority mishandled the subsequent multiplication by a negative, in most cases failing to reverse the inequality sign. Many omitted writing down a set of values.**

(b) **Simultaneous equations:** There was some improvement in this technique this year. However, weaker candidates proceeded by “squaring” the linear equation to get $x^2 + 4y^2 = 9$ or similar, or by “linearising” the second degree equation to get $x - y = 24$ or $2x - 2y = 24$.

(c) **Index equations:** Both parts were very poorly done. Candidates’ answering indicated that the laws of indices were not understood. Usually the only correct statement made was $\sqrt{3} = 3^{\frac{1}{2}}$. It was seldom known that $\frac{1}{\sqrt{3}} = 3^{-\frac{1}{2}}$.

Paper 1 Question 3

This question was not a popular choice, ranking sixth out of the eight questions (77% uptake). The average mark awarded was 28.

(a) **Substitution:** This was the only part in which most candidates scored highly. The vast majority substituted the numerical values and successfully solved for a .

(b)(i) **Multiplication with surds:** The involvement of the surd caused great problems for candidates. Most tried to multiply out and many had the x^2 term correct. However, few could handle $+x\sqrt{x} - x\sqrt{x}$ or $(\sqrt{x})^2$.

(b)(ii) **Equation:** Those who got $x^2 - x$ in (i) usually solved correctly in (ii). However, $x(x-1) = 6 \Rightarrow x = 6$ or $x - 1 = 6$ was common. Many ignored (ii) and others tested values on a trial and error basis.

(c)(i) **Cubic—find a and b :** Only overall high performers answered this well. Most candidates tried to substitute $x = 1$ and $x = 2$ but mistakes in signs, indices and in solving the equations were prolific. Other common errors included the use of $f(-1)$ and $f(-2)$ and even $f(x-1)$ and $f(x-2)$.

(c)(ii) **Solving the cubic:** This was very poorly answered. Many just wrote down the cubic, having substituted their a and b values, and stopped. However, algebraic divisions were well done. Also, the Factor Theorem was applied well, but when a and b were incorrect from (i) difficulties were likely. Those who used the Factor Theorem on the correct cubic usually succeeded in finding $x = 3$, but often made no mention of the other roots. In fact, leaving out one or more roots was common irrespective of the method chosen.

Paper 1 Question 4

With an uptake of 85% this question was the fourth most popular on this paper. The average mark was 36, making it the second most successfully answered question on paper 1.

(a)(i) Plot w : Well answered. Apart from reversing the axes occasionally, almost everyone plotted w correctly.

(a)(ii) Plot iw : Occasionally, candidates had difficulty simplifying $i(3 - 2i)$. The vast majority plotted iw , or whatever they calculated for iw , correctly.

(b) Equation—solve for x and y : This was reasonably well done, with most candidates choosing to multiply out the brackets as the first step and doing this successfully. Lower performing candidates tended to err when pairing off real and imaginary coefficients for the equations and/or solving the simultaneous equations.

(c)(i) Conjugates: This was generally well answered. Good multiplication skills were evident. Errors were mainly an incorrect interpretation of conjugate e.g. $-z$ taken as \bar{z} , $i^2 \neq -1$ and incorrect signs when multiplying.

(c)(ii) Moduli: This was not well answered. For many, only $|3 + 4i|$ was found correctly, but even here the i was sometimes included. Performing operations in an incorrect order was the most common difficulty observed and this led to incorrect statements such as $\sqrt{25} + \sqrt{169} = \sqrt{194}$.

Paper 1 Question 5

As in recent years, this question, which is dependent on knowing and being able to apply a small number of formulae, did not prove to be a popular choice for candidates. It ranked seventh in popularity on paper 1, with an uptake of 66%. The average mark was 28.

(a) $T_n = 813$, find n : This was almost always attempted. Most candidates found the values of a and d , with some stopping after that. Others listed all the terms and counted them. The correct answer, 101, was common without work being shown. Some candidates misinterpreted what was asked and evaluated T_{813} .

(b) GP: Answers to **(i)** and **(ii)** were often correct without work. Part **(iii)** was not well answered although the formula for S_n was usually correct. Quite a few candidates merely used a calculator to evaluate the given expression as 88572 and stopped.

(c) AP: S_1 and S_2 were almost universally correct among those who tried part **(i)**. However, almost everyone then took $d = S_2 - S_1 = 4$ (or -4 sometimes). Very few candidates worked out T_2 from $S_2 - S_1$ and then found $d = 8$. Part **(ii)** was poorly answered. The majority began by working out their own S_n expression and many generated one they were unable to solve. The small number who worked with $4n^2 - 8n$ fared much better.

Paper 1 Question 6

With an uptake of 83% this was the fifth most popular question on paper 1. The average mark was 30.

(a)(i) $g(2)$: Practically everyone could evaluate $g(x) = \frac{1}{5}$. However, many did not express their answer as a decimal, probably a sign that the question was not read carefully. Some gave $\frac{1}{5} = 0.5$, highlighting a serious lack of understanding.

(ii) $g(3)$: Few problems were encountered except for the occasional use of $3^2 = 6$.

(b)(i) $f(x)$ values: $f(2)$ and $f(5)$ were usually calculated correctly, but $f(-1)$ often suffered a sign error. Other errors arose from mishandling the index—for example, $2^3 = 6$ and $6(2)^2 = 12^2 = 144$.

(ii) $f'(x)$: This was well answered. Occasionally, the constant was mishandled.

(iii) Maximum and minimum: Many finished after finding the x values.

(iv) Graph: This was reasonably well answered. The vast majority constructed a full table from the start, ignoring previous parts, and drew their graph from it.

(v) Values of k : Most candidates ignored this part. Few seemed to understand what was required.

Paper 1 Question 7

This was the most popular question on paper 1. Its uptake was 93%. Candidates handled basic differentiation, including the product, quotient and chain rules, well. However, only the high achievers were able to apply their skills to the problem in part (c). The average mark was 29.

(a)(i) Differentiate $6x^5 + x^2$: This was well done.

(a)(ii) Differentiate $(x - 3)(x + 3)$: Most candidates scored full marks.

(b)(i) u/v : This was well answered despite errors in writing down the formula. It appeared that not many candidates referred to the Mathematical Tables (page 42) for the formula and relied on memory instead.

(b)(ii) Chain rule: The chain rule was well known. The brackets around $2x - 7$ were often omitted.

(c) Application of differentiation: Part (c) was not well answered. Candidates tended to differentiate, substitute values and/or equate derivatives to given values but often at the wrong time. Despite the ability of the majority to perform the techniques of differentiation, only a small number had the understanding necessary to apply these to the fireworks problem.

Paper 1 Question 8

With an uptake of just 61%, this was the least popular question on this paper and also, the one in which performance was poorest. The average mark was 22. It often became an excess question, not contributing to candidates' overall marks.

(a) $g(x)$: Both parts were well answered.

(b) First principles: Few got this part fully correct. Errors abounded, usually arising from mishandling the $-x$. The multiplication by 3, after getting $(x+h)^2$, also caused difficulties. Some candidates merely wrote $6x-1$, without work.

(c) $f(x)$: The three parts in **(c)** were poorly answered. In **(i)**, the quotient rule was usually used and once again, the formula often contained errors. The derivative of the constant 1 was frequently incorrect. Parts **(ii)** and **(iii)** were often omitted. For those who attempted them, very few brought the work to a successful conclusion.

Paper 2 Question 1

This was, as usual, the most popular question on paper 2. Its uptake was 98%. The average mark awarded was 30, placing it in fifth position in this regard in Section A.

(a) Length of track: This was attempted by all and answered moderately well. Many dealt with area instead of length. Others did not use π or an approximation for it.

(b) Simpson's Rule: This was attempted by almost everyone. It was fairly well answered. The top and bottom sections were usually dealt with separately. Many calculated only the top section. Some did not realise that the "first" and "last" were 0. There were many transposition errors. Confusion in handling the equation was widespread. Approximately half of candidates stopped at x , failing to calculate the length of the pipe, probably because the question was not read carefully.

(c) Sweets in rectangular box: Many did not know how to tackle this problem. All three parts were poorly answered. Often, the reference to spherical shapes in the first line seemed to act as a trigger which sent the candidate prematurely into complicated volume calculations and as a result, he/she lost touch with what was being asked. Such answering suggests that these candidates connected certain words and phrases to particular responses and consequently, they did not pay adequate attention to the information provided in the question before starting to work. This is a practice which is not recommended and may account for the surprisingly poor answers presented in part **(c)**.

Paper 2 Question 2

This was the third most popular question on paper 2. Its uptake was 94%. Its average mark was 35, making it the second most successfully answered on the paper.

(a) Find t : This was attempted by all. Transposition errors were the most common reason for loss of marks.

(b)(i) Perpendicularity: Well answered. The common approach was to use slopes.

(b)(ii) Length: Well answered, usually using the distance formula.

(b)(iii) Area of Triangle: This was not well answered. The short formula, $\frac{1}{2} |x_1.y_2 - x_2.y_1|$, was usually used but it was often presented with errors and furthermore, the necessary translation was frequently overlooked.

(b)(iv) Co-ordinates of h and g : This was badly answered and often omitted, even by strong candidates. Many, not knowing how to tackle this part, found slopes and stopped.

(b)(v) Equation of bc and show h is on line: Candidates were well able to find the equation and had few mistakes.

Paper 2 Question 3

This was the fourth most popular question on paper 2. Its uptake was 84%. The average mark awarded was 27, ranking it seventh in this regard.

(a)(i) Centre and radius: Usually candidates had no problem finding the length of the radius. They had more difficulty finding the centre. The correct answers were usually found by inspection.

(a)(ii) Find two values of k : This was badly answered. It was omitted by many indicating that candidates did not understand what was asked.

(b) Prove line is a tangent: This was well answered. Most candidates recognised the routine procedure required and made good attempts at carrying it out in spite of algebraic errors.

(c)(i) Equation of circle: This was attempted by most. The standard of answering was fair.

(c)(ii) Possible values of p : This was badly answered and often omitted. In answers of merit which involved testing of co-ordinates, $(0, 0)$ was seldom considered. Furthermore, those who arrived successfully at $p^2 < 13$ nearly always omitted the negatives when listing the values of p .

Paper 2 Question 4

An uptake of 21% placed this question in eighth position in terms of popularity. With an average mark of 19, it was the least successfully answered question in Section A, and second least successfully answered on Paper 2.

(a) Prove triangle is right-angled: This was answered well.

(b) Theorem: There were hardly any reasonable attempts at the proof, indicating that candidates were not prepared for this question.

(c)(i) Enlargements: Overall, answering was fair. However, diagrams were often drawn without a ruler and $\times k^2$ was not commonly used.

Paper 2 Question 5

This question was chosen by 48% of candidates ranking it seventh in popularity on paper 2. With an average mark of 32, it was the fourth best answered question.

(a) $\cos \theta$ and $\cos 2\theta$: Part **(i)** was reasonably well answered in spite of errors with sine, cosine and Pythagoras. Part **(ii)** was poorly done with many ignoring the given identity and reading $\cos 2\theta$ as $2 \cos \theta$.

(b)(i) Area: This was well answered. Many got full marks.

(b)(ii) Calculate side: This was well answered. Failure to round off was common.

(c)(i) Distance from t: This was attempted by most and reasonably well answered. Knowing the answer to expect clearly helped candidates.

(c)(ii) Distance from path to pillar: This was fairly well answered. Most used the sine rule rather than $\sin = \frac{\text{opp}}{\text{hyp}}$. A common error was taking $|\rho s|$ as the shortest distance. Many failed to round off.

Paper 2 Question 6

With an uptake of 72%, this was paper 2's fifth most popular question. The average mark was 34, making it the third most successfully answered.

(a) Birthdays: Part (i) was very well answered. Answering in (ii) was fair, with $\frac{1}{7} + \frac{1}{7}$ commonly occurring.

(b)(i) Letters of IRELAND: The first three parts were well done. Part (iv) was sometimes omitted and many incorrect answers were presented, with or without work.

(c)(i) Points on circle: This was reasonably well answered. Many candidates gave 8C_2 or 28, without work, for full marks.

(c)(ii) Find n: The standard of answering was only fair. Most seemed to use trial and error with the aid of a calculator. Very few solved a quadratic equation.

Paper 2 Question 7

With an uptake of 95%, this was the second most popular question. The average mark was 38 making it the most successfully answered question on this paper.

(a)(i) Mean: Very well answered.

(a)(ii) Standard deviation: The standard of answering was only fair. This routine procedure did not seem to be expected. Strong candidates tended to omit writing their answer correct to one decimal place.

(b)(i) Mean amount spent: The question was usually answered correctly but the following errors occurred:

- the upper limits rather than the mid-intervals were used
- the width of each interval was used, i.e. 8, 4, 4, 4, 12
- frequencies were ignored or 5 was used as the denominator.

(b)(ii) Cumulative frequency table: The standard of answering was fair. The table was usually completed without work.

(b)(iii) Ogive: The curve tended to be drawn very well although errors were sometimes made in scales.

(b)(iv) Estimate: Candidates knew how to approach this and work was of a reasonably high standard.

Paper 2 Question 8

This was the least popular question overall from both papers, with an uptake of just 4%. It was also the least successfully answered question overall, with an average mark of 14.

(a) Find measurement of two angles: Answers were usually written without work and were often incorrect.

(b) Theorem: Practically every candidate who attempted the question omitted this.

(c)(i) | px |: Well answered.

(c)(ii) | rs |: Badly answered, usually only guesswork.

(c)(iii) | pt |: Badly answered. When attempted, incorrect answers without work were commonly presented.

Paper 2 Question 9

With an uptake of 18%, this was the ninth most popular question on this paper and the second most popular question in section B. It was the eighth most successfully answered with an average mark of 25.

(a)(i) $\left| \vec{p} \right|$: This was well answered despite sign errors in the formula and mistakes such as $(-12)^2 = -144$.

(a)(ii) \vec{p}^\perp : The standard of answering was fair. The answer was usually written without work.

(b)(i) Find k and t : Answering was only fair. Even candidates who managed to set up the equations had difficulty solving them.

(b)(ii) Express in terms of \vec{a} and \vec{b} : This was badly answered with many candidates showing a lack of understanding of the concepts involved. Little use appeared to have been made of the diagrams given as a source of help. The answer was usually written without work. Errors in the use of the Triangular Law were common.

(c)(i) Dot product: The standard of answering was fair. Many could not reduce the expression to a scalar having multiplied out the brackets successfully. Errors such as $\vec{i} \cdot \vec{i} = -1$ were common.

(c)(ii) Measure of angle: This was badly answered. The definition of scalar product was not known for example, $\cos \theta$ was frequently omitted. Also, surds tended to be mishandled.

Paper 2 Question 10

With 14% uptake, this was the second least popular question on paper 2. The average mark of 21 made it this paper's ninth most successfully answered question.

(a) Expansions: This was poorly answered. The "hence" part was generally oversimplified using incorrect expansions. The binomial expansion was not used. Most candidates multiplied out using the distributive law. Sign errors were common.

(b) G.P.: Most candidates found the first three terms despite difficulties with fractions. Attempts to find an expression for S_5 were much less successful. Very few used the S_n formula and when the intention was there, an incorrect version was often written down. The sum to infinity was badly answered. Most did not seem to know that a formula exists. The main reasons for loss of marks were the wrong sign in the denominator of the formula, difficulties in handling fractions and incorrect substitution for a and/or r .

(c) Value of investment: The standard of answering was quite good. However, few managed to get the correct answer. Formulae were seldom used and, as a result, the work presented was unwieldy and often lacked accuracy.

Paper 2 Question 11

As usual, this was the most popular "option" question. Its uptake was 64%. Overall, it was the sixth most popular question on the paper and with an average mark of 29, it was also the sixth most successfully answered question.

(a) Graphing inequalities: This was reasonably well answered. However, much incomplete work was presented. For example, candidates often drew the three lines, not extending them enough to intersect, and did not deal with the half-planes. Some just found the point of intersection between lines $x + 2y = 8$ and $5x + y = -5$ and stopped.

(b)(i) Write and illustrate inequalities: The standard of candidates' answering was good for the inequalities. It was fair for the graph. Many candidates made errors dealing with the fractions, e.g. $\frac{1}{5}x + \frac{1}{10}y = 9$ became $5x + 10y = 90$. Others ignored the fractions and wrote $x + y \leq 9$. An incorrect number of zeros was frequently used for IR£4 million. These errors often led to unworkable graphs.

(b)(ii) Find number of houses: Those who found the point of intersection correctly used simultaneous equations. When solving the simultaneous equations many failed to multiply both sides and transposition errors were common. Quite a few tested all the points where the lines cut the axes for the number of buildings. Others tested only the point of intersection. Many did not explicitly state how many bungalows and semi-detached houses should be built.

4 Comments Arising from the 2001 Examination

Although the focus of this report has been on the levels of unsatisfactory performance in the examination, it is important to stress that a considerable amount of excellent work was presented by candidates. This can be seen from the fact that one candidate in seven (14%) was awarded an A grade. These candidates presented clear and accurate answers showing a solid understanding of all aspects of the syllabus.

The standard of answering of low-performing candidates, however, demonstrated that they experienced similar difficulties to those identified in previous years, but to a considerably greater extent. These difficulties relate to:

- Poor computational skills when negative numbers and fractions are involved
- Poor skills of manipulation, especially where indices and surds arise
- Poor algebraic skills, particularly basic transpositions, multiplication and factorisation
- Difficulty in solving equations, particularly quadratic equations and those involving fractions
- An inability to use the information given in questions to guide the work.

While there was an improvement in answers involving calculations this year, two difficulties in the use of the calculator remain evident: a reluctance to show intermediate work and a tendency towards overuse, even for the simplest of calculations. Students and teachers are advised to refer to *Calculators: Guidelines for Second-level Schools*, to be issued to schools in the near future, where issues relating to calculator use are dealt with in detail.

5 Issues Relating to Foundation Level Mathematics

5.1 Candidature Targets

It is useful to note first of all that mathematics is one of only two Leaving Certificate subjects that are offered at three levels: Higher, Ordinary and Foundation (the third level then known as the Ordinary Alternative was introduced to schools in 1990). The Foundation Level course (introduced in 1995), along with its predecessor, the Ordinary Alternative Level, were developed in recognition of the fact that there was a large body of students whose educational needs were not being met by the existing Ordinary Level course. One of the indicators of this problem was the high level of unsatisfactory performance at Ordinary Level at that time. It was considered that the needs of the full range of candidates could not be met by only two levels, and this led to the decision to develop a third level.

Given that now there are courses available to meet the needs of all students, it should be expected that a high level of satisfactory performance would exist at all levels. The reality, however, is that despite the availability of suitable courses, significant numbers of candidates are continuing to opt for courses that were not designed to meet their needs. It should be noted that, in designing the three Leaving Certificate courses, the NCCA course committee intended that the cohort should divide in the ratio 1:2:1. That is, the Higher Level course was designed to meet the needs of the strongest 25% of the cohort, the Ordinary Level to meet the needs of the next 50% and the Foundation Level to meet the needs of the remaining 25%.

The breakdown of candidature at the three levels indicates that this intention has not been realised in practice. The target of 25% for Higher Level has not been achieved, although considerable progress has been made. Far more significant, however, is the small number of candidates opting for the Foundation Level course. It is recognised that the Leaving Certificate Applied programme was introduced subsequent to the design of the three Leaving Certificate Mathematics courses and that this impacts on the number of students available for the three levels in the Leaving Certificate syllabus.

However, even taking this into account, it is apparent that in any given year there are substantial numbers of candidates presenting for examination at Ordinary Level who are properly part of the envisaged target group of the Foundation Level. This clearly has an impact on the number of low grades awarded at Ordinary Level.

Table 8: Breakdown of Candidature by Level, 1997 – 2001

Year	1997	1998	1999	2000	2001
Total Number of Leaving Certificate Mathematics Candidates (including Leaving Certificate Applied Mathematical Applications)	61,790	64,226	62,764	61,447	58,004
Number of Higher Level Candidates	11,042	10,723	10,696	10,645	9,938
Number of Ordinary Level Candidates	43,566	45,191	44,188	42,214	39,979
Number of Foundation Level Candidates	6,427	6,056	5,753	5,846	5,227
Number of Leaving Certificate Applied Mathematical Applications Candidates	755	2,256	2,127	2,742	2,860
% Higher Level (target—25%)	17.9	16.7	17.0	17.3	17.1
% Ordinary Level (target—50%)	70.5	70.4	70.4	68.7	68.9
% Foundation Level (target—25%)	10.4	9.4	9.2	9.5	9.0
% Leaving Certificate Applied	1.2	3.5	3.4	4.5	4.9

5.2 Standard Required in Examinations

While there have been suggestions to modify the standard required in the examination in order to give better grades to these low-performing candidates, the examination must remain a valid instrument for measuring the achievement of candidates in relation to the course as laid down in the syllabus. In order to do this, it must continue to take the performance criteria envisaged in the syllabus as its primary point of reference. Rather than attempting to distort the assessment instrument to perform a task for which it was not intended, the need to address the underlying reasons why candidates are not choosing the courses that were designed to meet their mathematics educational needs is recommended.

5.3 Addressing Inappropriate Choice of Level

No systematically collected evidence is available regarding the reasons why students for whom the Foundation Level course was designed continue instead to opt for Ordinary Level. However, there is ongoing anecdotal evidence that the decision is heavily influenced by the lack of appropriate recognition for achievement at Foundation Level by the providers of third level education and by employers.

Students choosing whether to opt for the Foundation or the Ordinary course may have legitimate aspirations for further study or other career paths. Their perception that pursuing the Foundation Level course shuts off many options is not unreasonable. A brief survey of entry requirements for a variety of third level courses in a range of institutions reveals that in very many cases, achievements on the Foundation Level course are not accorded the recognition they should clearly deserve and for many, do not satisfy basic entry requirements. The Department of

Education and Science should establish a system of proper equivalences between the different subject levels at Leaving Certificate to assist third level institutions and employers. This may go some way to addressing the problem of lack of recognition for Foundation Level Mathematics.

There are many third level courses with a significant mathematical content that builds directly on content-specific knowledge and skills of the Leaving Certificate Ordinary Level course. Clearly, the Foundation Level course does not prepare students adequately for such courses. However, there appears to be a vast array of courses which do not fall into this category and which nonetheless exclude Foundation Level candidates, irrespective of the grade achieved. The wisdom of the institutions concerned in making this determination is questionable. Foundation level students with high grades may be better prepared to cope with such courses than those with moderate or poor grades at Ordinary Level. They have demonstrated clearly their mastery of a significant body of mathematical knowledge; they are competent in the application of basic mathematical concepts and skills to a range of relevant situations; and they are generally well motivated and positively disposed towards further learning.

It is strongly recommended that such admissions policies be carefully reviewed, particularly in the light of very recent research that identifies poor competency in mathematics as a major reason for dropout from third level courses. Employers and their representative bodies should also review their policies in this area.

5.4 Syllabus Review

Given that the proportion of students taking the various levels has stabilised over the last few years at a ratio significantly out of line with that originally envisaged, it is now appropriate to undertake a comprehensive review of the syllabus. It is recommended that this be done, with a particular emphasis on the following issues:

- The need to determine clearly the appropriate target group for each of the three levels, taking cognisance of the situation pertaining in other subjects, of the existing reluctance of candidates to take the Foundation Level
- Once appropriate target groups are agreed, the need to ensure that the performance criteria, while remaining challenging, are achievable by the target groups.

6. General Recommendations

- Teachers and students are strongly advised to review the recommendations and advice in the Chief Examiner's Reports on the Leaving Certificate Mathematics Examinations, 1995 and 2000, and on the Junior Certificate Mathematics Examinations, 1996 and 1998. These reports were issued to all schools and are available on the Department's website: www.irlgov.ie
- Students are advised to guard against overuse of the calculator and omission of intermediate work and should refer to *Calculators: Guidelines for Second-level Schools*, which will be issued to schools in the near future, for direction in these and other calculator issues
- The NCCA should be requested to review the Leaving Certificate Mathematics syllabus, with a particular emphasis on the following issues:

- the need to determine clearly the appropriate target group for each of the three levels, taking cognisance of the situation pertaining in other subjects, of the existing reluctance of candidates to take Foundation Level
 - once appropriate target groups are agreed, the need to ensure that the performance criteria, while remaining challenging, are achievable by the target groups
- The current programme of In-Career Development for teachers of mathematics in Junior Cycle should be maintained. It should furthermore have a brief for advising upon and making available support materials for mathematics teaching
 - The Department of Education and Science should undertake to establish a system of proper equivalences between different levels of the subject at Leaving Certificate as a guide to third level institutions, employers, students and their parents
 - The admissions policies of third-level institutions regarding recognition of Foundation Level Mathematics should be carefully reviewed. So also should the policies of employers and their representative bodies
 - Research should be undertaken to determine accurately the extent and effects of the increase in part-time work being undertaken by second level students. If this research confirms significant changes in student lifestyle, the matters should be urgently addressed
 - A sample of principals and teachers should be surveyed to ascertain their perspectives regarding unsatisfactory performance in mathematics examinations and their suggestions for remediation.

Appendix 1

STATISTICS FOR MATHEMATICS – ORDINARY LEVEL: 1999, 2000, 2001

1999	A1	A2	B1	B2	B3	C1	C2	C3	D1	D2	D3	E	F	NG	TOTALS
All candidates	4518	3871	3913	3795	3711	3474	3357	3199	3031	2817	3093	3677	1562	170	44188
% of Total	10.2	8.8	8.9	8.6	8.4	7.9	7.6	7.2	6.9	6.4	7.0	8.3	3.5	0.4	
Female	2838	2324	2246	2187	2082	1852	1766	1636	1514	1370	1524	1796	710	67	23912
% of Female	11.9	9.7	9.4	9.1	8.7	7.7	7.4	6.8	6.3	5.7	6.4	7.5	3.0	0.3	
Male	1680	1547	1667	1608	1629	1622	1591	1563	1517	1447	1569	1881	852	103	20276
% of Male	8.3	7.6	8.2	7.9	8.0	8.0	7.8	7.7	7.5	7.1	7.7	9.3	4.2	0.5	

2000	A1	A2	B1	B2	B3	C1	C2	C3	D1	D2	D3	E	F	NG	TOTALS
All candidates	2942	3119	3491	3752	3709	3686	3614	3473	3092	2760	3205	3476	1670	225	42214
% of Total	7.0	7.4	8.3	8.9	8.8	8.7	8.6	8.2	7.3	6.5	7.6	8.2	4.0	0.5	
Female	1708	1834	1999	2147	2046	1978	1942	1838	1621	1463	1552	1728	781	86	22723
% of Female	7.5	8.1	8.8	9.4	9.0	8.7	8.5	8.1	7.1	6.4	6.8	7.6	3.4	0.4	
Male	1234	1285	1492	1605	1663	1708	1672	1635	1471	1297	1653	1748	889	139	19491
% of Male	6.3	6.6	7.7	8.2	8.5	8.8	8.6	8.4	7.5	6.7	8.5	9.0	4.6	0.7	

2001	A1	A2	B1	B2	B3	C1	C2	C3	D1	D2	D3	E	F	NG	TOTALS
All candidates	2643	3013	3369	3278	3327	3155	3146	2917	2774	2501	3238	4056	2228	330	39975
% of Total	6.6	7.5	8.4	8.2	8.3	7.9	7.9	7.3	6.9	6.3	8.1	10.1	5.6	0.8	
Female	1578	1825	1994	1926	1856	1692	1676	1509	1451	1269	1626	2016	1096	117	21631
% of Female	7.3	8.4	9.2	8.9	8.6	7.8	7.7	7.0	6.7	5.9	7.5	9.3	5.1	0.5	
Male	1065	1188	1375	1352	1471	1463	1470	1408	1323	1232	1612	2040	1132	213	18344
% of Male	5.8	6.5	7.5	7.4	8.0	8.0	8.0	7.7	7.2	6.7	8.8	11.1	6.2	1.2	

Appendix 2

**EXTRACT FROM LEAVING CERTIFICATE ORDINARY LEVEL
CHIEF EXAMINER'S REPORT, 2000**

4. OVERALL GENERAL COMMENT

A number of patterns are apparent from the detailed analysis presented in section 3.

Procedural Competence / Conceptual Understanding

It is clear from a range of questions throughout the two papers that candidates' strengths lie in the area of competent execution of routine procedures in familiar contexts. The evidence suggests, therefore, that fundamental objective B of the syllabus, (*instrumental understanding*), is being achieved quite well. It is arguably being achieved better than the lower order objective A (*recall*), as indicated by, for example, the notable difficulty with the accurate reproduction of standard formulae.

There is a significant weakness regarding sound conceptual understanding of much of the material, with corresponding weaknesses in its application in contexts which, though familiar, do not mimic well-rehearsed examples precisely, or do not contain the standard "trigger phrase". It is particularly noticeable when more than one idea or skill is involved. This indicates that objectives C and D, *relational understanding* and *application*, are not being as well achieved as might be hoped. This is the case not only with advanced material, but also with quite fundamental concepts and skills. Worthy of particular note here is the extent to which basic algebraic skills manifest as isolated mechanical procedures without underlying understanding or synthesis. Whereas this is often sufficient for survival with very familiar routine exercises, it is a serious disadvantage when any degree of higher order application is required.

The marking of mathematics examination papers has traditionally not placed a major emphasis on syllabus objective E (*psychomotor* and *communicative* skills). Perhaps understandably in that context, candidates frequently do not present work in a coherent fashion. Relevant supporting work is often not furnished and coherent explanations for answers offered, expressed in clear language, are rare.

The profile of skills described above is not entirely unexpected. The overall patterns of answering this year are generally in line with those of the past. Furthermore, international studies of achievement in mathematics, conducted with respect to younger age groups, have identified a tendency for Irish students to perform better than their international counterparts on routine execution of skills and worse on understanding of concept. This mirrors the above observations closely.

Changes in Strengths and Weaknesses

In terms of the generic skills displayed, there were no significant changes in the strengths and weaknesses displayed by the candidates in comparison with previous years.

Considered by topic, the relative strengths of the candidates in the various content areas remain for the most part unchanged. However, *Probability* and *Linear Programming* both showed improvement, whereas there was a marked decrease in the extent to which average and below average candidates knew the co-ordinate geometry formulae.

Examination Technique

A number of candidates did not perform as well as they might have, due to a poor approach to the paper, most especially in relation to the number of questions attempted. Whereas only 3% of candidates did not attempt six questions on paper 1, the corresponding figure for paper 2 was 12.9%. The examining team reported many instances of candidates scoring well on two, three or four questions and yet failing to attempt the full quota. The competence displayed by such candidates in the questions they attempted is therefore not reflected in their overall performance. This deficiency in the approach of candidates indicates either that large portions of the course were not covered at all, or that they were poorly prepared for the mechanics of sitting an examination of this type.

It is also common (indeed more so) to encounter scripts in which more than the required number of questions have been answered. On the sample of 3400 scripts (1680 paper 1 and 1720 paper 2), a total of 1415 excess questions was recorded. The circumstances in which it is appropriate for a candidate to attempt excess questions are not such as to justify this large a number. Examiners reported many cases in which the candidate's decision in this regard seemed inappropriate; quite often candidates attempted excess questions despite omitting parts of other questions. These candidates would have been better served by devoting the time to completing the required six.

In addition, there were some candidates who, on paper 2, answered six questions from section A and none from section B, or four from section A and two from section B.

5. RECOMMENDATIONS FOR TEACHERS AND STUDENTS

In order to achieve a high level of success, students need a solid conceptual understanding of the material on the syllabus. Teachers should not assume that because students can work through a set of similar closed-form exercises successfully that they will then have sufficient understanding to identify contexts where that knowledge is required, apply the mathematical tools appropriately and interpret the results correctly. This, of course, is not a problem that can easily be addressed at senior cycle in isolation from the earlier mathematical experiences of the students. Many of the difficulties arise from inadequate grasp of the fundamental concepts that are carried forward from Junior Cycle. They need to be addressed co-operatively by teachers on a whole school basis.

Whereas the rote memorisation of a procedure without addressing underlying concepts may appear to overcome a problem in the short term, it is ultimately unhelpful to the long-term strategy of maximising students' competence and interest in mathematics. Methodologies should be chosen with a view to developing understanding and students should be encouraged to question and discuss the mathematics they are working on. Insofar as possible, situations in which students are executing procedures the rationale for which they do not understand should be avoided. Also, when students are working on material that links conceptually to another area, or relies on skills developed therein, then these links should be made explicit and developed, both for reinforcement of understanding and so that students see mathematics as a coherent and sensible body of knowledge.

Students should be encouraged to apply a range of strategies to dealing with a particular problem. In class, if a variety of solutions to a problem are available, teachers should endeavour to draw as many solutions from the students as possible. Solutions that involve the application of knowledge from other content areas should be particularly encouraged and complimented. This approach encourages students to think more about the meaning of what they are doing, reinforces knowledge throughout the course and leaves the students better equipped to handle applications that are more complex, require multiple skills, or are set in a less familiar context. Also, teachers should avoid always phrasing questions and problems in a given topic in exactly the same way, as this can encourage an over-reliance on trigger phrases or words, at the expense of clear problem analysis.

Students should be encouraged always to present work in a coherent way and to communicate, both orally and in writing, the rationale behind what they are doing. Such clarity of communication will help both students and teachers to identify and rectify deficits in understanding.

It is clear, both from the continuing relatively high failure rate and from the type of work presented by the candidates who are failing, that there are significant numbers of candidates who are wholly unsuited to taking this examination. This should be evident to teachers quite far in advance of the examination. It is difficult to see what purpose is served by students continuing to follow a programme that quite evidently is not meeting their educational needs. The quality of the mathematics learning experiences such students will have had on leaving the system is not satisfactory. Where it is clear that students are not capable of engaging with the material in a meaningful way, such students should be actively encouraged at an early stage to follow a programme designed to meet their needs.

In addition to the general recommendations above, the following specific recommendations will assist teachers and students in preparing for the examination:

- Candidates should be encouraged to show all of their work as clearly as possible. They should be aware that most of the marks awarded to the majority of candidates are awarded for work that has not ultimately yielded the correct answer but nonetheless has considerable merit. *It should also be noted that correct answers without supporting work do not necessarily yield full marks.*
- Candidates should take special care in showing work when using calculators. There is a tendency to write down answers without indicating what calculation has led to them. Estimation and other error-tracking strategies appear to be lacking in many cases. Candidates would be well advised to:
 - write down what they are about to calculate before doing so
 - write down intermediate stages of calculations that are particularly complex
 - estimate the expected answer to a calculation before performing it, and critically evaluate any answer to a calculation
 - check answers by repeating calculations, by applying inverse operations, or through the use of other appropriate strategies
 - take care that the calculator is in the appropriate mode of angle measure for trigonometric work
 - use the same model of calculator in the examination as they are used to using in class.

- Candidates should not rub out or otherwise obliterate cancelled work. A single line drawn through it is sufficient. There is frequently more merit in work that candidates have cancelled than in their repeated efforts. In most cases this can be counted to the benefit of the candidate. For this reason, the use of pencil as the primary writing instrument should be discouraged.
- Candidates should be familiar both with the structure of the examination papers and with the style of question asked. In practising on past papers, note also the general recommendation above that there is significant learning potential in addressing a variety of solutions to a given problem.
- Candidates should have practised on questions from past papers under timed conditions. Such practice results in candidates being less likely either to rush questions, leading to errors or omissions, or to spend too much time on a few questions, resulting in an inability to attempt the required six.
- Candidates should always attempt the required number of questions. Excess questions should only be attempted under special circumstances.
- Having answered a question, candidates should read through it again to ensure that they have not omitted any parts.
- Candidates should not pre-select the questions they intend to do. The questions should all be read carefully at the start of the examination and a choice made thereafter. Time for reading through the paper carefully at the start and time to check solutions afterwards, as recommended in the previous bullet, should be budgeted for when planning the time to allocate to each question.
- Where diagrams are required, they should be drawn as accurately as possible, with a sharp pencil. Care should be taken in the positioning of axes for co-ordinate geometry and in the choice of an appropriate scale.
- Candidates should use sketches and diagrams to aid them in clarifying what is required in questions on visual topics. Frequently, the appropriate strategy required to solve an unfamiliar problem becomes much more apparent when a diagram is drawn.
- Candidates should ensure that they know all of the required formulae *accurately* by rote. This is particularly crucial in co-ordinate geometry, but also applies to the formulae required for sequences and series, the roots of a quadratic, and the trigonometric ratios in a triangle.
- Candidates should be familiar, if not with the detailed workings of the marking schemes, certainly with the general principles by which they operate. This will encourage them to follow the other recommendations above. Teachers should consider applying similarly designed schemes to their class and school based written assessments, as appropriate. As well as encouraging good practice, this may have the further benefit of improving the predictive validity of school tests regarding ultimate performance in the Leaving Certificate Examination.
- Teachers should use function notation when dealing with differentiation from first principles.

Appendix 3

Extract from Junior Certificate Ordinary Level Chief Examiner's Report, 1998

1. General Comments

The following key areas should be concentrated on in order to facilitate improvement in the candidates' understanding and answering of questions on examination papers:

- Basic arithmetic, especially division and decimal point manipulation
- Basic algebra, especially transposition, distributive law, multiplication and division of polynomials
- Solutions of simultaneous and quadratic equations
- Fractions and the concept of common denominator
- The concept of a function; functional notation
- Scientific notation
- Inequalities
- Questions based on a drawn graph, e.g. estimating answers to $f(x) = 1$, etc.
- The need for proper layout of answers.