LEAVING CERTIFICATE EXAMINATION

2000

MATHEMATICS

HIGHER, ORDINARY AND FOUNDATION LEVELS

CHIEF EXAMINER'S REPORT

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PREAMBLE

In 2000, the Department of Education and Science offered Leaving Certificate examinations in Mathematics at three levels - Higher, Ordinary and Foundation.

The current syllabuses in Mathematics for the Higher and Ordinary courses were introduced in September, 1992 and were examined for the first time in June 1994. The syllabus for the Foundation course was introduced in 1995 and examined in 1997. It replaced the Ordinary Alternative course which had been introduced in 1990 and was examined for five years, starting in 1992.

Table 1 below shows the numbers and percentages of candidates who took the three levels of Mathematics examinations from 1993 to 2000:

Year	1993	1994	1995	1996	1997	1998	1999	2000
Total Number of Candidates	58,922	60,524	63,200	53,246	61,035	61,970	60,637	58,705
Number of Higher Level Candidates	6,595	8663	10,639	9,470	11,042	10 723	10696	10,645
as a % of Total	11.2	14.3	16.8	17.8	18.1	17.3	17.6	18.1
Number of Ordinary Level Candidates	46,158	45,442	46,698	38,378	43,566	45,191	44,188	42,214
as a % of Total	78.3	75.1	73.9	72.1	71.4	72.9	72.9	71.9
Number of Ordinary Alternative/Foundation Level Candidates	6,169	6,419	5,863	5,398	6,427	6,056	5,753	5,846
as a % of Total	10.5	10.6	9.3	10.1	10.5	9.8	9.5	10.0

Table 1: Uptake of Leaving Certificate Mathematics Higher, Ordinary, Foundation Levels 1993-2000

The statistics for 1993 are included in Table 1 in order to illustrate the changes that were associated with the introduction of the new syllabuses. Of particular note is the increase of 34.1% in the number of Higher Level candidates who took the examination from 1993 to 1994.

Overall, the numbers of Higher Level candidates have increased significantly since the syllabuses were changed. In the first examination in 1994 there were 8663 Higher Level candidates. This increased to 10 645 in 2000. The proportion of Leaving Certificate Mathematics candidates who sat the Higher Level examination reached its highest in 2000 at 18.13%, slightly higher than 1997 when it was 18.09%. These changes are very much welcomed.

The significant decrease in the number of candidates who took Mathematics in 1996 was due to the increased uptake of the Transition Year Programme in 1994.

HIGHER LEVEL

1. INTRODUCTION

The examination consisted of two papers, each of two hours and thirty minutes duration.

A total of three hundred marks was allocated to each paper.

On paper 1, candidates were requested to attempt six out of a total of eight questions. Each question carried fifty marks.

On paper 2, candidates were requested to attempt five questions from Section A and one question from Section B. Section A consisted of seven questions each of which examined the core part of the syllabus. Section B consisted of four questions, one on each of the four options on the syllabus. Each question carried fifty marks.

2. PERFORMANCE OF CANDIDATES

Table 1 gives a summary of the results of the examination for the whole cohort, for females and for males. Table 2 shows the percentages of candidates who obtained A grades, high grades (that is, A, B or C), D grades and low grades (that is, E, F or NG) in 2000 and in the two preceding years. A detailed break-down of the examination statistics for 1998, 1999 and 2000 is provided in the Appendix at the end of this Report.

Mathematics – Higher Level - 2000								
Grade	**A**	**B**	**C**	**D**	**E**	**F**	**NG**	**Total Candidates**
Number	1496	3009	3403	2212	406	100	19	10 645
% of Total	14.0	28.3	31.9	20.7	3.8	0.9	0.2	
Total Female	655	1499	1611	950	157	36	5	4913
% Female	13.3	30.5	32.8	19.3	3.2	0.7	0.1	
Total Male	841	1510	1792	1262	249	64	14	5732
% Male	14.7	26.3	31.3	22.1	4.3	1.1	0.2	

Table 1: Summary of Results in 2000

	Mathematics – Higher Level			
	% A Grades	**% High Grades (A, B, C)**	**% D Grade**	**% Low Grades (E, F, NG)**
1998	18.8	83.7	13.2	3.0
1999	17.4	82.3	15.0	2.8
2000	14.0	74.2	20.7	4.9

Table 2: Percentages of A, High, D and Low Grades Awarded in 1998, 1999 and 2000

As evident from Table 1, Table 2 and the Appendix the results of the 2000 examination were lower than those of the 1998 and 1999 examinations. Two differences are particularly noticeable—the drop in the percentage of A grades awarded and the increase in the percentage who obtained E, F or NG. There were undoubtedly reasons for this slippage in the quality of candidates' work. The first step in the search for these reasons leads naturally to consideration of the level of difficulty of the questions asked and the manner in which candidates' work was marked. However, neither of these was considered to be significantly different to previous years. This suggests that other factors were at work.

One possible such factor relates to the increased proportion of the Mathematics cohort that sat the Higher Level examination this year. While this increase is very much welcomed it may have had some bearing on the profile of results. The number of candidates who sat Mathematics was almost 2000 lower than the 1999 figure. A proportionate change (3.2%) in the Higher Level candidature would have resulted in a drop of 341 candidates. However, the actual decrease was 51 candidates (0.5%). This suggests that there were approximately 300 candidates who took Higher Level in 2000 but who would have been likely to opt for Ordinary Level had they sat the examination in 1999. These candidates constituted almost 3% of the Higher Level cohort. It is reasonable to suggest that they were likely to have received modest marks in the examination and that this influx contributed to slippage in the overall grade profile. Support for this line of thinking lies in the lower A rate in the Ordinary Level examination this year - this reduced percentage probably reflects the absence of the before-mentioned group who would have performed strongly had they taken Ordinary level.

A further point worthy of some mention is the decrease in the number of school repeat candidates who took Higher Level Mathematics. This fell from 655 in 1999 to 585 in 2000. Although the drop is numerically small, the lower number of candidates taking the examination for the second time and after an extra year attending school is likely to have been partially accountable for the reduction in the high grades awarded.

It is recognised that there could be other factors that influenced the overall standard of candidates' answering. However, it would be speculative to posit reasons unsubstantiated by concrete evidence and this is not the purpose of this Report.

Comparison of the performance of males and females reveals some gender differences. The females outperformed the males in all but one respect namely, the award of the highest grade. A1 was achieved by 7.8% of males as opposed to 6.3% of females. Of particular note is the lower percentage of females who were awarded low grades - 4.0% of females compared with 5.8% of males received E, F or NG.

3. ANALYSIS OF PAPER

Question Popularity, Percentage Uptake and Average Mark

Table 3, Table 4 and Table 5 which follow show the ranking of questions in order of decreasing popularity. They also give the percentage uptake and the average of the marks awarded to those who chose each question. These figures are derived from an analysis of a sample of 4% of total scripts.

Order of Popularity	Question Number	Topic(s)	Percentage Uptake	Average Mark (out of 50)
1	1	Algebra	99%	41
2	2	Algebra	98%	37
3	3	Complex numbers	95%	35
4	6	Differential calculus	95%	36
5	8	Integral calculus	94%	37
6	7	Differential calculus	92%	36
7	5	Series, induction, logarithms	35%	32
8	4	Sequences and series	25%	30

Table 3: Order of Popularity, Percentage Uptake and Average Mark for Paper 1

Order of Popularity	Question Number	Topic(s)	Percentage Uptake	Average Mark (out of 50)
1	1	Co-ordinate geometry of circle	95%	40
2	3	Co-ordinate geometry of line/transformation geometry	93%	35
3	4	Trigonometry	90%	40
4	2	Vectors	78%	35
5	5	Trigonometry	75%	34
6	6	Probability and difference equations	72%	35
7	7	Combinations, probability and statistics	48%	25

Table 4: Order of Popularity, Percentage Uptake and Average Mark for Section A on Paper 2

Order of Popularity	Question Number	Topic(s)	Percentage Uptake	Average Mark (out of 50)
1	8	Further calculus and series	95%	28
2	10	Further probability and statistics	3%	38
3	9	Groups	2%	43
4	11	Further geometry	0%	-

Table 5: Order of Popularity, Percentage Uptake and Average Mark for Section B on Paper 2

Paper 1 Question 1

This question was both the most popular and the highest scoring on the paper (Table 3). It was attempted by 99% and the average mark awarded was 41.

(a) The most common approach to this generally well-answered question was to add by taking $(x-2)(2-x)$ as denominator—the fact that $(2-x)$ could be replaced by $-(x-2)$ was seldom recognised. Reducing the resulting fraction to a constant was often problematic. Many stopped at $\dfrac{-12+12x-3x^2}{-4+4x-x^2}$. Others invented an equation, for example $-3x^2+12x-12=0$ and proceeded to solve it.

(b) The proof of the factor theorem was well known. The usual approach was to show that $x-k$ is a factor of $f(x)-f(k)$ and, since $f(k)$ is zero, the result follows. A common mistake was to factorise x^3-k^3 incorrectly—for example, $(x-k)(x^2-k^2)$.
The long division method was not popular. The majority of candidates who took this approach were not fully successful.

(c) This part was attempted by most and was generally well answered. Dividing by $x^2-2tx+t^2$ and putting remainder equal to zero or simplifying $(x-t)^2(x+a)$ and equating coefficients were the two methods used. A frequently encountered blunder was: $(x-t)^2=x^2-t^2$.

Paper 1 Question 2

As Table 3 shows, this question was the second most popular. It was attempted by 98% of candidates. The average mark awarded was 37.

(a) The vast majority of candidates solved the three simultaneous equations successfully. Those who made arithmetical slips tended to end up with difficult calculations and travelled a much more torturous route than the smooth passage to integer solutions which careful arithmetic ensured.

(b) The majority chose to factorise the quadratic and to derive the roots. This was usually done successfully. The quadratic formula was also frequently used.
The "Hence" part
was inclined to cause problems. Trial and error solutions were often offered. Many multiplied out the given equation and arrived at $x^4-2x^3-16x^2-8x+16=0$ which they could not solve. Those who put $\left(x+\dfrac{4}{x}\right)=a$ tended to proceed well.

(c) Candidates' overall difficulties with factorisation and inequalities surfaced in this question. Part (i) was answered reasonably often but in general, with poor results. Recognising that $a^4-b^4=(a^2)^2-(b^2)^2$ escaped many. Very few could break the expression correctly into three factors. A common incorrect approach was: $a^4-b^4=(a-b)(a^3-b^3)=(a-b)(a-b)(a^2-b^2)$.

Only about a half of scripts carried attempts at part (ii) and results were poor. The fact that errors in factorising $(a^4 - b^4)$ were not penalised again was a benefit to many.
Showing the inequality was rarely attempted and seldom done so with success. Only those who transposed $a^4b + ab^4$ and viewed the required inequality as ">0" made worthwhile progress.

Paper 1 Question 3

This question, similar to question 6, was attempted by 95% of candidates (Table 3). Average mark was 35.

(a) Very good knowledge of getting inverses of matrices and matrix multiplication was evident. Mistakes were usually of minor nature and related to the determinant, arithmetical calculations and incorrect rearrangements of matrix entries.

(b) In part (i), candidates displayed a good understanding of dividing complex numbers using the conjugate. For those who opted for other methods, such as $\frac{-2+3i}{3+2i} = a + bi \Rightarrow (3+2i)(a+bi) = -2+3i$, then multiplied out the left hand side and equated "real with real" etc, the work was harder and carried much potential for error. Similarly, those who converted i to polar form in order to calculate i^9 tended to experience difficulties.
For part (ii), expanding and equating corresponding parts was the most common approach. This was well done as far as $a^2 - \frac{16}{a^2} = 15$. However, difficulties with algebraic manipulation manifested themselves then—for example: $a^4 - 15a^2 = 16 \Rightarrow a^2(a^2 - 15) = 16 \Rightarrow a^2 = 16$ or $a^2 - 15 = 16$ was often encountered.

(c) A correct statement of De Moivre's theorem was usually offered in part (i). However, difficulties in using this to prove the given trigonometric identity were widespread. A common incorrect approach was to use $\cos 3\theta = \cos(2\theta + \theta)$.
The standard of answering in part (ii) was good. Using an incorrect argument, $\frac{\pi}{6}$ for example, or omitting to cancel the $\frac{1}{2}$ with 2^{10} at the end were the most reasons for loss of marks.

Paper 1 Question 4

This question was attempted by 25% of candidates (Table 3). The average mark awarded was 30. It was both the least popular and the lowest scoring on the paper.

(a) This was poorly answered. Work presented pointed to a serious lack of knowledge of geometric sequences.

(b) Scoring in this part was high. The most common blunders related to incorrect handling of indices, $2.2^n = 4^n$, for example. Most who attempted the question did this part and little more.

(c) Marks awarded in (c) tended to be low. Few recognised the arithmetic-geometric series in (i). For those who did progress was often hindered by taking the wrong approach to arriving at the expression for $g(x)$ or by incorrect handling of the sum to infinity of the G.P.
Part (ii) was rarely attempted and very badly answered.

Paper 1 Question 5

This question was attempted by 35% of candidates (Table 3). The average mark awarded was 32. It was the second least popular and the second lowest scoring on the paper.

(a) This recurring decimal proved to be unattractive—only half of those who chose the question attempted writing it in the required form. Although some just wrote down the answers without showing any work most worthwhile attempts involved summing the G.P to infinity.

(b) This was attempted by almost all those who chose the question. Answering was remarkably good.

(c) Overall, part (i) showed the standard of answering involving logarithms to be higher than previous years—there was, for example, less tendency to "drop logs across". However, not rejecting the negative root (minus three) was a common cause for loss of marks. Incorrect factors of the quadratic, $x^2 - 15x - 54 = (x-6)(x-9)$ for example, appeared often.
Answering in (ii) was good. The most common approach was to substitute a dummy variable for e^x and to work with the resulting quadratic. A common cause for penalty was failure to replace the chosen variable by e^x and to give values for e^x as solutions instead of values for x. This type of omission is worthy of note as it was identified as a common occurrence.

Paper 1 Question 6

This question was attempted by 95% of candidates (Table 3). The average mark was 36.

(a) Part (i) was very well answered with most work meriting full marks. A few candidates expanded the cubed expression and differentiated term by term. The quotient rule tended to be applied directly in (ii). It was successfully completed in the majority of cases. Occasionally, the product rule with $(x-3)^{-1}$ was used.

(b) Candidates' answering in (i) indicated that this proof was not well known. In only very few cases was the work fully correct and complete. The most common approach was $f(x)$ and $f(x+h)$ etc. It was observed that those who

took the y and $y + \Delta y$ route tended to perform better indicating that, perhaps, students find this method easier to understand.
Part (ii) was also poorly done. The favoured approach was to differentiate using the formula from page 41 of the Mathematics Tables but the majority overlooked applying the chain rule. The implicit differentiation method appeared rarely.

(c) Answering in (i) was fair. Many who got to $\lim_{x\to\infty} y = 0$ did not give the asymptote to be $y = 0$ and consequently suffered a penalty. The most frequent award in (ii) was the attempt mark. There was much evidence of poor understanding of turning points and points of inflection. Part (iii) was seldom completed. Inability to proceed from $(x_1 + 1)^2 = (x_2 + 1)^2$ proved to be a definite stumbling block even in otherwise good answering.

Paper 1 Question 7

This question was attempted by 92% of candidates (Table 3). The average mark awarded was 36.

(a) There was wide variation in the standard of answering to part (i). In many answers the need for implicit differentiation was recognised and the method was worked through correctly. However, failure to use the product rule and misuse of the chain rule were common occurrences when answering was weak.

(b) Part (i) was fairly well answered. Lack of understanding of the chain rule often resulted in a loss of marks here. The substitution in (ii) was usually done successfully. However, the subsequent simplification to the required form was problematic for many, particularly for those who had made mistakes in (i).

(c) Part (i) was attempted by most but with widely varying degrees of success. Candidates were expected to find the one critical point by setting the derivative to zero, to identify this as a maximum and to find the value of the function at this point. Answers tended to be patchy and rarely were marks higher than attempt marks awarded. Part (ii) was ignored by the vast majority. The approach adopted in the small number of successful answers was to take the logarithm of both sides.

Paper 1 Question 8

This question was attempted by 94% of candidates (Table 3). The average mark awarded was 37 making it similar to question 2 in this respect.

(a) Answering was excellent in both (i) and (ii). Full marks were frequently awarded. A welcome difference this year was that the constant of integration was usually included in answers.

(b) Part (i) was both frequently attempted and well done. Work often earned full marks. It was noticeable that errors tended to be made when the formula for the integral of $\sin^2 x$ given on page 42 of the Mathematics Tables was used—for example, $\frac{1}{2}\left[3\theta - \frac{1}{2}\sin 6\theta\right]$ was a common result. Part (ii) was also well done. The merit in substituting $u = x^2 + 4$ was recognised by most. Again misuse of the formulae in the Mathematics Tables was evident—for example, results involving $\tan^{-1}\frac{x}{2}$ were not unusual.

(c) Answers in part (i) tended to merit either close to full marks or zero—those who started on the right track performed very well while token attempts at the integral were inclined to be worthless. Errors in completing the square, $x^2 - 4x + 5 = (x-2)^2 + 9$ for example, were common. Weak answers included: $\int \frac{dx}{x^2 - 4x + 5} = \int \frac{dx}{x^2} - \int \frac{dx}{4x} + \int \frac{dx}{5}$ and $\ln(x^2 - 4x + 5)$. The correct solution in (ii) was given in only a very small number of cases. The quality of answering suggested that the concept of using integration to find area was poorly understood. It was noticeable that hardly any attempts to solve the problem by using the area between the curve and the *y* axis were encountered.

Paper 2 Question 1

This was the most popular question on paper 2. It tied with question 4 as the highest scoring question on core material (Table 4).

(a) The majority of answers earned full marks here. Most used the slope of the normal to reach the required solution. There were some errors in finding the correct perpendicular slope. Occasionally, the work finished when the slope of the normal was found.

(b) The majority determined successfully the coordinates of the centre and radius of each circle. The only prevalent error was the placing of a plus sign before the *c* in the radius formula.
The condition for circles touching externally was well known. A point noted was that a surprisingly large number of candidates arrived correctly at $5 + \sqrt{136 - k} = 15$ and then squared each side of this to determine the value of *k*. While they usually arrived at the correct result it would have been easier and less time-consuming to move directly to $\sqrt{136 - k} = 10$ and simply solve for *k*.
In general, scoring was high with answers often deserving full marks.

(c) There was seldom any difficulty in finding *m*.
The most common approach to finding the two circles was to create three equations from the data given. The work presented was very disappointing. Many just substituted one point into the general equation and stopped. The lack of perseverance displayed was remarkable and surprising particularly as a very similar question was given in 1999.

Paper 2 Question 2

This question was attempted by 78% of candidates (Table 4). The average mark awarded was 35.

(a) This was very well done with most answers earning full marks. In a small number of cases only one value for *t* was offered.

(b) The correct value of *k* was found by the majority of candidates. Those who used $\vec{a} \perp \vec{b} \Rightarrow \vec{a}.\vec{b} = 0$ tended to be successful. Sometimes, $\vec{a} \perp \vec{b} \Rightarrow \vec{a}.\vec{b} = -1$ was taken as the basis of work and a penalty was incurred. Another approach, one that involved $\vec{a}^{\perp}$, was inclined to be unsuccessful. Those who took this route found $\vec{a}^{\perp} = -(2k+3)\vec{i} + 2\vec{j}$ correctly; equated $-(2k+3)$ with k^2 (which is incorrect as $\vec{a}^{\perp} = \lambda\vec{b}, \lambda \in \mathbf{R}$); solved the resulting quadratic and arrived at two values of *k*.

Part (ii) caused few difficulties except for those who had found two values of *k*. Candidates' answering in (iii) indicated a clear understanding of how to find the angle between two vectors. A surprisingly frequent misreading led to finding the angle between $\vec{a}$ and $\vec{b}$.

(c) Answering in (i) was very good. In a small number of cases $\vec{pq}$ was taken as $\vec{p}.\vec{q}$.
In (ii) work presented tended to merit either full marks or zero. Almost 70% of candidates received full marks. Cancelling vectors and expressing $\vec{p}.\vec{q}$ as a vector rather than a scalar were the main reasons that attempts were worthless.
Part (iii) tended to be left undone. Those who were successful used $\vec{s} = k\vec{r}$ to find *m*.

Paper 2 Question 3

This question attempted by 93% of candidates was the second most popular in Section A (Table 4). The average mark awarded was 35.

(a) Full marks were the usual award for this part. When errors occurred they related to the incorrect slope of *L* or the incorrect perpendicular slope.

(b) A striking feature of answering in (i) was the lack of familiarity with the ratio formula and with its correct application. In a number of cases, the ratio was ignored and the mid-point of [*ac*] was found.
The image points in (ii) were generally found without difficulty.
In (iii), working out $|f(a)f(b)|$ and $|f(b)f(c)|$ in surd was done very well.
However, many candidates were unable to reduce the ratio to 3 : 2. Answers sometimes contained the unnecessary steps of working out $|ab|$ and $|bc|$—this information was given in (i) and these calculations were wasteful of time and effort.

(c) Part (i) was well answered. The co-ordinate geometry area of triangle formula provided the usual route to success. As to be expected, errors occurred in the translation of points. Also, some used the $\frac{1}{2}|x_1y_2 - x_2y_1|$ formula without first bringing a vertex to (0, 0). In a significant number of cases the false assumption was made that triangle *rqu* was right angled with $|uq|$ as perpendicular height. Obviously this was deduced from observing the diagram. Candidates should be cautioned to check always that a solid basis is given in the question for any assumptions they make. The important piece of information that $k > 0$ did not register with many and the eventual result was that two values of *k* were given.

The majority attempted (ii) carrying with them their value of k. The proper approach to finding t was generally known. Incorrect values of k and errors in calculating the slope of sr often led to awkward arithmetic when solving the resulting equations. In a noticeable number of scripts work stopped when the x coordinate of t was found—the y coordinate was overlooked.

Paper 2 Question 4

This question, attempted by 90% of candidates, was the third most popular in Section A. The average mark was 40 making it and question 1 the highest scoring questions on core material (Table 4).

(a) This was well done although there was a tendency to insert π into the final answer.

(b) Performance was good in this part. Converting the equation into a quadratic in $\cos x$ and solving for $\cos x$ were generally done successfully. However, many went astray in finding x. About a third of candidates displayed insufficient understanding of the quadrants to find all the solutions.

(c) The proof of the $\cos(A+B)$ formula was well known by 75% of candidates. The unit circle was normally used. However, some simply wrote down the formula for $\cos(A-B)$ and replaced B by $-B$. In a few cases, a proof based on calculations of the area of a scalene triangle was presented. While the proof was correct it applied only to angles where $A+B<0^\circ$. This restriction in the applicability of the proof was treated sympathetically by the marking scheme and was classed as only a blunder.

Part (ii) was well answered. It was usually done by expanding $\cos(A+B)$ and $\sin(A+B)$. It was seldom recognised that the given expression was the expansion of $\cos[(A+B)-B]$.

Paper 2 Question 5

This was attempted by 75% of candidates (Table 4). The average mark was 34.

(a) The limit in (i) was successfully calculated by most candidates.
By contrast, answering to part (ii) was only fair. It carried much evidence of poor understanding of trigonometric limits. Sin$2x$ was surprisingly problematic—even those who replaced it by $2\sin x\cos x$ had difficulties cancelling and often, $\lim_{x\to 0} 2\cos x = 0$ appeared. L'Hospital's rule featured in about 5% of scripts and was usually applied correctly.

(b) Part (i) was well answered. The popular approach was to replace $\tan A$ by $\frac{\sin A}{\cos A}$ and correct work usually followed. Those who replaced $1+\tan^2 A$ by $\sec^2 A$ tended to be unable to finish.
Answering to (ii) was poor. Many failed to recognise the link with (i) and pursued unproductive routes—expanding $\tan(135^\circ - A)$ for example.

(c) Part (i) was attempted by most and was very well answered.
Many did not attempt (ii) and much poor work was presented. The basic point that $|qs| = |qr| - |sr|$ was often missed. In general, answers lacked structure and direction. Given the lead-in provided in (i), it was surprising that so many failed to use a tan function. Instead, they grasped at the sine rule or even treated *pqs* as right-angled.

Paper 2 Question 6

This question was attempted by 72% of candidates and the average mark was 35 (Table 4). In general, there was much evidence to suggest that candidates' understanding of and ability to apply the fundamental principle of counting was poor. In many cases, answers were written down in the form ${}^nP_r$ or ${}^nC_r$, incorporating some of the numbers given in the question, without any justification or supporting work.

(a) In very many cases the answers given to (i) were worthless—10!, ${}^{10}P_4$, ${}^{10}C_4$, 10.9.8.7 or 9.8.7.6. Part (ii) was also very poorly done—9.9.8.7 and 9.8.7.6 were common incorrect answers.

(b) The difference equation in (i) was solved with ease. The roots of the characteristic equation were usually found correctly and the method for arriving at the general solution was well known but slips in solving the simultaneous equations were prolific.
Finding u_3 in (ii) was very well done although a small number of candidates omitted this part.

(c) Part (i) was well answered. The most common error was to add the probabilities having first found separate probabilities for box A and box B—that is, $\frac{6}{10}\times\frac{5}{9}+\frac{5}{10}\times\frac{4}{9}$ was given.

Answering was poorer in (ii). The most common award was the attempt mark. Order tended not to be recognised—that is, that one could pick a red disc and then a green disc or a green followed by a red. The probability of two even numbered blue discs was often given as $\frac{5 \times 4}{10}$. Again, probabilities were sometimes added.

Very few got (iii) correct. Most attempts consisted of listing outcomes that totalled ten. Apart from this, little work of merit was presented.

Paper 2 Question 7

This question was attempted by 48% and the average mark was 25 (Table 4). It was the least popular question in section A and the lowest scoring on the entire paper.

(a) Answers to both (i) and (ii) were poor. There was widespread failure to recognise the problems posed as being combinations. Many worthless answers were given—for example, 6.5.4.3 or ${}^5C_3$ or ${}^5P_3$ appeared often in (i). This was the lowest scoring part (a) on the paper.

(b) Part (i) was rarely correct mainly because 3! was omitted—that is, the question was interpreted as meaning Jack-Queen-King only, giving the probability to be $\frac{1}{52} \times \frac{1}{51} \times \frac{1}{50}$ and in so doing excluding other permutations such as Queen-Jack-King.
Answers in (ii) were very good with most getting full marks.
Difficulties arising from order surfaced again in (iii). While black-black-diamond was considered a favourable outcome allowances were not made for black-diamond-black and diamond-black-black.

Part (iv) was fairly well answered. However, many calculated the probability for one colour only (red-red-red or black-black-black) without realising that "same colour" included both.

(c) Parts (i) and(ii) were omitted completely by many. The overall lack of attempts here supports an observation made during the marking of previous years' examinations that standard deviation appears to be an unattractive topic for candidates.

Those who attempted (i) performed well, usually gaining full marks. Only a small minority were successful in (ii). The fact that the result was given in the question proved to be of little assistance to candidates.

Paper 2 Question 8

This question was, by far, the most popular of the options (see Table 5). It was attempted by 95% of candidates. The average mark was 28.

(a) Answering showed an understanding of the ratio test but widespread difficulties were evident in handling the limit. Many evaluated $\lim_{n\to\infty}\left|\frac{n+3}{2}\right|$ to be $\frac{3}{2}$. Regardless of the value of the limit obtained few made any reference to the requirement for divergence or convergence so the work was effectively lacking conclusion.

(b) The first application of integration by parts in (i) was well handled. Full marks were obtained by many candidates. However, frequent errors in differentiation such as $\frac{d}{dx}(\cos x) = \sin x$ and $\frac{d}{dx}(e^{2x}) = e^{2x}$ occurred. In about two thirds of cases the substitution $u = \cos x$ was used and the remainder had $u = e^{2x}$. Those who opted for the former encountered fewer difficulties. An abundance of errors occurred in the working of the second integration by parts. As a result, the attempt mark was the most common award.

Candidates who completed (i) successfully had no difficulty in (ii). Those who had incorrect results from (i) usually obtained the attempt mark. This was also the fate of those who did not complete (i) but created a solution in order to proceed with (ii).

(c) A surprisingly high number of candidates made no attempt whatsoever at this part.
The answering presented in (i) was only fair. Many failed to recognise the need to use the theorem of Pythagoras. Some applied it but did so incorrectly. Others were unable to isolate y. A frequent and unexpected error at this level was: $y^2 = r^2 - x^2 \Rightarrow y = r - x$.
In general, those who managed (i) proceeded to express the area as $A = 4x\sqrt{r^2 - x^2}$. However, attempts to find $\frac{dA}{dx}$ were very poor. In many cases the rules of differentiation were badly broken. For example, A was commonly not taken as a product and the following, or a close variation, was offered: $\frac{dA}{dx} = 4.\frac{1}{2}(r^2 - x^2)^{-\frac{1}{2}}(-2x)$. Others attempted to rewrite A so as to apply the chain rule but erred as follows: $A = 4x\sqrt{r^2 - x^2} \Rightarrow A = 4\sqrt{xr^2 - x^3}$.
Only very few candidates, approximately 5%, presented correct solutions.

Paper 2 Question 9

This question was attempted by just 3% of candidates (see Table 5). The average mark awarded was 38. In general, those who opted for this question performed very well.

(a) This was very well answered. The most common error related to the "third" probability—the red or green—where either $\frac{7}{11}$ or $\frac{3}{11}$ was given instead of the correct $\frac{6}{11}$.

(b) Part (i) was very well answered. Converting to standard units caused no difficulties. Many candidates scored full marks.
Part (ii) was also well done. However, errors tended to occur in correctly determining the required area under the normal curve—handling $-0.5 < Z$ was the source of most difficulties—but the penalties incurred were relatively small. Overall, candidates scored well in (b).

(c) Part (i) was generally well answered. Notwithstanding this, choosing between 112 and 115 as the value for the mean caused some confusion and mistakes were made in calculating the standard error of the mean. There was also uncertainty about whether to use a one or two tailed test. Answering in part (ii) revealed much uncertainty about the confidence interval concept.

Paper 2 Question 10

This was attempted by only 2% of candidates (see Table 5). The average mark was 43.

(a) Full marks were usually awarded for part (a).

(b) Parts (i), (ii) and (iii) were very well answered with candidates displaying good understanding of composition and order. Part (iv) proved to be more testing but it was well handled. Some deterioration in answering was evident in (v) where the main cause of loss of marks was that the reasons for the isomorphism were not clearly demonstrated.

Paper 2 Question 11

No candidates attempted this question.

4. OVERALL GENERAL COMMENT

Lest the comments on the performance of candidates given earlier (section 2) be in any way misinterpreted it is important at the outset to stress that much excellent answering was presented. This statement is substantiated by the fact that one candidate in seven (14%) earned an A grade. The hallmark of high achieving scripts was clear accurate work that portrayed a sound understanding of all aspects of the syllabus. The solutions presented carried evidence of ongoing practice and thorough

recent revision. The candidates who produced such high quality work received rewards in terms of high grades for the commitment and effort that they obviously invested in preparing for the examination.

The purpose of this commentary is to highlight features of candidates' answering that may serve to improve standards in future years. It focuses firstly on concrete aspects of the work presented—the choices of questions candidates made, the strengths they displayed and areas in which improvements are necessary. It then considers some general features of the answering.

The pattern of question choice on both papers was in close keeping with that of previous years. On paper 1, algebra and complex numbers were the most popular. Series and sequences (question 4) along with induction and logarithms (question 5) continued to be the least attractive topics. Questions 4 and 5 were often attempted last when the adverse effects of time running out came to bear on the work produced. On paper 2, the majority of candidates depended on the circle, the line, vectors and trigonometry for their core questions. A drift away from discrete mathematics and statistics (questions 6 and 7) towards trigonometry has been noted in recent examinations and this year's answering was in keeping with this. Analysis of patterns in answering suggests that the choice of core questions was often made in advance of seeing the examination papers. This is a practice that is not recommended. It is, of course, unavoidable if all parts of the syllabus have not been studied. There was some evidence of the latter happening—for example, all candidates in some centres omitted the vectors question.

Regarding the options, calculus was again by far the most popular choice. It is noticeable that the average marks awarded for groups and for further probability (43 and 38, respectively) were considerably higher than that for calculus (28). For the first time since the current syllabus was examined in 1994 further geometry (question 11) was attempted by no candidates.

Analysis of the broad spectrum of work presented revealed a range of strengths attributable to the cohort as a whole. Strengths were most evident in procedural type questions where a fixed number of steps were required—for example:

- long division of algebra
- matrix multiplication
- multiplication and division of complex numbers
- application of De Moivre's theorem
- finding u_{n+1} given u_n
- proof by induction
- application of rules of logarithms
- straightforward differentiation
- finding asymptotes
- integration not requiring substitution
- proof of the factor theorem
- application of coordinate geometry formulae
- basic trigonometry
- solving difference equations
- getting images of points under transformations
- vector operations.

Compared with performance in 1999, an improvement in candidates' ability to handle logarithms and integration was noted and welcomed.

Turning to areas in need of improvement, weaknesses in answering were traceable to two broad sources. Firstly, foundation skills in mathematics were often not up to the standard required. Serious deficiencies were evident in algebra—for example, inability to factorise properly was specifically identified by examiners as a contributory factor to the lower profile of results. Also, the frequency with which mistakes in arithmetical calculations and with signs were made was disturbingly high. While it is true that the penalties for such slips are low it must be remembered that the knock-on effects commonly result in subsequent work being more difficult, or even impossible, to complete. So, the overall price paid in terms of marks lost can be significant. Secondly, weaknesses stemmed from inadequate understanding of mathematical concepts and underdeveloped problem-solving and decision-making skills. These inadequacies manifested themselves when questions unfolded in unfamiliar ways or required more than the mechanical application of routine and well-practised algorithms.

In summary, candidates' answering showed weaknesses in the following specific areas:

- handling fractions
- factorising
- handling inequalities
- getting the square root of a complex number
- recognising a recurring decimal as a geometric series
- proving the product rule from first principles
- differentiating inverse trigonometrical functions
- relating knowledge of differentiation to parallel tangents
- finding the coordinates of the maximum point of a logarithmic function
- applying integration to area problems
- remembering coordinate geometry formulae accurately
- evaluating limits
- reducing surds to simplest form
- squaring algebraic expressions
- interpreting probability questions properly
- applying integration by parts the second time
- establishing an equation using given data.

In the context of general observations and with the intention of providing helpful comment, five separate features of candidates' answering will now be identified. These relate more to the manner in which candidates handled the questions rather than the standard or quality of answering.

- Firstly, candidates displayed less perseverance than in previous years—they tended to stop and abandon work when they hit barriers, they rarely offered second attempts and showed weak will to tease out problems. The fact that straightforward early parts of questions were left unfinished and that candidates often wasted time over-explaining themselves suggests that shortage of time was not the root cause of this.
- Secondly, marks were often lost as a direct result of failing to read questions with due care—for example, stopping at two values of k in spite of the instruction to find "the value of k", expressing answers in a form other than that which the question specified and giving $a = \pm i$ even though the question stated $a + ib$ was a complex number (which meant that a was real) frequently incurred penalties for candidates.
- Thirdly, candidates' had some difficulty in building on work already done. In particular, there appeared to be a lack of understanding of the word "hence". This

meant that work was sometimes unnecessarily repeated or that a vital clue to the next part of a question offered by an answer just obtained was missed.

- Fourthly, candidates who presented clear well-structured scripts were usually rewarded for their efforts as an organised approach to answering minimises the risk of omitting necessary work and the consequent loss of marks. The importance of this cannot be over-emphasised.
- Finally, many scripts were completed in pencil. This is a practice that must be discontinued as it results in faint, unclear writing and increases the likelihood of work being misinterpreted.

In summary, scripts that were awarded high grades carried clear well-structured work that reflected a sound understanding of the syllabus. The lower grade profile overall was a result of a higher proportion of answers that displayed weak basic mathematical skills and a poor grasp of important concepts. A tendency to abandon questions that did not work out easily was a feature of this year's answering that set it apart from that of previous years.

5. RECOMMENDATIONS FOR TEACHERS AND STUDENTS

Arising from this year's answering the following advice is offered to those preparing for Higher Level Mathematics examinations:

- read the questions carefully
- do not decide in advance the questions you will answer—wait until you have read the paper
- start each new question at the beginning of a new page
- do not write in pencil
- check work when completed and ensure that your answer fits in with the requirements of the question—for example, it is not sufficient to stop at $x = 2$ if the question says "find the coordinates of t."
- test solutions
- use all the information given in the question—for example, if $k > 0$ is given it has a purpose and this may mean that one of two solutions to an equation must be rejected
- beware of careless mistakes particularly in addition, subtraction etc
- make sure you understand and know all the proofs on the syllabus
- when the word "hence" appears look for a link between question parts—do not start afresh
- double check formulae you write down
- where possible draw sketches and fill them in as you progress so as to be sure that your results make sense
- as you work through your solutions pretend that you are explaining what you are doing to someone else; this should ensure that you show all necessary work
- do not view the syllabus in terms of separate compartments—it is often possible to use knowledge associated with one topic to answer a question on another topic
- written practice is an essential part of examination preparation—relying on memory is not enough.

APPENDIX

Statistics for Leaving Certificate Mathematics
Higher Level 1998, 1999, 2000

1998	A1	A2	B1	B2	B3	C1	C2	C3	D1	D2	D3	E	F	NG	Total
Matamaitic		960	1148	1322	1317	1247	1113	820	593	442	382	272	45	7	10 723
Mathematics	9.8	9.0	10.7	12.3	12.3	11.6	10.4	7.6	5.5	4.1	3.6	2.5	0.4	0.1	
Total Female	410	462	529	639	639	615	544	373	254	202	140	106	14	0	4927
% Female	8.3	9.4	10.7	13.0	13.0	12.5	11.0	7.6	5.2	4.1	2.8	2.2	0.3	0.0	
Total Male	645	498	619	683	678	632	569	447	339	240	242	166	31	7	5796
% Male	11.1	8.6	10.7	11.8	11.7	10.9	9.8	7.7	5.8	4.1	4.2	2.9	0.5	0.1	

1999	A1	A2	B1	B2	B3	C1	C2	C3	D1	D2	D3	E	F	NG	Total
Matamaitic	947	910	1048	1226	1261	1277	1182	942	715	487	397	235	58	11	10 696
Mathematics	8.9	8.5	9.8	11.5	11.8	11.9	11.1	8.8	6.7	4.6	3.7	2.2	0.5	0.1	
Total Female	371	413	533	609	606	594	511	430	276	194	147	85	22	2	4793
% Female	7.7	8.6	11.1	12.7	12.6	12.4	10.7	9.0	5.8	4.0	3.1	1.8	0.5	0.0	
Total Male	576	497	515	617	655	683	671	512	439	293	250	150	36	9	5903
% Male	9.8	8.4	8.7	10.5	11.1	11.6	11.4	8.7	7.4	5.0	4.2	2.5	0.6	0.2	

2000	A1	A2	B1	B2	B3	C1	C2	C3	D1	D2	D3	E	F	NG	Total
Matamaitic	758	738	924	1015	1070	1152	1139	1112	877	702	633	406	100	19	10645
Mathematics	7.1	6.9	8.7	9.5	10.1	10.8	10.7	10.4	8.2	6.6	5.9	3.8	0.9	0.2	
Total Female	310	345	453	517	529	564	550	497	397	292	261	157	36	5	4913
% Female	6.3	7.0	9.2	10.5	10.8	11.5	11.2	10.1	8.1	5.9	5.3	3.2	0.7	0.1	
Total Male	448	393	471	498	541	588	589	615	480	410	372	249	64	14	5732
% Male	7.8	6.9	8.2	8.7	9.4	10.3	10.3	10.7	8.4	7.2	6.5	4.3	1.1	0.2	

ORDINARY LEVEL

1. INTRODUCTION

There are two papers in the examination, each lasting 2½ hours. On paper 1, candidates must answer any six of eight questions on core syllabus material. Paper 2 has two sections. Section A consists of seven questions on core syllabus material and Section B consists of one question on each of the four optional topics. Candidates must answer any five questions from Section A and any one from Section B.

Each question on each paper merits 50 marks, giving a total of 300 marks for each paper and hence an overall total of 600 marks.

The two papers test similar cognitive domains, although the more visual topics on the course (geometry, trigonometry, area and volume) appear on paper 2. All questions are designed to be of similar standard, with an internal grading of difficulty. This is typically achieved by questions with a three-part structure. The first part usually tests recall and basic understanding. The second usually tests application of routine procedures in relatively familiar contexts. The third tests less familiar applications or problem solving. All but one of the 19 questions on the two papers this year had such a three-part structure.

2. PERFORMANCE OF CANDIDATES

Table 1 gives a summary of the results of the examination for all candidates and also by gender. Table 2 shows the percentages of candidates who obtained A grades, and also the breakdown into high grades (A, B, C), D grades and low grades (E, F or NG) in 2000 and in the two preceding years. A detailed break-down of the examination statistics for 1998, 1999 and 2000 is provided in the Appendix at the end of this Report.

Mathematics – Ordinary Level - 2000								
Grade	**A**	**B**	**C**	**D**	**E**	**F**	**NG**	**Total**
Number	6061	10952	10773	9057	3476	1670	225	42214
%	14.4	25.9	25.5	21.5	8.2	4.0	0.5	
Total Female	3542	6192	5758	4636	1728	781	86	22723
% Female	15.6	27.2	25.3	20.4	7.6	3.4	0.4	
Total Male	2519	4760	5015	4421	1748	889	139	19491
% Male	12.9	24.4	25.7	22.7	9.0	4.6	0.7	

Table 1: Summary of Results in 2000

	Mathematics – Ordinary Level			
	% A Grades	**% High Grades (A, B, C)**	**% D Grade**	**% Low Grades (E, F, NG)**
1998	14.2	63.4	22.5	14.1
1999	18.8	67.5	20.2	12.2
2000	14.4	65.8	21.5	12.7

Table 2: Percentage of A/High/D/Low Grades Awarded in 1998, 1999 and 2000

This distribution of grades, although somewhat different from last year, is similar to the average distribution over the last number of years. Note that the effect on these results of a small shift in the proportion of students taking Higher Level, as mentioned in Higher Level Chief Examiner's report, is much less significant here than at that level, since there are more candidates at Ordinary Level.

3. ANALYSIS OF PAPER

The following three tables show the ranking of questions in order of decreasing popularity. They also give the percentage uptake and the average mark awarded on each question. These figures are derived from an analysis of a sample of approximately 4% of total scripts.

Order of Popularity	Question Number	Topic(s)	Percentage Uptake	Average Mark (out of 50)
1	2	Algebra	96%	29
2	1	Arithmetic	96%	39
3	7	Calculus	95%	29
4	4	Complex numbers	93%	33
5	3	Algebra	88%	31
6	6	Functions and calculus	83%	24
7	5	Sequences and series	64%	28
8	8	Functions and calculus	52%	19

Table 3: Order of popularity, percentage uptake and average marks – Paper 1

Order of Popularity	Question Number	Topic(s)	Percentage Uptake	Average Mark (out of 50)
1	1	Arithmetic (Area and Volume)	95%	36
2	2	Co-ordinate geometry of the line	93%	34
3	7	Statistics	90%	36
4	3	Co-ordinate geometry of the circle	78%	25
5	6	Discrete mathematics	75%	30
6	5	Trigonometry	72%	27
7	4	Geometry	48%	18

Table 4: Order of popularity, percentage uptake and average marks – Paper 2, Section A

Order of Popularity	Question Number	Topic(s)	Percentage Uptake	Average Mark (out of 50)
1	11	Linear Programming	64%	30
2	9	Vectors	20%	25
3	10	Further sequences and series	13%	18
4	8	Further geometry	3%	13

Table 5: Order of popularity, percentage uptake and average marks – Paper 2, Section B

An analysis of the answering on the individual questions is presented below.

Paper 1 Question 1

The second most popular question, at 96% uptake. It had the highest mean mark: 39.

(a) Good understanding of fractions and units of measure was displayed. Occasionally, simplification of the fraction was not completed.

(b) Candidates very often failed to show work. There was frequent use of an incorrect arithmetic operation resulting in a quite inappropriate answer, indicating a lack of intuitive grasp of currency conversions. One would expect that given the exchange rate, candidates would display understanding of whether the converted value should be a larger or smaller number than the value in the original currency, but this was often not the case. Failing to round appropriately was also common.

(c) The first part was a direct application of percentages in a familiar context and was well answered. The second part was a less direct application and hence required a greater level of understanding. This was not well answered. Most candidates found 46% of some quantity, rather than equating 46% to a quantity, indicating an inability to reason accurately at the appropriate level.

Paper 1 Question 2

The most popular question, at 96% uptake. It had the fourth highest mean mark: 39.

(a) The algebraic concept involved was well understood, but errors in the subsequent handling of the fractions were common.

(b) A disappointingly small proportion of candidates managed this routine algebraic procedure. Marks tended to be polarised, as those who were familiar with the procedure had no difficulty scoring well, whereas the remainder struggled to offer anything of value. The efforts of the latter group betrayed a poor understanding of fundamental underlying concepts. For example, the equation $x-3y=1$ was taken to be equivalent to $x^2-3y^2=1$ or $x^2-9y^2=1$, or the equation $x^2-y^2=0$ was taken to be equivalent to $2x-2y=0$.

(c) Candidates were able to handle the positive integer power in part (i), but $\sqrt{27}$ caused problems and most were not able to deal at all with the more involved work required by the rest of part (c).

Paper 1 Question 3

The fifth most popular question, at 88% uptake. It had the third highest mean mark: 31.

(a) Candidates in the past have frequently not performed well in the manipulation of formulae. This question was a little more straightforward than usual and was handled well by most candidates.

(b) **(i)** This routine procedure was executed well by almost all candidates. Some chose the indirect approach of dividing by the corresponding factor. The success rate among these latter candidates was not as high as among the rest.

(ii) The remaining quadratic factor was usually found by "long division". Sign errors were common, but the procedure was otherwise well executed.

(c) The algebraic procedures involved here were straightforward, but the question required clearer conceptual understanding in order to see how they are to be applied. Not unexpectedly, answering was generally very poor, indicating that candidates' grasp of concept lags significantly behind their procedural skills. Correct substitution was rare, with most candidates using trial and error or invented values. Even when substitution did lead to a pair of simultaneous equations, candidates failed to recognise this as such and solve.

Paper 1 Question 4

The fourth most popular question, at 93% uptake. It had the second highest mean mark: 33.

(a) This was well answered, indicating an ability to handle the simple arithmetic of complex numbers.

(b) The use of the Argand diagram to represent a complex number was well understood, in that the candidates were capable of plotting the correct points. Poor answering in part (ii), and the nature of the errors made, indicate a lack of an intuitive understanding of what the modulus represents. For example, $\sqrt{3^2 + (5i)^2}$ was a very common error. Part (ii) was a clearer trigger to a routine process and was handled well.

(c) Part (i), which again tested the routine arithmetic of complex numbers, was well answered. Part (ii), testing basic algebraic procedures applied to complex numbers, was much less so. Of the few candidates who did succeed in applying the procedures correctly, most were not capable of performing the additional manipulation of the surd form needed to give the answer in the required form.

Paper 1 Question 5

The seventh most popular question, at 64% uptake. It had the sixth highest mean mark: 28.

(a) Answered correctly by the great majority of candidates, indicating an ability to work with the sequence notation in a straightforward context.

(b) The answering here indicates an over-reliance on blind application of formulae at the expense of even a basic intuitive understanding of sequences. Both parts of the question are most easily done without resorting to T_n and S_n formulae. About half of the candidates took this approach to the first part, (i.e. repeated multiplication by the common ratio to evaluate the terms). However, having found each of the first four terms, few candidates chose to find their sum by simply adding them up. Among those who used formulae, a common error was to use those for an arithmetic rather than geometric progression.

(c) Parts (i) and (ii) were well answered here, usually better than part (b). This is despite the fact that (b) involved only basic concrete work with specific numerical terms, whereas this part required the creation of general formulae for the series, albeit in a very familiar context. This appears again to indicate that rote procedural work with formulae has dominated the candidates' learning.
Part (iii) was very rarely handled well, which is perhaps unsurprising in the context just outlined. The question required application of the formula with an understanding of context.

Paper 1 Question 6

The sixth most popular question, at 83% uptake. It had the seventh highest mean mark: 24.

(a) This part was well answered, indicating that candidates are capable of applying the basic learnt procedure to a straightforward function. The use of function notation remains less popular than $y + \Delta y$. This is despite the fact that the latter notation leaves candidates far more prone to error, as evidenced in the marking of the examination every year. The function notation is also considerably more conducive to the development of a conceptual understanding of the procedure. Also, the link it provides to the rest of this section of the syllabus results in mutual reinforcement. It is to be strongly recommended.

(b) The majority of the candidates were able to identify the period of the function. Understanding of the range was less evident, with many candidates offering the single value 10 as their answer. Reading the graph in order to evaluate the function at given values was very well handled, as indeed was the use of the periodic nature of the function to extrapolate beyond the given portion.

(c) This part was handled well by only a very small proportion of candidates. In recent years it has been observed that ordinary level candidates typically do not differentiate unless directed to do so. This indicates once more that execution of specified procedures is overemphasised at the expense of understanding and application. It is disappointing that even the stock phrase "slope of the tangent to the curve" did not provide a sufficient trigger to the candidates that differentiation was required. Even among those few who managed part (i), few correctly found the corresponding y co-ordinates required in order to find the equations.

Paper 1 Question 7

The third most popular question, at 95% uptake. It had the fifth highest mean mark: 29.

(a) This was very well answered. Candidates very capably differentiated the polynomials by rule, including the appropriate handling of constant and linear terms, which can often cause difficulties.

(b) Also very well handled, again indicating candidates' ability to apply familiar procedures accurately in a recognisable context.

(c) This part was not done well. This is a quite standard application, specifically mentioned in the syllabus and frequently examined. The nature of the work presented indicated a lack of understanding of how differentiation applies in this context. For example, even when candidates did differentiate s, they were not at all clear about whether s or its derivative was relevant to the question at hand. Also apparent was a more fundamental weakness in working with functions generally. For example, candidates were not clear at various points about whether to substitute for t or to equate and solve for t.

Paper 1 Question 8

The least popular question, at 52% uptake. It had the lowest mean mark: 19.

(a) This was not as well answered as might be expected. The algebraic manipulation required was well within the compass of the candidates, being, for example, significantly more straightforward than that required in the well-handled 3(a). However, this part also tested an ability to recognise a simple application of function notation. Many misunderstood what was required and, rather than solving the inequality, solved the corresponding equation.

(b) Answering here indicated a good ability to draw up a table and graph. However, a lack of intuitive understanding of the expected shape of the first graph was sometimes apparent, with candidates joining the two branches through the origin. Other candidates did not draw any portion of the graph in the interval $[-1, 1]$. This caused difficulties with part (iii), as one of the points of intersection did not then appear. Scaling of the axes also posed a problem for some, with 1/4, 1/3, 1/2 and 1 equally spaced on the y-axis. The linear graph caused little difficulty, though sometimes candidates used a separate diagram for it. A good number of candidates did not name or indicate the points of intersection even where they existed, indicating that they failed to see the connection between part (iii) and the graphs.

(c) This part tested the candidates' clarity of understanding of the relationship between the derivative of a function and the turning points of the corresponding curve, along with their ability to apply algebraic skills in that context. In the great majority of cases, such understanding was not displayed. Few candidates differentiated f. Many instead solved $f(-1) = 0$ for a. Irrespective of whether they had found the value of a correctly, very few were capable of distinguishing with justification between a maximum and minimum turning point. Furthermore, even having found some value for a, few exhibited a correct procedure for finding the remaining turning point.

Paper 2 Question 1

The most popular question in section A, at 95% uptake. It had the highest mean mark: 39.

(a) This part was answered moderately well. A common error was to omit the 1/2 when finding the area of the "missing" triangle.

(b) Candidates were well prepared for this question, displaying competence in the execution of this routine procedure. Errors that occurred were a failure to recognise that the measurements of the first and last offsets were 0 and a failure to convert the area in m^2 to hectares.

(c) The first part, a direct application of a formula, was very well handled. Whereas most were capable of applying the same skill again to part (ii), much fewer were capable of correctly finding the slant height first. The final part, requiring candidates to equate a formula to a known quantity and solve, was done well only by the A and B grade candidates.

Paper 2 Question 2

The second most popular question in section A, at 93% uptake. It had the third highest mean mark: 34.

(a) This direct application of a standard formula was not done as well as would be expected. The errors were due to poor knowledge of the required formula.

(b) The formula for distance was given accurately more often than that for the midpoint in part (a), though errors still occurred. Candidates handled parts (i) and (ii) well here, though a number of candidates did not recognise the terminology $|bc|$ for length or distance. Sign errors were quite common. The third part, requiring manipulation of numbers in surd form, was not well done.

(c) **(i)** Most candidates did not display an ability to find the slope of a line from its equation, nor an understanding of the numerical relationship between the slopes of perpendicular lines. Very few candidates used a diagram to aid them in their understanding, either here or in part (ii).

(ii) Only the very good candidates managed this part. Although the procedures involved are quite standard, the fact that there were distinct stages caused problems. Diagrams would have aided candidates considerably in clarifying what was required, but most do not seem to have the habit of drawing them.

Paper 2 Question 3

The fourth most popular question in section A, at 78% uptake. It had the sixth highest mean mark: 25.

(a) This part was well answered. Virtually all candidates displayed understanding of the relationship between the equation of a circle and the radius, and accurately justified the assertion that the given point is inside the circle.

(b) About a quarter of the candidates were unable to find the slope of the tangent. Many found the slope of the radius to the point of contact, but did not correctly

deduce the slope of the tangent. Most candidates who had a value for the slope were able to find the equation of the tangent.

(c) Although this part required reasoning beyond simple substitution into formulae, the application is not unfamiliar. Part (i), in particular, is relatively standard but was not handled particularly well. Again here it is evident that candidates do not use diagrams effectively to aid their understanding of what is being asked and how it might be solved. The understanding of fundamental ideas and procedures of co-ordinate geometry and algebra required for parts (ii) and (iii) was not at all apparent.

Paper 2 Question 4

The least popular question in section A, at 48% uptake. It had the lowest mean mark: 18.

In addition to being the least popular question, this question tended to be chosen by candidates whose answering in other questions indicated that they were poorly prepared for the paper overall.

(a) This posed little difficulty for candidates, who demonstrated clear competence in applying known geometrical results in straightforward numerical cases.

(b) Less than 10% of those attempting this question were able to prove the theorem. The most common outcome was the achievement of the attempt mark only, indicating that candidates either had little real understanding of what was required, or failed to perform even a cursory revision of the topic. In addition, the evidence provided by the answering of both this and part (c) supports the hypothesis that significant numbers of candidates have encountered this topic in Senior Cycle to a very limited extent or not at all, and are answering on the basis of recalled knowledge from Junior Cycle.

(c) The first part required only a rudimentary understanding of the concept of scale factor, but was not well answered, indicating a lack of familiarity with that concept. The second part was more often approached via the ratio results for triangles than via enlargements. With this question, that approach is sensible and gives a direct route to the required answer. However, the fact that candidates elsewhere failed to move beyond the routine frame of reference in order to find more efficient solutions, indicates that this is more likely to be a result of unfamiliarity with enlargements than an active decision between the two approaches. The poor answering to the third part also supports this assertion. In that part, there is no viable alternative to the enlargements approach and very few reasonable efforts were made, though some candidates attempted to find $|\angle acb|$ by trigonometric means.

Paper 2 Question 5

The sixth most popular question in section A, at 72% uptake. It had the fifth highest mean mark: 27. This question tended to be chosen more often by stronger candidates, as measured by their overall performance on the paper.

(a) Candidates handled this straightforward application of the formula for the area of a triangle very well. As usual, some candidates treated $\frac{1}{2}ab\sin C$ as $\frac{1}{2}|ab|\sin C$.

This problem is undoubtedly partly due to the practice of using lower case letters to denote points as well as lengths of segments. Nonetheless, if candidates are using formulae without a clear knowledge of what the variables in them represent, their understanding is clearly inadequate.

(b) This routine application of trigonometric methods to find unknown quantities was well answered. This is encouraging.

(c) The first part, a straightforward application of Pythagoras' Theorem was well answered. The second part, requiring either the calculation on an angle and the subsequent use of the cosine formula, or a more insightful use of Pythagoras' Theorem, was not well answered. Good answering often occurred in centre clusters, indicating that the teaching of the topic (either the approach or the depth of coverage) was a significant factor. The cosine formula was the more common approach, and some candidates chose a less direct route via the sine formula. Completion of the rectangle and application of Pythagoras' Theorem, although uncommon, was not confined to best candidates.

Paper 2 Question 6

The fifth most popular question in section A, at 75% uptake. It had the fourth highest mean mark: 30.

(a) This basic application of the Fundamental Principle of Counting and related listing was very badly handled. Most candidates did not apply the principle at all, but rather relied on an *ad hoc* listing of possibilities, which they subsequently counted. In listing their outcomes, candidates often listed single journeys only, rather than "to and from" combinations.

(b) Candidates usually did not offer any supporting work or rationale for their answers. Parts (i) and (ii), and to a lesser extent part (iii), were very well answered, indicating that candidates are capable of applying the relevant probabilistic principle in relatively straightforward cases. Part (iv) was far less well handled, with errors being made in identifying the total number of outcomes.

(c) The application of the Fundamental Principle of Counting in part (i) here was handled much better than that in part (a), despite the fact that it is significantly more complex. (The compound task consists of five subtasks rather than two; the number of ways of completing the second and subsequent tasks is not immediate, but needs to be reasoned out.) This, of course, is a very common type of question and so candidates were on familiar territory. The contrast between the competence displayed on the two items indicates that the candidates' ability to deal with such questions is based more on context than on concept.

Parts (ii) and (iii) were reasonably well handled also, indicating that in this familiar context, candidates are capable of making the appropriate adjustments to handle the constraints given.

Part (iv) was not at all well handled. The two simultaneous constraints required a switch from a standard repeated multiplication method to some form of case by case analysis. The nature of the incorrect work offered suggested that the well-drilled heuristic was the only tool the candidates could bring to bear. A systematic analysis of the task was very rarely apparent.

Paper 2 Question 7

The third most popular question in section A, at 90% uptake. It had the second highest mean mark: 36.

(a) This routine procedure was not as well answered as might be expected. Failure to multiply by the weights, and division by 4 rather than by the sum of the weights, were common errors.

(b) The histograms were neatly drawn on graph paper with labelled axes. The most common error was in incorrect height for the 40-80 rectangle. This rectangle was also sometimes drawn with incorrect width. Part (ii) tested the candidates' understanding of the nature of a grouped frequency table, by requiring a hypothesis about the possible pattern of the data described. This was not well answered. Answers, whether correct or incorrect, were usually offered without supporting work or rationale.

(c) Candidates displayed competence and accuracy in constructing the cumulative frequency table and ogive. When asked for the interquartile range, many candidates found the upper and lower quartiles but failed to subtract. Also, candidates in a case like this typically label the vertical axis in multiples of 10. In this case, the highest number actually plotted against the vertical axis, 52, was not a multiple of 10. Many candidates used the highest point on their vertical scale (usually 60), rather than 52, when calculating the quartiles. In other words, having scaled the vertical axis from 0 to 60, they read across from 15 and 45 to find the lower and upper quartiles. For part (iv), the majority of candidates used the ogive to identify correctly the number of weeks during which ≤56 calls were made, omitting the subtraction from 52 to finish.

Paper 2 Question 8

The least popular option, at 3% uptake. It had the lowest mean mark: 13. Of the few candidates who attempted this question, most were clearly ill-prepared.

(a) Most candidates managed part (i), which tested purely the recognition of a commonly understood known result. However, part (ii), requiring the numerical application of two results, was not so well done.

(b) The great majority made no attempt, or a very poor attempt, at proving the theorem. Those who did were clustered in centres. This indicates that, with appropriate preparation, ordinary level candidates are quite capable of handling geometry theorems, but that the other candidates had probably not prepared this option.

(c) Parts (i) and (ii) were straightforward numerical applications of the theorem just proved. They were nonetheless not well done. Part (iii), not surprisingly since it required application of other results, was handled even less well.

Paper 2 Question 9

The second most popular option, at 20% uptake. It had the second highest mean mark: 13.

(a) In part (i) the distributive law and "gathering" were usually, though not always, well applied. Part (ii), on the other hand, which required use of $\overrightarrow{xy} = \vec{y} - \vec{x}$, was not well done. Many candidates regarded the juxtaposition of x and y as indicating a product. Also common was a sign error in handling $-\vec{y}$.

(b) Answering was quite mixed. Many candidates were able to construct the scalar multiples $2\vec{s}$ and $2\vec{p}$, but not the required sums. Neither a visual nor a sound algebraic understanding of vector addition was apparent, and candidates were consequently unable to express $\vec{k} + \vec{m}$ in terms of $\vec{r}$.

(c) Sign errors were common in finding the related perpendicular vectors in part (i). Candidates were quite competent at finding the lengths in part (ii). Part (iii) was poorly answered. Many candidates simply substituted and stopped. Also common was the erroneous assertion $\left|\vec{a}^{\perp} + \vec{b}^{\perp}\right| = \left|\vec{a}^{\perp}\right| + \left|\vec{b}^{\perp}\right|$. Surds caused difficulty.

Paper 2 Question 10

The third most popular option, at 13% uptake. It had the third highest mean mark: 18.

(a) This part was not well done and frequently omitted. Most candidates used the binomial theorem to expand, rather than multiplying out. Candidates had difficulty with $(\sqrt{3})^3$. Common errors were $x^0 = 0$ and ${}^3C_0 = 0$. Many candidates did not use the first part in doing the second, but rather attempted an approximate verification by calculator.

(b) The straightforward sum to infinity of the geometric series was not well done. Many candidates found the first term and common ratio and stopped. Others substituted into a formula for the sum to n terms and stopped. Only very rarely was a reasonable attempt made on the follow-on question, which required an understanding of recurring decimals.

(c) Part (i) here was the most successfully answered part of the whole question. However, it would be incorrect to infer that high order skills were at work, since the method overwhelmingly used was to calculate repeatedly on an annual basis, rather than to apply the techniques for dealing with geometric progressions. As reliance on such direct computation techniques alone clearly undermines the role of this material in this section of the syllabus, it may be necessary to ensure in the future that questions are constructed that are impractical to handle in this way. Part (ii) was not well answered. Where a reasonable effort was made, the technique was usually to add the three amounts calculated at the intermediate stages of part (i).

Paper 2 Question 11

The most popular option, at 64% uptake. It had the highest mean mark: 30.

(a) The separation, as a distinct part, of the work involved in finding the equation of the line from that of identifying the inequalities was clearly of benefit to students. Most displayed competence in the straightforward procedure of finding the equation of the line given the two intercept points. Subsequent errors in the directions of inequalities were common, but candidates nonetheless displayed a good grasp of what was required.

(b) This part was well answered. Candidates were obviously well prepared in both the modelling skills and the mathematical techniques required for such applications. Inequalities were frequently simplified before proceeding, indicating a good algebraic awareness in this context, (although occasionally errors were made in simplifying). Sometimes the point of intersection at (12,10) was not found, or found graphically rather than algebraically. Candidates displayed a good ability to interpret the mathematics done in order to give the correct answers to the worded questions.

4. OVERALL GENERAL COMMENT

A number of patterns are apparent from the detailed analysis presented in section 3.

Question Grading

The internal grading of questions from lower to higher order skills was, for the most part, reflected in the level of success achieved by candidates in dealing with the various parts. The (a) parts were almost all handled very well, (b) parts quite well and (c) parts less so. Notable exceptions to this progression were:

- Paper 1, Question 5, where an over-reliance on formula techniques appears to have led to lack of basic intuitive understanding, with straightforward numerical work being handled less well than abstract work.
- Paper 2, Question 4, where candidates were clearly ill-prepared for proofs of theorems, so that what should have been routine was not known at all.
- Paper 2, Question 6, where the simple application of the Fundamental Principle of Counting in part (a) was handled far worse than the significantly more complex but context-familiar application in part (c).
- Paper 2, Question 8, where the application of geometric series to financial matters, by virtue of involving only a few terms, was treated by candidates without any reference to series methods at all (not unreasonably). Hence, it did not test the intended higher order skill.

Procedural Competence / Conceptual Understanding

It is clear from a range of questions throughout the two papers that candidates' strengths lie in the area of competent execution of routine procedures in familiar contexts. The evidence suggests, therefore, that fundamental objective B of the syllabus, (*instrumental understanding*,) is being achieved quite well. It is arguably being achieved better than the lower order objective A (*recall*), as indicated by, for example, the notable difficulty with the accurate reproduction of standard formulae.

There is a significant weakness regarding sound conceptual understanding of much of the material, with corresponding weaknesses in its application in contexts which, though familiar, do not mimic well-rehearsed examples precisely, or do not contain the standard "trigger phrase". It is particularly noticeable when more than one idea or skill is involved. This indicates that objectives C and D, *relational understanding* and *application*, are not being as well achieved as might be hoped. This is the case not only with advanced material, but also with quite fundamental concepts and skills. Worthy of particular note here is the extent to which basic algebraic skills manifest as isolated mechanical procedures without underlying understanding or synthesis. Whereas this is often sufficient for survival with very familiar routine exercises, it is a serious disadvantage when any degree of higher order application is required.

The marking of mathematics examination papers has traditionally not placed a major emphasis on syllabus objective E (*psychomotor* and *communicative* skills). Perhaps understandably in that context, candidates frequently do not present work in a coherent fashion. Relevant supporting work is often not furnished and coherent explanations for answers offered, expressed in clear language, are rare.

The profile of skills described above is not entirely unexpected. The overall patterns of answering this year are generally in line with those of the past. Furthermore, international studies of achievement in mathematics, conducted with respect to younger age groups, have identified a tendency for Irish students to perform better than their international counterparts on routine execution of skills and worse on understanding of concept. This mirrors the above observations closely.

Changes in Strengths and Weaknesses

In terms of the generic skills displayed, there were no significant changes in the strengths and weaknesses displayed by the candidates in comparison with previous years.

Considered by topic, the relative strengths of the candidates in the various content areas remain for the most part unchanged. However, *Probability* and *Linear Programming* both showed improvement, whereas there was a marked decrease in the extent to which average and below average candidates knew the co-ordinate geometry formulae.

Examination Technique

A number of candidates did not perform as well as they might have, due to a poor approach to the paper, most especially in relation to the number of questions attempted. Whereas only 3% of candidates did not attempt six questions on paper 1, the corresponding figure for paper 2 was 12.9%. The examining team reported many instances of candidates scoring well on two, three or four questions and yet failing to attempt the full quota. The competence displayed by such candidates in the questions they attempted is therefore not reflected in their overall performance. This deficiency in the approach of candidates indicates either that large portions of the course were not covered at all, or that they were poorly prepared for the mechanics of sitting an examination of this type.

It is also common (indeed more so) to encounter scripts in which more than the required number of questions have been answered. On the sample of 3400 scripts (1680 paper 1 and 1720 paper 2), a total of 1415 excess questions was recorded. The

circumstances in which it is appropriate for a candidate to attempt excess questions are not such as to justify this large a number. Examiners reported many cases in which the candidate's decision in this regard seemed inappropriate; quite often candidates attempted excess questions despite omitting parts of other questions. These candidates would have been better served by devoting the time to completing the required six.

In addition, there were some candidates who, on paper 2, answered six questions from section A and none from section B, or four from section A and two from section B.

5. RECOMMENDATIONS FOR TEACHERS AND STUDENTS

In order to achieve a high level of success, students need a solid conceptual understanding of the material on the syllabus. Teachers should not assume that because students can work through a set of similar closed-form exercises successfully that they will then have sufficient understanding to identify contexts where that knowledge is required, apply the mathematical tools appropriately and interpret the results correctly. This, of course, is not a problem that can easily be addressed at senior cycle in isolation from the earlier mathematical experiences of the students. Many of the difficulties arise from inadequate grasp of the fundamental concepts that are carried forward from Junior Cycle. They need to be addressed co-operatively by teachers on a whole school basis.

Whereas the rote memorisation of a procedure without addressing underlying concepts may appear to overcome a problem in the short term, it is ultimately unhelpful to the long-term strategy of maximising students' competence and interest in mathematics. Methodologies should be chosen with a view to developing understanding and students should be encouraged to question and discuss the mathematics they are working on. Insofar as possible, situations in which students are executing procedures the rationale for which they do not understand should be avoided. Also, when students are working on material that links conceptually to another area, or relies on skills developed therein, then these links should be made explicit and developed, both for reinforcement of understanding and so that students see mathematics as a coherent and sensible body of knowledge.

Students should be encouraged to apply a range of strategies to dealing with a particular problem. In class, if a variety of solutions to a problem are available, teachers should endeavour to draw as many solutions from the students as possible. Solutions that involve the application of knowledge from other content areas should be particularly encouraged and complimented. This approach encourages students to think more about the meaning of what they are doing, reinforces knowledge throughout the course and leaves the students better equipped to handle applications that are more complex, require multiple skills, or are set in a less familiar context. Also, teachers should avoid always phrasing questions and problems in a given topic in exactly the same way, as this can encourage an over-reliance on trigger phrases or words, at the expense of clear problem analysis.

Students should be encouraged always to present work in a coherent way and to communicate, both orally and in writing, the rationale behind what they are doing. Such clarity of communication will help both students and teachers to identify and rectify deficits in understanding.

It is clear, both from the continuing relatively high failure rate and from the type of work presented by the candidates who are failing, that there are significant numbers of

candidates who are wholly unsuited to taking this examination. This should be evident to teachers quite far in advance of the examination. It is difficult to see what purpose is served by students continuing to follow a programme that quite evidently is not meeting their educational needs. The quality of the mathematics learning experiences such students will have had on leaving the system is not satisfactory. Where it is clear that students are not capable of engaging with the material in a meaningful way, such students should be actively encouraged at an early stage to follow a programme designed to meet their needs.

In addition to the general recommendations above, the following specific recommendations will assist teachers and students in preparing for the examination:

- Candidates should be encouraged to show all of their work as clearly as possible. They should be aware that most of the marks awarded to the majority of candidates are awarded for work that has not ultimately yielded the correct answer but nonetheless has considerable merit. *It should also be noted that correct answers without supporting work do not necessarily yield full marks*.
- Candidates should take special care in showing work when using calculators. There is a tendency to write down answers without indicating what calculation has led to them. Estimation and other error-tracking strategies appear to be lacking in many cases. Candidates would be well advised to:
  - write down what they are about to calculate before doing so
  - write down intermediate stages of calculations that are particularly complex
  - estimate the expected answer to a calculation before performing it, and critically evaluate any answer to a calculation
  - check answers by repeating calculations, by applying inverse operations, or through the use of other appropriate strategies
  - take care that the calculator is in the appropriate mode of angle measure for trigonometric work
  - use the same model of calculator in the examination as they are used to using in class.
- Candidates should not rub out or otherwise obliterate cancelled work. A single line drawn through it is sufficient. There is frequently more merit in work that candidates have cancelled than in their repeated efforts. In most cases this can be counted to the benefit of the candidate. For this reason, the use of pencil as the primary writing instrument should be discouraged.
- Candidates should be familiar both with the structure of the examination papers and with the style of question asked. In practising on past papers, note also the general recommendation above that there is significant learning potential in addressing a variety of solutions to a given problem.
- Candidates should have practised on questions from past papers under timed conditions. Such practice results in candidates being less likely either to rush questions, leading to errors or omissions, or to spend too much time on a few questions, resulting in an inability to attempt the required six.
- Candidates should always attempt the required number of questions. Excess questions should only be attempted under special circumstances.
- Having answered a question, candidates should read through it again to ensure that they have not omitted any parts.

- Candidates should not pre-select the questions they intend to do. The questions should all be read carefully at the start of the examination and a choice made thereafter. Time for reading through the paper carefully at the start and time to check solutions afterwards, as recommended in the previous bullet, should be budgeted for when planning the time to allocate to each question.
- Where diagrams are required, they should be drawn as accurately as possible, with a sharp pencil. Care should be taken in the positioning of axes for co-ordinate geometry and in the choice of an appropriate scale.
- Candidates should use sketches and diagrams to aid them in clarifying what is required in questions on visual topics. Frequently, the appropriate strategy required to solve an unfamiliar problem becomes much more apparent when a diagram is drawn.
- Candidates should ensure that they know all of the required formulae *accurately* by rote. This is particularly crucial in co-ordinate geometry, but also applies to the formulae required for sequences and series, the roots of a quadratic, and the trigonometric ratios in a triangle.
- Candidates should be familiar, if not with the detailed workings of the marking schemes, certainly with the general principles by which they operate. This will encourage them to follow the other recommendations above. Teachers should consider applying similarly designed schemes to their class and school based written assessments, as appropriate. As well as encouraging good practice, this may have the further benefit of improving the predictive validity of school tests regarding ultimate performance in the Leaving Certificate Examination.
- Teachers should use function notation when dealing with differentiation from first principles.

APPENDIX

Statistics for Mathematics –Ordinary Level, 1998, 1999, 2000

1998	A1	A2	B1	B2	B3	C1	C2	C3	D1	D2	D3	E	F	NG	TOTALS
All candidates	3093	3312	3705	3772	3684	3760	3759	3555	3406	3037	3721	4379	1789	219	45191
% of Total	6.8	7.3	8.2	8.3	8.2	8.3	8.3	7.9	7.5	6.7	8.2	9.7	4.0	0.5	
Female	1872	1894	2175	2130	2050	2064	2018	1942	1759	1532	1886	2121	862	86	24391
% of Female	7.7	7.8	8.9	8.7	8.4	8.5	8.3	8.0	7.2	6.3	7.7	8.7	3.5	0.4	
Male	1221	1418	1530	1642	1634	1696	1741	1613	1647	1505	1835	2258	927	133	20800
% of Male	5.9	6.8	7.4	7.9	7.9	8.2	8.4	7.8	7.9	7.2	8.8	10.9	4.5	0.6	

1999	A1	A2	B1	B2	B3	C1	C2	C3	D1	D2	D3	E	F	NG	TOTALS
All candidates	4518	3871	3913	3795	3711	3474	3357	3199	3031	2817	3093	3677	1562	170	44188
% of Total	10.2	8.8	8.9	8.6	8.4	7.9	7.6	7.2	6.9	6.4	7.0	8.3	3.5	0.4	
Female	2838	2324	2246	2187	2082	1852	1766	1636	1514	1370	1524	1796	710	67	23912
% of Female	11.9	9.7	9.4	9.1	8.7	7.7	7.4	6.8	6.3	5.7	6.4	7.5	3.0	0.3	
Male	1680	1547	1667	1608	1629	1622	1591	1563	1517	1447	1569	1881	852	103	20276
% of Male	8.3	7.6	8.2	7.9	8.0	8.0	7.8	7.7	7.5	7.1	7.7	9.3	4.2	0.5	

2000	A1	A2	B1	B2	B3	C1	C2	C3	D1	D2	D3	E	F	NG	TOTALS
All candidates	2942	3119	3491	3752	3709	3686	3614	3473	3092	2760	3205	3476	1670	225	42214
% of Total	7.0	7.4	8.3	8.9	8.8	8.7	8.6	8.2	7.3	6.5	7.6	8.2	4.0	0.5	
Female	1708	1834	1999	2147	2046	1978	1942	1838	1621	1463	1552	1728	781	86	22723
% of Female	7.5	8.1	8.8	9.4	9.0	8.7	8.5	8.1	7.1	6.4	6.8	7.6	3.4	0.4	
Male	1234	1285	1492	1605	1663	1708	1672	1635	1471	1297	1653	1748	889	139	19491
% of Male	6.3	6.6	7.7	8.2	8.5	8.8	8.6	8.4	7.5	6.7	8.5	9.0	4.6	0.7	

FOUNDATION LEVEL

1. INTRODUCTION

The examination consisted of two papers, each of two hours and thirty minutes duration.

A total of three hundred marks was allocated to each paper.

On paper one, candidates were requested to attempt question 1 which consisted of ten parts, each part carried ten marks and four other questions from a choice of six questions. These six questions carried fifty marks each.

On paper two, candidates were requested to attempt six questions out of a total of eight questions. Each question carried fifty marks.

2. PERFORMANCE OF CANDIDATES

Table 1 gives a summary of the results of the examination for the whole cohort, for females and for males. Table 2 shows the percentages of candidates who obtained A grades, high grades(that is, A, B or C), D grades and low grades (that is, E, F or NG) in 2000 and in the two preceding years. A detailed break-down of the examination statistics for 1988, 1999 and 2000 is provided in the Appendix at the end of this Report.

Mathematics—Foundation Level—2000								
Grade	**A**	**B**	**C**	**D**	**E**	**F**	**NG**	**Total Candidates**
Number	470	936	1953	1101	280	91	15	5846
%	8.0	3.1	33.4	18.8	4.8	1.6	0.3	
Total Female	251	1005	999	555	119	34	6	2969
% Female	8.5	33.9	33.7	18.7	4.0	1.1	0.2	
Total Male	219	931	954	546	161	57	9	2877
% Male	7.6	32.4	33.1	19.0	5.6	2.0	0.3	

Table 1: Summary of Results in 2000

	Mathematics—Foundation Level			
	% A Grades	**% High Grades (A, B, C)**	**% D Grade**	**% Low Grades (E, F, NG)**
1998	8.3	73.3	20.0	6.7
1999	9.4	75.3	17.7	6.8
2000	8.0	74.5	18.8	6.7

Table 2: Percentages of A/High/D/Low Grades Awarded in 1998, 1999 and 2000

As evident from Table 2, the profile of results has varied little over the last three years.

Since 1998 there has been a 3.5% drop in the number of candidates who took Foundation Level. Candidature was 6056 in 1998, 5753 in 1999 and 5846 in 2000.

3. ANALYSIS OF PAPER

Table 3 and Table 4 show the ranking of questions in order of decreasing popularity. They also give the percentage uptake and the average of the marks awarded to those who chose each question. These figures are derived from an analysis of a sample of 4% of total scripts.

Order of Popularity	Question Number	Topic(s)	Percentage Uptake	Average Mark
1	1	Computation/use of Calculator	100%	74
2	6	Graphs (interpretation of)	95%	40
3	2	Arithmetic	88%	35
4	3	Arithmetic	73%	28
5	5	Algebra	64%	28
6	7	Functions and Graphs	39%	28
7	4	Algebra	33%	25

Table 3: Order of Popularity, Percentage Uptake and Average Mark for Paper 1

Order of Popularity	Question Number	Topic(s)	Percentage Uptake	Average Mark
1	1	Areas and Volumes	95%	39
2	2	Areas and Volumes	91%	36
3	7	Statistics and Probability	88%	29
4	6	Statistics and Probability	84%	34
5	4	Geometry	81%	30
6	3	Geometry	78%	31
7	5	Trigonometry	52%	23
8	8	Constructions and Enlargements	34%	17

Table 4: Order of Popularity, Percentage Uptake and Average Mark for Paper 2

Paper 1 Question 1

This compulsory question was attempted by 100% of candidates. The average mark awarded was 74 out of 100 marks.

Candidates appeared to be well adapted to this type of question. For most it was the first question to be attempted. Answering displayed a good level of skill in the use of the calculator. Candidates showed more steps in their work than previous years and consequently were often awarded marks for incorrect answers as examiners were able to follow the reasoning that led to the answers. The importance of showing all necessary work is emphasised.

While the level of accuracy of candidates in this question was quite good, rounding off answers to required levels of accuracy was a significant problem. It was either ignored completely or done incorrectly by the majority of candidates. Areas of particular difficulty were where a decimal of the form 0.012 was required to a stated number of significant figures. Candidates were more successful at rounding off a straightforward number such as 1.234 to a specific number of decimal places. However, the slightest twist in the number or context seemed to present real difficulties. Many candidates appeared content to ignore the round-off and to suffer the loss of marks as a result.

(i) This part was very well answered apart from the round-off. The answer commonly given was 8.0 instead of 8.1

(ii) This part was generally well answered with the majority of candidates being awarded 9 or 10 marks.

(iii) This part was also very well answered. Even those who did not get full marks were generally awarded 6 or 7 marks because they showed their intermediate work.

(iv) Candidates found this part more testing. While generally the square was found correctly, the reciprocal was less well managed. Quite a few candidates ignored it and subtracted 0.0025 from 392.04.

(v) This was reasonably well answered. Some candidates divided IR£25.18 by 16. Rounding the answer to the nearest penny was frequently omitted or incorrect.

(vi) This part was also reasonably well answered. Candidates who converted each fraction to a decimal before adding were generally correct apart from difficulties with an early or incorrect round-off. Attempts to add the fractions before converting quite often led to problems - 7/20 was given as an answer reasonably often. For those who succeeded in getting 67/99, many divided 99 by 67 or did not divide at all. Candidates did not appear to be using the fraction function available on many calculators.

(vii) This part was also well answered. The most common error was to multiply by the conversion factor rather than to divide.

(viii) This part of the question was, without doubt, the part candidates found most difficult. The reason for this was probably that it combined the concepts of the 24 hour clock with average speed. It was common for candidates to divide the 36 by either 25 or 0.25. Another frequent error was to multiply 36 and 25.

(ix) The majority of candidates dealt effectively with this part of the question. Quite a significant percentage got the correct answer. The most popular approach was to convert the scientific notation form to natural numbers. There were candidates who interpreted 2.45×10^6 as 2.45×60 but this blunder was not as frequent as in previous years. Blunders in precedence also occurred - the denominator was sometimes divided into only one element of the numerator, for example, 16 000 was divided into 18 000.

(x) Again this part was well answered. Rounding off the number to two significant figures was the most challenging part for some candidates – the concept of significant figures did not seem to be well understood. A common error was to round the denominator back to 0.66 before dividing. A number of candidates worked the numerator and denominator but did not finish the question and divide the resulting fraction.

In question 1, parts ***(viii)***, ***(vii)*** and ***(vi)*** proved to be the most difficult for candidates. The highest scoring parts were ***(i)***, ***(ii)***, and ***(iii)***.

Paper 1 Question 2

This question was attempted by 88% of candidates. The average mark was 35.

This was a well answered question. Generally candidates answered part (a) well and gained high marks but were less successful with part (b).

(a) The first and second sections presented little difficulty with candidates calculating the correct answers apart from a few who were not quite clear how to deal with the tax free allowance. Section (iii) was generally that which caused some difficulty with candidates unclear on the amount to use for the calculation of the PRSI. Section (iv) was generally correctly done.

(b) This part proved testing for many candidates. While most could calculate the average speed correctly in hours, converting the answer to hours and minutes caused difficulties. It was quite common to find 1 hour 6 minutes given as the answer after 1.06 hours was calculated as the average speed. The final part of the question proved difficult also with many candidates working with the incorrect distance or interpreting the time as 1.45 hours.

Paper 1 Question 3

This question was attempted by 73% of candidates. The average mark was 28.

This question was the third most popular question of the 50 mark questions after question 6 and question 2. In general, candidates achieved reasonably good marks on it.

(a) Calculating the error did not present problems to candidates. However, converting this error to a percentage error was a significant hurdle for many. Dividing the error by the estimate rather then the true value and dividing the true mass by the estimated mass were common errors. Candidates also experienced difficulty in expressing a number such as 2.5 as a percentage of a much larger number such as 67.5 - they tended to divide the smaller number into the larger number quite often.

(b) The most usual way of answering this question was to calculate the depreciation on a year by year basis. The fact that this process was carried out correctly in the majority of cases, even though the calculation was for four years, suggests it is being taught this way rather than by using the formula. However, it was quite usual for candidates who worked this way to round off at each stage in the process, thereby simplifying the work for themselves slightly.

There was, however, a sizeable number of candidates who successfully substituted into the formula and calculated it out correctly.

A frequent error was not to compound the depreciation but to calculate the correct depreciation for the first year, to multiply it by 4 and either to give that as the answer or to subtract that amount from the initial cost.

(c) Candidates coped reasonably well with this part of the question. In particular those who understood the meaning of proportion tackled the problem with confidence and got a correct answer. The most common errors arose from a lack of understanding of the concept of proportion rather than from any particularly difficult calculation. The most common way of dealing with the last part of the question was to increase the prize fund by 10% and recalculate the proportions.

Paper 1 Question 4

This question was attempted by 33% of candidates. The average mark was 25.
This was the least popular of all the questions on the paper.

(a) This part was very poorly answered, maybe because it was being asked for the first time or maybe because candidates have difficulty with scientific notation.

(b) Candidates seemed to be well prepared for these simultaneous equations. Those attempting it generally did well although a significant minority stopped after finding one variable. Efforts at solving by trial and error were less frequent than in other years. The usual blunders when multiplying an equation by a number occurred. Problems with signs when either adding or subtracting equations were also common.

(c) Section *(i)* of this part posed little problems apart from candidates writing $x = 4$ as the answer having calculated $x \leq 4$.
Section *(ii)* was more difficult because it involved a minus sign on the x. In the inequality $-2x \leq 11 - 5$ the minus was generally ignored. In only a few cases was the sign retained and the direction of the inequality reversed.

Section *(iii)* was sometimes viewed by candidates as a repeat of the first two sections rather than as a deduction from them. There was seldom a listing of the elements satisfying the inequalities. Only in the rarest of instances an intersection of sets was found.

Paper 1 Question 5

This question was attempted by 64% of candidates. The average mark was 28.

This question, although not a popular choice, was the more popular of the two algebra questions on the paper. Those candidates who tackled the question scored reasonable marks on it, particularly in parts (a) and (b).

(a) This part was generally well answered. Slips in signs were the most common type of error. A minority of candidates who got $7x = 14$ either stopped or wrote the answer as $14 - 7$.

(b) Despite appearing consistently on papers at this level, solving a quadratic equation presented a lot of difficulty for candidates. Generally, candidates could readily identify the values for *a, b* and *c* and substitute them correctly into the formula. Thereafter, errors frequently arose. In particular, $\sqrt{16-12}$ for $\sqrt{16+12}$ was common, resulting in $\sqrt{4}$ which significantly simplified the answer. Dividing by 6 before finding the square root was also frequent as was

dividing only one term of the numerator by 6. Another error was to take $a = 3^2$. In other cases, the x remained in the working. Slips and blunders in the calculation after substitution were very common. Many failed to finish the question having substituted into the formula.

(c) In spite of their many efforts in a lot of cases, the percentage of candidates who successfully wrote the information as an equation in x was small although probably not as small as in previous years. Most candidates ignored the question completely.
In the second section of this part, the persistent candidate often calculated the correct number without any reference to the equation in x and showed that his or her answer satisfied the given conditions.

Paper 1 Question 6

This question was attempted by 95% of candidates. The average mark was 40.

Apart from question 1, this was the most popular question. It was attempted by almost all candidates. In general, scoring was high.

(i) This part was generally answered correctly.
(ii) This part was also well done with most candidates writing down the correct answer. The most common incorrect response given was 30.
(iii) This part was again well answered.
(iv) In this part the most common mistake was to give the answer as 30, the time the car was travelling before reaching its maximum speed.
(v) This proved to be the most difficult part of the question with a variety of incorrect answers although those candidates who had parts *(iii)* and *(iv)* correct often went on to answer this part correctly.
(vi) A common error in this part was to divide the distance by 30 rather than 60.

Paper 1 Question 7

This question was attempted by 39% of candidates. The average mark was 28.
It was the second least popular question. In general, it was reasonably well answered by those who attempted it.

Constructing the table presented some problems for candidates. The $-3x$ line presented problems, mainly associated with signs. The constant was often interpreted as x. Also, in adding to calculate $f(x)$, the original values of x were included in some cases.

The small number of candidates who did not use the table format and attempted to find the values of $f(x)$ by direct substitution into the expression for the function generally became confused in the calculations. The graph was generally well drawn. Blunders in the scales were the most common problem.

Candidates who attempted this question frequently finished having drawn the graph and ignored the questions on the graph. However, those who persisted were usually successful in answering parts *(i)* and *(ii)*. Part *(iv)* was generally incorrect. The most common error was to confuse x values with $f(x)$ values.

Paper 2 Question 1

This question was attempted by 95% of candidates. The average mark was 39.

The quality of the answering and the presentation of the work were both satisfactory. Candidates displayed little difficulty in understanding the requirements of the question.

(a) The main errors that occurred in the answering were not multiplying by 2 and in subtracting the length 8.1 from the given area of 10.125 cm^2.

(b) The application of Simpson's Rule by the candidates was generally correct. The omission or inversion of $\frac{5}{3}$ was the only error of note. Where other mistakes occurred, they were mainly in interchanging "odds" with "evens" and in the value of h. Again, in the last part, rounding off the answer correct to the nearest metre, was sometimes omitted.

Paper 2 Question 2

This question was attempted by 91% of candidates. The average mark was 36.

(a) Maximum marks were frequently awarded for calculating the volume of the block/ blocks. However, evaluating the unknown quantity k as a measurement of the large block proved to be difficult for some. In these cases, k tended to be incorrectly evaluated as $8 \times 9 \times 30$ or $720 \div 30$ or $24 \div 30$ or $720 - 72$.

(b) Attempt marks were very common for this part of the question. Incorrect formulae were often used. The radius of the sphere was incorrectly calculated by $r = \frac{Area}{\pi} = 324$. Omitting to calculate the square root, or to calculate the square root of the numerator only also occurred. Calculating the height h of the cylinder was usually not attempted.

Paper 2 Question 3

This question was attempted by 78% of candidates. The average mark was 31.
High marks were common in parts (a) and (b). Low marks were regularly awarded for part (c) which was not attempted at all by many.

(a) While the vast majority answered this part correctly, mistakes that occurred were in identifying incorrect orientation of the isosceles triangle.

(b) The few errors that did occur were in thinking alternate or corresponding angles sum to 180° and that opposite angles were complementary.

(c) Many candidates were unable to distinguish between angle measure, distance and area.
Some of the answers for angle measure included $|\angle abc| = 6 + 8 = 14$ or $|\angle abc| = 6 \times 8 = 48$.

The length of the diameter [ac] was often calculated as $\sqrt{(6^2 + 8^2)} = \sqrt{6^2} + \sqrt{8^2}$.

Also the incorrect radius tended to be used when calculating the area of the circle, $r = 6$ or 7or 8, for example. Not dividing by 2 in finding the area of the triangle was also common.

Paper 2 Question 4

This question was attempted by 81% of candidates. The average mark was 30.

(a) Full marks were common for this part. Where marks were lost, this was mostly due to candidates misreading co-ordinates, not plotting the points, confusing the axes and not finding the midpoint.

(b) Where candidates took the co-ordinates of q as the second for substitution in the formula for the length and slope of [pq], $2 - (-3) = -1$ was frequently written down and then doubled instead of squared for the length of the x/y co-ordinates.

Substituting for x and y as well as for x_1 and y_1 in the equation of the line formula, $y - y_1 = m(x - x_1)$, also occurred in some solutions.

(c) The slope of the line L was generally found correctly, but not so for the perpendicular slope.

Some candidates simply copied the formula for the slope of a line from the formula sheet and stopped. Others copied the equation of a line formula and did not proceed any further.

Paper 2 Question 5

This question was attempted by 52% of candidates. The average mark was 23.

Often, part (a) only was attempted. It seemed that some of these candidates had not studied this topic.

(a) Giving $\tan A = \frac{3}{4}$ encouraged students to attempt this part.

The errors that occurred were in finding the sin, cos and tan of 3°, 4° and 5° and using inverted ratios.

(b) Most attempts were purely arithmetical with no trigonometrical functions being used - e.g. 58×5, $58 \div 5$.

Incorrect trigonometric functions were used, e.g. $\tan 32° = \frac{y}{5}$.

Incorrect transposition occurred and RAD or GRAD mode were used on the calculator.

(c) This part was rarely attempted. There were some graphical attempts, including measurement from the diagram. Sometimes, h was calculated as $49° \times 20$.

Paper 2 Question 6

This question was attempted by 84% of candidates. The average mark was 34.
The most common loss of marks in this question was, surprisingly, in part (a) where 2 + 3+ 4 was a very common answer. Otherwise, candidates gained high marks in this question.

(a) As stated above, marks were often lost in this part of the question. Typical of some of the incorrect answers were $2 + 3 + 4 = 9$ or $3!$ or $2! + 3! + 4! = 32$.

(b) Some candidates did not divide by the total number of possible outcomes in this part. For example in parts *(i)* and *(ii)*, common answers included
$\frac{1}{9}$, $\frac{1}{11}$, $\frac{1}{31}$ and $\frac{11}{29}$, $\frac{9}{31}$.
Also the probability of a green ticket was frequently incorrectly given as
$$\text{P(green)} = \text{P(not red)} + \text{P(not white)} = \frac{49}{40}.$$

(c) Giving the first three permutations in the problem enabled candidates to find the remaining five permutations. However, the phrase " at least " caused difficulties. Some did not know how to proceed with that phrase. Often, "at least two… boys" was interpreted as "two boys".

Paper 2 Question 7

This question was attempted by 88% of candidates. The average mark was 29.
Once again the candidates found part (a) more difficult than part (b). Very few managed to produce a correct histogram.

(a) Instead of drawing a histogram, some candidates gave a bar chart or trend graph. When the histogram was drawn, a common error was not doubling the length of the base in the last rectangle and not halving the height. Axes were also reversed on occasions.

(b) Giving part of the cumulative frequency table on the question paper seemed to help candidates to complete the table correctly. Where errors did occur, they were in the areas of axes reversed, scale errors in axes, not cumulating the numbers in the table correctly.

(c) The majority of candidates found the correct mean of the array. However, most stopped at this stage and made no attempt at calculating the standard deviation. Only rarely was the statistics mode on the calculator used to determine these quantities.
The main areas of difficulty were:
- not dividing by 4 when calculating the mean; division by 2 occurred frequently
- neglecting to calculate the square root when finding the standard deviation.

Paper 2 Question 8

This question was attempted by 34% of candidates. The average mark was 17.
It was the least popular question on the examination paper. In the constructions for parts (a) and (c), there was very little evidence of the use of compasses or protractors. The vast majority of those who attempted this question showed little knowledge of the concept of enlargement.

(a) In constructing the triangle *abc*, the lengths of the sides drawn were at times incorrect and the required angle $\angle abc$ was not measured.

(b) The attempt mark was the most common award for all three items here. All possible combinations of the values given in the diagram were used to determine the scale factor (for example, $k = \frac{8.3}{3.3}$ or $\frac{8.8}{5}$); to find distance $|od'|$ (for example, $|od'| = 1.6 \times 3.3$) and to find the area of the triangle.

(c) The quality of the answering was poor for this part. Some candidates produced a circumcircle. A freehand incircle was common and arcs were usually not shown when bisecting angles.

4. OVERALL GENERAL COMMENT

The quality of the answering on both papers was similar to that of previous years. Since the first examination at Foundation Level in June 1997, candidates presenting have become increasingly more skilled in the use of the calculator. This is evident from marks awarded in question 1 on Paper 1. These reflect candidates' mastery in accurate and efficient use of the calculator. Candidates' answering on Paper 1 provides evidence of good understanding of mathematical knowledge that is of immediate applicability and usefulness. Algebra, however, continues to be unpopular with students at this level and it is an area of weakness.

On Paper 2, there has been an improvement in the standard of answering over the past number of years. It was noted that candidates who did not manage to reach a D grade usually did not attempt the required number of questions. It also seemed from the quality of answering of the questions on trigonometry and constructions/enlargements, that some candidates had either not studied these topics at all or had not practised them enough in preparation for the examination.

5. RECOMMENDATIONS FOR TEACHERS AND STUDENTS

(i) The importance of attempting the required number of questions on each paper should be emphasised to students presenting for the examination.

(ii) Students should show as much work as possible in answering questions.

(iii) Teaching methods which foster and deepen students' understanding of algebra and trigonometry are strongly recommended.

APPENDIX

Statistics for Mathematics - Foundation Level 1998, 1999, 2000

1998	A1	A2	B1	B2	B3	C1	C2	C3	D1	D2	D3	E	F	NG	Total
Number	183	320	534	704	736	751	659	553	513	374	323	299	94	13	6056
%	3.0	5.3	8.8	11.6	12.2	12.4	10.9	9.1	8.5	6.2	5.3	4.9	1.6	0.2	
Female	83	140	243	347	355	399	375	290	274	196	148	142	34	5	3031
% of Female	2.7	4.6	8.0	11.4	11.7	13.2	12.4	9.6	9.0	6.5	4.9	4.7	1.1	0.2	
Male	100	180	291	357	381	352	284	263	239	178	175	157	60	8	3025
% of Male	3.3	6.0	9.6	11.8	12.6	11.6	9.4	8.7	7.9	5.9	5.8	5.2	2.0	0.3	

1999	A1	A2	B1	B2	B3	C1	C2	C3	D1	D2	D3	E	F	NG	Total
Number	191	351	543	704	743	690	601	516	410	322	286	289	89	18	5753
%	3.3	6.1	9.4	12.2	12.9	12.0	10.4	9.0	7.1	5.6	5.0	5.0	1.5	0.3	
Female	105	202	271	380	396	353	313	270	205	169	123	138	37	5	2967
% of Female	3.5	6.8	9.1	12.8	13.3	11.9	10.5	9.1	6.9	5.7	4.1	4.7	1.2	0.2	
Male	86	149	272	324	347	337	288	246	205	153	163	151	52	13	2786
% of Male	3.1	5.3	9.8	11.6	12.5	12.1	10.3	8.8	7.4	5.5	5.9	5.4	1.9	0.5	

2000	A1	A2	B1	B2	B3	C1	C2	C3	D1	D2	D3	E	F	NG	Total
Number	147	323	534	656	746	745	641	567	441	363	297	280	91	15	5846
%	2.5	5.5	9.1	11.2	12.8	12.7	11.0	9.7	7.5	6.2	5.1	4.8	1.6	0.3	
Female	73	178	302	321	382	380	329	290	231	196	128	119	34	6	2969
% of Female	2.5	6.0	10.2	10.8	12.9	12.8	11.1	9.8	7.8	6.6	4.3	4.0	1.1	0.2	
Male	74	145	232	335	364	365	312	277	210	167	169	161	57	9	2877
% of Male	2.6	5.0	8.1	11.6	12.7	12.7	10.8	9.6	7.3	5.8	5.9	5.6	2.0	0.3	

H346