Coimisiún na Scrúduithe Stáit
State Examinations Commission

LEAVING CERTIFICATE EXAMINATION 2005

MATHEMATICS

CHIEF EXAMINER'S REPORT

HIGHER, ORDINARY AND FOUNDATION LEVELS

CONTENTS

1. GENERAL INTRODUCTION

This chapter deals with the overall context of the examination and with matters that are related to all three levels. Chapters 2, 3 and 4 deal with the three levels separately and may be read independently of each other.

This report does not include "Exemplars of Standard", as often appear in other subjects. The nature of the subject and the manner in which the examination is marked render this unnecessary. The marking schemes for mathematics are very comprehensive and, to any reader familiar with the subject, will give a clear indication as to the standard and quality of work required to achieve particular grades. These marking schemes are available in the "Examination Material Archive" section of the State Examinations Commission's website: www.examinations.ie.

1.1 SYLLABUS DETAILS AND PARTICIPATION RATES

The State Examinations Commission offers Leaving Certificate examinations in Mathematics at three levels – Higher, Ordinary and Foundation.

Despite not being a compulsory subject for Leaving Certificate, mathematics continues to be extremely popular, taken by over 96% of all Leaving Certificate candidates. Without doubt, this is due, in no small measure, to the fact that the vast majority of third-level institutions require applicants to have studied the subject to Leaving Certificate level, and any Irish employer looking at academic achievement is likely to have the same expectation, if not requirement. This reflects the high level of respect that achievement in mathematics has traditionally enjoyed in society, and has resulted in the subject being treated as effectively compulsory in schools.

The current syllabuses in Mathematics for the Higher and Ordinary courses were introduced in September 1992 and were examined for the first time in June 1994. The syllabus for the Foundation course was introduced in 1995 and examined for the first time in 1997. It replaced the Ordinary Alternative course which had been introduced in 1990 and was examined for five years, starting in 1992. Full details of the aims, objectives and content of the syllabuses are contained in the relevant syllabus documents, which are available at the website of the Department of Education and Science – www.education.ie. It is nonetheless worth referring briefly to the rationale and style of the courses at the three different levels, as described in the following extracts from the syllabus documents.

> **[Higher]**
> The Higher course is aimed at the more able students. Students may choose it because it caters for their needs and aspirations as regards careers or further study, or because they have a special interest in mathematics. … The course offers opportunities for [the students] to deepen their understanding of mathematical ideas, to encounter more of the powerful concepts and methods that have made mathematics important…. Particular emphasis can be given to aims concerned with problem-solving, abstracting, generalising and proving. … Due attention should be given to maintenance of the more basic skills…. However, it may be assumed that some of the aims regarding the use of mathematics in everyday life and work have been achieved in the Junior Cycle; they are therefore less prominent at this level.
>
> **[Ordinary]**
> For many of [these students], mathematics is essentially a service subject – providing knowledge and techniques that will be needed in future study of scientific, economic, business and technical subjects. For others, [the course] may provide their last formal encounter with mathematics. … The course moves gradually from the relatively concrete and practical to more abstract and general

concepts. … Particular emphasis can be given to aims concerned with the use of mathematics. Due attention should be given to maintenance of the more basic skills, especially in application of arithmetic and algebra.

[Foundation]
The Foundation course is intended to equip students with the knowledge and techniques required in everyday life and in various kinds of employment. It is also intended to lay the groundwork for students who proceed to further education and training where specialist mathematics is not required. … Basic knowledge is maintained and enhanced by being approached in an exploratory and reflective manner. … Computational work is balanced by emphasis on the visual and spatial. … Particular emphasis can be placed on the aims concerned with the use of mathematics in everyday life and work – especially as regards intelligent and proficient use of calculators – and with the recognition of mathematics in the environment.

The Higher and Ordinary Level syllabuses each consist of a core and four options, with the intention that students will study all of the core and one of the options. The Foundation Level syllabus does not include such optional material – it is intended that students of this course study should cover all of the syllabus material.

1.2 UPTAKE AT THE VARIOUS LEVELS

In light of the NCCA's current review of mathematics education at second level, it is useful to provide an overview of uptake and achievement over the period of the current syllabuses. Table 1.1 below shows the numbers and percentages of candidates who took the three levels of Mathematics examination from 1993 to 2005. The percentage figures are illustrated in Figure 1.1 on the next page.

		Number at each level			Percentage at each level		
Year	Total Maths Candidature	Foundation*	Ordinary	Higher	Foundation*	Ordinary	Higher
1993	*58922*	*6169*	*46158*	*6595*	*10.5*	*78.3*	*11.2*
1994	60524	*6419*	45442	8663	*10.6*	75.1	14.3
1995	63200	*5863*	46698	10639	*9.3*	73.9	16.8
1996	53246†	*5398*	38378	9470	*10.1*	72.1	17.8
1997	61035	6427	43566	11042	10.5	71.4	18.1
1998	61970	6056	45191	10723	9.8	72.9	17.3
1999	60637	5753	44188	10696	9.5	72.9	17.6
2000	58705	5846	42214	10645	10.0	71.9	18.1
2001	55149	5227	39984	9938	9.5	72.5	18.0
2002	53658	5296	38932	9430	9.9	72.6	17.6
2003	54256	5702	39101	9453	10.5	72.1	17.4
2004	53052	5832	37794	9426	11.0	71.2	17.8
2005	52178	5562	36773	9843	10.7	70.5	18.9

Table 1.1: Uptake of Leaving Certificate Mathematics at the three levels, 1993 – 2000.

The statistics for 1993 (the last year of the previous syllabuses) are included in Table 1.1 in order to illustrate the changes that were associated with the introduction of the new syllabuses. Of particular note is the increase in the proportion of Higher Level candidates over this period. This grew rapidly over the first four years of the syllabus before stabilising over the following years. The proportion has increased again somewhat in 2005 and is now greater than it has been at any time over the lifetime of this or the previous syllabus. It is notable that the proportion of the candidature opting for Higher Level is now almost 70% greater than it was in 1993, the last year of the previous syllabus.

Whereas this development is very welcome, it must also be recognised that the uptake at the various levels does not match the uptake anticipated by the original course committee. The courses were designed in anticipation of the cohort dividing between the three levels in the ratio 25:50:25. Although the uptake at Higher Level has come a considerable way towards

* The figures in the *Foundation* column from 1993 to 1996 refer to the *Ordinary Level Alternative* course.

† The significant dip in the total number of candidates who took Mathematics in 1996 was due to the increased uptake of the Transition Year Programme in 1994.

meeting this expectation, it is clear that the proportion of candidates following the Foundation Level course has remained far below the expected 25%, (see Figure 1.1 below). This has serious implications for the effective assessment of candidates at both Ordinary and Foundation levels. In the case of the Ordinary Level, it means that the examination is struggling to cater not only for its intended target group, but for a further group of approximately 7,500 candidates who are properly part of the target group for the Foundation Level (not to mention the 3,000 or so candidates who are properly part of the Higher Level target group). This inevitably results in tensions between the intended standards of achievement to be measured and the need to have a reasonable grade distribution for the examination.

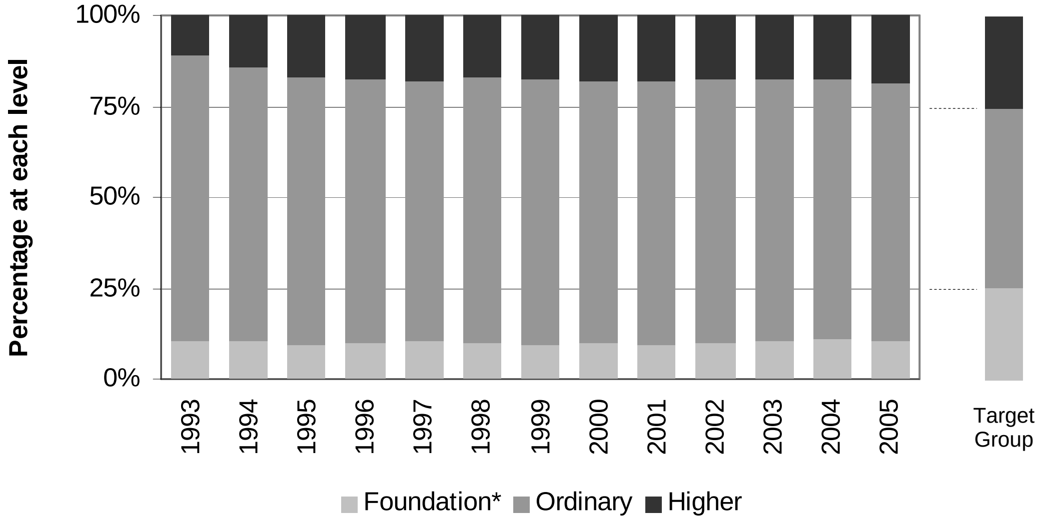

Figure 1.1: Percentage uptake at each level, 1993 – 2005.

Similarly, the Foundation Level examination is being taken by less than half of its intended cohort, and this group can safely be assumed to be strongly weighted towards the lower achieving end of that intended cohort. This means that hardly any of those who were intended to be the A and B grade candidates at this level are actually present in the *de facto* candidature. Here again, this leads to inevitable tensions between the intended standards and the need to have a reasonable grade distribution for the examination.

The reasons for inappropriate choice of level and some recommendations for dealing with this are dealt with in the *Leaving Certificate Ordinary Level Chief Examiner's Report 2001*, and it is not intended to repeat them here. Nonetheless, it is worth noting that the situation has not improved in the intervening period. In light of the continuing policy of many higher education institutions and employers to decline to give Foundation Level Mathematics the level of recognition recommended by the Department of Education and Science, it must be accepted that a large portion of the target group will continue to reject it, and serious consideration should, therefore, be given to how best to deal with this.

More specific issues related to the level of candidate achievement in the various competencies specified in the syllabuses are dealt with in the body of this document. On a more general level, and on the presumption that syllabus revisions will follow the current mathematics review, the Chief Examiner suggests that it will be vital in such circumstances to specify outcomes and standards that have a realistic expectation of being achieved by the envisaged proportion of the overall cohort. It will then be equally vital to encourage students into the appropriate course and to support the achievement of the envisaged standards.

1.3 GENDER DIFFERENCES

There have historically been significant differences between boys and girls regarding both the level chosen and the grades achieved. As regards choice of level, boys are more likely to opt for Higher Level than girls, and boys are more likely to opt for Foundation Level than girls. It is noticeable, however, that the former of these two phenomena is becoming less pronounced over time (Table 1.2 and Figure 1.2). Whereas in 1995, boys were 42% more likely than girls to take Higher Level*, this dropped to 27% in 2000 and to 14% in 2005. Indeed, almost all of the growth in the Higher Level population referred to in section 2.1 above is accounted for by the increased uptake among girls. The difference between boys and girls in the uptake at Foundation Level shows some variation from year to year, but is remaining essentially stable over time.

		Percentage of the gender cohort at each level		
		Foundation	*Ordinary*	*Higher*
1995	Female	8.8	77.2	14.0
1995	Male	9.8	70.4	19.8
2000	Female	9.7	74.2	16.1
2000	Male	10.2	69.4	20.4
2005	Female	9.9	72.4	17.6
2005	Male	11.5	68.3	20.2

Table 1.2: Percentage uptake at each level, by gender.

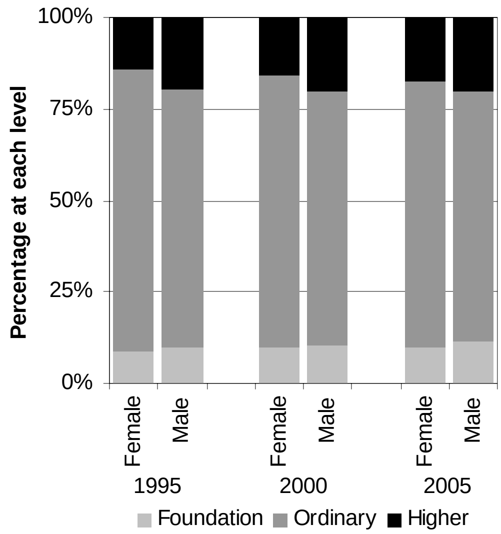

Figure 1.2: Percentage uptake at each level, by gender.

The gender differences in choice of level militate against a proper analysis of gender differences in grade achievement, (as indeed do gender differences in retention rates). In comparing the grade distributions of boys and girls within any level, one is not comparing equivalent portions of the gender cohort. Nonetheless, the 2005 data seem to indicate that the higher performing boys at Higher Level are outperforming the higher performing girls, and at all other points of the achievement spectrum, girls are outperforming boys. (See Appendix A for grade distributions by gender).

* That is, the proportion of the male candidature opting for Higher Level was 42% greater than the proportion of the female candidature opting for Higher Level.

2. FOUNDATION LEVEL

2.1 INTRODUCTION

The examination consists of two papers, each of two and a half hours' duration. Three hundred marks are allocated to each paper, giving a total of 600 marks.

On paper 1, candidates attempt question 1 and any four from a range of six further questions. Question 1 carries 100 marks and consists of ten short parts testing the candidates' ability to use a calculator accurately in a range of straightforward contexts. There is a certain level of internal grading of difficulty within this question, in that the earlier parts are somewhat more straightforward than the later.

The remaining questions on paper 1 carry fifty marks each. All of these questions have an internal grading of difficulty, usually but not necessarily structured as an **(a)**, **(b)** and **(c)** part. There is no internal choice within questions.

On paper 2, candidates attempt any six from a range of eight questions. Each question carries fifty marks. All questions have an internal grading of difficulty, usually but not necessarily structured as an **(a)**, **(b)** and **(c)** part. There is no internal choice within questions.

2.2 PERFORMANCE OF CANDIDATES

Tables 2.1 to 2.3 below summarise the results of the Foundation Level examination for the whole cohort. Note that Table 2.3 is a cumulative table, indicating that, for example, 38.7% of candidates achieved at least a B3 grade in 2005.

	A1	A2	B1	B2	B3	C1	C2	C3	D1	D2	D3	E	F	NG
2002	181	299	438	597	643	647	616	470	402	320	306	260	103	14
2003	273	423	558	675	757	702	607	430	406	276	270	245	70	10
2004	214	366	506	673	767	733	608	522	415	358	289	286	84	11
2005	**136**	**283**	**453**	**614**	**666**	**678**	**629**	**557**	**440**	**334**	**341**	**319**	**102**	**10**

Table 2.1: Number achieving each grade – Foundation Level, 2002 to 2005

	A1	A2	B1	B2	B3	C1	C2	C3	D1	D2	D3	E	F	NG
2002	3.4	5.6	8.3	11.3	12.1	12.2	11.6	8.9	7.6	6.0	5.8	4.9	1.9	0.3
2003	4.8	7.4	9.8	11.8	13.3	12.3	10.6	7.5	7.1	4.8	4.7	4.3	1.2	0.2
2004	3.7	6.3	8.7	11.5	13.2	12.6	10.4	9.0	7.1	6.1	5.0	4.9	1.4	0.2
2005	**2.4**	**5.1**	**8.1**	**11.0**	**12.0**	**12.2**	**11.3**	**10.0**	**7.9**	**6.0**	**6.1**	**5.7**	**1.8**	**0.2**

Table 2.2: Percentage achieving each grade – Foundation Level, 2002 to 2005

	A1	A2	B1	B2	B3	C1	C2	C3	D1	D2	D3	E	F	NG
2002	3.4	9.1	17.3	28.6	40.7	53.0	64.6	73.5	81.1	87.1	92.9	97.8	99.7	100
2003	4.8	12.2	22.0	33.8	47.1	59.4	70.1	77.6	84.7	89.6	94.3	98.6	99.8	100
2004	3.7	9.9	18.6	30.2	43.3	55.9	66.3	75.3	82.4	88.5	93.5	98.4	99.8	100
2005	**2.4**	**7.5**	**15.7**	**26.7**	**38.7**	**50.9**	**62.2**	**72.2**	**80.1**	**86.1**	**92.3**	**98.0**	**99.8**	**100**

Table 2.3: Percentage achieving at or above each grade – Foundation Level, 2002 to 2005

Question Popularity, Percentage Uptake and Average Mark

Tables 2.4 and 2.5 show the ranking of questions in order of decreasing popularity. The tables also give the percentage of candidates who attempted each question, and the average mark awarded for each question. These figures are estimates derived from an analysis of a sample of 4% of the scripts.

Order of Popularity	*Question Number*	*Topics (Syllabus section and subsection)*	*% Attempts*	*Average Mark (out of 50)*
1	1	Arithmetic – calculator skills (compulsory)	100%	71 (out of 100)
2	6	Functions and graphs, 4	98%	42
3	3	Arithmetic 1, 3, 4	84%	24
4	2	Arithmetic 3, 5, 9	82%	23
5	5	Number systems 2, Algebra 1	81%	25
6	4	Algebra 1, 2	62%	28
7	7	Functions and graphs 1, 2	43%	27

Table 2.4: Popularity and Average Mark of Questions on Paper 1

Order of Popularity	*Question Number*	*Topics (Syllabus section and subsection)*	*% Attempts*	*Average Mark (out of 50)*
1	1	Area and volume 1, 2	96%	38
2	2	Area and volume 1	95%	31
3	6	Statistics & Probability 1, 2	90%	38
4	7	Statistics & Probability 3	87%	27
5	4	Geometry 1	86%	27
6	5	Trigonometry 1, 2	67%	23
7	3	Geometry 2	66%	28
8	8	Geometry 3, 4, 5	21%	23

Table 2.5: Popularity and Average Mark of Questions on Paper 2

It may be noted that popularity and average mark are only moderately correlated.

2.3 ANALYSIS OF CANDIDATE PERFORMANCE

Paper 1, Question 1

Attempts: 100% (compulsory) **Average Mark: 71 (out of 100)**

The question was well answered by most, with less than 2.5% of candidates scoring below 40 marks on this question and over 30% scoring above 80 marks. This question is essentially a test of calculator skills, and candidates in general showed competence in this area. However, it should be noted that the level of accuracy and competence that was envisaged when the syllabus was designed has never been achieved.

There was no evidence that the candidates made any estimate in advance of their calculations regarding what the answers should be, with the result that most errors went unnoticed.

Rounding answers to a specified degree of accuracy (especially in the case of significant figures) posed problems. Many candidates ignored the rounding requirement.

(i) This part was very well answered, apart from the rounding.

(ii) This part was very well answered, apart from failure to write the answer to the nearest whole number. Some candidates multiplied by 3, or treated the cube as a square.

(iii) There were a lot of errors in precedence here.

(iv) A lot of errors occurred in this part, and it would appear that fractions are not well understood. Dividing the denominator by the numerator, precedence errors and sign errors were common. There was also considerable evidence of lack of understanding of how the candidates' own calculators deal with decimals, fractions and precedence.

(v) This part was well answered generally, apart from the rounding. However, many candidates displayed difficulty dealing with percentages.

(vi) This part was quite well answered, but dividing by 1.25 was a common error. Again, rounding was frequently omitted or incorrect

(vii) Fractions again caused problems for many. Many did add the fractions correctly, but did not convert the result to a decimal. Errors included adding numerators and denominators respectively (to yield $1\frac{4}{15}$).

(viii) This part was fairly well answered by those candidates who scored well overall. However, ratio appears not to be well understood by many. A large number correctly divided by 14 to get 8 and did not finish. Some just divided 112 by 2, by 5 and by 7.

(ix) Precedence errors occurred here, and candidates also failed to round to 2 significant figures. Very commonly, the answer was left as $\frac{71.34}{34.6}$. It is conceivable that some of these candidates misunderstood "2 significant figures" to mean that two numbers should be left in the answer, causing them not to finish from here.

(x) The most common approach was to expand each number. Many did this and finished the question successfully. There was no evidence of the 'EXP' function on the calculator being used. Some candidates misinterpreted 10^5 as 50, and so on. The lower scoring candidates tended to omit this part.

Paper 1, Question 2

Attempts: 82% **Average Mark: 23**

Syllabus Section(s): Arithmetic 3, 5, 9

Overall the question was poorly answered.

(a) This part was reasonably well answered. Common errors included failure to convert litres to cm^3, or using an incorrect conversion factor. Sometimes the answer was given correctly in litres (2.25).

(b) **(i)** The system of tax credits was not well understood. Subtracting the tax credit first was very common, as in the old system of Tax Free Allowances. There were some errors in getting 20% also. Although scoring well in many cases, few candidates got full marks in what was a very straightforward part.

(ii) This part was quite well answered. A small number of candidates misused the tax credits again.

(c) **(i)** This part was very poorly answered. Many got to 1.3(33…) hours but failed to convert correctly to hours and minutes. Candidates frequently took 1.33…hours to be 1 hour 33mins.

(c) **(ii)** There were very few meaningful attempts here at all. At best, attempts were made at using the speed/distance/time formula, but the vast majority of candidates did not know how to marshal the information given.

Paper 1, Question 3

Attempts: 84% **Average Mark: 24**

Syllabus Section(s): Arithmetic 1, 3, 4

(a) **(i)** This part was very well done. Almost all candidates who attempted this question gave the correct answer.

(ii) This section was poorly answered. Various combinations of 3 with 35, 38 and 100 were offered, but percentages again caused difficulties. Many candidates are unable to form the relevant fraction and then convert it to a percentage. That is, the idea of a percentage as being an equivalent expression for a fraction or ratio was clearly not well understood, even in this familiar context.

(b) **(i)** This was either fully correct, often without work shown, or very poorly answered, with candidates just making some attempt at finding the cost of 1g or 50g.

(ii) This was poorly answered. There were many fully correct answers, but even some of those who correctly answered part **(i)** made errors in this part. Other than the correct answer, there was a huge range of unusual attempts and answers, most of which were without merit.

(c) **(i)** Here again finding a percentage caused great difficulty. Many gave €3000 as the final answer, or then asserted that this was equal to 30%.

(ii) This section was very poorly answered. Only a small number of candidates completed it successfully. Those who did so used a year-by-year method rather than using the supplied formula. The majority did not compound the depreciation. €8000 was a very common incorrect answer. (This results from assuming a depreciation of €3000 each year.)

Paper 1, Question 4

Attempts: 62% **Average Mark: 28**

Syllabus Section(s): Algebra 1, 2

Although this was not a popular question, many of those who attempted it scored well.

(a) This part was well answered. There were some transposition errors.

(b) This part was reasonably well answered. Generally those attempting to solve the simultaneous equations knew what was required. The most popular method was to multiply one equation by 2 and then cancel the terms containing y (or x). There were, however, errors in multiplying by 2, sign errors, and transposition errors. A number of candidates did not attempt to find the value of the second variable. Those who attempted to do so generally did it successfully.

(c) **(i)** This part was well answered. Sometimes the inequality sign was dropped towards the end.

(ii) This part caused more difficulties, as the x-term was negative. Very few candidates handled the direction of the inequality sign correctly. There were also transposition errors.

(iii) Most candidates did not seem to relate this part to the work done in the previous parts. A generously awarded attempt mark was the most common outcome.

Paper 1, Question 5

Attempts: 81% **Average Mark: 25**

Syllabus Section(s): Number systems 2, Algebra 1

(a) Part **(i)** was very well answered and part **(ii)** less so, with many candidates taking “prime numbers” to mean “odd numbers”.

(b) **(i)** This part was poorly answered. The most common method was to use the quadratic formula, despite the fact that the expression was very easy to factorise. Errors in substitution and calculation then followed. For those candidates who did factorise, errors in getting the factors occurred often, although the correct method (equating each factor to zero) was almost invariably used to finish.

(ii) This part was also poorly answered. As in part **(i)**, errors in using the quadratic formula were frequent. Most of the errors were sign errors in substitution and calculation, rather than incorrect substitution *per se*. The error: -4(5)(-3) = -60 was common. Precedence errors also occurred, such as $\frac{11 \pm 13.45}{10} = 11 \pm 1.345$ or $\frac{11 \pm \sqrt{181}}{10} = 11 \pm \sqrt{18 \cdot 1}$. Other errors were due to rounding incorrectly, or not rounding at all.

(c) This part was generally well answered. The easily understood nature of the question led to the problem being answered using informal arithmetic strategies, without reference to 'x' in many cases.

(i) Reasonable attempts were made at forming an equation. Using x^2 instead of $2x$ was a common error. Often one term was omitted.

(ii) This part was well answered. It was often worked out without the x being used: 74 – 8 = 66 ÷3 = 22, or similar. A common error was $2x = 66 \Rightarrow x = 33$.

(iii) This part was very well answered.

(iv) This part was not well answered. As in other questions, percentages caused difficulties. Answers, when found, were frequently not rounded correctly.

Paper 1, Question 6

Attempts: 98% **Average Mark: 42**

Syllabus Section(s): Functions and graphs, 4

Apart from Question 1, which is compulsory, this was by far the most popular question and was well answered by almost all candidates.

(i) This part was very well answered. There was some misreading of the chart (20 instead of 21) or confusion of the line for DVDs with that for videos.

(ii) This part was also very well answered. Again some misreading of the chart occurred, 10 – 5 being the most common. Less frequently, 10 + 6 was given.

(iii) This part was well answered. Errors that occurred were: giving the number of days rather than naming the days; omitting one or more days; and giving Saturday only.

(iv) This part was reasonably well done. The concept of the *mean* seems to have been understood. The total number of videos was usually found correctly, although there were occasional slips. Occasionally, the average of the DVDs was found.

(v) This part was poorly answered. Once again, percentages caused problems. The total number of DVDs was usually found correctly, but the most common next step was to calculate the average as in part **(iv)**. Attempts at finding a percentage had similar errors to other questions on percentages.

Paper 1, Question 7

Attempts: 43% **Average Mark: 27**

Syllabus Section(s): Functions and graphs 1, 2

The spread of marks was widest in this question, with 25% of attempts scoring at least 40 marks almost 17% scoring below 15 marks

Graph:

The most common approach was to construct and use a table to calculate couples. While many candidates did this correctly, there were also many errors in the table: $2x^2$ was treated as $4x$, or as $(2x)^2$, and $3x$ was sometimes treated as $x + 3$. There were also many errors in signs and in totting. The plotting of the graph itself was quite well done on the basis of the candidate's couples. Some errors in scale occurred, and points were sometimes inaccurately drawn. However, most managed a reasonably smooth curve.

Few used direct substitution into $f(x)$ to calculate the couples, and usually with limited success. Nonetheless, it would seem that the table method is learned by rote without any real understanding of what is happening. In general, methods which are computationally more efficient but less obviously related to underlying concepts may not ultimately be in the best educational interests of students. Accordingly, they be should be treated with caution, especially at this level. The method of direct substitution into $f(x)$, if dealt with properly and given due time, facilitates sounder understanding, and is therefore to be recommended.

The majority of candidates ignored the follow-on questions on the graph.

(i) Poorly answered. The meaning of "roots" was evidently not understood.

(ii) This part was answered quite well.

(iii) Reasonably well answered. A common error was reading $f(x) = 1.5$.

(iv) Very poorly answered. Some drew the line $f(x) = 1$ but gave one value only. Others found $f(1)$. Many ignored this part completely.

Paper 2, Question 1

Attempts: 96% **Average Mark: 38**

Syllabus Section(s): Area and volume 1, 2

(a) Not as well done as would be expected. Common errors included: taking 10 cm^2 to mean $(10\ cm)^2$ and multiplying 10 by 8. For those who set up an equation, the usual errors with transposing terms occurred. Incorrect formulae were frequently used.

(b) Well answered. The first and last ordinates being equal to zero caused problems for some. Many have difficulty handling the $\frac{h}{3}$ in the formula.

Paper 2, Question 2

Attempts: 95% **Average Mark: 31**

Syllabus Section(s): Area and volume 1

(a) Perimeter was frequently given instead of area. Many could not identify the radius of the circle from the information given.

(b) **(i)** Well answered, although many did not give the answer in terms of π.

(ii) Many ignored h or inserted a value for h.

(iii) Many did attempt to use $h = \frac{3(vol)}{\pi r^2}$, but correct substitution was rare.

Paper 2, Question 3

Attempts: 66% **Average Mark: 28**

Syllabus Section(s): Geometry 2

(a) Poorly done. Common errors were $x = y = 130^o$ or $x = y = \frac{130}{2} = 65^o$.

(b) The value of A was usually correct. The other angles were rarely correct, with 40° a common answer for any of the angles B, C or D.

(c) **(i)** The right-angle was not recognised.

(ii) Well answered.

(iii) Very few attempted to use Pythagoras' theorem, and many of these failed to do so correctly.

Paper 2, Question 4

Attempts: 86% **Average Mark: 27**

Syllabus Section(s): Geometry 1

(a) Marks were lost due to candidates misreading coordinates and confusing the axes.

(b) Errors occurred in substitution and in handling signs in such cases as 3 – (-5).

(c) **(i)** Well answered.

(ii) The technique for finding the slope of a line was not understood.

(iii) Only the A-grade candidates persevered this far, and with limited success.

Paper 2, Question 5

Attempts: 67% **Average Mark: 23**

Syllabus Section(s): Trigonometry 1, 2

Candidates showed little understanding of the basics of trigonometry. Numbers were combined in apparently random ways, without any reference to trigonometric understanding, and these spurious outcomes offered as answers. Among the few who offered meaningful work, many had their calculators in the wrong mode. Very few could manage part **(c)**.

Paper 2, Question 6

Attempts: 90% **Average Mark: 38**

Syllabus Section(s): Statistics & Probability 1, 2

All parts were very well answered by a large number of the candidates.

Paper 2, Question 7

Attempts: 87% **Average Mark: 27**

Syllabus Section(s): Statistics & Probability 3

(a) Most candidates calculated the *mean* instead of giving the *mode*.

(b) Errors occurred in compiling the cumulative table. Also common were reversing of the axes, and errors in the scaling of the axes.

(c) The *mean* was successfully calculated, but very few knew all the steps in finding the *standard deviation*. There was little use of the statistical facilities on the calculator.

Paper 2, Question 8

Attempts: 21% **Average Mark: 23**

Syllabus Section(s): Geometry 3, 4, 5

(a) The diagonals of the rectangle were usually given as axes of symmetry.

(b) Very few candidates did any meaningful work on this part.

(c) Some produced the circumcircle. Some produced the incircle without any evidence of having bisected the angles. Freehand drawings were the most common.

2.4. CONCLUSIONS

Patterns of Question Choice

The patterns of answering summarised in section 2.2 above, in terms of the popularity of the various questions, are similar to those of previous years. A large number of the candidates attempted excess questions on paper 1, with very few failing to attempt the required number. This phenomenon was not especially evident on paper 2, where fewer candidates attempted excess questions, and a substantial number failed to attempt the required number.

Standard of Answering; Candidate Strengths and Weaknesses

Candidates showed some skill in the use of the calculator, as evidenced by their performance in question 1 on paper 1 in particular, although understanding precedence of arithmetic operations is poor. Even the weaker candidates have developed reasonable calculator skills, as evidenced by the fact that, of the candidates who failed to obtain a grade D or better on paper 1, the vast majority had scored well on question 1 but accumulated very few further marks from the other questions. However, it is clear that the ability to apply the acquired skills is lacking.

Percentages continue to cause significant difficulties for candidates at this level. This is of particular concern considering the importance of percentages in the numeracy skills required for daily life and work. The related area of fractions is also problematic.

Algebra continues to present a lot of difficulty for candidates, particularly at a conceptual level, although certain basic algebraic techniques were well executed by many candidates. Transposing accurately and with understanding is certainly a difficulty.

Questions involving basic arithmetic in very straightforward contexts were well handled, and many candidates were able to display some proficiency in routine procedural work, such as the application of Simpson's rule, and calculations of the mean of a set of numbers. Many candidates also displayed a quite satisfactory level of numerical proficiency in, for example, solving the numerical problem in question 5(c) on paper 1 without using algebra. Signs, however, cause particular problems.

Strengths that displayed understanding on the part of candidates include their ability to interpret graphical displays of concrete situations, such as the line graph describing video and DVD sales in question 6 on paper 1. This contrasts sharply with the poor ability to interpret graphs presented in a more abstract context, such as question 7 on the same paper. Candidates have the procedural competence to construct such graphs when asked in the standard format, but there is little evidence of much accompanying understanding or appreciation. The questions on probability at this level are quite straightforward, but it should nonetheless be noted that they are handled well, and candidates appear to understand the underlying concept.

Trigonometry and geometry are content areas of particular weakness. Competent and accurate use of drawing instruments is also rare, although there is no reason for this to be the case, since the development of such skills is well within the capacity of virtually all candidates.

The overall average standard of answering continues to fall short of the originally envisaged standards, but this must be considered in the context of this course failing to attract a large proportion of its intended cohort, as discussed in section 1.2 above.

2.5. RECOMMENDATIONS FOR TEACHERS AND CANDIDATES

The following advice is offered to teachers preparing candidates for the Foundation Level examination:

- Build on methodologies advocated for the revised Junior Certificate courses, which encourage the understanding of concepts.
- Select such techniques and use such language as continually reinforce underlying concepts. A lack of conceptual understanding is particularly debilitating in algebra.
- Encourage estimation as a routine part of numerical work. This will help in avoiding obvious errors (such as precedence or other errors when using the calculator). It also develops an important life skill.
- In their classwork and homework, encourage students to show as much work as possible in answering questions.
- Encourage discussion and investigation of all the mathematics encountered, in order to develop confidence, appreciation and positive attitudes.
- Cover the syllabus fully to give candidates maximum potential in the examination.

The following advice is offered to candidates preparing for the Foundation Level examination:

In advance of the examination

- Make sure you *understand* all of the mathematical methods you employ. Continually consider *why* the particular steps in answering a question are followed, and ask your teacher to explain when you cannot figure this out. Following procedures that are meaningless to you will not be of much use to you in the future.
- Work on your basic skills in arithmetic and algebra, paying heed to accuracy. This will stand to you in all content areas.
- Close to the examination, practise doing standard types of questions from past examination papers, and become familiar with the format, terminology and overall structure of the paper.

In the examination

- Show all your work.
- Read the questions carefully.

- Pay attention to specific instructions regarding rounding your answer, or giving your answer in a particular format.
- Estimate what your answer should be before you start any calculation or similar problem; this will help you see if you are approaching the question correctly.
- Make sure you attempt the required number of questions.

3. ORDINARY LEVEL

3.1 INTRODUCTION

The examination consists of two papers, each of two and a half hours' duration. Three hundred marks are allocated to each paper, giving a total of 600 marks.

On paper 1, candidates attempt any six from a range of eight questions. All questions relate to material on the *Core* part of the syllabus. Each question carries fifty marks.

On paper 2, candidates attempt any five questions from the seven available in Section A, and one question from the four available in Section B. All questions in Section A relate to material on the *Core* part of the syllabus. Section B consists of one question on each of the four *Options* on the syllabus. Each question carries fifty marks.

On both papers, all questions have an internal grading of difficulty, usually but not necessarily structured as an **(a)**, **(b)** and **(c)** part. There is no internal choice within questions.

3.2 PERFORMANCE OF CANDIDATES

Tables 3.1 to 3.3 below summarise the results of the Ordinary Level examination for the whole cohort. Note that Table 3.3 is a cumulative table, indicating that, for example, 40.5% of candidates achieved at least a B3 grade in 2005.

	A1	A2	B1	B2	B3	C1	C2	C3	D1	D2	D3	E	F	NG
2002	2478	2803	3032	3187	3275	3202	3271	3102	2942	2777	3248	3675	1713	227
2003	1583	2698	3224	3526	3634	3556	3494	3385	3163	2921	3436	3164	1198	119
2004	2619	3318	3513	3679	3653	3391	3176	2823	2501	2216	2583	2893	1239	190
2005	**2055**	**2831**	**3298**	**3372**	**3331**	**3341**	**3216**	**3039**	**2745**	**2404**	**2723**	**2946**	**1290**	**182**

Table 3.1: Number achieving each grade – Ordinary Level, 2002 to 2005

	A1	A2	B1	B2	B3	C1	C2	C3	D1	D2	D3	E	F	NG
2002	6.4	7.2	7.8	8.2	8.4	8.2	8.4	8.0	7.6	7.1	8.3	9.4	4.4	0.6
2003	4.0	6.9	8.2	9.0	9.3	9.1	8.9	8.7	8.1	7.5	8.8	8.1	3.1	0.3
2004	6.9	8.8	9.3	9.7	9.7	9.0	8.4	7.5	6.6	5.9	6.8	7.7	3.3	0.5
2005	**5.6**	**7.7**	**9.0**	**9.2**	**9.1**	**9.1**	**8.7**	**8.3**	**7.5**	**6.5**	**7.4**	**8.0**	**3.5**	**0.5**

Table 3.2: Percentage achieving each grade – Ordinary Level, 2002 to 2005

	A1	A2	B1	B2	B3	C1	C2	C3	D1	D2	D3	E	F	NG
2002	6.4	13.6	21.4	29.5	38.0	46.2	54.6	62.5	70.1	77.2	85.6	95.0	99.4	100
2003	4.0	10.9	19.2	28.2	37.5	46.6	55.5	64.2	72.3	79.8	88.5	96.6	99.7	100
2004	6.9	15.7	25.0	34.7	44.4	53.4	61.8	69.2	75.9	81.7	88.6	96.2	99.5	100
2005	**5.6**	**13.3**	**22.3**	**31.4**	**40.5**	**49.6**	**58.3**	**66.6**	**74.0**	**80.6**	**88.0**	**96.0**	**99.5**	**100**

Table 3.3: Percentage achieving at or above each grade, Ordinary Level, 2002 to 2005

Question Popularity, Percentage Uptake and Average Mark

Tables 3.4 to 3.6 show the ranking of questions in order of decreasing popularity. The tables also give the percentage of candidates who attempted each question, and the average mark awarded for each question. These figures are estimates derived from an analysis of a sample of 4% of the scripts.

Order of Popularity	*Question Number*	*Topics (Syllabus section and subsection)*	*% Attempts*	*Average Mark (out of 50)*
1	7	Functions and Calculus 2	94%	31
2	2	Algebra 1, 2	92%	28
3	1	Arithmetic 1, 2, 3	91%	31
4	4	Algebra 4	91%	35
5	3	Algebra 1, 3	85%	34
6	6	Functions and Calculus 1, 2	85%	30
7	8	Functions and Calculus 1	80%	28
8	5	Finite Sequences and Series	56%	34

Table 3.4: Popularity and Average Mark of Questions on Paper 1, Ordinary Level

Order of Popularity	*Question Number*	*Topics (Syllabus section and subsection)*	*% Attempts*	*Average Mark (out of 50)*
1	1	Arithmetic 2, 4	99%	36
2	2	Geometry 2	96%	30
3	7	Discrete Mathematics & Statistics 3	92%	33
4	3	Geometry 2	87%	32
5	6	Discrete Mathematics & Statistics 1, 2	70%	32
6	5	Trigonometry	58%	29
7	4	Geometry 1, 3	22%	19

Table 3.5: Popularity and Average Mark of Questions on Section A of Paper 2, Ordinary Level

Order of Popularity	*Question Number*	*Topics (Syllabus section and subsection)*	*% Attempts*	*Average Mark (out of 50)*
1	11	Linear Programming	69%	34
2	9	Vectors	19%	32
3	10	Further Sequences and Series	10%	16
4	8	Further Geometry	2%	13

Table 3.6: Popularity and Average Mark of Questions on Section B of Paper 2, Ordinary Level

It may be noted that popularity and average mark are well correlated on paper 2, but not on paper 1[*].

[*] In fact, correlation coefficient is negative on paper 1: –0.3.

3.3 ANALYSIS OF CANDIDATE PERFORMANCE

Paper 1, Question 1

Attempts: 91% **Average Mark: 31**

Syllabus Section(s): Arithmetic 1, 2, 3

Question 1 is usually one of the most popular and best answered questions on the paper. While it retained its popularity this year, the performance of candidates was weaker than usual.

(a) Most candidates answered this straightforward question correctly. The most common error was taking 1 m = 1000 cm. Inverting the fraction also occurred, but the vast majority were awarded full or nearly full marks.

(b) **(i)** Well answered, indicating a good understanding of percentage error calculations. The most frequent error was taking the estimate rather than the true value as the denominator, or taking 4000/4029 and not finishing correctly. Inversion, decimal errors and rounding errors, also occurred, and some just calculated the error itself to be 29 and stopped.

(ii) Poorly answered. Many tried to use the lowest common denominator in the first part, but did not follow up properly in the second part. Few found the simplified 6:4:3 and even some of those followed up by dividing by 12 instead of 13. All the weaker candidates, and many others, simply found ½, ⅓ and ¼ of 325. Inverting the ratios to get 2:3:4 was also a common error.

(c) Weak candidates made little or no progress in this part. Even those who scored reasonably well only did so by starting with the incorrect assumption that $P = 2000$. This indicates that most candidates seem to have difficulty bringing a basic algebraic concept (use of a letter to represent an unknown quantity) into their arithmetic work. They needed to begin with a concrete number.

Paper 1, Question 2

Attempts: 92% **Average Mark: 28**

Syllabus Section(s): Algebra 1, 2

The second most popular question, but also scoring worse than usual. There was an apparent inability to cope with fractions and square roots in an algebraic setting (part **(c)**).

(a) Very well answered, by the vast majority, indicating a good understanding of the basic concept of an algebraic expression.

(b) Quite well answered. Better candidates answered well and often got full or high marks. Using the linear equation to isolate x and substitute into the nonlinear equation was the most commonly used method (especially by successful candidates). Isolating y was far less prevalent. Other attempted methods generally ended in failure. Some simply ignored the "offending y" in the second equation and continued as for two linear equations. Other errors included factorising incorrectly, or finding only one root of the quadratic, or (having found the values of the first variable) substituting incorrectly. Overall, though, the standard of answering was quite high.

(c) Almost all of the candidates struggled severely with this part. Candidates rarely achieved more than the attempt marks, indicating poor algebraic manipulation skills when fractions and square roots are involved.

Paper 1, Question 3

Attempts: 85% **Average Mark: 34**

Syllabus Section(s): Algebra 1, 3

(a) Well answered, most getting at least 7marks. Manipulation of formulae is usually not a strong topic among candidates, but this was a straightforward question with no hidden dangers and most candidates were thus able to show what they knew.

(b) Part **(i)** was well answered, part **(ii)** somewhat less well, and part **(iii)** quite poorly. In part **(ii)**, it was the need to change the inequality when changing the sign that caused difficulty more often than coping with the fraction. Answering in part **(iii)** was frequently hampered by errors in the previous parts.

(c) Part **(i)** was very well answered, with most scoring full marks, although some failed to execute the calculation fully. There were surprisingly few errors in evaluation.
Part **(ii)**, however, was poorly answered. Some merely evaluated $f(3)$ again, occasionally along with other efforts at trial and error. Many were impatient to use the quadratic formula and made efforts to apply it prematurely, to say the least. In some cases, the constant was dropped (temporarily) from the cubic and then the expression was divided by x; in other cases, x was factorised out of the first three terms and then discarded as the candidate worked on with the quadratic part; some even differentiated the cubic to reduce it to a quadratic.

Nevertheless, many candidates made a good effort to divide the cubic by $x - 3$ and factorise the result, and many (especially the better ones) earned high marks.

Paper 1, Question 4

Attempts: 91% **Average Mark: 35**

Syllabus Section(s): Algebra 4

A popular choice, this question also had the highest average mark on this paper. Unusually, part **(c)** was answered better than part **(b)**.

(a) Very well answered, with most candidates scoring either the full or almost full marks. Very few erred calculating $u - 4$, apart from a few who had $u - 4 = 2i$. The plotting of points was of a high standard, but plotting $-2i$ at $+2i$ was seen occasionally. Sometimes, axes were reversed, with points then correctly plotted in relation to those non-standard axes.

(b) **(i)** This straightforward simplification was not well answered. All substituted correctly, but some gained no further marks. Not as many finished correctly as one would expect given the frequency of similar questions on previous papers.

(ii) Also very poorly handled, which is surprising, since the modulus is tested almost every year. Many candidates tried to verify without evaluating any of the moduli. Others worked out the left-hand side, but were unable to cope with the right. Of those who appeared to understand what was required, relatively few got all three moduli correct; of those that did, very few spotted that $\sqrt{20} \neq \sqrt{10} + \sqrt{10}$.

(c) Part **(i)** was excellently answered, although some seemed to suspect a more complicated answer was called for and proceeded to work on beyond the required value. Sign errors and juxtaposition of real and imaginary parts were among the (uncommon) errors. Performances in part **(ii)** were varied. Almost all gained marks for substitution and removal of brackets, and, while many stopped at this stage, many others continued to form one or both equations correctly. A respectable number finished correctly. Common errors were doubling before squaring in $2z^2$, squaring errors in $(1 - 2i)^2$, incorrect formation of equations from real and imaginary parts, and taking $4i^2$ to be imaginary.

Paper 1, Question 5

Attempts: 56% **Average Mark: 34**

Syllabus Section(s): Finite Sequences and Series

The question on Finite Sequences and Series continues to be the least popular question on this paper. As in previous years, those attempting it tend to be either good candidates who have little or no trouble with the question, or very weak candidates with limited skills and techniques at their disposal. That is, the candidates in the middle of the achievement range tend not to attempt this question. Very few confused arithmetic with geometric sequences this year, which is a clear improvement on the situation prevailing in other years.

(a) Both parts were very well answered, indicating a good understanding of the basic idea of an arithmetic sequence.

(b) Answering in this part indicated considerable confusion between the *n*th term of a sequence or series and the sum of the first n terms. For example, the majority of candidates used the difference between S_2 and S_1 as the common difference d of the series. This did not prevent candidates from correctly finding the first term, or from displaying knowledge of how to generate the terms of an arithmetic series from a known first term and common difference.

(c) The first two parts were generally well done. Candidates favoured more intuitive techniques over the use of relevant formulae. Some calculation errors occurred, perhaps not surprisingly since fractions were involved, but candidates mostly displayed a reasonable understanding of what was going on. Obviously, those who were unable to handle part **(i)** were unable to manage part **(ii)**. Part **(iii)** was poorly answered, and very few candidates had correct answers or indeed any work of merit. The few who made a good effort did so by switching to decimals and writing all the terms until they got a suitably small one. Very few used the formula for the *n*th term of a GP to calculate the answer.

Paper 1, Question 6

Attempts: 85% **Average Mark: 30**

Syllabus Section(s): Functions and Calculus 1, 2

(a) Candidates had little difficulty evaluating the function at the two given values, but remarkably many of them failed to add the answers, as required. One can only guess that perhaps many interpreted "+" as "and", which is common in everyday informal communications, but is obviously a wholly inappropriate interpretation in a mathematical context.

(b) This differentiation from first principles was quite well answered, although the minus sign in $3x - x^2$ did cause some difficulties. Some candidates re-arranged the terms as $-x^2 + 3x$ but still found the sign confusing and it was often placed on the wrong side of the brackets as the work continued. Other common errors included incorrect expansion of $(x + h)^2$, errors in multiplying a bracketed expression by 3, and errors in limits, including forcing the result. As in previous years, the left-hand side (stating what step is being done at each stage) was sometimes left out, partially or totally.

There was little evidence of any great difference in either the relative popularity of the two methods, or the relative performance of candidates using them. This contrasts somewhat with previous years, when it was evident that the candidates using "deltas" were far more prone to errors.

(c) Answering indicated that candidates can competently execute the technique of differentiating by rule, (as evidenced by success in part **(i)**), but are not able to apply their knowledge with any degree of understanding, (as evidenced by their failure to engage meaningfully with part **(ii)**).

In part **(iii)**, although candidates did appear to know how to find the equation of a line, the vast majority failed to recognise any connection between the derivative of the function and the slope of the tangent.

Paper 1, Question 7

Attempts: 94% **Average Mark: 31**

Syllabus Section(s): Functions and Calculus 2

This was the most popular question on the paper, as is often the case.

(a) Very well answered. Candidates displayed a good knowledge of the technique for differentiating a polynomial by rule. Errors were few and candidates routinely scored full marks in this part.

(b) Both parts here were well answered, with candidates displaying competence in executing the relevant techniques. The product rule was the preferred method in part **(i)**, rather than multiplying out, although both methods were encountered. A good number of candidates either omitted or made errors in the evaluation portion of part **(ii)**.

(c) This part was also quite well answered. Candidates understood that the speed formula would be given by the derivative of the distance formula, although many failed to follow through by making the connection between these formulae and the situation presented. That is, many failed to make the connection between the car passing p and the value $t = 0$ in the first part, and more failed to make the connection between the car stopping and the speed being equal to 0 in the second part. Candidates who answered part **(ii)** successfully usually proceeded to answer part **(iii)** correctly also. Others, not surprisingly, offered little of merit for part **(iii)**.

Paper 1, Question 8

Attempts: 80% **Average Mark: 28**

Syllabus Section(s): Functions and Calculus 1

This was an unpopular question, as is usually the case, and did not score particularly well on this occasion.

(i) This was well answered in the sense that the vast majority of candidates knew what to do, but there was an alarming lack of competence in evaluating the fractions involving the decimal values of x.

(ii) The graph was poorly answered. Many candidates made no connection with part **(i)**, instead drawing up a fresh table. There were also significant problems in scaling the vertical axis. Candidates by and large conveyed the impression that they had no idea what the curve should look like. The two branches of the graph were frequently linked up, displaying lack of understanding of the behaviour of the graph in the region of the vertical asymptote. No candidate used method II in the scheme, (sketching the curve by relating it to its asymptotes).

(iii) Well answered, although usually drawn from a full table with 5 points (again displaying poor understanding of graph properties). Many nonetheless scored full marks here. A minority drew the graph on a separate diagram, despite the explicit instruction to the contrary, making part **(iv)** impossible to complete. Some had difficulty using their existing inaccurate scales on the y-axis from **(ii)**. Points were generally plotted correctly, but, in cases where one or more were incorrectly plotted, the graph was usually drawn as a number of line segments, indicating a lack of understanding that the graph should be a single straight line.

(iv) This was poorly answered. Many had at most one intersection point. Even among the small minority who drew correct graphs, many did not extend the graph of f sufficiently close to the asymptote to show a second intersection point. Some simply marked the intersection point without finding the corresponding value for x. Candidates generally scored the attempt mark, if anything.

(v) Not attempted very often. Some did set up the equation but then stopped. Only the very best candidates could solve the equation, despite the fact that it was not especially difficult. One would, by contrast, have expected good efforts at it in the context of an algebra question. This accordingly indicates poor transfer of knowledge across topics.

Paper 2, Question 1

Attempts: 99% **Average Mark: 36**

Syllabus Section(s): Arithmetic 2, 4

(a) The vast majority of candidates had no difficulty with the first part – a straightforward calculation of the area of a rectangle. However, a good number of candidates had difficulty with the application of Pythagoras' theorem in the second part. In particular, a number of candidates were unable to correctly determine which side was the hypotenuse.

(b) **(i)** Candidates were well prepared for this question and executed this routine procedure with confidence. The most common errors were: failure to recognise that the last offset was 0; repeated use of 22 both as the last offset and as an even offset; and division instead of multiplication by 6 (which was $\frac{h}{3}$), this latter error being made particularly by weaker candidates.

(ii) Many candidates continued on correctly and directly from **(i)** to also get a correct answer to this part, although failure to round was a common omission. However, many of the weaker candidates failed to use the answer to part **(i)** and tried to find an average by, for example, adding the offsets and dividing by some number.

(c) **(i)** The most striking feature of this part was the almost universal failure to note that the two measurements were given in different units and that a conversion was therefore required. Apart from this, the question was well answered with a direct substitution into a correct formula. A minority of candidates addressed the different units, but often had 30 m = 300 cm. Some candidates dropped π in the answer, while others substituted a value for π. Just a few candidates used the formula for a cone or sphere.

(ii) This part was answered reasonably well. Early rounding which affected the answer was common, while quite a few candidates found the volume of a sphere but did not know how to continue. Some candidates omitted the π in the answer to part **(i)** but did not omit it in calculating the volume of the sphere.

(iii) This part was badly answered. Only the very best candidates made a good attempt at it. The work was seldom laid out as an equation from the start. Successful candidates more often just divided the volume of steel by 225000 and then divided this answer by $^4/_3\,\pi$. For those who got this far, finishing caused no difficulty. Incorrect efforts were common, since some attempt was made by most candidates.

Paper 2, Question 2

Attempts: 96% **Average Mark: 30**

Syllabus Section(s): Geometry 2

(a) This direct application of a standard formula was answered very well. Almost all candidates had the formula with substitution and worked it out correctly. Mistakes were made with signs in the formula or interchanging x's and y's in the substitution. Some errors were also made in squaring.

(b) **(i)** This part was generally answered well although some candidates substituted 0 for y to find a and 0 for x to find b. Many candidates made mistakes transposing. Having found x and y values some candidates failed to appreciate that the points were distinct, and offered the answer (-4, 3). Weaker candidates tended to substitute some value other than 0 for x or y. Occasionally, candidates made a mistake with signs when writing down the equation.

(ii) This part was answered well by about half of the candidates. Generally, candidates did not have a problem with drawing the line L. Taking the point of intersection at the intercept on the wrong axis was a common mistake, and quite a few candidates drew K as the image of L under the axial symmetry in the y-axis. An approach adopted by quite a few candidates was to do part **(iii)** first, (usually making a mistake in the slope) and then plotting that line K. Candidates are clearly not used to using diagrams effectively to assist with their work.

(iii) This part was answered reasonably well by most candidates. However, many either had a poor understanding of the relationship between the slopes of perpendicular lines, or made errors putting such understanding into practice. Some candidates substituted the wrong point, and quite a few made errors in simplifying K (often in the application of the distributive law).

(iv) The standard of answering of the very best candidates here was good, but the moderate and weaker candidates had difficulty with this somewhat less direct question. Among those who approached correctly, some errors were made in isolating t.

(c) **(i)** This part was attempted by most candidates but was badly answered, with very few getting full marks. Candidates appeared to be unable to deal with the unknown k. For those who succeeded in eliminating y and isolating x, many wrote $-3k$ for $3 - k$. Transposing errors were also common. Many candidates achieved only the attempt mark for some relevant work.

(ii) This part was also badly answered and often omitted. Many substituted 0 for x in one or both of the given equations and stopped.

Paper 2, Question 3

Attempts: 87% **Average Mark: 32**

Syllabus Section(s): Geometry 2

(a) **(i)** This part was very well answered. However, quite frequently only the centre **or** the radius were written, rather than both.

(ii) This part was well answered with almost all candidates justifying the assertion that the given point was outside the circle. However, some failed to give an proper conclusion. Also, the error $(-5)^2 = -25$ was common.

(b) **(i)** This part was attempted by most candidates with a reasonable success rate. The weaker candidates had difficulty with the algebra. The fact that *y* was already isolated in the linear equation certainly helped candidates. However, having solved for *x*, many substituted this into the equation of the circle (rather than the line) and failed to complete successfully. Incorrect squaring of the brackets was common, and candidates frequently tried to "force" factors on expressions which, due to errors, did not admit factors.

(ii) Candidates who drew the circle first were generally successful. Many other candidates, however, plotted the points and the line but then had a problem with the circle. They either used the wrong centre, (using, for example, [*ab*] as diameter) or else they omitted the circle entirely. Occasionally, the scale on the *x*-axis was different from the scale on the *y*-axis and this caused a further problem when drawing the circle. However, the majority of candidates got most or all of the marks for this part. Examiners noted with surprise that many candidates who correctly found *a* and *b* in part **(i)** did not draw a diagram at all in part **(ii)**. This indicates, as in question 2, that there are apparently many candidates who do not routinely use diagrams in their work in co-ordinate geometry, a somewhat bizarre approach that certainly cannot help conceptual understanding.

(c) **(i)** There was a good standard of answering to this part, which was attempted by most candidates. The most common errors were to give (4, –3) as the centre, or to have one of the signs incorrect. Weaker candidates often just wrote down a circle formula and stopped.

(ii) There was a good standard of answering by the average and good candidates. A common error was the use of a wrong translation, often $(2, 3) \rightarrow (0, 0)$ or $(-4, 3) \rightarrow (2, 3)$.

(iii) Only the more able candidates attempted this part, and their most common approach was to substitute, expand $(y - 3)^2$ and solved the quadratic equation using factors. Errors in expanding $(y - 3)^2$ were common. Many candidates made mistakes trying to factorise their expressions and/or were unable to correctly apply the quadratic roots formula. Those who did not expand $(y - 3)^2$ often found only one value for y. The fact that so few of even the better candidates can solve $(y - 3)^2 = 36$ without resorting to expanding the left-hand side is a disappointing indicator of the limited range of algebraic techniques and understanding that they have at their disposal.

Paper 2, Question 4

Attempts: 22% **Average Mark: 19**

Syllabus Section(s): Geometry 1, 3

(a) Both parts were well answered by those who did this question. Most candidates got the correct answer to part **(i)** while fewer got the correct answer to part **(ii)**.

(b) This part was very badly answered, with only a small percentage of those attempting this question making any effort to prove this standard theorem from the specified list in the syllabus. Those who did make an attempt at the proof usually secured only the attempt mark. The evidence is that most candidates have little real understanding of geometry and do not learn the geometry theorems.

(c) **(i)** This part was well answered. Almost every candidate drew the square accurately. However, the vertices were often either incorrectly labelled or not labelled.

(ii) This part was poorly answered. Those who had an image of side 2 usually placed it at the centre of the first square or in some other incorrect location, often with no regard for the point o. Most of the candidates who did have o as the centre used 0·75 or 1·25 as the scale factor, instead of 0·25.

(iii) This part was well answered using candidates' images from part **(ii)**. The most common approach was |side| × |side| = area. Candidates' answers were generally either fully correct or entirely without merit; partial credit was rarely awarded.

(iv) This part was answered fairly well by the better candidates. A common mistake was to offer $^{100}/_{8}$ (area of image square / side of original square).

Paper 2, Question 5

Attempts: 58% **Average Mark: 29**

Syllabus Section(s): Trigonometry

Trigonometry is one of the more polarised topics; good candidates do very well and weak candidates do very badly.

(a) The standard of answering was relatively low. Errors in the formula, particularly the fraction, were very common and many of the weaker candidates progressed no further than writing $^{135}/_{360}$. Attempts to find the area of the minor sector (rather than the length of the arc) were reasonably common.

(b) **(i)** This part was poorly answered with only the very good candidates having any significant success. It was the least well answered part of this question. Many of the better candidates got as far as $\sin|\angle abc| = 0{\cdot}829$ and stopped. As is usual when the formula for the area of a triangle is involved, quite a few took *ab* to mean |*ab*| in the formula (length of one side rather than product of two sides). The convention of using lower case letters for points in geometry makes this error almost inevitable, so teachers and candidates need to be alert to it.

(ii) Candidates who had found some value for $|\angle abc|$ did reasonably well in this part. Without a value, however, candidates were unable to score beyond the attempt mark. Many of the candidates attempting this question took $|\angle abc| = 90^{\circ}$ and used Pythagoras' theorem to find $|ac| = \sqrt{89}$. Some tried to use the Sine Rule, without success. Quite a few rounded off $55{\cdot}99^{\circ}$ to 60°.

(c) **(i)** This part was well answered by the better trigonometry candidates. Most used the Sine Rule to find $|\angle q| = 30^{\circ}$, but only a few continued to give the bearing correctly. Some did not relate the sides correctly to their corresponding angles in the Sine Rule while others treated sin 41·3° as just 41·3 (i.e. they ignored the trigonometric function). Many of the less able candidates took triangle *qph* to be right-angled.

(ii) The standard of answering in this part was moderate. Both the Sine Rule and the Cosine Rule were used as techniques to find $|pq|$. Strangely, it was the requirement to find the time taken that caused most difficulty, with even the very strong candidates experiencing considerable difficulty. This certainly indicates poor transfer of knowledge, since finding the time taken for a journey given the speed and distance would cause little difficulty in another context. Less able candidates used $41{\cdot}3^{o}$ where they should have used $108{\cdot}7^{o}$, and here again many took $|\angle phq| = 90^{o}$ and used Pythagoras' theorem.

Paper 2, Question 6

Attempts: 70% **Average Mark: 32**

Syllabus Section(s): Discrete Mathematics & Statistics 1, 2

(a) (i) This part was very well answered with almost all candidates getting the correct answer. However, some candidates did not work the multiplication.

(ii) This part was also well answered, although not as well as part **(i)**. Again, some candidates did not finish the calculation, while the weaker candidates worked aimlessly with relevant integers.

(b) Throughout this section the standard of answering was moderate, with the majority of candidates failing to offer any supporting work or rationale for their answers.

(i) Quite a number of candidates paired each team twice (home and away) giving 50 as the answer, while other candidates gave 5 + 5 as the answer.

(ii) Some candidates worked well, giving answers consistent with their work in part **(i)**, e.g. $^{2}/_{50}$. While most candidates gave some fraction as an answer, it was usually difficult to see where the fraction had come from without supporting work.

(iii) In this part, $^{1}/_{10}$ was a common incorrect answer. Although many candidates worked with relevant integers, generally the answers were incorrect.

(iv) In this part $^{9}/_{10}$ was a common incorrect answer.

(c) Again in this part there was a noticeable absence of supporting work, to the detriment of the candidates.

- **(i)** This part was handled very well, with almost all candidates getting the correct answer.
- **(ii)** A common mistake in this part was to use a combination when a permutation was required. Nevertheless, the part was reasonably well answered.
- **(iii)** The reverse of the error in part **(ii)** was frequently made in this part. Incorrect answers, but with some relevance, were common.
- **(iv)** This part was badly answered. Again, the lack of supporting work militated against the candidates. Common incorrect answers were 21, 1/42 or 1/7, while the most common answer offered was 2/7.

Paper 2, Question 7

Attempts: 92% **Average Mark: 33**

Syllabus Section(s): Discrete Mathematics & Statistics 3

(a) This part was not as well answered as one would have expected for such a routine procedure. Common errors were: failure to multiply by the weights; and division by 3 rather than the sum of the weights. This suggests that perhaps candidates are not familiar with the concept of weighted mean.

(b) Candidates seem reluctant to rely on competent use of their calculators, favouring the more tedious long-hand calculations. Certainly they need to be able to perform the calculation long-hand if required, but the ability to do statistical calculations with ease on a calculator is at least as valuable a skill to develop, if not more so.

- **(i)** The standard of answering to this part was considerably higher than that for part **(a)**, despite the fact that it was procedurally more difficult. This reinforces the view that candidates do not appear to be familiar with the concept of weighted mean. Most candidates dealt successfully with the mid-intervals and went on to correctly calculate the mean, apart from a few numerical slips. Errors in finding the mid-intervals seemed to be fewer than in previous years. Division by 5 (the number of intervals) was an error that occurred from time to time.
- **(ii)** This was the least well answered part of this question. This standard deviation question was not answered nearly so well as in previous years, and this may be

accounted for by the fact that the mean was not an integer. (This obviously would have made no difference to those using the statistical features of their calculators.) Common errors included confusion between the variable data and the frequency data, failure to square the deviations or to multiply the squared deviations by the frequencies. Candidates who may have been a little unsure rounded the mean to a whole number before proceeding.

(c) In general, candidates did better on this part than on the previous two parts.

(i) Some candidates misinterpreted the question as requiring a cumulative frequency table from a frequency table and so added the number of people in the hall at the various times instead of subtracting. This suggests that candidates are not able to read and interpret information clearly, or have poor understanding of the meaning of the various types of tables and charts they have studied, but are merely operating "on autopilot" in response to standard cues. Candidates' answers to subsequent parts would seem to bear this out.

(ii) This part was well answered, even by those who worked from incorrect frequency tables in part **(i)**. Answers were often written without work. However, some did explain why they chose the particular interval, while others drew an ogive and indicated the interval before answering the question. Some candidates gave a time within the interval without naming the interval.

(iii) This part was also well answered. Candidates wrote down the correct interval, or one consistent with their table in part **(i)**, again without showing work. The interval 7.50 – 8.00 was probably the most common incorrect and inconsistent interval given as an answer. Occasionally, the time of 7.40 was given.

(iv) This part was reasonably well answered. The correct answer of 0 was often written without supporting work, while 30 was given almost as frequently as 0. Some candidates gave 1 as the answer, without offering justification.

Paper 2, Question 8

Attempts: 2% **Average Mark: 13**

Syllabus Section(s): Further Geometry

This question was the least popular question in section B and in the entire examination. It was also the lowest scoring. This shows that even the few candidates who did attempt this question were very poorly prepared to do so.

(a) This part tested the recognition and direct application of commonly known results. Part **(i)** was well answered with many of the few candidates taking this question giving the correct answer without work. Part **(ii)**, however, which required the application of a second result, was more testing and candidates were less successful. Generally, work was not shown and incorrect answers, therefore, yielded no marks.

(b) The theorem was hardly ever attempted. The very few who knew the theorem proved it reasonably well, although they only ever offered the proof of one direction. A correct diagram with no further work was occasionally drawn.

(c) All parts of this section were reasonably straight-forward applications of well-known results. Nevertheless, all parts were badly answered except by the very rare able candidates who chose the question. Answers were generally offered without work. Some candidates clearly did not realise that $oa \perp ap$.

Paper 2, Question 9

Attempts: 19% **Average Mark: 32**

Syllabus Section(s): Vectors

As regards popularity of questions in section B, this question comes a distant second to question 11. The average or good candidates who opted for this question had clearly prepared the topic well.

(a) **(i)** Most candidates answered this part well. However, the weaker ones were at a loss. Answers were usually written without work. Some candidates left the answer as $\overrightarrow{oa} + \overrightarrow{ab}$ while others did not start at o and wrote $\overrightarrow{ab} = \overrightarrow{c}$ and stopped.

(ii) This part was not answered as well as part **(i)**. Answers without work were reasonably common, and common errors were $\vec{a} + ½\vec{c}$ and $\vec{m} = ½\vec{cb}$. Weak candidates often showed lack of understanding with answers such as $\overrightarrow{mcb}$.

(b) **(i)** This part was answered reasonably well. However, a significant number of candidates displayed an inability to distinguish between $\vec{pq}$ and the vectors $\vec{q}$ and $\vec{p}$, with many trying some form of multiplication, or else simply writing $\vec{pq} = \vec{p} + \vec{q}$. Sign errors in dealing with $-\vec{p}$ were common.

(ii) This part was well answered with many candidates showing they were able to construct the scalar multiples of vectors and add the components. The most common mistakes in this part were with signs. Some candidates who got the correct answer went on to divide out the common factor 2.

(iii) Candidates made a reasonable attempt at this question, although many were unable to follow through to completion, having difficulty equating the different components. Sign errors were again common.

(c) **(i)** While many candidates could not write the related perpendicular vector correctly, finding the modulus did not present serious difficulties. Weaker candidates, however, did not appear to understand the notation.

(ii) Reasonable attempts were made at this part. However, sign errors were again common, while many candidates displayed a lack of understanding of the dot product, or appeared unsure as to how to cope when addition and subtraction of vectors were involved as well.

Paper 2, Question 10

Attempts: 10% **Average Mark: 16**

Syllabus Section(s): Further Sequences and Series

This question was the third most popular question in section B with an uptake of 10% and a mean mark of just 16. The pattern of answering indicates that many weaker candidates, who have not studied the topic at all, are often attracted to answering it by the part dealing with the application of geometric series to compound interest or depreciation, believing that they can attempt this as a regular application of basic arithmetic. Teachers should explicitly direct such candidates away from this course of action, as it is not in their interest.

(a) This part was badly answered. Examiners regularly reported not finding one correct expansion in their batch of scripts. While a majority of the candidates attempted to use the binomial theorem, a significant number tried to expand by long multiplication, and a few used Pascal's triangle. There were various mistakes in the use of the binomial theorem and also in algebra. Attempts at differentiation of the expression appeared with alarming regularity. In the end, many candidates succeeded in getting only one or two terms correct.

(b) **(i)** The straightforward sum to infinity of the geometric series was not well done. Quite a few candidates wrote the first term and stopped, while others found the common ratio and some went as far as writing the fourth term (which was not required). Substitution into a correct formula was rare.

(ii) This part was also very badly answered, being related to part **(i)**. Most candidates made no effort at it, while a few tried some substitution into the formula for the sum to *n* terms of a geometric series.

(c) **(i)** This part was attempted by all who opted for this question. It was the best answered part of the question with a reasonable number of good attempts. Most of the candidates calculated the value on a year-by-year basis and many who endured to the end failed to round correctly. Weak candidates were content once they had calculated 15% of €25 000 = €3750, and then either multiplied it by 12 or stopped.

(ii) This part was very badly answered. Despite the wording of the question most candidates tried to do it on a year-to-year basis. This did not answer the question asked, and candidates were fortunate to receive an attempt mark for it. Treating the question as a geometric series as instructed in the question was very rare. Candidates seemed to be unaware as to how to even write the question in the form of a series.

Paper 2, Question 11

Attempts: 69% **Average Mark: 34**

Syllabus Section(s): Linear Programming

This question was, as usual, by far the most popular question in section B.

(a) **(i)** This part was very well answered, with almost all candidates successful. A few made sign errors finding the slope, some made a transposing error when simplifying K, and an occasional candidate used the mid-point rather than one of the given points. A few candidates wrote $4x + 8y = 0$ without any other work.

(ii) This part was answered reasonably well by most candidates but some candidates had a difficulty determining the half-plane. Errors in the directions of the inequalities were common. The more successful candidates showed work involving testing points, but many wrote the inequalities without showing work. Some weak candidates just wrote the coordinates of the three vertices and stopped, while some wrote $x = 0$ and stopped. Often this part was omitted by weaker candidates. Candidates who made a mistake in part **(i)** had difficulty in getting the corresponding inequality here.

(b) This part was well answered. Candidates were well prepared in identifying the inequalities. Answers were usually given without showing work. Some, however, grouped all the information given about the constraints together before writing the inequalities. A few wrote only one inequality, usually $2x + 5y \leq 800$, and omitted the other.

Candidates showed competence in the techniques for plotting the inequalities. Inequalities were frequently simplified before proceeding to find points of intersection with the axes. Good, accurate graphs were frequently drawn and the appropriate set of feasible points indicated clearly. Almost all found the point of intersection (150, 100) algebraically, but the accuracy of the graphs tempted some to note the co-ordinates from the diagram. Weaker candidates tended to draw the lines and omit indicating the half-plane.

The more able candidates displayed a good ability to interpret the mathematics to give the correct expression for the income, but the profit proved a little more challenging. Almost all candidates worked with $20x + 40y$, and the more successful candidates made out a table as in the marking scheme. A common error was to use the point (0, 150) instead of (0, 160). Those who omitted finding the intersection of the two lines usually omitted parts **(ii)** and **(iii)** as well.

3.4. CONCLUSIONS

Patterns of Question Choice

The pattern of question choice on both papers was in close keeping with that of previous years. On paper 1, question 7 (on *Functions and Calculus 2*) continues to be the most popular, followed closely by questions 2, 1, and 4 (on *Algebra* and *Arithmetic*) with uptakes of over 90% each. The least popular question continues to be question 5 (on *Finite Sequences and Series*) with an uptake of only 56%.

Typically, candidates start with Q1 and proceed numerically through the paper, leaving out some questions. That is, they do not necessarily start with their strongest question and work towards their weakest. This is generally not in their best interests. On paper 1, for example, questions 4 and 7 often gain the most marks, but are rarely done first. Most teachers instruct candidates to read through the entire paper first and begin with what they know they can answer well. It would appear that most candidates do not take this advice.

It is relatively common on both papers for candidates to attempt more than the required number of questions. Quite often, candidates attempted such excess questions despite not completing other questions fully, which is obviously of dubious value.

As is usually the case, there were many candidates who did not attempt the required number of questions and this clearly reduces their chances of getting a good grade. In light of the internal grading within questions, it is clearly in the best interest of all candidates to attempt the required number of questions, since the initial marks in any question are the easiest to obtain. In relation to paper 2, it is worth noting that examiners regularly encounter candidates who answer a surplus question in section A with reasonable success, but fail to attempt any question from section B, as required. The competence displayed by such candidates in the questions they attempted is, therefore, not reflected in their overall performance.

Standard of Answering

The internal grading of questions from lower to higher order skills was, for the most part, reflected in the level of success achieved by candidates in dealing with the various parts. The (a) parts were almost all handled very well, (b) parts quite well and (c) parts less so. Notable exceptions to this progression were:

- Paper 1, Question 1(b)(ii), where candidates appeared unable to handle routine arithmetic work with fractions in a ratio context.

- Paper 1, Question 4(b), where some straightforward with complex numbers was surprisingly poorly handled.
- Paper 1, Question 8(ii), where candidates were clearly unfamiliar with the structure of one of the few standard graph types listed in the syllabus.
- Paper 2, Question 7, where the straightforward (a) part was handled much worse than the procedurally similar but more challenging (b) part.

The standard of answering this year was reasonably good, particularly in relation to the routine application of familiar procedures. The (c) parts, which tend to test the higher order skills, naturally have a lower level of success, but it was disappointingly low in Q1(c) and Q2(c) on paper 1, which are popular questions and normally handled better.

Presentation of work in the scripts is sometimes of a very poor quality and displays undisciplined working techniques on the part of the candidate concerned. This in turn often leads to mistakes being made, with the result that the solutions presented do not reflect the real potential of the candidate.

As indicated in the overall introduction to this report, the proportion of candidates who fail to achieve a D grade conceals a sizeable number of candidates who are clearly unsuited to studying this syllabus and should have taken the Foundation Level examination. Many of those who failed to reach this standard did so by such a wide margin that they must have known well beforehand of the likely consequences of opting for this level. It would appear that students are more content to follow an Ordinary Level syllabus from which they are deriving little educational benefit than to follow what would clearly be a far more suitable and rewarding syllabus at Foundation Level.

Candidate Strengths

An overview of the work presented reveals a range of strengths displayed by the cohort. As mentioned above, strong performance was most evident in procedural questions where a definite sequence of familiar steps was required. Strengths included:

- Arithmetic, (except for fractional ratios).
- Substitution into algebraic expressions and functions
- Solving quadratic equations, especially by factors
- Algebraic division (by better candidates).

- Plotting complex numbers on an Argand diagram, naming the conjugate of a complex number, and substituting into expressions or equations involving a complex numbers.
- Working with an arithmetic progression, as in 5(a) and 5(b)
- Mechanical aspects of differentiation
- Routine parts of co-ordinate geometry, apart from drawing the diagrams
- Trigonometry (in the case of the relatively few who attempted it)
- Linear programming technique
- Application of Simpson's rule.

When the basic routines above needed to be applied to less familiar problems, the success rate was considerably lower.

Candidate Weaknesses

Weaknesses, by and large, relate to inadequate understanding of mathematical concepts and a consequent inability to apply familiar techniques in anything but the most familiar of contexts and presentations.

Specific candidate weaknesses on these papers included the following:

- Ratios involving fractions
- Percentages, other than routine operations
- Handling simultaneous equations where one is of 1st and the other of 2nd degree.
- Manipulation of fractions, both in arithmetic and algebra
- Handling of surds
- Inequalities, especially handling a change of sign
- Division of complex numbers, and modulus of complex numbers
- Geometric progressions involving fractions
- Applications of differentiation
- Graph of the form $y = \frac{1}{x+a}$
- Synthetic geometry
- Further sequences and series (paper 2 option) – concepts such as sum to infinity for a convergent series are not understood
- Average candidates experience difficulty with all but the most basic of algebraic manipulations and can cope only with basic routines in solving equations. Usually they fail to exploit quicker solutions than those offered by the practiced routine.

3.5. RECOMMENDATIONS FOR TEACHERS AND STUDENTS

The following advice is offered to teachers and candidates preparing for the examination at Ordinary Level. The thrust of the advice will be familiar to teachers from previous Chief Examiner's reports in mathematics, and teachers are reminded that a report dealing specifically with issues related to Ordinary Level Mathematics was issued in 2001.

In advance of the examination

- Be adequately prepared for all core syllabus areas. Restricting the choice available on day is not advisable.
- Practise to gain comfort and accuracy in your basic skills: dealing with signs (+ and -); algebraic manipulation, (such as expanding and factorising expressions, transposing, solving equations, etc.); working with fractions; arithmetic of complex numbers; drawing co-ordinate geometry diagrams; using formulae accurately in co-ordinate geometry and in area & volume; statistical techniques; and standard calculus techniques.
- Get into the habit of checking your work for errors. Practise checking answers by referring them back to the question to see if they are correct and sensible, and practise techniques for finding errors quickly and calmly in your own work, including getting to know your own weaknesses.
- You should carry out a planned, detailed and effective revision. Ask your teacher for a list of what you should know in each topic, and for other guidance.
- Close to the examination, practise on questions from previous examination papers under timed conditions, and get feedback on your work. Simulate examination conditions as far as possible and get into the proper examination habits listed below.
- Become familiar with the structure of the papers and, in particular:
  - Know what topics are normally examined in each question
  - Understand the internally graded structure of the questions
  - Know how many questions are required to be answered in section A and in section B on paper 2.
- Revise all formulae, making sure you know them accurately by rote. Ensure also that you understand all relevant notation and symbols.
- Practicing questions is at the core of successful revision in mathematics. However, it is suggested that students may spend insufficient time *thinking* about mathematics and *discussing* mathematics. These are key elements in building sound understanding.

- Make sure you understand the general principles by which the marking schemes operate. This will help you to understand the rationale for much of the further advice below, and encourage you to follow it.

In the examination

- Read the whole paper carefully before you begin. This will both assist you in making effective question choices, and will also prompt your subconscious mind to begin retrieving relevant knowledge as you work.
- Choose your questions carefully during the exam, not beforehand.
- Begin with what you think is the best question on the day, on a topic you know well. Take your time and do it well, and check the question after each part (as there may be parts you have missed). Then do your next best question, and so on.
- When finished answering a question, read it again to ensure that all parts are finished fully and that answers are given in a particular format, if required, or to the required accuracy.
- Monitor the time so as to allow you to answer six questions and check over them when finished. Make sure you try all parts of them.
- Read each question carefully. Questions with a lot of text should be read carefully several times.
- Number each question part clearly, e.g. 7 (c)(i) etc., and take particular care to draw attention to any one-line answers.
- Support your answers with necessary work, even when using a calculator, and do your 'Rough Work' close to the question, not at the back of the answerbook. Note that most candidates get most of their marks from work that may not yield the correct answer. Furthermore, even if an answer is correct, it may not get full marks without the necessary supporting work.
- Take special care in showing work when using calculators. It is tempting to write down answers without indicating what calculation has led to them. Use estimation and other error-tracking strategies as appropriate. Make a point of doing the following:
  - write down what you are about to calculate before doing so
  - write down intermediate stages of calculations that are particularly complex
  - estimate the expected answer to a calculation before performing it, and critically evaluate any answer to a calculation

- check answers by repeating calculations, by applying inverse operations, or through the use of other appropriate strategies
- take care that the calculator is in the appropriate mode of angle measure for trigonometric work
- use the same model of calculator in the examination as you are used to using.

- Write in blue or black ink (other than for diagrams – see below).

- Do not rub out or otherwise obliterate cancelled work. A single line drawn through it is sufficient. There is frequently more merit in cancelled work than in repeated efforts. In most cases this can be counted to your benefit.

- Watch out for links between minor parts of a question: e.g. (b)(i) may help you when attempting (b)(ii). However, major parts (a), (b) and (c) are usually not related unless there is an explicit statement to that effect.

- Use sketches and diagrams to help clarify what is required in questions on visual topics. Frequently, the appropriate strategy required to solve an unfamiliar problem becomes much more apparent when a diagram is drawn.

- Where diagrams are specifically required, they should be drawn as accurately as possible, with a sharp pencil. Care should be taken in the positioning of axes and in the choice of an appropriate scale for co-ordinate geometry and graphs.

- Except under unusual circumstances, you should not attempt more than the required number of questions. Make sure you have exhausted all potential in your first six questions before attempting an extra one. Bear in mind that the extra question will need to overtake the weakest previous one before you start gaining marks, and this is rarely a good use of time. Instead, make sure you have attempted *all* parts of existing questions, and spend the time revisiting and improving these, or making fresh attempts at parts that have not yet worked out well.

- If answering through Irish, avoid all use of English in your work, since otherwise you risk not being awarded the relevant bonus.

4. HIGHER LEVEL

4.1 INTRODUCTION

The examination consists of two papers, each of two and a half hours' duration. Three hundred marks are allocated to each paper, giving a total of 600 marks.

On paper 1, candidates attempt any six from a range of eight questions. All questions relate to material on the *Core* part of the syllabus. Each question carries fifty marks.

On paper 2, candidates attempt any five questions from the seven available in Section A, and one question from the four available in Section B. All questions in Section A relate to material on the *Core* part of the syllabus. Section B consists of one question on each of the four *Options* on the syllabus. Each question carries fifty marks.

On both papers, all questions have an internal grading of difficulty, usually but not necessarily structured as an **(a)**, **(b)** and **(c)** part. There is no internal choice within questions.

4.2 PERFORMANCE OF CANDIDATES

Tables 4.1 to 4.3 below summarise the results of the Higher Level examination for the whole cohort. Note that Table 4.3 is a cumulative table, indicating that, for example, 47.3% of candidates achieved at least a B3 grade in 2005.

	A1	A2	B1	B2	B3	C1	C2	C3	D1	D2	D3	E	F	NG
2002	649	596	782	906	978	1118	1077	959	792	581	575	318	85	14
2003	572	685	859	919	1064	1047	1032	1027	786	548	509	334	59	12
2004	834	700	842	952	1029	1012	987	941	717	560	459	300	83	10
2005	**763**	**762**	**942**	**1078**	**1109**	**1105**	**1045**	**879**	**717**	**566**	**456**	**327**	**83**	**11**

Table 4.1: Number achieving each grade – Higher Level, 2002 to 2005

	A1	A2	B1	B2	B3	C1	C2	C3	D1	D2	D3	E	F	NG
2002	6.9	6.3	8.3	9.6	10.4	11.9	11.4	10.2	8.4	6.2	6.1	3.4	0.9	0.1
2003	6.1	7.2	9.1	9.7	11.3	11.1	10.9	10.9	8.3	5.8	5.4	3.5	0.6	0.1
2004	8.8	7.4	8.9	10.1	10.9	10.7	10.5	10.0	7.6	5.9	4.9	3.2	0.9	0.1
2005	**7.8**	**7.7**	**9.6**	**11.0**	**11.3**	**11.2**	**10.6**	**8.9**	**7.3**	**5.8**	**4.6**	**3.3**	**0.8**	**0.1**

Table 4.2: Percentage achieving each grade – Higher Level, 2002 to 2005

	A1	A2	B1	B2	B3	C1	C2	C3	D1	D2	D3	E	F	NG
2002	6.9	13.2	21.5	31.1	41.5	53.3	64.8	74.9	83.3	89.5	95.6	99.0	99.9	100
2003	6.1	13.3	22.4	32.1	43.4	54.4	65.4	76.2	84.5	90.3	95.7	99.2	99.9	100
2004	8.8	16.3	25.2	35.3	46.2	57.0	67.4	77.4	85.0	91.0	95.8	99.0	99.9	100
2005	**7.8**	**15.5**	**25.1**	**36.0**	**47.3**	**58.5**	**69.1**	**78.1**	**85.3**	**91.1**	**95.7**	**99.0**	**99.9**	**100**

Table 4.3: Percentage achieving at or above each grade – Higher Level, 2002 to 2005

Question Popularity, Percentage Uptake and Average Mark

Tables 4.4 to 4.6 show the ranking of questions in order of decreasing popularity. The tables also give the percentage of candidates who attempted each question, and the average mark awarded for each question. These figures are estimates derived from an analysis of a sample of 4% of the scripts.

Order of Popularity	*Question Number*	*Topics (Syllabus section and subsection)*	*% Attempts*	*Average Mark (out of 50)*
1	1	Algebra 1, 2	96%	35
2	2	Algebra 1, 3	95%	39
3	6	Functions & Calculus 2	92%	35
4	3	Algebra 4, 6	90%	30
5	7	Functions & Calculus 2	89%	35
6	8	Functions & Calculus 3	85%	35
7	5	Algebra 1, 5; Sequences & Series	45%	32
8	4	Algebra 1; Sequences & Series	35%	34

Table 4.4: Popularity and Average Mark of Questions on Paper 1

Order of Popularity	*Question Number*	*Topics (Syllabus section and subsection)*	*% Attempts*	*Average Mark (out of 50)*
1	1	Geometry 1	93%	35
2	3	Geometry 1, 3	92%	37
3	4	Trigonometry	85%	40
4	2	Geometry 2	75%	38
5	5	Trigonometry	72%	32
6	6	Discrete Maths & Statistics 1,2, 4	68%	42
7	7	Discrete Maths & Statistics 1,2, 3	57%	32

Table 4.5: Popularity and Average Mark of Questions on Section A of Paper 2

Order of Popularity	*Question Number*	*Topics (Syllabus section and subsection)*	*% Attempts*	*Average Mark (out of 50)*
1	8	Further Calculus and Series	95%	37
2	9	Further Probability and Statistics	3%	42
3	10	Groups	2%	34
4	11	Further Geometry	0%*	-

Table 4.6: Popularity and Average Mark of Questions on Section B of Paper 2

It may be noted that popularity and average mark are not at all strongly correlated.

* Four candidates in the full cohort.

4.3 ANALYSIS OF CANDIDATE PERFORMANCE

Paper 1, Question 1

Attempts: 96% **Average Mark: 35**

Syllabus Section(s): Algebra 1, 2

(a) This routine procedure was correctly executed by virtually all candidates. Very few candidates tested their answers, (which was not required but is good practice).

(b) **(i)** This part, which required only basic arithmetic manipulation of powers, was extremely poorly done. It is noteworthy that candidates regularly handle much more complex work with powers when dealing with, for example, calculus, induction, or difference equations. Candidates floundered completely in their attempts here, making distributive law errors, confusing multiplication with addition, and treating $2^{\frac{1}{4}}$ as $2\frac{1}{4}$.

(ii) This was well answered by nearly all the candidates. They had no difficulty finishing out correctly. The vast majority followed the first method listed in the marking scheme (collecting like powers), which is not surprising in light of the fact that this is the commonly used approach to proving the factor theorem.

(c) This part was well answered, with candidates displaying a sound understanding of the underlying concepts and a clearheaded execution of the algebraic procedures required. Most used the first method listed in the scheme, (expanding $(x-p)^2$ and then dividing it into $f(x)$). The candidates who actually wrote down the remainder after the division did not make as many mistakes as those who tried to "read" the remainder directly into the next stage of the work. The few candidates who used the second method in the scheme generally did not do very well, and mostly did not proceed beyond multiplying out the factors.

Paper 1, Question 2

Attempts: 95% **Average Mark: 39**

Syllabus Section(s): Algebra 1, 3

(a) This basic procedure was generally well handled, although the great majority chose the more complicated and error-prone method of squaring both sides over the more straightforward and obvious method, (*viz.* $-7 < x - 1 < 7 \Rightarrow -6 < x < 8$).

(b) This was well answered, with most candidates showing a good understanding of the technique required to solve an equation of this type. Systematic testing to find the integer root posed little difficulty, although some candidates abandoned the work when one or two trials did not yield the root. Candidates rarely used the fact that an integer root must divide the constant co-efficient. Such understanding would increase the searching efficiency somewhat.

There were very few attempts at factorising $(4x^2 - 2x - 1)$, indicating that candidates understood the assertion in the question that the remaining roots were irrational, (assisted in some cases, no doubt, by the reference to surd form).

(c) Both parts were well answered and candidates scored well here. A good number had some difficulty working with $f\left(\frac{k}{m}\right)$ in part **(i)**, but most managed to finish out this part in any case. The factorisation required to successfully finish the second part caused difficulty for some.

Paper 1, Question 3

Attempts: 90% **Average Mark: 30**

Syllabus Section(s): Algebra 4, 6

(a) This straightforward matrix calculation was well answered. Most candidates got full marks here.

(b) Although routine, this procedure requires some clarity of thought regarding what one is doing, and care in execution. The great majority used the quadratic formula, as expected, and did so competently. For some, arithmetic errors led to a square root that was difficult to evaluate correctly, and those candidates usually did not manage to finish. Even those who did not err in the calculations often lost marks through failure to finish, stopping at $\frac{-3+i}{2i}$ and $\frac{-3-3i}{2i}$. A small number of candidates took $z = a + bi$ as root, and substituted this into the original equation, yielding: $(-4ab+6a-2b+3)+(2a^2-2b^2+6b+2a-6)i=(0)+(0)i$. Equating like coefficients here yields a linear and a quadratic equation in *a* and *b*, which can be successfully solved. However, the work involved is a little complicated, and only very few arrived at correct roots.

A relatively common incorrect approach was to treat *z* as real and to gather real parts and imaginary parts in the original equation, proceeding to solve them separately. Some other candidates tried various forms of manipulation of the equation, usually without making any useful progress.

(c) Candidates did not handle this section well, despite the fact that it is a relatively standard type of application of De Moivre's theorem.

(i) When candidates applied the theorem directly to z^n and z^{-n}, the solution was straightforward and they almost invariably obtained full marks. However, most candidates, having substituted z^n in both fractions, brought the two together as a single fraction, which made the work much more difficult. These candidates were generally unable to finish.

(ii) Full marks were very rare here. Most candidates who attempted this part ignored the direction given in the question and used the trigonometric formulae from the tables. After a lot of work they obtained the correct expression, but, not having answered the question asked, received no marks.

Paper 1, Question 4

Attempts: 35% **Average Mark: 34**

Syllabus Section(s): Algebra 1; Sequences & Series

(a) This part was well handled by all attempting the question, although many neglected to write the recurring decimal as an infinite geometric series.

(b) (i) This part was badly answered; the binomial expansion proved a significant obstacle and this is a perennial difficulty. Once over the binomial hurdle, candidates were able to make good progress. The incorrect assertion: $(kx)^2 = kx^2$ was common.

(ii) The candidates found u_{n+1} and u_{n+2} readily enough, although in some cases brackets were abused very much. Candidates had more difficulty in substituting back into the equation, but once this was achieved, candidates showed a good knowledge of indices to finish out here. (Note the contrast with question 1(b).)

(c) **(i)** Candidates had little difficulty providing a proof that would be satisfactory if it were given that $(a+b)$ is positive. Very few candidates considered the case $(a+b) < 0$, or made reference to the implications of squaring an inequality.

(ii) Clear and direct solutions were uncommon, but candidates generally managed to muddle through. Many candidates just repeated the work they had done in **(c)(i)**.

Paper 1, Question 5

Attempts: 45% **Average Mark: 32**

Syllabus Section(s): Algebra 1, 5; Sequences & Series

(a) This was well answered, with many candidates getting full marks here. The only widespread error was a failure to test the solutions, leading to a failure to eliminate the extraneous value. Candidates were fortunate that only a light penalty was applied on this occasion.

(b) This was handled somewhat more competently than has been the norm for induction questions over the last few years, but nonetheless remains very disappointing.
Candidates have little difficulty verifying $P(1)$, or writing down what the inductive hypothesis $P(k)$ is. However, it is rare to see any clear understanding as to what is required in the deduction of $P(k+1)$, which is at the heart of the matter.
On the evidence of the last few years, the great majority of candidates have little or no understanding of the principle of induction and its application.

(c) The first part was generally well answered, requiring as it did only the direct application of the change-of-base rule. The second part appeared more difficult than it was, which doubtless put some candidates off. Nonetheless, those who were prepared to try, and who applied the result from part **(i),** were usually successful.

There were some intriguing cases of candidates who proved the result in part **(c)(ii)** by induction, despite displaying little or no competence in the technique in part **(b)**.

Paper 1, Question 6

Attempts: 92% **Average Mark: 35**

Syllabus Section(s): Functions & Calculus 2

(a) **(i)** This simple application of the chain rule was well executed by almost all candidates. Some expanded the given expression, yielding a straightforward polynomial and thereby avoiding the chain rule.

(ii) Most candidates were able to read the answer directly from the tables, with the substitution $a = 5$, and had little difficulty. Candidates using other methods were more likely to make errors, most commonly in applying the chain rule.

(b) Candidates did not handle this part well. Whereas they managed the differentiation without great difficulty, there were significant problems with the trigonometric manipulation required. The candidates who did the trigonometric manipulation first, (simplifying y to $\left(\tan\frac{x}{2}\right)^2$ before differentiating,) were more likely to score full marks than those who differentiated first.

Candidates showed little perseverance, and, in general, were quick to abandon their work when they encountered difficulties.

(c) Only the routine algebraic portion of part **(c)** was well done.

(i) This part was quite well handled. Candidates recognised the relevance of the derivative and found it correctly. Nonetheless, candidates tend to be poor at expressing clear and correct conclusions from their work. A good number simply stopped after finding the derivative, and many others struggled to express properly what they apparently knew.

(ii) The asymptotes were found and plotted, but candidates were not at all adept at sketching the graph, despite that it is one of only a very few forms whose shape they are expected to be familiar with. Many unnecessarily calculated lots of points on the curve, and even having done this, many still produced a curve of the wrong form. Various shapes of graphs were encountered, intersecting the asymptotes or drawn in the wrong quadrants. About half of the candidates drew a correct sketch.

(iii) Only very few candidates got full marks here. Most recognised that (1, 1) was the point of intersection of the asymptotes and therefore needed to be shown to be the centre of symmetry. Frequently, the image of (2, 2) was shown to be (0, 0) without any other relevant work. There were virtually no reasonable attempts at an algebraic proof, and candidates rarely achieved more than the attempt mark.

Paper 1, Question 7

Attempts: 89% **Average Mark: 35**

Syllabus Section(s): Functions & Calculus 2

(a) Candidates displayed a high level of competence in the process of differentiating from 1^{st} principles. The candidates knew the procedure to be followed, and completed all the necessary steps. There was no trouble in expanding $(x+h)^2$. Most candidates used the $f(x+h)$ approach (as opposed to $y+\Delta y$, etc.).

(b) **(i)** This was answered reasonable well, although there was some mishandling of the chain rule in finding $\frac{dx}{dt}$ and/or $\frac{dy}{dt}$. Generally, $\frac{dy}{dx}$ was correctly found, and $\sqrt{2}$ was well handled.

(ii) As might be expected, virtually all candidates approached this via implicit differentiation. That is, virtually none approached this question by isolating x. Most problems arose in differentiating xy^2. However, candidates were well able to isolate $\frac{dy}{dx}$ and to finish out. Sign errors were relatively common, as was the expected manifestation of a perennial error: $\frac{d}{dx}(6)=6$.

(c) Other than part **(i)**, this was very poorly handled by the majority, despite the fact that establishing an iterative rule for approximating a square root is a standard application of the Newton-Raphson method, and has been dealt with before on examination papers. The widespread inability to handle this question stands in sharp contrast to the fact that candidates normally have little difficulty applying the same method to find a specific approximation of a specific polynomial (a well-rehearsed routine algorithm). This indicates that candidates are not achieving the stated aim of the syllabus that they should be able to apply their knowledge in the context of "the ability to solve problems, abstract and generalise".

Paper 1, Question 8

Attempts: 85% **Average Mark: 35**

Syllabus Section(s): Functions & Calculus 3

(a) The vast majority of candidates achieved full marks here. The candidates showed a good grasp of basic integration techniques, including the need to insert the constant of integration.

(b) **(i)** This was very well answered by most candidates. They recognised the integral to be of the form $\int \frac{f'(x)}{f(x)}dx$, and consequently completed correctly by substitution. In most cases, the integral was expressed in terms of the original variable before substituting the original limits. Candidates who did this had fewer errors with limits than those who did not return to the original variable.

(ii) There were two commonly used approaches to solving this problem. The second method listed in the scheme (using trigonometric formulae to transform the integrand) normally led to a correct solution. Perhaps paradoxically, the more direct approach using the formula on page 42 of the Tables for integrating $\sin^2 x$ led more often to an incorrect solution. The errors arose from mishandling the substitution of 2θ for *x*, (or *u*).

(c) **(i)** Many candidates either did not recognise the form of the integral or were not familiar with the required technique for dealing with integrals of this form. They accordingly tried various substitutions that had no prospect of success. Of those who did apply the required technique, the majority executed it poorly. Most of the errors arose from work in trying to complete the square.

(ii) This was very badly answered, considering that the tightly circumscribed specification of this material in the syllabus renders it very straightforward to cover comprehensively. Not very many achieved full marks here.

Paper 2, Question 1

Attempts: 93% **Average Mark: 35**

Syllabus Section(s): Geometry 1

(a) This was very well answered, with most candidates scoring full marks.

(b) **(i)** Well answered by nearly all candidates. Finding the slope of the normal and hence the equation of the tangent was the preferred method. (Very few used differentiation instead.) Many failed to clearly justify the last step in the proof, (i.e. to state or explain why $x_1^2 + y_1^2 = r^2$).

(ii) About 70% of candidates scored full marks on this part. The most popular method was to use perpendicular distance, (second method in the scheme). Only the better candidates recognised (5, *b*) as an element of circle, and these answered without error or difficulty (first method in the scheme). Many of those who used the perpendicular distance method were prone to simplification errors in determining *b*. The weaker candidates attempted to solve between the line and the circle without success.

(c) The majority of candidates were unable to reach a correct final solution to this part. The usual approach was to use the general equation of a circle and then substitute the two given points, resulting in two equations in *g*, *f* and *c*. The majority failed to use the third piece of information appropriately, and hence failed to proceed further. Many contrived a third piece of information to proceed, such as taking the point (7, 6), (the mid-point of the given chord of the circle,) as the centre, or taking (–1, 0) as the point of tangency.

Paper 2, Question 2

Attempts: 75% **Average Mark: 38**

Syllabus Section(s): Geometry 2

(a) This part was answered extremely poorly, with only about 30% of candidates scoring full marks. Most candidates either did not attempt it or offered an answer that indicated little or no understanding of what was required. This is quite extraordinary in light of the basic understanding of the concept of a vector that was being tested. Candidates at Ordinary Level rarely have any great difficulty with questions of this type, which test a level of understanding that is often deemed too trivial to warrant inclusion on a

Higher Level paper. Unfamiliarity with this as an examination question is hardly a comforting explanation for poor performance, since it is difficult to understand how students can gain any meaningful understanding of the topic as a whole if they are unable to handle such a basic task.

(b) **(i)** This part was well answered, with nearly all candidates scoring full marks. Occasionally, candidates offered $-\vec{q}^{\perp}$ for $\vec{q}^{\perp}$.

(ii) This part was also well answered with many scoring full marks. Although there were some algebraic or arithmetic errors, candidates showed an understanding of the question and conceptual errors were rare.

(c) Part **(i)** was a straightforward and familiar procedure and was well executed by the great majority of candidates. Part **(ii)** was somewhat more conceptually challenging, but candidates nonetheless made a good effort. However, various algebraic errors prevented most from arriving at a solution. Often, $\vec{r}$ was not expressed correctly, due to minor errors.

Paper 2, Question 3

Attempts: 92% **Average Mark: 37**

Syllabus Section(s): Geometry 1, 3

(a) This part was very well answered with virtually all candidates scoring full marks. Most candidates found the point of intersection correctly and proceeded correctly to a solution. A minority of candidates used the method $L_1 + \lambda L_2 = 0$.

(b) Parts **(i)** and **(ii)** were very well handled, although transposition errors led to incorrect co-ordinates for the intercepts. Part **(iii)** was poorly answered. Errors included: modulus omitted or mishandled in finding the area of the triangle; failure to take account of the given information $m > 0$; and taking the point (−4, 6) as a triangle vertex.

(c) **(i)** By far the most commonly used methods here involved inverting the transformation. Most were successful in doing this much. Many stopped at this stage and found no image lines. Others created specific lines and found their images. Some who found correct general image lines failed to establish that they were parallel. Only the better candidates proceeded to a fully correct solution.

Most candidates failed to understand that they needed to work on two parallel lines like $ax+by+c=0$ and $ax+by+d=0$ (or equivalent). Use of matrices was rare. Use of parametric form was not encountered, nor was proof by contradiction.

(ii) Well answered. Candidates clearly understood that *oc* would be parallel to *ab*, and, therefore, that they needed to find *c*. Most did this correctly.

Paper 2, Question 4

Attempts: 85% **Average Mark: 40**

Syllabus Section(s): Trigonometry

(a) Unexpectedly, this part was poorly answered, with candidates showing little ability to deal correctly with limits. This is in sharp contrast to previous years, in which this type of question was handled much better. Candidates by and large knew the correct answer but were unable to derive it correctly. The second method in the scheme (L'Hôpital's rule) was seldom used.

(b) **(i)** This straightforward proof was done correctly by virtually all candidates. However, the method of proof was usually far more complicated than necessary.

(ii) The majority of candidates correctly used the link with the previous part to form a quadratic and hence found a solution. This part was well handled by most. An incorrect or missing solution was the only common error.

(c) **(i)** Well answered. Most candidates were able to find the length of [*ec*] and hence of [*ab*]. The most usual errors here were in finding $|dc|$ or $|de|$. Occasionally, candidates misused Pythagoras' theorem.

(ii) Most of the candidates' difficulties here surrounded determining the angles in the sectors. Candidates failed to made use of the right-angled triangle *edc* to determine $|\angle edc|$ or $|\angle ecd|$, and thus were also unable to find $|\angle dcb|$.

Paper 2, Question 5

Attempts: 72% **Average Mark: 32**

Syllabus Section(s): Trigonometry

(a) In the main, very well answered. Once candidates recognised that each angle was 60º, there were no problems. Some weaker candidates took the triangle as right angled, or mishandled the basic algebra (e.g. $a \times a = 2a$).

(b) Part **(i)** was well handled by most candidates. Few errors occurred, except occasionally in cross-multiplication. Part **(ii)**, however, was poorly answered. Many candidates failed to recognise the need to apply the relevant formula to $\sin 2\beta$. Of the candidates who found $\cos\beta = \frac{5}{6}$, many were unable to continue from here to find $\tan\beta$.

(c) Very poorly answered. Many did express $\tan\theta$ and $\tan 2\theta$ in terms of h and x, but made little effort at proceeding much further.

Paper 2, Question 6

Attempts: 68% **Average Mark: 42**

Syllabus Section(s): Discrete Maths & Statistics 1,2, 4

(a) Both parts were very well answered. Most scored full marks in the part **(i)**. Occasionally in part **(ii)**, the incorrect answer 5×5×5 was offered.

(b) The first part was handled very well, as indeed questions of this type have been in previous years, with most scoring full marks. Such errors as occurred were mainly numerical slips. Part **(ii)** was very poorly answered, however. Candidates generally found u_2 correctly from their formula for u_n, but failed to verify it. This would seem to indicate a lack of understanding of how the sequence is generated by its original definition. This is worrying, since without such an understanding it is hard to see how students can appreciate what has been achieved when a closed-form expression for u_n is derived. The role of this topic on the syllabus is significantly impoverished by such a lack of appreciation.

(c) Parts **(i)** and **(ii)** involved relatively standard counting techniques in a familiar context. Nonetheless they required the some clarity of thought, and in this context were very well answered, with most candidates scoring full marks. Part **(iii)** was poorly answered. Candidates seemed unable to identify favourable outcomes and many did not even attempt the question. It would appear that, other than with certain well-rehearsed question types, candidates lack the capacity to systematically list and/or systematically count outcomes satisfying particular criteria. Even the basic generic skills for this topic, such as exploring the situation by looking at examples of outcomes that satisfy or do not satisfy relevant criteria, were lacking.

Paper 2, Question 7

Attempts: 57% **Average Mark: 32**

This was the least popular question in Section A.

Syllabus Section(s): Discrete Maths & Statistics 1,2, 3

(a) This straightforward question on combinatorics was very well answered, although candidates sometimes added instead of multiplying in part **(ii)**, (i.e. gave ${}^5C_2 + {}^5C_2$).

(b) Parts **(i)** and **(ii)** were very well answered by most candidates. Parts **(iii)** and **(iv)**, however, caused much more difficulty than ought to be expected at this level. It is worth noting that the majority of candidates approached these parts using multiplicative laws for probability, which are not on the core course. Such candidates (unless they have developed their understanding substantially through studying the material on the option) tend to favour a blind application of rules over clear thinking, and generally suffer the consequences in all but the most basic of situations. In this case, they made errors related to ordering. As might be expected, the candidates who stuck to the basic principle for dealing with all situations involving equally likely outcomes (i.e., count the outcomes and put favourable over possible) fared much better than those attempting to use the more complex rules.

(c) This question tested basic understanding of the concepts of mean and standard deviation in a familiar real-life context. Candidates generally displayed a good understanding of the mean and its properties, but a relatively poor understanding of standard deviation and its properties. In part **(i)**, candidates gave the correct values for both statistics, but were often not able to articulate their reason effectively in the case of the standard deviation. Answers to part **(ii)** were usually correct, indicating a good understanding of the properties of the mean in this situation. Answers to part **(iii)** were much worse, and those who did give the correct answer generally offered no valid explanation, indicating, perhaps, that many were simply guessing. Candidate responses here, combined with patterns of answering in previous years, indicate that candidates have generally acquired competence in calculating the standard deviation, but have no real understanding of what this statistic is actually measuring.

Paper 2, Question 8

Attempts: 95% **Average Mark: 37**

Syllabus Section(s): Further Calculus and Series

Candidates scored well in this question. Only part **(b)(ii)** caused a problem and to lesser extent part **(b)(iii)**.

(a) Very well answered, with the great majority scoring full, or close to full, marks. Very few errors were encountered. Omitting the constant of integration was the most likely error, and this was not as prevalent as in previous years.

(b) Candidates had no difficulty at all with the expansion in part **(i)**, but were surprisingly poor at the standard application required in part **(ii)**. Less than a quarter of the candidates were able to recognise and correctly execute the step required (i.e., make $(1 + x)$ correspond to $\frac{11}{10}$ and hence set x to be $\frac{1}{10}$). In part **(iii)** the general term of the series was identified somewhat better than in previous years, but still far less well than ought to be expected. The work related to the ratio test displayed little understanding. Even in the few cases where reasonable work was being presented, the modulus was often omitted or misused.

(c) This part was well answered by candidates. Most candidates expressed r^2 in terms of h correctly. Differentiation was usually correct, and candidates had little difficulty in determining the appropriate value of h.

Paper 2, Question 9

Attempts: 3% **Average Mark: 42**

Syllabus Section(s): Further Probability and Statistics

As is usually the case, some candidates who clearly had not studied this option attempted it on the basis of their core probability knowledge. Needless to say, they scored very poorly.

Those candidates who had genuinely studied this topic as an option scored very well. The comments below refer to the genuine attempts only.

(a) This question, testing the basic procedure of handling the normal distribution tables, was well answered, with nearly all scoring full marks.

(b) **(i)** This caused no difficulty, with candidates generally scoring full marks.

(ii) Also well answered, with most scoring full marks. Occasionally, the binomial coefficient was omitted, with candidates offering $\left(\frac{4}{5}\right)^3\left(\frac{1}{5}\right)^1$ as the solution.

(iii) This was surprisingly poorly handled. In general, candidates only calculated one relevant probability (usually $P(9)$ or $P(10)$). Candidates appeared unable to handle the idea of scoring "at least eight" penalties.

(c) This part was very well answered. Candidates found the standard error of the mean without difficulty. They had a clear understanding of the procedure for finding the confidence interval and generally proceeded accurately to a correct solution.

Paper 2, Question 10

Attempts: 2% **Average Mark: 34**

Syllabus Section(s): Groups

Candidates scored well in parts **(a)** and **(b)** but showed lack of relevant knowledge in part **(c)**.

(a) This part was well answered, with most candidates scoring full marks. Candidates created a correct Cayley table and gave correct identity and inverses.

(b) Candidates had little difficulty listing the elements of the group, as required in part **(i)**. Part **(ii)** was generally well done, also, although some candidates did include incorrect elements or omit correct ones. There appeared to be some guessing on the part of candidates who were not familiar with the meaning of "centralizer".

(c) This part proved problematic for many candidates, and very few were successful. Candidates generally displayed little understanding of the symmetries of a tetrahedron. The usual score was the attempt mark, or no marks.

Paper 2, Question 11

Attempts: 0% (4 candidates) **Average Mark: [not available]**

Syllabus Section(s): Further Geometry

A total of four candidates (from different centres) took this question as an option.

(a) Well answered.

(b) The candidates made a good effort at this part and scored well.

(c) The candidates make a worthwhile effort at part **(i)**, but there was little clear understanding of what was required in part **(ii)**.

4.4. CONCLUSIONS

Patterns of Question Choice

The pattern of question choice on both papers was in close keeping with that of previous years. On paper 1, algebra and complex numbers continue to be the most popular. Sequences and series, along with induction and logarithms (questions 4 and 5) continued to be the least attractive topics. Questions 4 and 5 were often attempted last, when the adverse effects of time running out came to bear on the work produced. On paper 2, the majority of candidates depended on the circle, the line, vectors and trigonometry for their core questions. The drift identified in the last report, away from discrete mathematics and statistics and towards trigonometry, has stabilised. It has reversed slightly this year, but not to a great extent. Analysis of patterns in answering again suggests that the choice of core questions is often made in advance of seeing the examination papers. This is a practice that is not recommended. It is, of course, unavoidable if all parts of the syllabus have not been studied. There was some evidence of the latter happening (such as all candidates in some centres omitting a particular topic).

As regards the optional material, *Further Calculus and Series* continues to be an almost universal selection, despite the fact that it consistently delivers a relatively moderate return compared with the other options, although it did hold up better than usual this year. Anecdotal evidence indicates that it is perceived to be easier or faster to cover than the other options. This is not necessarily the case, and the more conceptually difficult aspects of the topic are very poorly handled, indicating that perhaps insufficient time is being devoted to them.

Standard of Answering

Despite the fact that the analysis in section 4.3 above identifies deficits in understanding on the part of many candidates, it is important to state that much excellent answering was also presented. The scripts of candidates achieving high scores were characterised by clear, accurate work that portrayed competence in all aspects of the syllabus. The solutions presented carried evidence of solid conceptual understanding and thorough revision. The candidates who produced work of such quality naturally received high grades, a just reward for the commitment and effort that they obviously invested in preparing for the examination.

Nonetheless, it would be remiss not to express the concerns of the examining teams that there has been a noticeable slippage over a relatively short period of time in the quality of work being presented by the cohort at this level. In particular, two aspects of the work cause concern. Firstly, the examining team on paper 1 reports that the candidates appear to be less comfortable than before with the basics of algebraic manipulation that form the foundation for the more substantive work involved at this level.

Secondly, examiners have been commenting on a noticeable decline in the capacity of candidates to engage with problems that are not of a routine and well-rehearsed type. Whereas in the relatively recent past it was common to see two, three, or more attempts at a problem with which a candidate was struggling, it is now more common to see the work abandoned after the first attempt fails to yield rapid success. Also more prevalent is the inclination of quite good candidates to not even attempt parts of questions that do not resemble well rehearsed examples from class. This is not caused by difficulties with the time available, as evidenced by the fact that there has been an increase in the number of surplus questions attempted. This diminution in the level of perseverance was first identified in the Chief Examiner's Report of 2000, and has continued unabated. That is, there is evidence of an ongoing decline in the candidates' willingness to struggle for success.

A further concern also deserves mention. Although related to the previous points, it can hardly be described as new. As identified in various previous Chief Examiners' reports, candidates' conceptual understanding of the mathematics they have studied is inferior to that which one would hope for and expect at this level. Whereas procedural competence continues to be adequate, any question that requires the candidates to display a good understanding of the concepts underlying these procedures causes unwarranted levels of difficulty.

Candidate Strengths

An overview of the work presented reveals a range of strengths displayed by the cohort. As mentioned above, strong performance was most evident in procedural questions where a definite sequence of familiar steps was required. Strengths included:

- Solving linear simultaneous equations
- Applying the factor theorem
- Solving a linear inequality
- Use of quadratic formula
- Manipulating 2×2 matrices
- Solving quadratic equation with complex coefficients
- Writing a recurring decimal as a fraction

- Differentiating by rule
- Differentiating from first principles
- Evaluating indefinite and definite integrals of standard and easily recognised form, including integration by substitution
- Accurate manipulation of fractions (presumably using a calculator)
- Basic co-ordinate geometry
- Knowledge of formulae
- Applying linear transformations in co-ordinate geometry
- Evaluating dot product and modulus of vectors
- Proof of trigonometric formulae
- Solving trigonometric equations
- Solving difference equations
- Basic probability
- Permutations and combinations
- Certain algebraic skills: factors and solving quadratics
- Integration by parts
- Deriving MacLaurin series
- Application of calculus to max/min problem.

The above observations are in line with previous years, although it is worth stating that there was some improvement in trigonometry generally, and also in the more straightforward aspects of probability.

Candidate Weaknesses

As mentioned earlier, weaknesses in answering were traceable to two broad sources. Firstly, foundation skills in mathematics were often not up to the standard required. Deficiencies were evident in algebra, for example, leading to candidates not being in a position to progress to displaying other competencies. Secondly, weaknesses continue to stem from inadequate understanding of mathematical concepts and underdeveloped problem-solving and decision-making skills. These inadequacies become significant when questions unfold in unfamiliar ways or require more than the mechanical application of routine and well-practised algorithms. Candidates then further hamper their own cause by being too quick to abandon their efforts.

Specific candidate weaknesses identified on this occasion include the following:

- Improper handling of brackets, and similar basic algebraic errors

- Incorrect cancelling in algebraic fractions.
- Poor manipulation of expressions involving indices
- Poor application of De Moivre's theorem
- Inability to expand such expressions as $\left(z + \frac{1}{z}\right)^4$
- Proof by induction: most are completely unable to handle the substantive step.
- Inability to express $\cos x$ and $\sin x$ in terms of $\tan \frac{x}{2}$
- Failure to consider of positive and negative cases when working with inequalities
- Poor curve sketching, and inability to deal with properties of a curve
- Poor handling of integration in cases where the form is not immediately obvious (in particular, recognising the need to complete the square, and being able to do so)
- Inability to apply the Newton-Raphson method to a less familiar problem
- Inability to establish the formula for a cone using integration techniques.

4.5. RECOMMENDATIONS FOR TEACHERS AND CANDIDATES

The following advice is offered to teachers and candidates preparing for Higher Level Mathematics examinations:

In advance of the examination

- Be adequately prepared for all core syllabus areas. Restricting the choice available on day is not advisable.
- Close to the time of the examination, practise questions from previous examination papers. However, do not rely too much on examination papers during the main period of learning. A much richer variety of source material is required in order to build the required problem solving skills, particularly if aiming for a high grade.
- Get into the habit of persevering with questions that cause difficulty, trying different approaches where necessary, and discussing strategies, errors and problems with other students and your teacher. If you are not willing and able to tackle unfamiliar or difficult problems and persevere with them, then you will not achieve a high grade.
- Ensure, through practise, that you are able to apply the knowledge and skills gained in one content area to problems tackled in another. Compartmentalising your knowledge may help to keep it organised, but it risks restricting your ability to cope with the unexpected.
- Practise finding as many different approaches as possible to a problem. Building up your arsenal of techniques while in familiar territory will pay dividends in the less familiar.
- Practise to gain comfort and accuracy in your basic skills: algebraic manipulation, (such as expanding and factorising expressions, solving equations, completing the square, etc.,) trigonometric manipulation, and standard calculus techniques.
- Learn the required proofs – there are not that many of them. Ensure that you understand the logic of the proof you are learning, and the meaning and power of the result. It is extremely difficult to learn a proof that you do not understand clearly, or whose significance you do not understand. In the latter respect, consider the question: what difference would it make to this topic if this result were *not* true?
- Get into the habit of checking your work for errors. Practise checking answers by referring them back to the question to see if they are correct and sensible, and practise

techniques for finding errors quickly and calmly in your own work, including getting to know your own weaknesses.

In the examination

- Read the whole examination paper carefully, and choose questions with care. Most candidates benefit from attempting their best question first, followed by their next best, and so on.
- Read individual questions carefully, taking heed of key words (such as “hence”). Make sure you answer the question asked.
- Questions often contain clues regarding the nature of the answer. For example, “Find the value of x for which…” indicates that there is only one such value, whereas “Find the values of x for which…” indicates that there is more than one. Similarly, take heed of indications that numbers are positive, negative, rational, etc.
- Concentrate while answering – good candidates can lose a lot of marks through carelessness. It is often not so much the penalty for the error itself as the consequences for your further work that cause the greatest loss of marks and time.
- In the case of **(c)** parts in particular, expect the unexpected, and stay calm. Remember that in order to determine which candidates merit the top grades, the **(c)** parts of the examination are designed to test how you can apply your knowledge to unfamiliar problems and in less familiar contexts, and to test your level of conceptual understanding of the course content. If these parts were familiar to candidates, the examination would not be doing what it is supposed to be doing. The less familiar the territory, the more credit will be awarded for attempts at applying appropriate strategies, and it is always advisable to make an attempt.
- Whenever possible, keep all parts of a question together in your answerbook. Do not, for example, answer all of the **(a)** parts followed by all of the **(b)** parts, etc. Although there are no penalties for this, it causes difficulties for the examiner trying to follow your work and keep track of your marks. The risk of your omitting parts is also increased. If you wish to tackle questions in this way, then allow sufficient space to come back and fill in the omitted parts later. Likewise, try to keep any rough work together with the main body of work, rather than relegating it to the back of the answerbook – it often helps the examiner to follow your train of thought.

- Show all necessary steps clearly. Remember that the objectives being tested include communicative skills, and the onus is on you to communicate effectively with the examiner.
- Except under unusual circumstances, you should not attempt more than the required number of questions. Unless you are abandoning a question *very* soon after starting it, make sure you have exhausted all potential in your first six questions before attempting an extra one. Bear in mind that the extra question will need to overtake the weakest previous one before you start gaining marks, and this as rarely a good use of time. Instead, make sure you have attempted *all* parts of existing questions, and spend the time revisiting and improving these, or making fresh attempts at parts that have not yet worked out well.
- If answering through Irish, avoid all use of English in your work, since otherwise you risk not being awarded the relevant bonus.

APPENDIX A: GRADES ACHIEVED, DISAGGREGATED BY GENDER

Tables A.1 to A.9 below summarise the results of the three levels of the examination, disaggregated by gender. Note that Tables A.3, A.6 and A.9 are cumulative tables, indicating in Table A.3, for example, that 41.0% of female Foundation-Level candidates achieved at least a B3 grade. Note also that comparisons of performance within each level should be treated with caution, since patterns of uptake at the different levels differ by gender (c.f. section 1.3).

	A1	A2	B1	B2	B3	C1	C2	C3	D1	D2	D3	E	F	NG
Female	77	150	232	314	337	322	309	256	218	163	153	136	37	4
Male	59	133	221	300	329	356	320	301	222	171	188	183	65	6
Total	136	283	453	614	666	678	629	557	440	334	341	319	102	10

Table A.1: Number of males and females achieving each grade – Foundation Level, 2005

	A1	A2	B1	B2	B3	C1	C2	C3	D1	D2	D3	E	F	NG
Female	2.8	5.5	8.6	11.6	12.4	11.9	11.4	9.5	8.1	6.0	5.6	5.0	1.4	0.1
Male	2.1	4.7	7.7	10.5	11.5	12.5	11.2	10.5	7.8	6.0	6.6	6.4	2.3	0.2
Overall	2.4	5.1	8.1	11.0	12.0	12.2	11.3	10.0	7.9	6.0	6.1	5.7	1.8	0.2

Table A.2: Percentage of males and females achieving each grade – Foundation Level, 2005

	A1	A2	B1	B2	B3	C1	C2	C3	D1	D2	D3	E	F	NG
Female	2.8	8.4	16.9	28.5	41.0	52.9	64.3	73.7	81.8	87.8	93.5	98.5	99.9	100
Male	2.1	6.7	14.5	25.0	36.5	49.0	60.2	70.7	78.5	84.5	91.1	97.5	99.8	100
Overall	2.4	7.5	15.7	26.7	38.7	50.9	62.2	72.2	80.1	86.1	92.3	98.0	99.8	100

Table A.3: Percentage of males and females achieving at or above each grade – Foundation Level, 2005

	A1	A2	B1	B2	B3	C1	C2	C3	D1	D2	D3	E	F	NG
Female	1253	1707	1899	1954	1856	1850	1756	1584	1407	1196	1301	1379	551	63
Male	802	1124	1399	1418	1475	1491	1460	1455	1338	1208	1422	1567	739	119
Total	2055	2831	3298	3372	3331	3341	3216	3039	2745	2404	2723	2946	1290	182

Table A.4: Number of males and females achieving each grade – Ordinary Level, 2005

	A1	A2	B1	B2	B3	C1	C2	C3	D1	D2	D3	E	F	NG
Female	6.3	8.6	9.6	9.9	9.4	9.4	8.9	8.0	7.1	6.1	6.6	7.0	2.8	0.3
Male	4.7	6.6	8.2	8.3	8.7	8.8	8.6	8.6	7.9	7.1	8.4	9.2	4.3	0.7
Overall	5.6	7.7	9.0	9.2	9.1	9.1	8.7	8.3	7.5	6.5	7.4	8.0	3.5	0.5

Table A.5: Percentage of males and females achieving each grade – Ordinary Level, 2005

	A1	A2	B1	B2	B3	C1	C2	C3	D1	D2	D3	E	F	NG
Female	6.3	15.0	24.6	34.5	43.9	53.2	62.1	70.2	77.3	83.3	89.9	96.9	99.7	100
Male	4.7	11.3	19.5	27.9	36.5	45.3	53.9	62.4	70.3	77.4	85.7	95.0	99.3	100
Overall	5.6	13.3	22.3	31.4	40.5	49.6	58.3	66.6	74.0	80.6	88.0	96.0	99.5	100

Table A.6: Percentage of males and females achieving at or above each grade – Ordinary Level, 2005

	A1	A2	B1	B2	B3	C1	C2	C3	D1	D2	D3	E	F	NG
Female	322	367	464	552	582	568	526	441	365	260	204	136	24	3
Male	441	395	478	526	527	537	519	438	352	306	252	191	59	8
Total	763	762	942	1078	1109	1105	1045	879	717	566	456	327	83	11

Table A.7: Number of males and females achieving each grade – Higher Level, 2005

	A1	A2	B1	B2	B3	C1	C2	C3	D1	D2	D3	E	F	NG
Female	6.7	7.6	9.6	11.5	12.1	11.8	10.9	9.2	7.6	5.4	4.2	2.8	0.5	0.1
Male	8.8	7.9	9.5	10.5	10.5	10.7	10.3	8.7	7.0	6.1	5.0	3.8	1.2	0.2
Overall	7.8	7.7	9.6	11.0	11.3	11.2	10.6	8.9	7.3	5.8	4.6	3.3	0.8	0.1

Table A.8: Percentage of males and females achieving each grade – Higher Level, 2005

	A1	A2	B1	B2	B3	C1	C2	C3	D1	D2	D3	E	F	NG
Female	6.7	14.3	24.0	35.4	47.5	59.3	70.2	79.4	87.0	92.4	96.6	99.4	99.9	100
Male	8.8	16.6	26.1	36.6	47.1	57.7	68.1	76.8	83.8	89.9	94.9	98.7	99.8	100
Overall	7.8	15.5	25.1	36.0	47.3	58.5	69.1	78.1	85.3	91.1	95.7	99.0	99.9	100

Table A.9: Percentage of males and females achieving at or above each grade – Higher Level, 2005